Introductory note

Combinatorics is the study of countable structures. On the the GMAT, when faced with a combinatorics problem, your main concern should be determining the most efficient and accurate approach for determining the correct answer. Many students read combinatorics questions and fall into the trap of applying a C or P formula. The truth is, on most problems, you do not even need to use one of these formulas. Keep this in mind while reviewing this section.

Fundamentals

Enumeration: the number of ways n items can be arranged in a row.

Example: "How many ways can you arrange 6 students?"

Answer: =6! = 720 ways

Anagram Grid: a very useful tool for solving potentially difficult questions.

Example: "How many ways can you arrange the letters in the word ROOST?"

Answer: R O O S T

Count the total number of letters = 5

Count the total number repeated letters (O,O) = 2

= 5! / 2!

= 60

Slot Method: a great method for organizing outcomes that are highly constricted. This method considerers the number of options available to each individual decision.

Example: "Annie has the choice of 6 toppings and 5 flavors of ice cream for her sundae. She also has the option of whipped cream or no whipped cream. How many ways can she create a sundae?

Answer: Decision 1 * Decision 2 * Decision 3

= 6 * 5 * 2

= 60

Combination formula: a combination is an *unordered* collection of k objects taken from a set of n objects.

$$= c_k^n = \frac{n!}{k!(n-k)!}$$

Example: "How many ways can I pick 3 people from a group of 6?"

Answer: $= c_3^6 = \frac{6!}{3!(6-3)!} = 5 * 4 = 20$

Permutation formula: a permutation is an *ordered* collection of k objects taken from a set of n objects.

$$= \mathrm{P}_k^n = \frac{n!}{(n-k)!}$$

Example: "How many ways can a committee choose a president, a vice president, and a treasurer from a set of 8 qualified individuals? An individual can only serve one position at a time."

Answer: $= \mathrm{P}_3^8 = \frac{8!}{(8-3)!} = 8 * 7 * 6 = 336$ (using formula)

Answer: Decision 1 * Decision 2 * Decision 3 (using slot method)

= 8 * 7 * 6 = 336

Answer: Pres. | V.P. | Treas. | Ind. | Ind. | Ind. | Ind. | Ind. (anagram)

Count the total number of "slots" = 8

Count the total number of repeated "slots" = 5

= 8! / 5! = 336

Special cases:

Circular arrangements: a circular arrangement is similar to enumeration, except the number of objects are being arranged in a circle, not a line.

Example: "How many ways can you arrange 6 objects in a circle?"

Answer: = (6-1)! = 5! = 120

“Next to each other”: this is a stem that confuses a lot of people. To solve these problems you must treat the objects that are “next to each other” as one unit.

Example: “Jim and Mary are going to the movies with 3 additional friends. If Jim and Mary must sit next to each other, how many ways can the friends be arranged?”

Answer: Treat Jim and Mary as one person. Therefore there are now a total of 4 people attending the movies (instead of 5)

= 4! (total number of people attending) * 2! (number of possibly Jim and Mary arrangements)

= 4! * 2! = 48

“To the left/right of”: this another question stem that often confuses a lot of people. The solution is very simple and this same solution can be applied to question stems containing “before/after” also. In situations like this, apply the symmetry rule.

Example: “Jim and Mary are going to the movies with 3 additional friends. If Jim must always be to the left of Mary, how many ways can the friends be arranged”?

Answer: The total number of people attending the movies is 5.

= 5! / 2 (the symmetry rule results in an answer divided in half)

= 60

Problem ID:

Combinatoric questions will almost always contain the following verbiage in the question stem, which can be used to quickly identify the problem and possibly even dictate your methodology:

“How many ways can...”

“How many different arrangements of…”

“Next to each other” - apply glue method

“To the left/right of” or “before/after” - apply the symmetry rule