Types of Polygon

Regular A polygon with all sides and interior angles the same. Regular polygons are always convex.

Convex All interior angles less than 180°, and all vertices 'point outwards' away from the interior. The opposite of concave. Regular polygons are always convex.

Definitions, Properties and Tips

• **Sum of Interior Angles** $180(n-2)$ where n is the number of sides

• For a regular polygon, the total described above is spread evenly among all the interior angles, since they all have the same values. So for example the interior angles of a pentagon always add up to 540°, so in a regular pentagon (5 sides), each one is one fifth of that, or 108°. Or, as a formula, each interior angle of a regular polygon is given by: $\frac{180(n-2)}{n}$, where n is the number of sides.

• The apothem of a polygon is a line from the center to the midpoint of a side. This is also the inradius - the radius of the incircle.

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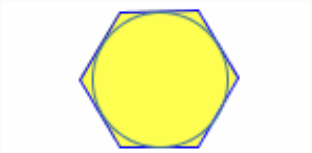

polyincircle.gif [2.68 KiB | Viewed 5111 times]

• The radius of a regular polygon is a line from the center to any vertex. It is also the radius of the circumcircle of the polygon.

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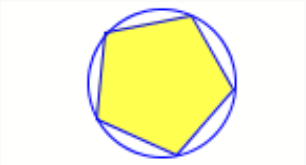

polycircumcircle.gif [2.91 KiB | Viewed 5101 times]

GMAT is dealing mainly with the following polygons:

Quadrilateral *A polygon with four 'sides' or edges and four vertices or corners.*

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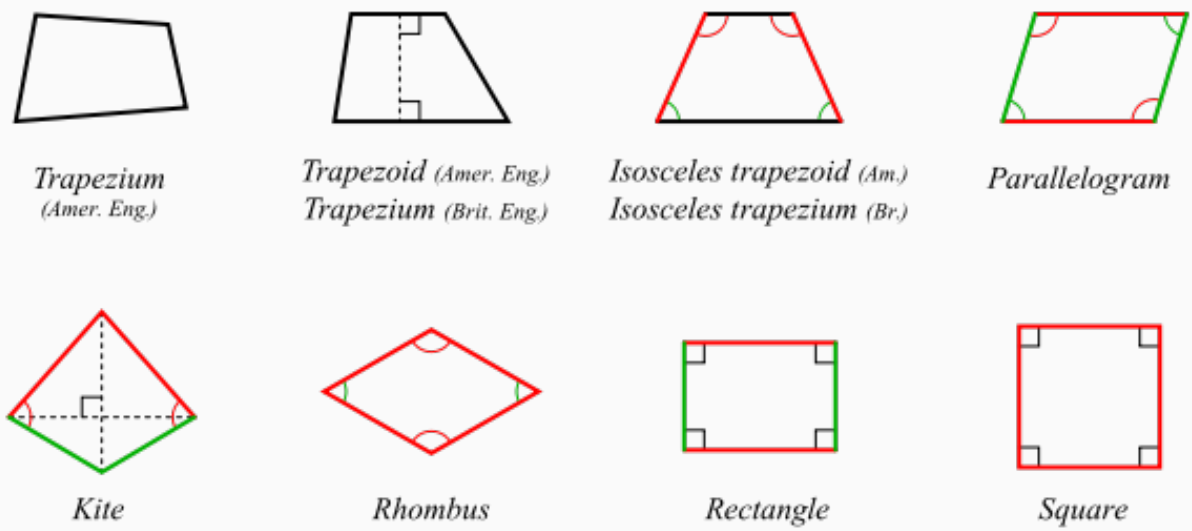

661px-Quadrilaterals.svg.png [24.04 KiB | Viewed 6077 times]

Types of quadrilateral:

Square All sides equal, all angles 90°. See Definition of a square.

Rectangle Opposite sides equal, all angles 90°. See Definition of a rectangle.

Parallelogram Opposite sides parallel. See Definition of a parallelogram.

Trapezoid Two sides parallel. See Definition of a trapezoid.

Rhombus Opposite sides parallel and equal. See Definition of a rhombus.

Parallelogram *A quadrilateral with two pairs of parallel sides.*

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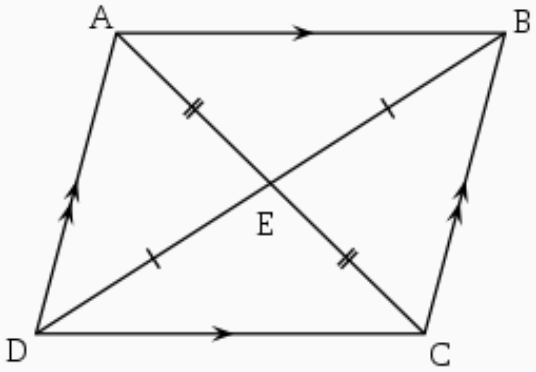

255px-Parallelogram.svg.png [6.3 KiB | Viewed 5114 times]

Properties and Tips

- **Opposite sides of a parallelogram are equal in length.**
- **Opposite angles of a parallelogram are equal in measure.**
- Opposite sides of a parallelogram will never intersect.
- **The diagonals of a parallelogram bisect each other.**
- **Consecutive angles are supplementary, add to 180°.**
- **The area**, A, of a parallelogram is $A = bh$, where b is the base of the parallelogram and h is its height.
- The area of a parallelogram is twice the area of a triangle created by one of its diagonals.

A parallelogram is a quadrilateral with opposite sides parallel and congruent. It is the "parent" of some other quadrilaterals, which are obtained by adding restrictions of various kinds:

- A rectangle is a parallelogram but with all angles fixed at 90°
- A rhombus is a parallelogram but with all sides equal in length
- A square is a parallelogram but with all sides equal in length and all angles fixed at 90°

Rectangle *A 4-sided polygon where all interior angles are 90°*

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250px-Rectangle_.png [752 Bytes | Viewed 5084 times]

Properties and Tips

• Opposite sides are parallel and congruent

• The diagonals bisect each other

• The diagonals are congruent

• A square is a special case of a rectangle where all four sides are the same length.

• It is also a special case of a parallelogram but with extra limitation that the angles are fixed at 90°.

• The two diagonals are congruent (same length).

• Each diagonal bisects the other. In other words, the point where the diagonals intersect (cross), divides each diagonal into two equal parts.

• Each diagonal divides the rectangle into two congruent right triangles. Because the triangles are congruent, they have the same area, and each triangle has half the area of the rectangle.

• $Diagonal = \sqrt{w^2+h^2}$ where: w is the width of the rectangle, **h** is the height of the rectangle.

• The **area** of a rectangle is given by the formula $Width*Height$.

A rectangle can be thought about in other ways:

• A square is a special case of a rectangle where all four sides are the same length. Adjust the rectangle above to create a square.

• It is also a special case of a parallelogram but with extra limitation that the angles are fixed at 90°.

Squares *A 4-sided regular polygon with all sides equal and all internal angles*

90°

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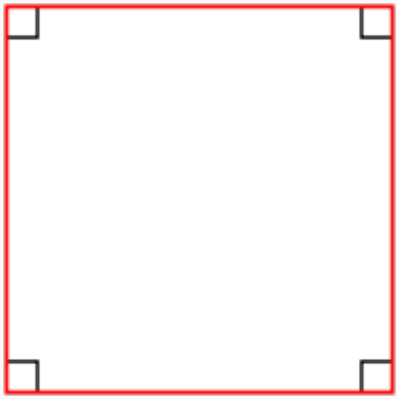

220px-Square_(geometry).svg.png [1.01 KiB | Viewed 5076 times]

Properties and Tips

• If the diagonals of a rhombus are equal, then that rhombus must be a square. The diagonals of a square are (about 1.414) times the length of a side of the square.

• A square can also be defined as a rectangle with all sides equal, or a rhombus with all angles equal, or a parallelogram with equal diagonals that bisect the angles.

• If a figure is both a rectangle (right angles) and a rhombus (equal edge lengths), then it is a square. (Rectangle (four equal angles) + Rhombus (four equal sides) = Square)

• If a circle is circumscribed around a square, the area of the circle is $\frac{\pi}{2}$ (about 1.57) times the area of the square.

• If a circle is inscribed in the square, the area of the circle is $\frac{\pi}{4}$ (about 0.79) times the area of the square.

• A square has a larger area than any other quadrilateral with the same perimeter.

• Like most quadrilaterals, the **area** is the length of one side times the perpendicular height. So in a square this is simply: $area = s^2$, where s is the length of one side.

• **The "diagonals" method.** If you know the lengths of the diagonals, the area is half the product of the diagonals. Since both diagonals are congruent (same

length), this simplifies to: $area = \frac{d^2}{2}$, where d is the length of either diagonal
• Each diagonal of a square is the perpendicular bisector of the other. That is, each cuts the other into two equal parts, and they cross and right angles (90°).
• **The length of each diagonal** is $s\sqrt{2}$ where s is the length of any one side.

A square is both a rhombus (equal sides) and a rectangle (equal angles) and therefore has all the properties of both these shapes, namely:
The diagonals of a square bisect each other.
• The diagonals of a square bisect its angles.
• The diagonals of a square are perpendicular.
• Opposite sides of a square are both parallel and equal.
• All four angles of a square are equal. (Each is 360/4 = 90 degrees, so every angle of a square is a right angle.)
• The diagonals of a square are equal.

A square can be thought of as a special case of other quadrilaterals, for example
• a rectangle but with adjacent sides equal
• a parallelogram but with adjacent sides equal and the angles all 90°
• a rhombus but with angles all 90°

Rhombus *A quadrilateral with all four sides equal in length.*

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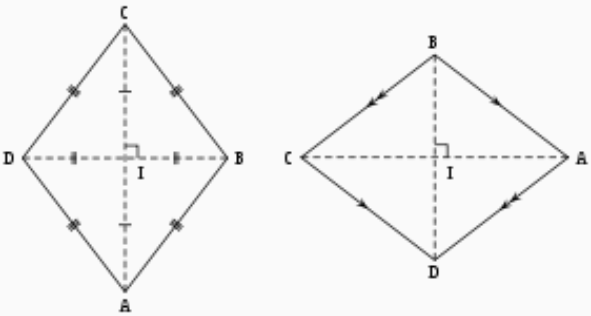

280px-Rhombus.svg.png [5.78 KiB | Viewed 5077 times]

Properties and Tips

• **A rhombus is actually just a special type of parallelogram.** Recall that in a parallelogram each pair of opposite sides are equal in length. With a rhombus, all

four sides are the same length. **It therefore has all the properties of a parallelogram.**

• **The diagonals of a rhombus always bisect each other at 90°.**

• There are several ways to find the **area of a rhombus**. The most common is: $base*altitude$.

• **The "diagonals" method.** Another simple formula for the area of a rhombus when you know the lengths of the diagonals. The area is half the product of the diagonals. As a formula: $\frac{d1*d2}{2}$, where $d1$ is the length of a diagonal $d2$ is the length of the other diagonal.

Trapezoid *A quadrilateral which has at least one pair of parallel sides.*

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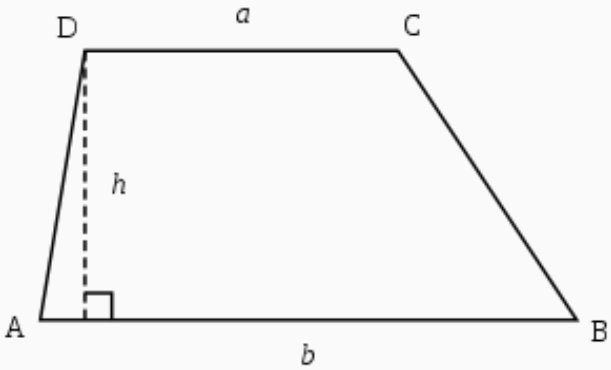

292px-Trapezoid.svg.png [2.66 KiB | Viewed 5103 times]

Properties and Tips

• **Base** - One of the parallel sides. Every trapezoid has two bases.

• **Leg** - The non-parallel sides are legs. Every trapezoid has two legs.

• **Altitude** - The altitude of a trapezoid is the perpendicular distance from one base to the other. (One base may need to be extended).

• If both legs are the same length, this is called an isosceles trapezoid, and both base angles are the same.

• **If the legs are parallel, it now has two pairs of parallel sides, and is a parallelogram.**

• **Median** - The median of a trapezoid is a line joining the midpoints of the two legs.

• **The median line is always parallel to the bases.**

- **The length of the median is the average length of the bases, or using the formula:** $\frac{AB+DC}{2}$
- **The median line is halfway between the bases.**
- **The median divides the trapezoid into two smaller trapezoids each with half the altitude of the original.**
- **Area** - The usual way to calculate the area is the average base length times altitude. The area of a trapezoid is given by the formula $h*\frac{a+b}{2}$ where a, b are the lengths of the two bases h is the altitude of the trapezoid
- The **area** of a trapezoid is the $altitude*median$.