| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14 | 16 | 17 | 18 | 19 | 21 | 22 | 23 | 24 |
| 28 | 32 | 34 | 36 | 38 | 42 | 44 | 46 | 48 |
| 42 | 48 | 51 | 54 | 57 | 63 | 66 | 69 | 72 |
| 56 | 64 | 68 | 72 | 76 | 84 | 88 | 92 | 96 |
| 70 | 80 | 85 | 90 | 95 | 105 | 110 | 115 | 120 |
| 84 | 96 | 102 | 108 | 114 | 126 | 132 | 138 | 144 |
| 98 | 112 | 119 | 126 | 133 | 147 | 154 | 161 | 168 |
| 112 | 128 | 136 | 144 | 152 | 168 | 176 | 184 | 192 |
| 126 | 144 | 153 | 162 | 171 | 189 | 198 | 207 | 216 |
| 140 | 160 | 170 | 180 | 190 | 210 | 220 | 230 | |
| | | | | | | | | 240 |

| $x$ | $x^2$ |
|---|---|
| 13 | 169 |
| 14 | 196 |
| 15 | 225 |
| 16 | 256 |
| 17 | 289 |
| 18 | 324 |
| 19 | 361 |
| 21 | 441 |
| 22 | 484 |
| 23 | 529 |
| 24 | 576 |
| 26 | 676 |
| 27 | 729 |
| 28 | 784 |
| 29 | 841 |

| $x$ | $x^3$ |
|---|---|
| 3 | 27 |
| 4 | 64 |
| 5 | 125 |
| 6 | 216 |
| 7 | 343 |
| 8 | 512 |
| 9 | 729 |
| 11 | 1331 |
| 12 | 1728 |

$\sqrt{2}$= 1.41 $\sqrt{3}$= 1.73 $\sqrt{5}$= 2.236

$\sqrt{6}$=2.449 $\sqrt{7}$= 2.646 $\sqrt{8}$=2.828

$\sqrt{10}$=3.16

$2^5 = 32$ $2^6 = 64$ $2^7 = 128$ $4^3 = 64$

$4^4 = 256$ $5^3 = 125$ $3^3 = 27$ $3^4 = 81$

$3^5 = 243$ $6^3 = 216$ $7^3 = 343$ $8^3 = 512$

$9^3 = 729$ $11^3 = 1331$ $12^3 = 1728$

**Standard Deviation:**

1. |Median-Mean| <= SD.

2. Variance is the square of the standard deviation.

3. If Range or SD of a list is 0, then the list will contain all identical elements. And vise versa: if a list contains all identical elements then the range and SD of a list is 0. If the list contains 1 element: Range is zero and SD is zero.

4. SD is always >=0. SD is 0 only when the list contains all identical elements (or which is same only 1 element).

5. Symmetric about the mean means that the shape of the distribution on the right and left side of the curve are mirror-images of each other.

6. If we add or subtract a constant to each term in a set: Mean will increase or decrease by the same constant. SD will not change.

7. If we increase or decrease each term in a set by the same percent: Mean will increase or decrease by the same percent.SD will increase or decrease by the same percent.

8. Changing the signs of the element of a set (multiplying by -1) has no effect on SD.

9. The SD of any list is not dependent on the average, but on the deviation of the numbers from the average. So just by knowing that two lists having different averages doesn't say anything about their standard deviation - different averages can have the same SD.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2},$$

* If a series is in Airthmetic Progression then the series will have **Mean = Median**

*************
General

1. In order for *x* and *y* to be consecutive perfect squares, given that *x* is greater than *y*, it would have to be true that $\sqrt{x} = \sqrt{y} + 1$.

2. not as tall as means may be shorter or taller ..i.e. exactly not the same height as

3. **A prime number is always +ve and > 1**. 1 is not a prime number
first 26 prime numbers are:
2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,69,71,73,

3. if it is difficult to find solve inequality with modes then square the inequality ... this makes things easier.

4. all squares of even numbers must be multiples of 4

5. if a number is of 2digit then consider .. 10x+y kind of equation and so on

6.in triangles with given base and given perimeter area is maximum for the isosceles triangle on the base.

7. sum of integers that are formed by the permutations of n digits
sum is given by equation:

= (sum of digits)*(n-1)!*(111... n times)
if repetition is not allowed.

= (sum of digits)*$(n)^{(n-1)}$*(111... n times)
if repetition is allowed.

i.e What is the sum of all 3 digit positive integers that can be formed using the digits 1, 5, and 8, if the digits are allowed to repeat within a number?

A. 126 B. 1386 C. 3108 D. 308 E. 13986

here n = 3 , sum of digits = 14 thus for 3 digits we have to take 111 only.

so sum = 14*3^2*(111)=13986

if not repetition then sum = 14 *2!*111 =3108

** if a+b+c = constant then $a^2b^3c^4$ have maximum values when a, b and c are in ratio 2:3:4

e.g volume of cylinder V = $\Pi r^2 h$, and given that r+h=9 then maximum value of V will be when

r and h are in ratio 2:1

**some maths solving strategy:**

1.think for 5sec before starting calculation

2.picking numbers is almost always the best to attack even/ odd questions.

3.generally answers are arranged in ascending order, if picking number is the strategy then checking answer choice C, so it will be clear whether we need to check D&E or A&B

4. to check smaller or greater of two values >1 and are raised to power.. first make all of them to equal roots by taking LCM

5. first 10 +ve multiple of 5 are: 5,10,15,20,25,30,35,40..........( this is example to show what is multiple of some number means )

6.when answer choices contains variable then check answer option from E-->A

7. range of difference of two range: lower limit = min of X -max of Y and upper limit = max of X - min of Y
e.g. if -2<=Y<=2 & 3<=X<=8 then range of X-Y will be 1<=(X-Y)<=10

8.I. if |x| <= b & b>0 then -b <= x <= b...that is x lies between -b & b, inclusive
II. if |x| >= b & b>0 , then either x <= -b or x >= b...that is x lies outside the range -b to b, exclusive.

**General info:**

1. a leap year has 366 days .. and in a year there are 52 weeks and reaming 1 or 2days can be any of the seven days..

**compound interest:**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where,

- P = principal amount (initial investment)
- r = annual nominal interest rate (as a decimal)
- n = number of times the interest is compounded per year
- t = number of years
- A = amount after time t

Example usage: An amount of $1500.00 is deposited in a bank paying an annual interest rate of 4.3%, compounded quarterly. Find the balance after 6 years.
A. Using the formula above, with P = 1500, r = 4.3/100 = 0.043, n = 4, and t = 6:

$$A = 1500\left(1 + \frac{0.043}{4}\right)^{4\times 6} = 1938.84$$

So, the balance after 6 years is approximately $1,938.op

**Geometry**

1. In case of triangles, an equilateral triangle has maximum area.
For a given area equilateral triangle has the smallest perimeter.

2. Triangles of equal heights have areas proportional to their corresponding bases.

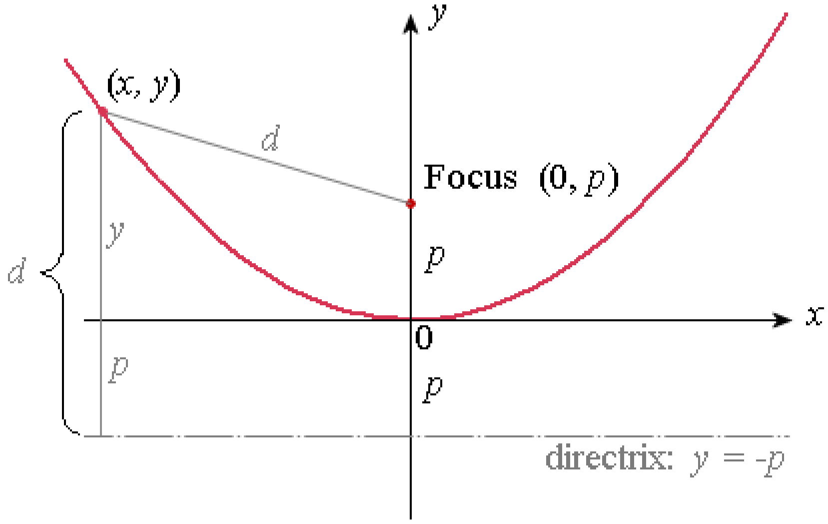
3. Triangles of equal bases have areas proportional to their corresponding heights.

4. Areas of two triangles are equal if they have the same base and lie between the same parallel line.

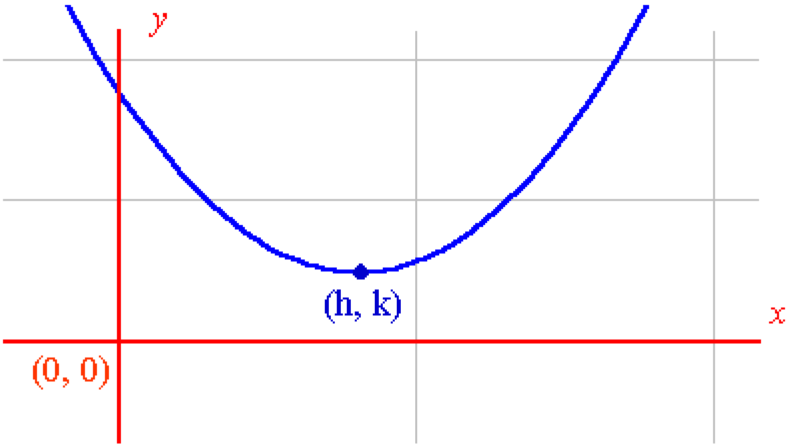
5. In two similar triangles, the ratio of their areas is the square of the ratio of their sides.

6. angle between two lines:

$$\tan \phi = \frac{m_2 - m_1}{1 + m_1 m_2}$$

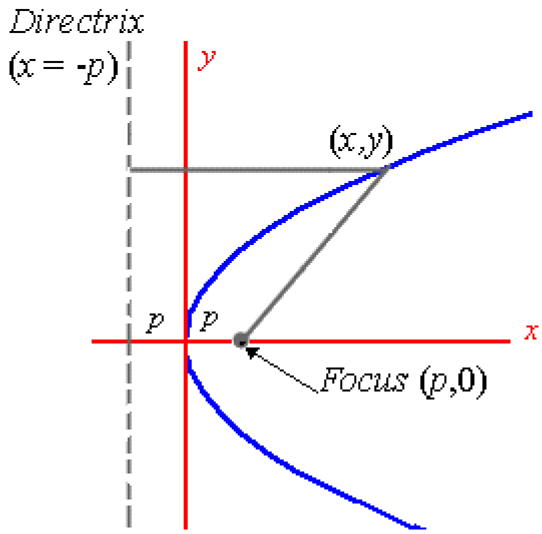



here equation for parabola is form $x^2 = 4ay$




if parabola is offset by(h,k) then equation will be ($X-h)^2 = 4a(Y-k)$

for the following type of parabola the equation will be $Y^2 = 4aX$

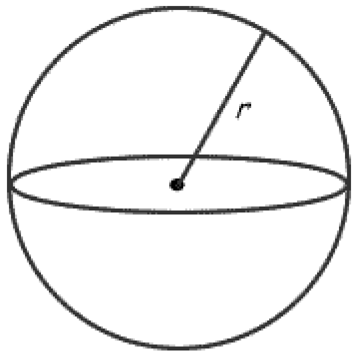



if parabola is offset by(h,k) then equation will be ($Y-k)^2 = 4a(X-h)$

## Sphere

**Surface Area**

$A = 4\pi r^2$




**Volume**

$V = \frac{4}{3}\pi r^3$

## Cone

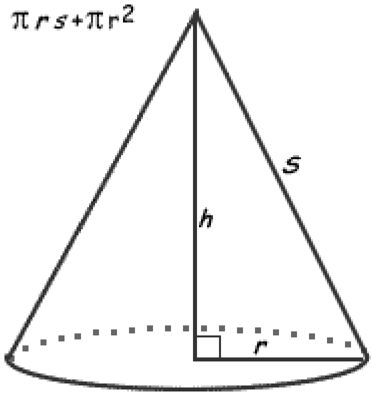
**Surface Area**

We will need to calculate the surface area of the cone and the base.

Area of the cone is $\pi r s$

Area of the base is $\pi r^2$

Therefore the Formula is: $SA = \pi r s + \pi r^2$




**Volume**

$V = \frac{1}{3}\pi r^2 h$

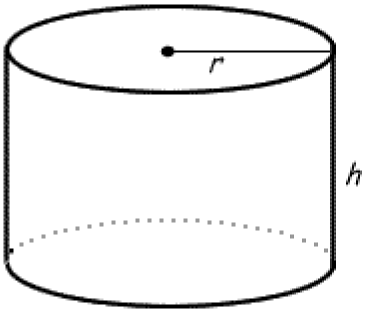
Cylinder

<u>Surface Area</u>

We will need to calculate the surface area of the top, base and sides.

Area of the top is $\pi r^2$
Area of the bottom is $\pi r^2$
Area of the side is $2\pi r h$

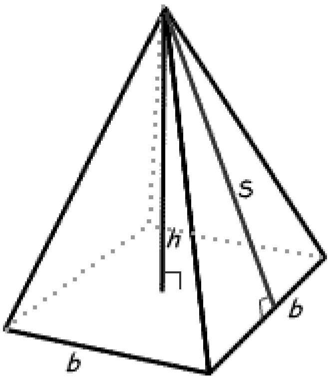
Therefore the Formula is: $A = 2\pi r^2 + 2\pi r h$




<u>Volume</u> $V = \pi r^2 h$

## Square Based Pyramid

Surface Area $A = 2bs + b^2$




Volume $V = \frac{1}{3} b^2 h$

## Sector

Area The area of a sector of a circle can be calculated by degrees or radians. $\left(\frac{\pi}{2} \text{ radians} = 90^\circ\right)$

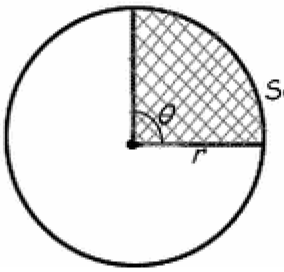
A: Area r

r: radius r

$\theta$: central angle

Formula $\frac{\theta}{2} r^2$ (in radians)

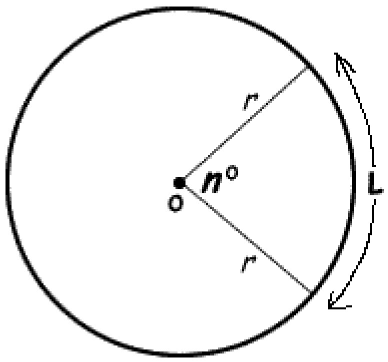
$\frac{\theta}{360} \pi r^2$ (in degrees)



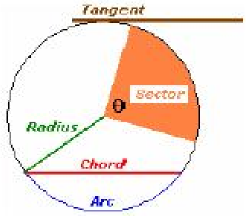

Sector is the shaded area

Length of an Arc Formula

$$\textit{Length} = \frac{n^\circ}{360^\circ} \times 2\pi r$$



A circle is a set of all points in a plane that lie at a constant distance from a fixed point. The fixed point is called the center of the circle and the constant distance is known as the radius of the circle.




**Arc:** An arc is a curved line that is part of the circumference of a circle. A **minor arc** is an arc less than the semicircle and a **major arc** is an arc greater than the semicircle.

**Chord:** A chord is a line segment within a circle that touches 2 points on the circle.

**Diameter:** The longest distance from one end of a circle to the other is known as the diameter. It is equal to twice the radius.

**Circumference:** The perimeter of the circle is called the circumference. The value of the circumference = $2\pi r$, where r is the radius of the circle.

**Area of a circle:** Area = $\pi \times (\text{radius})^2 = \pi r^2$.

**Sector:** A sector is like a slice of pie (a circular wedge).

**Area of Circle Sector:** (with central angle θ) Area = $\frac{\theta}{360} \times \pi \times r^2$

**Length of a Circular Arc:** (with central angle θ) The length of the arc = $\frac{\theta}{360} \times 2\pi \times r$

**Tangent of circle:** A line perpendicular to the radius that touches ONLY one point on the circle

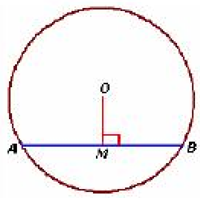
**If 45° arc of circle A has the same length as 60° arc of circle B, find the ratio of the areas of circle A and circle B.**

**Answer:** Let the radius of circle A be $r_1$ and that of circle B be $r_2$.

$\Rightarrow \frac{45}{360} \times 2\pi \times r_1 = \frac{60}{360} \times 2\pi \times r_2 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow$ Ratio of areas $= \frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{9}$.

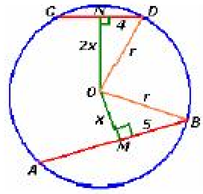
**RULE!**

The perpendicular from the center of a circle to a chord of the circle bisects the chord. In the figure below, O is the center of the circle and OM ⊥ AB. Then, AM = MB.




Conversely, the line joining the center of the circle and the midpoint of a chord is perpendicular to the chord.

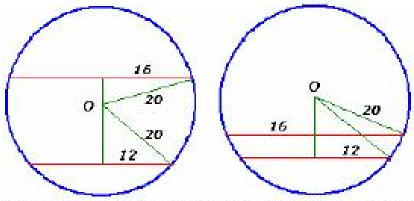
**In a circle, a chord of length 8 cm is twice as far from the center as a chord of length 10 cm. Find the circumference of the circle.**




**Answer:** Let AB and CD be two chords of the circle such that AB = 10 and CD = 8. Let O be the center of the circle and M and N be the midpoints of AB and CD. Therefore OM ⊥ AB, ON ⊥ CD, and if ON = 2x then OM = x.
$BM^2 + OM^2 = OB^2$ and $DN^2 + ON^2 = OD^2$.
OB = OD = r → $(2x)^2 + 4^2 = r^2$ and $x^2 + 5^2 = r^2$.
Equating both the equations we get, $4x^2 + 16 = x^2 + 25$
Or $x = \sqrt{3}$ → $r = 2\sqrt{7}$. Therefore circumference = $2\pi r = 4\pi\sqrt{7}$.

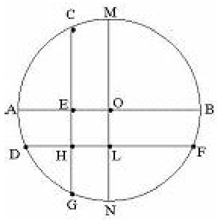
**What is the distance in cm between two parallel chords of length 32 cm and 24 cm in a circle of radius 20 cm? (CAT 2005)**
1. 1 or 7 2. 2 or 14 3. 3 or 21 4. 4 or 28

**Answer:** The figures are shown below:




The parallel chords can be on the opposite side or the same side of the centre O. The perpendicular (s) dropped on the chords from the centre bisect (s) the chord into segments of 16 cm and 12 cm, as shown in the figure. From the Pythagoras theorem, the distances of the chords from the centre are $\sqrt{20^2 - 16^2} = 12$ and $\sqrt{20^2 - 12^2} = 16$, respectively. Therefore, the distances between the chords can be 16 + 12 = 28 cm or 16 – 12 = 4 cm.

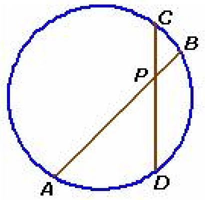
**In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2, and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is (CAT 2005)**



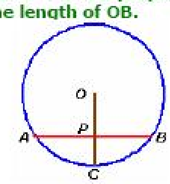

**Answer:** In the above figure, AB = MN = 3 cm and AE: EB = NL: LM = 1: 2
⇒ AE = NL = 1 cm. Now AO = NO = 1.5 cm ⇒ OE = HL = OL = 0.5 cm. Join O and D
⇒ $OD^2 = OL^2 + DL^2 \Rightarrow DL^2 = \sqrt{OD^2 - OL^2} = \sqrt{1.5^2 - 0.5^2} = \sqrt{2} \Rightarrow DH = DL - HL = \sqrt{2} - \frac{1}{2} = \frac{2\sqrt{2} - 1}{2}$

**RULE!**
Equal chords are equidistant from the center. Conversely, if two chords are equidistant from the center of a circle, they are equal.

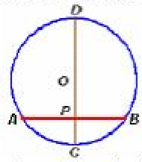
**RULE!**
In the following figure, two chords of a circle, AB and CD, intersect at point P. Then, AP × PB = CP × PD.




**In the following figure, length of chord AB = 12. O-P-C is a perpendicular drawn to AB from center O and intersecting AB and the circle at P and C respectively. If PC = 2, find the length of OB.**




**Answer:** Let us extend OC till it intersects the circle at some point D.



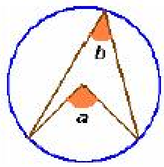

CD is the diameter of the circle. Since OP is perpendicular to AB, P is the midpoint of AB. Hence, AP = PB = 6. Now DP × PC = AP × PB → DP = 18. Therefore, CD = 20 → OC = 10. OB = OC = radius of the circle = 10.

**RULE!**

In a circle, equal chords subtend equal angles at the center.

**RULE!**

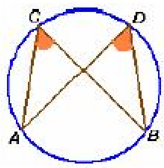
The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circumference.




In the figure shown above, a = 2b.

**RULE!**

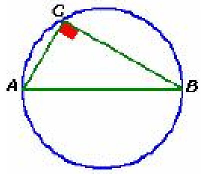
Angles inscribed in the same arc are equal.




In the figure angle ACB = angle ADB.

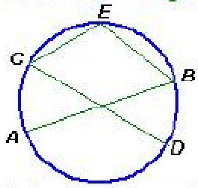
**RULE!**

An angle inscribed in a semi-circle is a right angle.

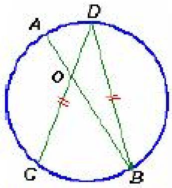



Let angle ACB be inscribed in the semi-circle ACB; that is, let AB be a diameter and let the vertex C lie on the circumference; then angle ACB is a right angle.

**In the figure AB and CD are two diameters of the circle intersecting at an angle of 48°. E is any point on arc CB. Find angle CEB.**



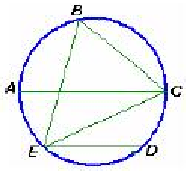

**Answer:** Join E and D. Since arc BD subtends an angle of 48° at the center, it will subtend half as many degrees on the remaining part of circumference as it subtends at the center. Hence, angle DEB = 24°. Since angle CED is made in a semicircle, it is equal to 90°. Hence, angle CEB = angle CED + angle DEB = 90° + 24° = 114°.




**In the above figure, AB is a diameter of the circle and C and D are such points that CD = BD. AB and CD intersect at O. If angle AOD = 45°, find angle ADC.**

**Answer:** Draw AC and CB. CD = BD $\Rightarrow \angle DCB = \angle DBC = \theta$ (say). $\angle ACB = 90° \Rightarrow \angle ACD = 90° - \theta$.
$\angle ABD = \angle ACD = 90° - \theta \Rightarrow \angle ABC = \theta - (90° - \theta) = 2\theta - 90$. In $\Delta OBC$, $45° + 2\theta - 90 + \theta = 180° \Rightarrow 3\theta = 225° \Rightarrow \theta = 75°$.
$\angle ADC = \angle ABC = 2\theta - 90 = 60°$.

**In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If angle CBE = 65°, then what is the value of angle DEC? (CAT 2004)**

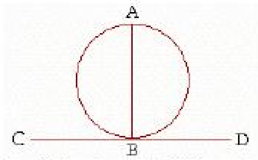



1. 35° 2. 55° 3. 45° 4. 25°

**Answer:** $\angle ABC = 90° \Rightarrow \angle ABE = 90 - \angle EBC = 25°$. $\angle ABE = \angle ACE = 25°$. $\angle ACE = \angle CED = 25°$ (alternate angles)

**RULE!**

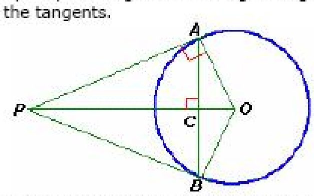
The straight line drawn at right angles to a diameter of a circle from its extremity is tangent to the circle. Conversely, If a straight line is tangent to a circle, then the radius drawn to the point of contact will be perpendicular to the tangent.




Let AB be a diameter of a circle, and let the straight line CD be drawn at right angles to AB from its extremity B; then the straight line CD is tangent to the circle.

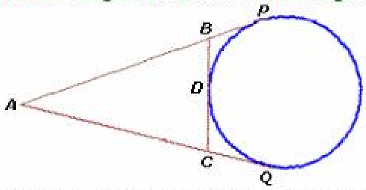
**RULE!**

If two tangents are drawn to a circle from an exterior point, the length of two tangent segments are equal. Also, the line joining the exterior point to the centre of the circle bisects the angle between the tangents.




In the above figure, two tangents are drawn to a circle from point P and touching the circle at A and B. Then, PA = PB. Also, ∠APO = ∠BPO. Also, the chord AB is perpendicular to OP.

**In the following figure, lines AP, AQ and BC are tangent to the circle. The length of AP = 11. Find the perimeter of triangle ABC.**

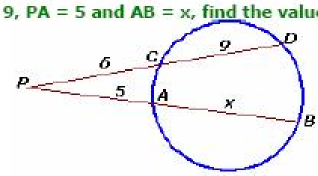



**Answer:** let AB = x and BP = y. Then, BD = BP because they are tangents drawn from a same point B. Similarly CD = CQ and AP = AQ. Now perimeter of triangle ABC = AB + BC + CA = AB + BD + DC + AC = AB + BP + CQ + AC = AP + AQ = 2AP = 22.

**RULE!**

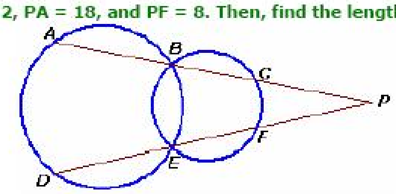
From an external point P, a secant P-A-B, intersecting the circle at A and B, and a tangent PC are drawn. Then, PA × PB = $PC^2$.

**In the following figure, if PC = 6, CD = 9, PA = 5 and AB = x, find the value of x**




**Answer:** Let a tangent PQ be drawn from P on the circle. Hence, PC × PD = $PQ^2$ = PA × PB → 6 × 15 = 5 × (5 + x) → x = 13

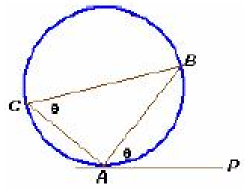
**In the following figure, PC = 9, PB = 12, PA = 18, and PF = 8. Then, find the length of DE.**



Answer: In the smaller circle PC × PB = PF × PE → PE = $12 \times \frac{9}{8} = \frac{27}{2}$. In the larger circle, PB × PA = PE × PD → PD = $12 \times 18 \times \frac{2}{27}$ = 16.
Therefore, DE = PD – PE = 16 – 13.5 = 2.5

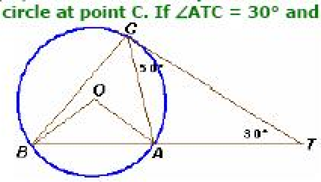
**RULE!**

The angle that a tangent to a circle makes with a chord drawn from the point of contact is equal to the angle subtended by that chord in the alternate segment of the circle.




In the figure above, PA is the tangent at point A of the circle and AB is the chord at point A. Hence, angle BAP = angle ACB.

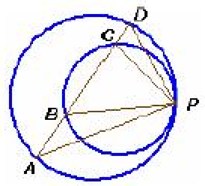
**In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is (CAT 2003)**



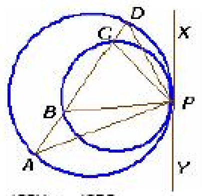

1. 100° 2. 150° 3. 80° 4. not possible to determine

**Answer:** Tangent TC makes an angle of 50° with chord AC. Therefore, ∠TBC = 50°. In triangle TBC, ∠BCT = 180° – (30° + 50°) = 100°. Therefore, ∠BCA = ∠BCT – ∠ACT = 100° – 50° = 50°. ∠BOA = 2∠BCA = 100°.

**Two circles touch internally at P. The common chord AD of the larger circle intersects the smaller circle in B and C, as shown in the figure. Show that , ∠APB = ∠CPD.**



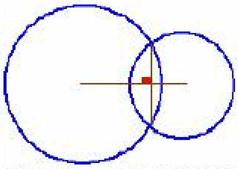

**Answer:** Draw the common tangent XPY at point P.




Now, for chord DP, ∠DPX = ∠DAP, and for chord PC, ∠CPX = ∠CBP
⇒ ∠CPD = ∠CPX – ∠DPX = ∠CBP – ∠DAP. In triangle APB, ∠CBP is the exterior angle
⇒ ∠CBP = ∠CAP + ∠APB
⇒ ∠CBP – ∠CAP = ∠APB
⇒ ∠CPD = ∠CPX – ∠DPX = ∠CBP – ∠DAP = ∠APB

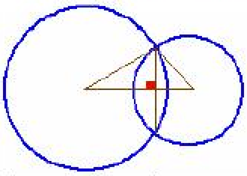
**RULE!**

When two circles intersect each other, the line joining the centers bisects the common chord and is perpendicular to the common chord.

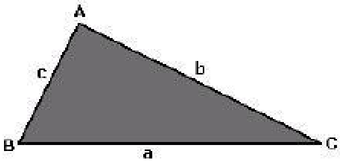


In the figure given above, the line joining the centers divides the common chord in two equal parts and is also perpendicular to it.

**Two circles, with diameters 68 cm and 40 cm, intersect each other and the length of their common chord is 32 cm. Find the distance between their centers.**



**Answer:** In the figure given above, the radii of the circles are 34 cm and 20 cm, respectively. The line joining the centers bisects the common chord. Hence, we get two right triangles: one with hypotenuse equal to 34 cm and height equal to 16 cm, and the other with hypotenuse equal to 20 cm and height equal to 16 cm. Using Pythagoras theorem, we get the bases of the two right triangles equal to 30 cm and 12 cm. Hence, the distance between the centers = 30 + 12 = 42 cm.

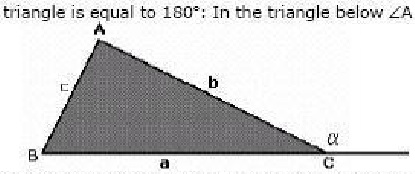



**General Properties of Triangles:**

1. The sum of the two sides is greater than the third side: $a + b > c$, $a + c > b$, $b + c > a$

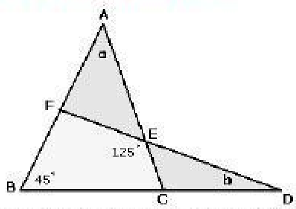
   **Problem: The two sides of a triangle are 12 cm and 7 cm. If the third side is an integer, find the sum of all the values of the third side.**
   Answer: Let the third side be of x cm. Then, $x + 7 > 12$ or $x > 5$. Therefore, minimum value of x is 6. Also, $x < 12 + 7$ or $x < 19$. Therefore, the highest value of x is 18. The sum of all the integer values from 6 to 18 is equal to 156.

2. The sum of the three angles of a triangle is equal to 180°: In the triangle below $\angle A + \angle B + \angle C = 180°$

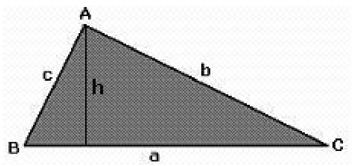



   Also, the exterior angle α is equal to sum the two opposite interior angle A and B, i.e. $\alpha = \angle A + \angle B$.

   **Problem: Find the value of a + b in the figure given below:**




   Answer: In the above figure, $\angle CED = 180° - 125° = 55°$. $\angle ACD$ is the exterior angle of $\Delta ABC$. Therefore, $\angle ACD = a + 45°$.
   In $\Delta CED$, $a + 45° + 55° + b = 180° \Rightarrow a + b = 80°$




3. **Area of a Triangle:**

   Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times h$

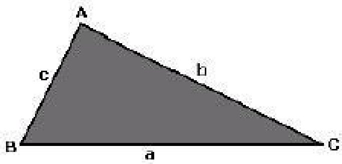
   Area of a triangle $= \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$

   Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

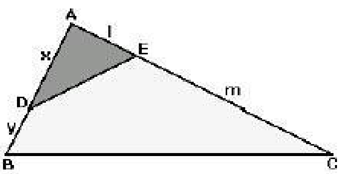
   Area of a triangle $= \frac{abc}{4R}$ where R = circumradius

   Area of a triangle $= r \times s$ where r = inradius and $s = \frac{a+b+c}{2}$

4. **More Rules:**




   - Sine Rule: $\frac{\text{Sin}A}{a} = \frac{\text{Sin}B}{b} = \frac{\text{Sin}C}{c}$
   - Cosine Rule: $\text{Cos}A = \frac{b^2 + c^2 - a^2}{2bc}$, $\text{Cos}B = \frac{a^2 + c^2 - b^2}{2ac}$, $\text{Cos}C = \frac{b^2 + a^2 - c^2}{2ab}$




   Let D and E be on sides AB and AC of triangle ABC such that $\frac{AD}{DB} = \frac{x}{y}$ and $\frac{AE}{EC} = \frac{l}{m}$. Then, area triangle ADE $= \frac{1}{2} lx \sin A$ and area triangle ABC $= \frac{1}{2}(l+m)(x+y)\sin A$ .Therefore, $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{lx}{(x+y)(l+m)}$

5. **Medians of a triangle:**