‘Work’ Word Problems Made Easy

NOTE: Incase you are not familiar with translating word problems into equations please go through this post first : word-problems-made-easy-87346.html

What is a ‘Work’ Word Problem?

1) It involves a number of people or machines working together to complete a task.

2) We are usually given individual rates of completion.

3) We are asked to find out how long it would take if they work together.

Sounds simple enough doesn’t it? Well it is!

There is just one simple concept you need to understand in order to solve any ‘work’ related word problem.

The ‘Work’ Problem Concept

STEP 1 : Calculate how much work each person/ machine does in one unit of time (could be days,

hours, minutes, etc).

How do we do this? Simple. If we are given that A completes a certain amount of work in X hours, simply reciprocate the number of hours to get the per hour work. Thus in one hour, A would complete (1/X) of the work.
But what is the logic behind this? Let me explain with the help of an example.

Assume we are given that Jack paints a wall in 5 hours.
This means that in every hour, he completes a fraction of the work so that at the end of 5 hours, the fraction of work he has completed will become 1 (that means he has completed the task).
Thus, if in **5** hours the fraction of work completed is **1**,
Then in **1** hour, the fraction of work completed will be **(1*1)/5**

STEP 2 : Add up the amount of work done by each person/machine in that one unit of time.

This would give us the total amount of work completed by both of them in one hour. For example, if **A completes (1/X)** of the work in one hour and **B completes (1/Y)** of the work in one hour, then **TOGETHER**, they can complete **[(1/X) + (1/Y)]** of the work **in one hour**.

STEP 3 : Calculate total amount of time taken for

work to be completed when all persons/machines are working together.

The logic is similar to one we used in STEP 1, the only difference being that we use it in reverse order. Suppose [(1/X) + (1/Y)] = (1/Z). This means that **in one hour**, A and B **working together** will complete **(1/Z)** of the work. **Therefore, working together, they will complete the work in Z hours.**

My advice here would be : DON'T go about these problems trying to remember some formula. Once you understand the logic underlying the above steps, you will have all the information you need to solve any 'work' related word problem. (You will see that the formula you might have come across can be very easily and logically deduced from this concept).

Now, lets go through a few problems so that the above-mentioned concept becomes crystal clear.

Lets start off with a simple one :

Example 1.

Jack can paint a wall in 3 hours. John can do the same job in 5 hours. How long will it take if they work together?

Solution 1.

This is a simple straightforward question wherein we must just follow steps 1 to 3 in order to obtain the answer.

STEP 1 : Calculate how much work each person does in one hour.

Jack → (1/3) of the work

John → (1/5) of the work

STEP 2 : Add up the amount of work done by each person in one hour.

Work done in one hour when both are working together = (1/3) +(1/5) = (8/15)

STEP 3 : Calculate total amount of time taken when both work together.

If they complete **(8/15)** of the work in **1** hour,
then they would complete **(1)** job in **(1*1)/(8/15)** hours.

Therefore answer is (15/8) hours.

Simple, wasn't it? Now lets move onto one that is slightly trickier.

Note: As we move on to trickier problems, we will refrain from using any specific approach (Eg. Step 1, Step 2, Step 3, etc.) since that might serve only to confuse us when the problem involves a lot of twists and turns. Armed with a clear understanding of the concept, we should be able to tackle any

'work' related word problem, no matter how convoluted, by applying our knowledge to the information at hand.

Example 2.

Working, independently X takes 12 hours to finish a certain work. He finishes 2/3 of the work. The rest of the work is finished by Y whose rate is (1/10) of X. In how much time does Y finish his work?

Solution 2.

Now the only reason this is trickier than the first problem is because the sequence of events are slightly more complicated. The concept however is the same. So if our understanding of the concept is clear, we should have no trouble at all dealing with this.

'Working, independently X takes 12 hours to finish a certain work'
This statement tells us that in one hour, X will finish (1/12) of the work.

'He finishes 2/3 of the work'
This tells us that (1/3) of the work still remains.

'The rest of the work is finished by Y whose rate is (1/10) of X'
Y has to complete (1/3) of the work.
His rate is (1/10) that of X.

We have already calculated rate at which X works to be (1/12). Therefore, rate at which Y works is (1/10)*(1/12) = (1/120)

'In how much time does Y finish his work?'
If Y completes **(1/120)** of the work in **1** hour,
Then he will complete **(1/3)** of the work in **[(1/3)*1]/(1/120)** = 40 hours.

Therefore answer is 40 hours.

So as you can see, even though the question might have been a little difficult to follow at first reading, the solution was in fact quite simple. We didn't use any new concepts. All we did was apply our knowledge of the concept we learnt earlier to the information in the question in order to answer what was being asked.

Example 3.

Working together, printer A and printer B would finish a task in 24 minutes. Printer A alone would finish the task in 60 minutes. How many pages does the task contain if printer B prints 5 pages a minute more than printer A?

Solution 3.

This problem is interesting because it tests not only our knowledge of the concept of word problems, but also our ability to 'translate English to Math'

'Working together, printer A and printer B would finish a task in 24 minutes'
This tells us that A and B combined would work at the rate of (1/24) per minute.

'Printer A alone would finish the task in 60 minutes'
This tells us that A works at a rate of (1/60) per minute.

At this point, it should strike you that with just this much information, it is possible to calculate the rate at which B works.
Rate at which B works = (1/24) - (1/60) = (1/40)

'B prints 5 pages a minute more than printer A'
This means that the difference between the amount of work B and A complete in one minute corresponds to 5 pages.
So let us calculate that difference. It will be = (1/40) - (1/60) = (1/120)

'How many pages does the task contain?'
If **(1/120)** of the job consists of **5** pages,
Then the **1** job will consist of **(5*1)/(1/120)** = 600 pages

Therefore answer is 600 pages.

Hopefully you're getting the hang of it by now.

Lets move on to a really tough one.

Example 4.

Machine A and Machine B are used to manufacture 660 sprockets. It takes machine A ten hours longer to produce 660 sprockets than machine B. Machine B produces 10% more sprockets per hour than machine A. How many sprockets per hour does machine A produce?

Solution 4.

You wont come across many problems tougher than this on the GMAT. But as we will see, even the toughest of problems can be solved with relative ease if we employ the concept discussed above.

'It takes machine A ten hours longer to produce 660 sprockets than machine B'

Let machine A produce 660 sprockets in 'x' hours.

Therefore, machine B will produce 660 sprockets in 'x - 10' hours.

With this information, we can calculate the amount of work machine A and B do per hour respectively.

Rate at which machine A works = [1/x] per hour

Rate at which machine B works = [1/(x - 10)] per hour

'Machine B produces 10% more sprockets per hour than machine A'

If machine A produces [1/x] sprockets an hour, then machine B will produce (1/x) +(10/100)*(1/x) = (11/10x)

But we already know that rate at which machine B works = [1/(x - 10)] per hour. Therefore, equating it to (11/10x) we get the following equation:

(11/10x) = 1/(x-10) → x = 110 hours

'How many sprockets per hour does machine A produce?'
If in **110** hours A produces **660** sprockets,
Then in **1** hour it will produce **(660*1)/110** = 6

Therefore answer is 6.

As you can see, the main reason the 'tough' problems are 'tough' is because they test a number of other concepts apart from just the 'work' concept. However, once you manage to form the equations, they are really not all that tough.

And as far as the concept of 'work' word problems is concerned - it is always the same!

You can go here to practice questions on 'work' word problems : work-rate-questions-i-collected-from-gmat-club-73382.html

Feel free to discuss any question or doubt you might have in this post.