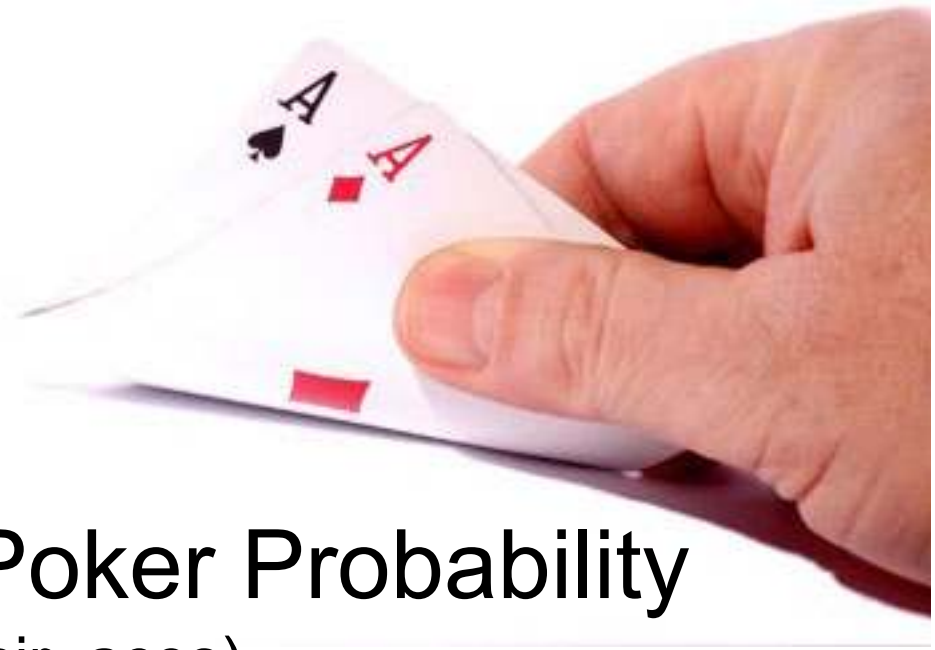


Combinations + Poker Probability

(pocket pair, aces)

by Zeke Lee

GMAT PILL

www.GMATPILL.com

In a kickball competition of 9 teams, how many possible matches can each team play with each other?

(A) 9

(B) 16

(C) 24

(D) 36

(E) 54

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

In a kickball competition of 9 teams, how many possible matches can each team play with each other?

(A) 9

(B) 16

(C) 24

(D) 36

(E) 54

① $9C2 = \frac{9!}{2!\,7!} = \frac{9\times 8\times \cancel{7!}}{2!\,\cancel{7!}} = \frac{72}{2!} = 36$

② $\frac{9C1 \times 8C1}{2!} = \frac{9\times 8}{2} = \frac{72}{2} = 36$

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

How many ways can you pick something?

Each spot choose from among REMAINING options (use nCr formula)

Each spot choose among ALL options
Use Exponent ^

Variations

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

How many ways can you pick something?

Each spot choose among ALL options
Use Exponent ^

Each spot choose from among REMAINING options (use nCr formula)

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

Variations

Password

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Use Exponent ^

Combinations
(Order Does NOT Matter)

<

Variations

How many ways can a slot machine display a set of 3 slots if each slot can have 7 possibilities?

= 7^3 = 343

How many password combinations does a decryption software need to go through to decrypt an 8-letter password assuming only lower-cased alphabetic letters.

How many ways can you roll 3

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Each spot choose among ALL options
Use Exponent ^

REMAINING options (use nCr formula)

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

Variations

How many ways can a slot machine display a set of 3 slots if each slot can have 7 possibilities?

How many password combinations does a decryption software need to go through to decrypt an 8-letter password assuming only lower-cased alphabetic letters.

How many ways can you roll 3 dice?

How many distinct ways can 3 guys vote for those 3 girls?

$= 6^3$

$= 216$

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

A How many ways can you pick a team of 3 out of 10 people?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

3 starting players are selected from a team of 10, each with a jersey letter. How many 3-l e created when th up togethe

A How many ways can you pick a team of 3 out of 10 people?

B How many ways can you roll 3 identical dice and get 3 different numbers?

All counts as 1 combination

1 3 5
1 5 3
3 5 1
3 1 5
5 1 3
5 3 1

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

an you pick a
l people?

3 starting players are selected from a team of 10, each with a jersey letter. How many 3-letter sequences can be created when the 3 selected players line up together.

1 3 5
1 5 3
3 5 1
3 1 5
5 1 3
5 3 1

Counts as 6 combinations

How many ways can you roll 3 differently colored dice and get 3 different numbers?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

nong ALL options
se Exponent ^

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

3 starting players are selected from a team of 10, each with a jersey letter. How many 3-letter sequences can be created

Diamond ->Spade

Spade -> Diamond

SAME - Counts as 1
(order does NOT matter)

y ways can you roll 3

marble?

marble?

D How many ways can you get a pocket pair?

y distinct ways can 3 guys
hose 3 girls?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

nong ALL options
se Exponent ^

Combinations
(Order Does NOT Matter)

<

Permutations
(Order Matters = More Possibilities)

3 starting players are selected from a team of 10, each with a jersey letter. How many 3-letter sequences can be created

Diamond ->Spade

Spade -> Diamond

DIFFERENT
Counts as 2

y ways can you roll 3

marble?

marble?

D How many ways can you get a pocket pair?

If you are dealt a pocket pair, how many different ways could that pair have been individually dealt to you?

y distinct ways can 3 guys
hose 3 girls?

How many ways can you pick something?

Each spot choose among ALL options
Use Exponent ^

Each spot choose from among REMAINING options (use nCr formula)

Variations

How many ways can a slot machine display a set of 3 slots if each slot can have 7 possibilities?

How many password combinations does a decryption software need to go through to decrypt an 8-letter password assuming only lower-cased alphabetic letters.

How many ways can you roll 3 dice?

How many distinct ways can 3 guys vote for those 3 girls?

	Combinations (Order Does NOT Matter)	<	Permutations (Order Matters = More Possibilities)
A	How many ways can you pick a team of 3 out of 10 people?	↔	3 starting players are selected from a team of 10, each with a jersey letter. How many 3-letter sequences can be created when the 3 selected players line up together.
B	How many ways can you roll 3 identical dice and get 3 different numbers?	↔	How many ways can you roll 3 differently colored dice and get 3 different numbers?
C	How many ways can you distribute 5 marbles in 3 identical baskets such that each basket has at least 1 marble?	↔	How many ways can you distribute 5 different colored marbles in 3 distinct baskets such that each basket has at least 1 marble?
D	How many ways can you get a pocket pair?	↔	If you are dealt a pocket pair, how many different ways could that pair have been individually dealt to you?
E	How many potential dates can happen between 3 guys and 3 girls?		How many different ways can 3 guys and 3 girls pair up for a group date of 6 people? (In calculation – order matters)

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

A

How many ways can you pick a team of 3 out of 10 people?

= 10C1 * 9C1 * 8C1 / 3!
= 720 / 6 = **120**

10C3 = 720 / 6 = **120**

Permutations
(Order Matters = More Possibilities)

3 starting players are selected from a team of 10, each with a jersey letter. How many 3-letter sequences can be created when the 3 selected players line up together.

10C1 * 9C1 * 8C1 = **720**

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

B

How many ways can you roll 3 identical dice and get 3 different numbers?

Divide by 3!

6C1*5C1*4C1 / 3! = 20

Permutations
(Order Matters = More Possibilities)

How many ways can you roll 3 differently colored dice and get 3 different numbers?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

B

How many ways can you roll 3 identical dice and get 3 different numbers?

6C1*5C1*4C1 / 3! = **20**

Some combinations are effectively duplicates

Permutations
(Order Matters = More Possibilities)

How many ways can you roll 3 differently colored dice and get 3 different numbers?

6C1*5C1*4C1 = **120**

Every permutation counts (order MATTERS)

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

C

How many ways can you distribute 5 marbles in 3 identical baskets such that each basket has at least 1 marble?

3_1_1:
(5C3*2C1*1C1)/(2!) = 10

2_2_1:
(5C2*3C2*1C1)/(2!) = 15

= 10 +15 = 25

How many ways can you distribute 5 different colored marbles in 3 distinct baskets such that each basket has at least 1 marble?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

C

How many ways can you distribute 5 marbles in 3 identical baskets such that each basket has at least 1 marble?

3_1_1:
(5C3*2C1*1C1)/(2!) = 10

2_2_1:
(5C2*3C

= 10 +1

Note both
3C2 = 3
3C1 = 3

How many ways can you distribute 5 different colored marbles in 3 distinct baskets such that each basket has at least 1 marble?

3_1_1: (3C1)(5C3*2C1*1C1)=60
2_2_1: (3C2)(5C2*3C2*1C1)=90
= 60+90= 150

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D How many ways can you get a pocket pair?

Out of 13 values (A, 2, 3, ..J, Q, K)

choose 1

Method 1

(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

$$\boxed{{}_{13}C_1} * ({}_4C_1 * {}_3C_1) / 2!$$
$$= 13 * 6 = \mathbf{78}$$

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D How many ways can you get a pocket pair?

Method 1
(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

Out of 4 suits, choose 1

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D How many ways can you get a pocket pair?

Method 1
(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

Out of 3 remaining suits, choose 1

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D How many ways can you get a pocket pair?

Method 1

(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

D How many ways can you get a pocket pair?

Method 1

(If order does not matter, divide by r!)

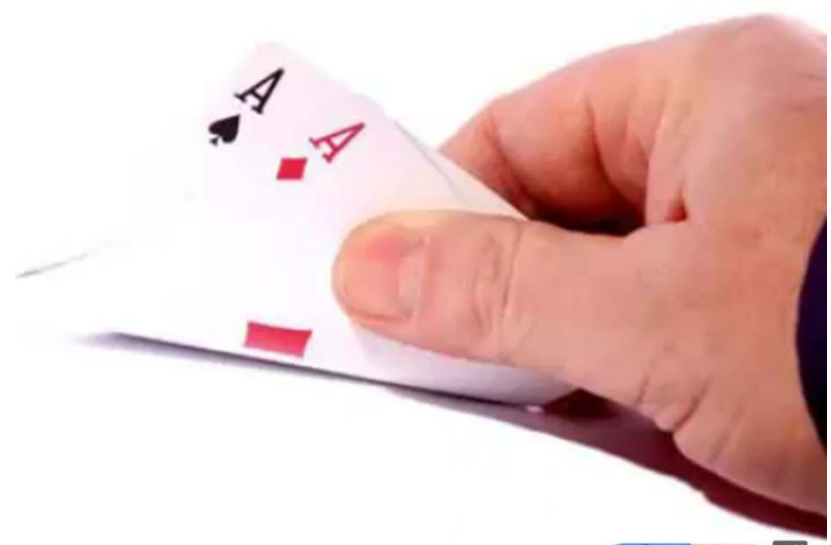
Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

Method 2

13C1 * 4C2 = **78**



#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

D How many ways can you get a pocket pair?

Method 1

(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

Method 2

13C1 * 4C2 = **78**

Out of 4 suits, choose 2

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D

How many ways can you get a pocket pair?

If you are dealt a pocket pair, how many different ways could that pair have been dealt to you?

1) Spade, then heart
2) Heart, then spade

Method 1

(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

$$13C1 * (4C1 * 3C1) / 2!$$
$$= 13 * 6 = \mathbf{78}$$

Method 2

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

D

How many ways can you get a pocket pair?

Method 1

(If order does not matter, divide by r!)

Break down into sequences.
Choose 1st card, then choose 2nd card.

Since order does not matter, divide by (2!)

13C1 * (4C1 * 3C1) / 2!
= 13 * 6 = **78**

Method 2

If you are dealt a pocket pair, how many different ways could that pair have been dealt to you?

Break into sequences.

Choose 1st card, then choose 2nd card

Since order matters, **do not divide by (2!)**

13C1 * (4C1 * 3C1)
= 12 * 13 = **156**

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

How many potential dates can happen between 3 guys and 3 girls?

How many different ways can 3 guys and 3 girls pair up for a group date of 6 people?

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Warmup | Variations | Combinations & Permutations | Picking Teams | Dice | Distributing Marbles | Pocket Pair | Dating

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

How many potential dates can happen between 3 guys and 3 girls?

3C1 + 3C1 + 3C1 = 9

How many different ways can 3 guys and 3 girls pair up for a group date of 6 people?

3C1 * 2C1 = 6

G1 G2 G3

Combinations
(Order Does NOT Matter)

Permutations
(Order Matters = More Possibilities)

How many potential dates can happen between 3 guys and 3 girls?

$3C1 + 3C1 + 3C1 = 9$

How many different ways can 3 guys and 3 girls pair up for a group date of 6 people?

$3C1 * 2C1 = 6$

Since order matters, do not divide by 2!

Given that there are 5 basketball players per team, how many ways can you select 2 basketball players from 3 teams if no more than one player can be selected from each team?

(A) 15

(B) 30

(C) 60

(D) 75

(E) 90

#1 Framework: How many ways? | #2 Apply to GMAT | #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

Given that there are 5 basketball players per team, how many ways can you select 2 basketball players from 3 teams if no more than one player can be selected from each team?

(A) 15

(B) 30

(C) 60

(D) 75

(E) 90

A B C

$3C2 \times (5C2)^2$

$\frac{3!}{1!.2!}$ $\frac{5!}{1!.4!}$

$3 \times (5)^2 = 75$

$3C2$

$\frac{3!}{2!.1!}$

$7C3$

$\frac{7!}{3!.4!}$

Learn to Recognize Easy Calculations Like 3C2 and 5C1

Answer in <2 min | Thought Process

#1 Framework: How many ways? | #2 Apply to GMAT | #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

Given that there are 5 basketball players per team, how many ways can you select 2 basketball players from 3 teams if no more than one player can be selected from each team?

(A) 15

(B) 30

(C) 60

(D) 75

(E) 90

A B C

$3C2 \times (5C2)^2$

$\frac{3!}{1!.2!}$ $\frac{5!}{1!.4!}$

$3C2$

$\frac{3!}{2!.1!}$

$7C3$

$\frac{7!}{3!.4!}$

$3 \times (5)^2 = 75$

Answer in <2 min | Thought Process & Concepts | Recap

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$8C1 \times 7C1 \times 5C1 \times 4C1 \times 3C1 \times 2C1$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$$\frac{\overset{M}{8C1 \times 7C1}}{2!} \times \frac{\overset{W}{5C1 \times 4C1 \times 3C1 \times 2C1}}{4!}$$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

2M4F $\frac{56}{2} \times 5 = 28 \times 5 = \underline{140}$

3M3F $\frac{8C1 \times 7C1 \times 6C1}{3!} \times \frac{5C1 \times 4C1 \times 3C1}{3!}$

$= 56 \times \frac{60}{6} = 56 \times 10 = \underline{560}$

Total $= 140 + 560 = 700$

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$$\frac{{}^{M}8C1 \times 7C1}{2!} \times \frac{{}^{W}5C1 \times 4C1 \times 3C1 \times 2C1}{4!}$$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

2M4F $\frac{56}{} \times 5 = 28 \times 5 = \underline{140}$

3M

=

Total = 140 + 560 = 700

If ORDER DOES NOT MATTER, then divide by r!

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$$\frac{{}^{M}8C1 \times 7C1}{2!} \times \boxed{\frac{{}^{W}5C1 \times 4C1 \times 3C1 \times 2C1}{4!}}$$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

2M4F $\frac{56}{2} \times 5 = 28 \times 5 = \underline{140}$

3M3F $\frac{8C1 \times 7C1 \times 6C1}{3!} \times \frac{5C1 \times 4C1 \times 3C1}{3!}$

$= 56 \times \frac{60}{6} = 56 \times 10 = \underline{560}$

Total $= 140 + 560 = 700$

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$(8C2 \times 5C4) + (8C3 \times 5C3)$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

Method #2: Write it all out, cancel, and calculate

#1 Framework: How many ways? | #2 Apply to GMAT | #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$(8C2 \times 5C4) + (8C3 \times 5C3)$

$\frac{8!}{2!\,6!} \times \frac{5!}{4!\,1!} + \frac{8!}{5!\,3!} \times \frac{5!}{3!\,2!}$

$\frac{8 \times 7}{2!} \times 5 +$

$= 28 \times 5 +$

$= 140 + 560$

$= 700$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

Easy to set up, but have to write out more factorials

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

A team of 6 cooks is chosen from 8 men and 5 women. The team must have at least 2 men and at least 3 women. How many ways can this team be created?

2M 4F
3M 3F
~~4M 2F~~

$8C2 \times 5C4$

$\frac{8C1 \times 7C1}{2!} \times \frac{5C1 \times 4C1 \times 3C1 \times 2C1}{4!}$

(A) 140

(B) 320

(C) 560

(D) 700

(E) 840

Recall that anything Choose 1 equals itself:

?C1 = ?
8C1 = 8

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

Peter took a 52-card deck of cards and removed all small cards, all spades, and all clubs. The only remaining cards were 10, Jack, Queen, King, and Ace in the suits of heart and diamond. Of these 10 cards, he deals himself 5 cards. What is the probability that Peter deals himself at least on pair of cards?

$\frac{1}{2}$ $\frac{1}{5}$

$1 - P(\text{no Pair})$

(A) 7/10

(B) 17/33

(C) 55/63

(D) 66/165

(E) 18/25

P(At least) = 1 - P(none)

Answer in <2 min | Thought Process & Concepts | Recap

#1 Framework: How many ways? | #2 Apply to GMAT | #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

Peter took a 52-card deck of cards and removed all small cards, all spades, and all clubs. The only remaining cards were 10, Jack, Queen, King, and Ace in the suits of heart and diamond. Of these 10 cards, he deals himself 5 cards. What is the probability that Peter deals himself at least on pair of cards?

(A) 7/10

(B) 17/33

(C) 55/63

(D) 66/165

(E) 18/25

$$1 - P(\text{no Pair})$$

$$1 - \left[\frac{\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1}{5!} \times ({}^2C_1)^5}{{}^{10}C_5}\right]$$

$=1$

Answer in <2 min | Thought Process & Concepts | Recap

#1 Framework: How many ways? | #2 Apply to GMAT | #3 Poker Probability

Combinatorics #1 | Combinatorics #2A | Combinatorics #2B | Combinatorics #3

Peter took a 52-card deck of cards and removed all small cards, all spades, and all clubs. The only remaining cards were 10, Jack, Queen, King, and Ace in the suits of heart and diamond. Of these 10 cards, he deals himself 5 cards. What is the probability that Peter deals himself at least on pair of cards?

(A) 7/10

(B) 17/33

(C) 55/63

(D) 66/165

(E) 18/25

$$1 - P(\text{no Pair})$$

$$1 - \left[\frac{\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1}{5!} \times ({}^2C_1)^5}{{}^{10}C_5}\right]$$

$$1 - \frac{32}{\frac{10!}{5!\,5!}} = 1 - \frac{32}{\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}} = 1 - \frac{8}{9 \times 7}$$

$$\frac{63-8}{63} = \boxed{55/63}$$

Answer in <2 min | Thought Process & Concepts | Recap

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Pocket Pair vs Pocket Aces | Given pocket aces, 3-of-a-kind? | Exactly 1-pair by 5th flop card | 1-pair, 2-pair, 3-of-a-kind, straight | Flush, full house, royal, nothing special

$$\text{Probability} = \frac{\text{\# of ways}}{\text{Total possibilities}}$$

Since we are dividing by total possibilities, we should count all permutations. Calculate using "Order Matters"

What is P(pocket pair) ?

What is P (pocket aces)?

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What is P(pocket pair) ?

What is P (pocket aces)?

$$\frac{\text{Value * Suit Variations}}{\text{Total possibilities}}$$

$$\frac{\text{Value * Suit Variations}}{\text{Total possibilities}}$$

Total Possibilities = 52C2

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Since we are dividing by total possibilities, we should count all permutations. Calculate using "Order Matters"

What is P(pocket pair) ?

What is P (pocket aces)?

$$\frac{\text{Value * Suit Variations}}{\text{Total possibilities}}$$

$$\frac{(13C1) * (4C2)}{52C2}$$

$$= 78 / 1326$$

P (Pocket Pair) = 5.882%

$$\frac{\text{Value * Suit Variations}}{\text{Total possibilities}}$$

$$\frac{(4C2)}{52C2} = 6 / 1326 = .45\%$$

P (Pocket Ace) = 0.45%

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Given pocket ACES, what is P(3-of-a-kind) ?

= P (3rd ace) + P (3-of-a-kind on the flop)

Your Pocket Aces

1st card | 2nd card | 3rd card } The Flop

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Given pocket ACES, what is P(3-of-a-kind) ?

= P (3rd ace) + P (3-of-a-kind on the flop)

Out of remaining 2 aces, pick 1

Out of remaining 12 values, pick 2 for 2nd and 3rd card

For each of those 2 picked values, out of 4 suits, pick 1. Do this twice: once for 2nd card, once for 3rd card

1st card | 2nd card | 3rd card — The Flop

OR
For all 3 cards, out of remaining 12 values pick 1

For each value, out of 4 suits, pick 3 for all 3 cards in the flop.

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

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Given pocket ACES, what is P(3-of-a-kind) ?

= P (3rd ace) + P (3-of-a-kind on the flop)

Out of remaining 2 aces, pick 1 (2C1)

Out of remaining 12 values, pick 2 for 2nd and 3rd card (12C2)

For each of those 2 picked values, out of 4 suits, pick 1. Do this twice: once for 2nd card, once for 3rd card $(4C1)^2$

1st card | 2nd card | 3rd card — The Flop

OR
For all 3 cards, out of remaining 12 values pick 1 (12C1)

For each value, out of 4 suits, pick 3 for all 3 cards in the flop. (4C3)

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

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= P (3rd ace) + P (3-of-a-kind on the flop)

1st card | 2nd card | 3rd card } The Flop

$$(2C1)\ (12C2)\ (4C1)^2 + (12C1)\ (4C3)$$

$$= \frac{(2) * (66) * (16) + (12) * (4)}{50C3}$$

$$= \frac{2{,}160}{19{,}600} = 11.020\%$$

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P (exactly one pair by the time the last 5th card comes out, if no pocket pair) = ?

The Flop			Turn	River
1st card	2nd card	3rd card	4th card	5th card

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P (exactly one pair by the time the last 5th card comes out, if no pocket pair) = ?

For each of the 2 cards you have, out of 3 remaining suits, pick 1

Out of remaining 11 values, choose any 4 values

1st card	2nd card	3rd card	4th card	5th card

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$2 * (3C1)$

Out of remaining 11 values, choose any 4 values

$(11C4)$

For each of those 4 picked values, out of 4 suits, pick 1. Do this four times: once for each of the 4 cards

$(4C1)^4$

1st card	2nd card	3rd card	4th card	5th card

Or 2 of them can be a pair, and the other 3 are different.
Out of remaining 11, choose 1 value to be the pair.
Then out of remaining 10, choose 3 to be remaining cards

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Or 2 of them can be a pair, and the other 3 are different.
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$$2 * (3C1)\ (11C4)\ (4C1)^4 + (11C1)(4C2) * (10C3)(4C1)^3$$

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Or 2 of them can be a pair, and the other 3 are different.
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Then out of remaining 10, choose 3 to be remaining cards

$$\frac{2 * (3C1)\,(11C4)\,(4C1)^4 + (11C1)(4C2) * (10C3)(4C1)^3}{50C5}$$

$$= \frac{506{,}880 + 506{,}880}{2{,}118{,}760} = 47.84685\%$$

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Pocket Pair vs Pocket Aces | Given pocket aces, 3-of-a-kind? | Exactly 1-pair by 5th flop card | 1-pair, 2-pair, 3-of-a-kind, straight | Flush, full house, royal, nothing special

Poker Probability II

7♠	8♣	J♥	K♦	A♦

1) P(1 pair)?	1,098,240	42.257%
2) P(2 pair)?	123,552	4.754%
3) P(3 of a kind)?	54,912	2.113%
4) P(straight)?	10,200	0.392%
5) P(flush)	5,108	0.197%
6) P(full house)?	3,744	0.144%
7) P(4 of a kind)?	624	0.024%
8) P(straight flush)?	36	0.001%
9) P(royal flush)?	4	0.000%
10) P(nothing special)?	1,302,540	50.118%
	2,598,960	100%

$_{52}C_5$

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Poker Probability II

7♠	8♣	J♥	K♦	A♦

1) P(1 pair)?

2) P(2 pair)?

3) P(3 of a kind)?

4) P(straight)?

$$\text{Probability} = \frac{\text{\# of ways}}{\text{Total Possibilities}}$$

For Total Possibilities, Apply nCr formula:

Total Possibilities = 52 Choose 5

$$= \frac{52}{5!\,(47!)}$$

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7♠	8♣	J♥	K♦	A♦

Answers (divide by 52C5)

1) P(1 pair)? [13C1 x 4C2] x [12C3 x 4C1 x 4C1 x 4C1]

2) P(2 pair)? [13C2 x 4C2 x 4C2] x [11C1 x 4C1]

3) P(3 of a kind)? [13C1 x 4C3] x [12C2 x 4C1 x 4C1]

4) P(straight)? [10C1 x 4C1 x 4C1 x 4C1 x 4C1 x 4C1]

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

Pocket Pair vs Pocket Aces | Given pocket aces, 3-of-a-kind? | Exactly 1-pair by 5th flop card | 1-pair, 2-pair, 3-of-a-kind, straight | Flush, full house, royal, nothing special

Poker Probability II

7♠	8♣	J♥	K♦	A♦

Answers (divide by 52C5)

1) P(1 pair)? [13C1 x 4C2] x [12C3 x 4C1 x 4C1 x 4C1]

13C1: **Out of 13 number/face cards**
(A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)
---choose 1 Value

$$13C1 = \frac{13!}{1!\,(12!)}$$

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Answers (divide by 52C5)

1) P(1 pair)? → [13C1 x 4C2] x [12C3 x 4C1 x 4C1 x 4C1]

13C1: **Out of 13 number/face cards**
(A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)
---choose 1 Value

$$13C1 = \frac{13!}{1!\,(12!)}$$

Out of 4 suits of ACES, choose 2 of them

$$4C2 = \frac{4!}{2!\,(2!)}$$

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Poker Probability II

7♠ 8♣ J♥ K♦ A♦

Remaining 3 cards

1) P(1 pair)? [13C1 x 4C2] x [12C3 x 4C1 x 4C1 x 4C1]

2) P(2 pair)?

3) P(3 of a kind)?

4) P(straight)?

12 remaining Values

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Poker Probability II

7♠ 8♣ J♥ K♦ A♦

Remaining 3 cards

1) P(1 pair)?

2) P(2 pair)?

3) P(3 of a kind)?

4) P(straight)?

[13C1 x 4C2] x [12C3] 4C1 x 4C1 x 4C1

12 remaining Values

Each remaining value still has 4 suits

(diamonds, hearts, clubs, spades)

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Poker Probability II

7♠ 8♣ J♥ K♦ A♦

1) P(1 pair)? [13C1 x 4C2] x [12C3] x 4C1 x 4C1 x 4C1

2) P(2 pair)? [13C2 x 4C2 x 4C2] x [11C1 x 4C1]

3) P(3 of a kind)? [13C1 x 4C3] x [12C2 x 4C1 x 4C1]

4) P(straight)? [10C1 x 4C1 x 4C1 x 4C1 x 4C1 x 4C1]

Straight Possibilities

A, 2, 3, 4, 5

....

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Poker Probability II

7♠ 8♣ J♥ K♦ A♦

5) P(flush)

6) P(full house)?

7) P(4 of a kind)?

8) P(straight flush)?

9) P(royal flush)?

10) P(nothing special)?

Answers

[4C1 x 13C5]

Suits

Flush = All 5 cards have same suit

(diamonds, hearts, clubs, spades)

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Poker Probability II

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Answers

5) P(flush) [4C1 x 13C5]

6) P(full house)? [13P2 x 4C3 x 4C2]

Use permutation because...

3 Jacks + 2 sevens
is different from
3 sevens + 2 Jacks

--even though they comprise of the same 2 value/face cards

Full House = 3 of a kind + 2 of another kind (pair)

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Poker Probability II

7♠	8♣	J♥	K♦	A♦

Answers

5) P(flush) [4C1 x 13C5] Triplet

6) P(full house)? [13P2 x 4C3 x 4C2] Pair

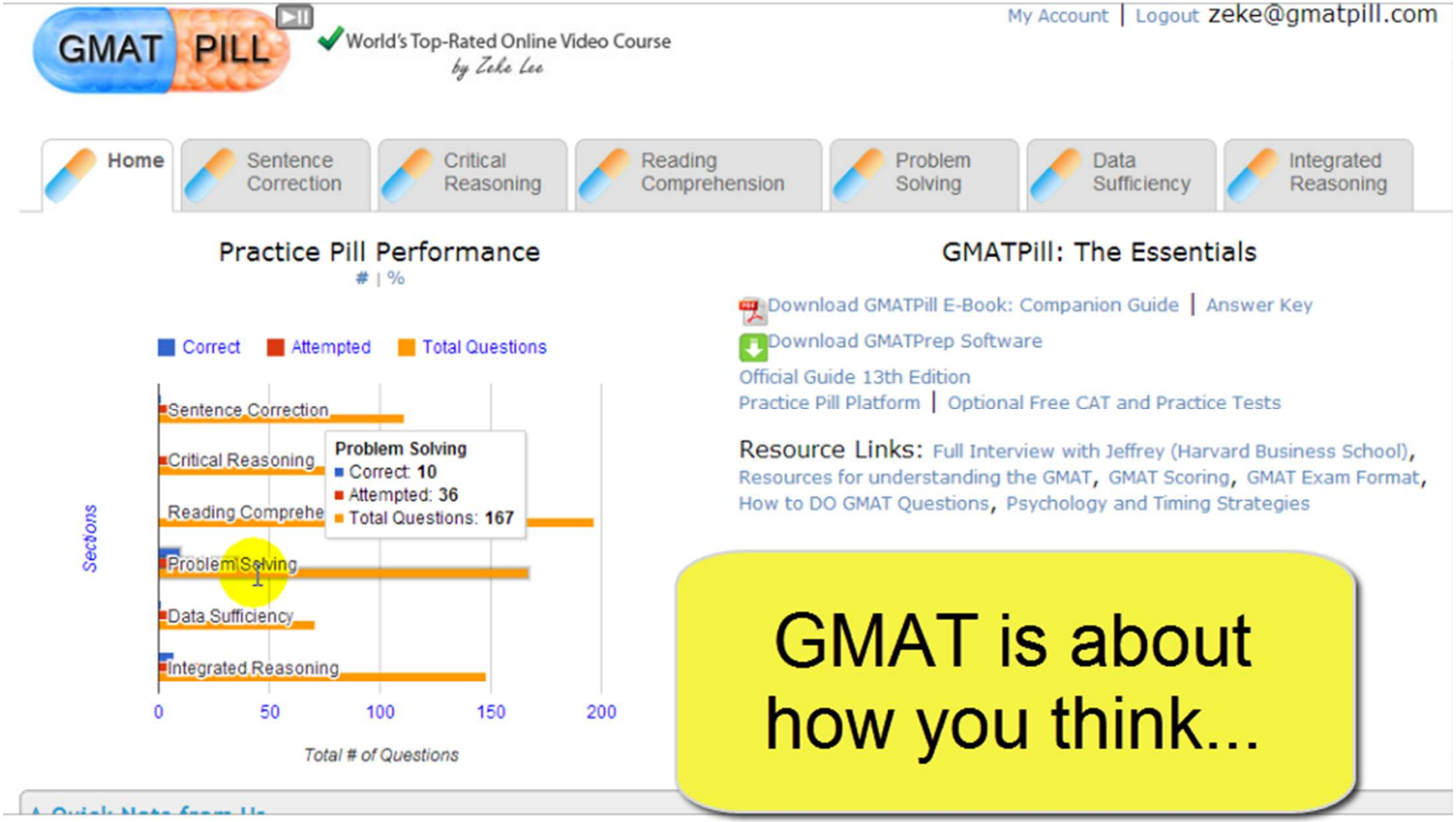
Use permutation because...

3 Jacks + 2 sevens
is different from
3 sevens + 2 Jacks

--even though they comprise of the same 2 value/face cards

Full House = 3 of a kind + 2 of another kind (pair)

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability



GMAT is about how you think...

#1 Framework: How many ways? #2 Apply to GMAT #3 Poker Probability

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