GMAT Quant Notes & Tips

Bunuel→gmatclub.com

Formula for counting the sum of the first n positive perfect squares is: $\frac{N(N+1)(2N+1)}{6}$
Suppose N=15, then
$n=15$, $Sum=1240$

(2) TIP:
When odd number n is doubled, 2n has twice as many factors as n.
That's because odd number has only odd factors and when we multiply n by two we remain all these odd factors as divisors and adding exactly the same number of even divisors, which are odd*2.

(When even number is doubled, 2n has 1.5 more factors as n.)

Rate Problems:

General formula for calculating the time needed for two workers A and B working simultaneously to complete one job:

Given that a and b are the respective individual times needed for A and B workers (pumps, ...) to complete the job, then time needed for A and B working simultaneously to complete the job equals to $T_{(A\&B)}=\frac{a*b}{a+b}$ hours, which is reciprocal of the sum of their respective rates ($\frac{1}{a}+\frac{1}{b}=\frac{1}{t}$).

General formula for calculating the time needed for three A, B and C workers working simultaneously to complete one job:

$T_{(A\&B\&C)}=\frac{a*b*c}{ab+ac+bc}$ hours.

Also for rate problems it's good to know that:

TIME to complete one job=Reciprocal of rate. eg 6 hours needed to complete one job (time) --> 1/6 of the job done in 1 hour (rate).

Time, rate and job in work problems are in the same relationship as time, speed (rate) and distance.

Time*Rate=Distance
Time*Rate=Job

<u>Scoowhoop →beatthegmat.com</u>

This is a little thing I put together, I haven't seen it anywhere on here so I figured I'd share....

In this method we use reverse logic by finding all the numbers that arn't prime and eliminating them. Since all of these primes (11-109) can only end in 1,3,7,or 9 we create a table for all possible outcomes. We notice a repeating pattern: (21, 51, 81), (33,63,93), (27,57,87), and (39,69,99) are all NOT prime and can be eliminated. This only leaves 49,77,and 91 as the other non-primes. Draw this matrix at the beginning of the test to help reference prime numbers. I've attached a better diagram if you have a hard time visualizing below. Hope this helps!

11 13 17 19
21 23 **27** 29
31 **33** 37 **39**
41 43 47 ***49***
51 53 **57** 59
61 **63** 67 **69**
71 73 ***77*** 79
81 83 **87** 89
91 **93** 97 **99**
101 103 107 109

~Drew

Attachments

Primes from 11-109.JPG

11	13	17	19	(+10)
	23		29	
31		37		
41	43	47	~~49~~	
	53		59	
61		67		
71	73	~~77~~	79	
	83		89	
~~91~~		97		
101	103	107	109	

<u>Brian@VeritasPrep-->BTG's Reply</u>

That's pretty neat, Drew - thanks for posting that!

Because my nature with regard to the GMAT is to always ask "why", I looked at this and my first inclination was to determine why that's the case. For anyone interested, it's this:

The pattern that Drew shows here with 21,51, 81; 33, 63, 93; 27, 57, 87; and 39, 69, 99 is that all of those are multiples of 3. By taking the base multiple of 3 (21, 33, 27, etc.) and adding 30s, you're just creating more multiples of 3 (3*x + 3*10 = 3 * (x+10).

Once those are out, and we've already taken out even numbers (anything ending in 2, 4, 6, 8, or 0) and multiples of 5 (anything ending in 5), we've already accounted for numbers divisible by prime factors 2, 3, and 5.

The next one up is 7, which brings us to 49 (7*7), 77 (7*11) and 91 (7*13). Essentially, these other numbers to "memorize" are the products of 7 and another prime number. Any other multiple of 7 has already been accounted for, as it's a multiple of 2, 3, or 5. And, in this range, we don't need to worry about any primes above 11, because 11^2 is 121; if that's the upper limit of our range, then any number larger than 11 would need to be multiplied by something less than 11 in order to be less than 121.

One other note on primes - I had a student once mention that she had been taught in school to "look for numbers next to multiples of 6" when seeking out primes. Why does that work? 6 is a multiple of the two smallest prime numbers, 2 and 3. So, any multiple of 6 won't be prime, and any number two away from a multiple of 6 will be a multiple of 2 (every second number - 2, 4, 6, 8, etc.) is a multiple of 2) and any number three away from a multiple of 6 will be a multiple of 3 (every third number - 3, 6, 9, 12, etc. is a multiple of 3). So, the only numbers that could be prime, once you get past 2 and 3, are one away from a multiple of 6: 5, 7, 11, 13, 17, 19, 23, 29, 31, 37...

To summarize this post (and thanks, readers, for indulging me), you can often learn the mechanics behind these shortcut tricks, and in doing so you'll internalize them that much more. If you ask yourself "why" when you see tricks and shortcuts posted here, you'll likely be able to apply those concepts even broader and more effectively on the GMAT.

Pkw209→BTG:

Any "X" consecutive positive integers will always include exactly one multiple of "X."

True or False?

<u>Mohit11's reply</u>

Answer is YES, since a number is multiple of itself.

1 2 3 , x = 3 and 3 is a multiple of itself.

2 3 4 5 6 7 , x = 6, 6 is a multiple of itself.

- **GMAT Math Formulae**

Permutation and Combination

nPr = n! / (n-r)!
nPn = n!
nCr = n! / (n-r)! r!
nCn = 1
nP0 = 1
nC0 = 1

Arithmetic Progression

An = a + (n-1)d
Sn = n/2[2a + (n-1)d]

Geometric Progression

An = ar(n-1)
Sn = a(rn – 1)/ (r-1)
S(infinite series) = a/(1-r)

Metrics

1 mile = 1760 yards
1 yard = 3 feet
1 mile^2 = 640 acres
I gallon = 4 quarts
1 quart = 2 pints
1 pint = 2 cups
1 cup = 8 ounces
1 pound = 16 ounces
1 ounce = 16 drams
1 kg = 2.2 pounds

Triangle

- 30-60-90 triangle -> 1:sqrt(3):2 sides
- 45-45-90 triangle -> 1:1:sqrt(2) sides
- In a triangle with sides of measure a, b and c, a-b<c<a+b
- In a triangle, if the sum of two angles = third angle, then it is a right angled triangle.

Numbers and Fractions

- a^3>b^3 -> a>b
- If A than B => not B than not A
- Zero divided by any nonzero integer is zero.
- Division by 0 is undefined.
- Dividend = Divisor * Quotient + Remainder
- LCM * HCF = Product of 2 numbers.
- 1 + 2 + 3 ………………..n = n * n+1 / 2
- Sum of squares of 1st n natural numbers = n (n+1)(2n+1) / 6
- Sum of cubes of 1st n natural numbers = [n (n+1)/2]^2
- The number of integers from A to B inclusive is = B - A +1
- Average of consecutive numbers:Ex from 14 to 26 = (14+26)/2 = 20
- 0^0 = undefined
- If a-b = odd => a + b = odd
- If z1, z2, z3 … zn are consecutive positive integers and their average is an odd integer => n is odd => sum of series is odd
- Fraction > (Fraction)^2 for all positive fractions
- Fraction > sqrt(Fraction) for all positive fractions
- If the difference between the largest and the smallest divisor of a number is X, the number is X + 1

- Every number raised to power 5 has the number itself as unit digit

- If a + b + c = Z, than the largest of a, b, and c cannot be greater than the mean of the other two.

Speed, Distance and Time

Average speed = Total distance / Total Time

Case1: When equal distances are covered in different speed then
Average Speed = 2ab / a + b

Case 2: For Different distances in same time
Average Speed is = a + b / 2

Currency

1 Nickel = 5 cents
1 dime = 10 cents
1 quarter = 25 cents
1 half = 50 cents
1 dollar = 100 cents

Geometry

- Area of Equilateral triangle = sqrt(3/4)* a^2

- Area of trapezium = ½ (Height * Sum of parallel sides)

- Slope = (change in y)/(change in x)

- Measure of an angle of a cyclic polygon = 180 – 360/n , where n is the number of sides of the polygon.

- Degree measure of one angle in a regular polygon with n sides ={(n-2)*180 }/n

- Sum of interior angles of a polygon with n sides = (n-2)*180

- If area of a rectangle is known, diagonal(d) is known, perimeter can be found

- a^2 + b^2
  =diagonal^2
  Add
  2ab

a^2 + b^2 + 2ab = diagonal^2+ 2ab
(a + b)^2 = diagonal^2+ 2*Area

- **To find last digit of power of a Number: amitdgr→gmatclub.com**

Remember:
1) Numbers 2,3,7 and 8 have a cyclicity of 4
2) Numbers 0,1,5 and 6 have a cyclicity of 1 (i.e.) all the powers will have the same unit digit. e.g. 5^245 will have "5" as unit digit, 5^2000 will also have "5" as unit digit. Same holds for 0,1 and 6
3) If 4 is the number in the unit place of the base number then the unit digit will be "4" if the power is odd and it will be "6" if the power is even. e.g. 4^123 will have unit digit of 4 since 123 is odd.
4) Similarly, for 9 the unit digit will be "9" for odd powers and "1" for even powers. e.g. 9^234 has unit digit as "1" since 234 is even.

Let's take an example:

3^1=3,
3^2=9,
3^3=27,
3^4 =81,
3^5=243,
3^6=729, and so on .....

Now it is not humanly possible to remember all the numbers till 3^{27}

If you have noticed in the above series the last digit repeats after every 4 terms

the last digit is same for $3^5$and 3^1
the last digit is same for $3^6$and 3^2

If 3 is the unit digit of a number then the unit digit repeats every fourth consecutive term.For our convenience here, lets call it cyclicity. So 3 has a cyclicity of 4.

To find the unit digit of a number having 3 as its last digit and raised to a positive power, divide the power by 4 and find the remainder.

If the remainder is 1 then the unit digit is same as of the unit digit of 3^1
If the remainder is 2 then the unit digit is same as of the unit digit of $3^2$and so on.....

Note that if the remainder is "0" then the unit digit is same as 3^4 since the cyclicity is 4.

Also remember that the numbers 2, 3, 7 and 8 have cyclicity of 4

in our problem above we have 3^27

3 has a cyclicity of 4 so divide the number 27 by 4. We get a remainder of 3. Now as per cyclicity the last digit of 3^27 is same as that of 3^3. 3^3 is 27 so the last digit of 3^27 is 7.

Lets take another example

Find the last digit of 122^94

A. 2
B. 4
C. 6
D. 8
E. 9

Now the last digit of 122 is 2. We require only this number to determine the last digit of 122 raised to a positive power.

so the problem is essentially reduced to find the last digit of 2^94.

Now we know 2 has a cyclicity of 4. So we divide 94 by 4. The remainder for 94/4 is 2.

So last digit of 2^94 is same as that of 2^2 which is 4.

So last digit of 122^94 is 4

Here's another way of looking at it !

Here the given number is $(xyz)^n$
z is the last digit of the base.
n is the index

To find out the last digit in $(xyz)^n$, the following steps are to be followed.
Divide the index (n) by 4, then

Case I
If remainder = 0
then check if z is odd (except 5), then last digit = 1
and if z is even then last digit = 6

Case II
If remainder = 1, then required last digit = last digit of the base (i.e. z)
If remainder = 2, then required last digit = last digit of the base $(z)^2$
If remainder = 3, then required last digit = last digit of the base $(z)^3$

Note : If z = 5, then the last digit in the product = 5

Example:
Find the last digit in (295073)^130

Solution: Dividing 130 by 4, the remainder = 2
Referring to Case II, the required last digit is the last digit of $(z)^2$, ie $(3)^2$= 9 , (because z = 3)

<u>Onceatsea's view:</u>

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<u>Bunuel's view:</u>

RULE: The order of operation for **exponents**: **x^y^z=x^(y^z)** and **not** (x^y)^z. **The rule is to work from the top down.**

3^3^3=3^(3^3)=3^27

Cycle of 3 in power is four. The units digit of 3^27 is the same as for 3^3 (27=**4***6+**3**) --> 7.

- **When to use Permutation Formula**

By a gmatclub.com member→I3igDmsu

I was struggling with when to use the permutation formula and when to use the combination formula in various problems. The definitions of permutations and combinations are are sometimes not clear to beginners. The easiest way that I have found to use the permutation formula is when the question states that order matters. Looking for key words in the sentence like:
*Rank
*Arrange
*Place
*Chose
*Pick
and many others signify that order matters. If order matters then use the permutation formula.

Examples:
*In how many ways can 8 CD's be arranged on a shelf?

*If a softball league has 10 teams, how many different end of the season rankings are possible? (Assume no ties).

*How many different arrangements can be made using two of the letters of the word TEXAS if no letter is to be used more than once?

* A teacher has 15 students and 5 are to be chosen to give demonstrations. How many different ways can the teacher choose the demonstrators given the following conditions.

*In how many ways can a sorority of 20 members select a president, vice president and treasury, assuming that the same person cannot hold more than one office.

All of these questions are answered using the permutation formula: $nPr = \frac{n!}{(n-r)!}$

This is something that has helped me, hopefully it helps others.

- **How to calculate simple interest**

Simple interest is another topic that would be tested in **GMAT problem solving section**. A clear understanding of percentages is essential before cracking Simple interest questions.

Simple interest is the amount that a borrower has to pay the lender during a particular period of time (For which the money has been borrowed). **The principal** (or money borrowed remains the same during the period of time)

Example: A borrower borrows $1000 at 10% per annum from a lender for four years. Calculate the total interest that the borrower has to pay for the f**our years.**

Year	Principal	Simple Interest
1	1000	100 (10 % of 1000)
2	1000	100 (10 % of 1000)
3	1000	100 (10 % of 1000)
4	1000	100 (10 % of 1000)

Total Simple Interest for 4 years = 400

P.S

Principal = Amount borrowed

Rate of Interest = Rate of Interest per year

Time = Time is calculated in years or 12 month period or in 356 days

Formula for Simple Interest = (Principal*Rate of Interest/annum*Time)/100
S.I = (1000*10* 4) /100
= $400

What if we have to calculate simple interest per quarter or per half year?

For quarter time = 3 months

In years = 3/12 = 1/4

For half-year time = 6 months

In years = 6/12 = 1/2

S.I (per quarter) = (1000 * 10 * 1/4) /100

= $25

S.I (per half year) = (1000* 10 * 1/2) /100
= $50

What if we have to calculate simple interest for 300 days?

S.I (300 days) = (1000* 10 * (300/365))/100
= $82.19

P.S: Rate of interest (R) in Simple interest is always calculated per year or its equivalent . So no operations on rate of interest is required while calculating Simple interest per quarter or per half year or for 300 days.

Q) An investment yields an interest payment of $228 each month. If the simple annual interest rate is 9%, what is the amount of the investment?

(a) $28,300
(b) $30,400
(c) $31,300
(d) $32,500
(e) $35,100

S.I = Principal × Interest × Time
228 = Principal × .09 × (1/12)
Principal = $30,400
The correct answer is B.

How to calculate compound interest

- compound interest

Compound interest is not only important in GMAT but also in all your investments. With Simple Interest, we invest X amount of money and get 'I' amount of interest after n period(years , months or days) but with Compound Interest we invest X amount of money and depending on the period of calculation, the Interest 'I' gets added to your principal amount 'X'. Sounds Confusing?

A= P x (1 + R)^N
P is the principal amount (Initial amount borrowed or invested)
R is the annual rate of interest (In Percentage)
N is the number of times the interest is paid
A is the amount of money accumulated after N period
C.I (Compound Interest) = A - P

Case 1: Interest paid Annually = P × (1 + R)
Case 2: Interest paid Quarterly = P × (1 + R /4) ^4
Case 3: Interest paid Monthly = P × (1 + R /12) ^12

How much interest will $2,400 earn at an annual rate of 8% in one year if the interest is compounded every 4 months?

(a) $141
(b) $150
(c) $197
(d) $234
(e) $312

Compounded every 4 months (N) = 12/4 = 3

$A = P \times (1 + R/3)^3$
$= \$2,400 \times (1 + .026)^3 = 2597$
C.I = A – P = 2597 – 2400 = $197
Correct Answer is C

How much interest will an investment of $2,400 earn every 4 months if simple annual interest rate is 8%?

(a) $ 68
(b) $ 64
(c) $ 70
(d) $ 74
(e) $ 78

Correct Answer is B

S.I = Principal × Interest × Time
$= \$2,400 \times .08 \times (4/12)$
S.I = $64

- **How to identify a large prime number**

Prime Numbers: A prime number is a natural number, which can be divided by exactly 2 distinct natural number 1 and itself

Ex: 2 have exactly two factors 2 and 1 so it is prime.

The other positive integers are composite and they have three or more factors.

For example, 9 has factors 1, 3 and 9 (3 factors), so is composite.
2 has factors 1, 2, 3, 4, 6, 12 (6 factors), so is composite.

Ex: 2,3,5,7,11...

Properties of Prime Number

1) 2 and 3 are the lowest even and odd prime numbers respectively.
2) 2 is the only even prime number
3) Prime numbers between 1 and 100 are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
There are 25 prime numbers between 1 and 100. Memorize this series(This would be useful for GMAT)
4) There are no negative prime numbers
5) All prime numbers except 2 and 5, end in 1, 3, 7 or 9, since numbers ending in 0, 2, 4, 6 or 8 are multiples of 2 and numbers ending in 0 or 5 are multiples of 5
6) All prime numbers above 3 are of the form 6n ? 1 or 6n + 1, because all other numbers are divisible by 2 or 3 where n>=1

How to check whether a number is prime or not

Let us take a number 2,321

1) Find the square root of the number The Square root of 2,321 is 48.176......
2) Remember that if the square root results in an integer it is automatically composite
3) If your number ends in a 0,2,4,5,6,8 it is NOT PRIME 2,321 ends in a 1, so go to next step
4) Add up the digits of your number, if the sum is divisible by 3, your number is composite -> 2+3+2+1= 8
8 is not divisible by 3
5) Divide the number by all the prime numbers less than the square root (you can skip 2, 3, and 5)
Since the square root of 2,321 is 48.176... Divide 2,321 by primes less than 48 (7,11,13,17,19,23,29,31,37,41,43,47)
Since 2,321 is divisible by 11, it is NOT prime and therefore composite
6) If a number is not divisible by any of the prime numbers less than the square, it is PRIME otherwise it is COMPOSITE

- **Circular permutation**

Before go we go into circular permutation, let us see how we can arrange objects in a linear arrangement.

A B C D

Suppose we shift each element to the right and the right-most element takes the leftmost spot. Then we have the following arrangements:

A B C D
D A B C
C D A B
B C D A

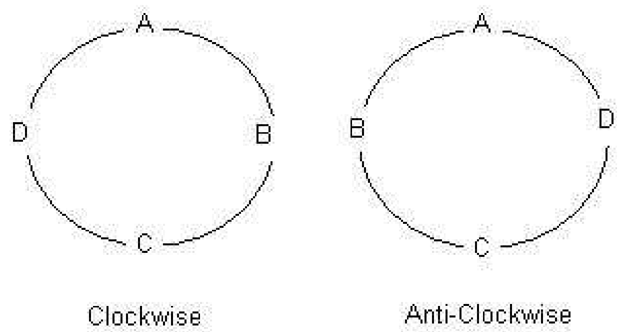
Now let us consider the same scenario for a circular arrangement. We see that for a circle there is no start and end for such an arrangement.

Therefore for four elements the total number of permutations in a linear arrangement is (n) ! = 4! = 24

But for a circular arrangement the total number of permutations for 4 elements would be (n)!/n = 4!/4 = 6 = (n – 1) !

Clockwise and Anti-Clockwise Permutations

There are two types of circular permutations: clockwise and anti-clockwise. The arrangements are distinct based on the type of objects in the arrangements.



Distinct Arrangement: When we consider an arrangement of objects with different identity then the arrangement is distinct.

For Example: A circular arrangement of people with different names.
For distinct objects the **total number of permutations = (n – 1) !**

Non-Distinct Arrangement: When the objects in the arrangement cannot be identified separately then the arrangement is non-distinct.

For Example: A circular arrangement of beads in a necklace.
For non-distinct objects, both clockwise and anti-clockwise arrangement would be the same. Therefore the **total number of permutation = (n-1)!/2**

- **Profit and Loss**

In order to understand **GMAT Profit and loss** questions a fair idea of percentages is a must.

- **Profit :** if an entity is sold for a price that is greater than the cost of the entity then the difference in Selling Price(SP) and Cost Price(CP) is the Profit.

Profit = SP-CP

- **Loss:**if an entity is sold for a price that is less than the cost of the entity then the difference in Cost Price(CP) and Selling Price(SP) is the loss.

Loss= CP-SP

Profit and Loss can be represented in currency($) or as a percentage.

Q) A shopkeeper bought a cupboard for a $ 300 and sold it for $450 .Find the profit or loss percentage?

A: Since SP=$450
CP=$300
and SP>CP

Profit($) = SP-CP
= $150

Profit % = (Profit/CP)*100 = 150/300
= 50%

Note: Percentage profit and loss are always calculated with CP as denominator unless otherwise stated

Other Shortcuts to remember :

SP= (CP*(100+Profit%))/100

CP=(100*SP)/(100+Profit%)

Profit % = (Profit/CP)*100

Loss% = (Loss/CP)*100

Marked Price

Generally shopkeepers add a margin to the cost price and the resulting price is called marked up price.

CP + Mark Up(based on margin) = Marked Price

If the shopkeeper sells the product at marked up price then

Marked-up price = Selling price

However if the shopkeeper provides a discount on the marked-up price then

Discounted marked-up price = selling price

Q) If a merchant offers a discount of 30% on the marked price of his goods and thus ends up selling at cost price, what was the % mark up?

A: Let marked price = MP
New Marked Price = MP-0.3MP = 0.7MP
0.7MP=CP
% MP = (MP-CP)/CP
= 0.3MP/0.7MP
= 42.85%

Common Profit and Loss GMAT Questions

Q) A shopkeeper sells two T-Shirts at the same price.If he sells one of the T-shirts at a profit of 10% and the other for a loss of 10%,find the percentage profit and loss.

A: Remember that questions which has an equal % profit and %loss would result in a loss.

Formula to remember:(x/10)^2 % where x is the percentage loss and profit on two different entities.

Here in this question, x=10%

So Net Loss % = (10/10)^2% = 1%

Q) On Selling 10 apples if a fruit vendors recovers the cost price of 15 mangoes. Find the profit percentage.

A: 10* x = Selling price of 10 apples where x is the SP of one apple
15* y = Cost price of 15 mangoes where y is the CP of one mango

10x=15y

Note : When the SP and CP are equated the formula to remember is

Profit percentage = (Number of items Left)*100/(Number of Items Sold)
= 5*100/10 = 50 %

Another commonly tested GMAT Question is

Q) A dishonest shopkeeper uses 800 gram weights in place of 1 kg weights. Find the profit percentage if he sells per kg at the same price as he buys a kilogram.

A) Profit percentage = (Number of items Left)*100/(Number of Items Sold)
= 200*100/800 = 25 %

- **How to calculate standard deviation**

Standard Deviation of a probability distribution **measures the distribution of data from the mean value**. Probability distribution maps the probability of a variable within a particular range of values.

For Example: Suppose we toss a coin two times

All possible outcomes: HH, HT, TH, TT
Total number of outcomes = 4
Let a variable, X measures the number of heads in this experiment
Probability of n heads = (Outcomess with n heads)/ (Total number of outcomes)

Number of Heads (X)	**Probability (Probability of n heads)**
0	0.5
1	0.25
2	0.25

For a probability distribution, plot **number of Heads(X)** on the x-axis and **the probability of n heads** on the y-axis.

Range of values in this case = 0-2

Consider a data set with following value

2, 2, 4, 5, 3, 6, 13

Step 1: Count the number of elements in the data set
N = 7

Step 2: Find the mean or average of the data set

Arithmetic Mean (average) = (2 + 2 + 4 + 5 + 3 + 6 + 13)/7

= 35/7 = 5

Step 3: Subtract Mean from each element in the data set and square the result

(2 – 5) = -3 : (-3) ^ 2 = 9
(2 – 5) = -3 : (-3) ^ 2 = 9
(4 – 5) = -1 : (-1) ^ 2 = 1
(5 – 5) = 0 : (0) ^ 2 = 0
(3 – 5) = -2 : (-2) ^ 2 = 4
(6 – 5) = 1 : (1) ^ 2 = 1
(13 –5) = 8 : (8) ^ 2 = 64

For Sample Set

Step 4: Divide the sum of the resulting values with one less than the number of elements in the data set and find the square root

SD(Standard Deviation) = Sqrt ((9 + 9 + 1 + 0 + 4 + 1 + 64)/6)
= 3.82

Variance is the square of standard deviation = SD ^ 2 = 14.59

For Complete Set

Step 4: Divide the sum of the resulting values with the number of elements in the data set and find the square root

SD(Population Standard Deviation) = Sqrt ((9 + 9 + 1 + 0 + 4 + 1 + 64)/7)
= 3.54

Population variance is the square of standard deviation = SD ^ 2
= 12.25

Example of Sample Set: Sample voters in an exit poll
Example of Complete Set: Set of all the voters in an election

- **Absolute Value and Inequality - 1**

Absolutely positive: That is what everyone knows about an absolute value expression. /3/ = 3 and /-3/ = 3. But why do absolute value bars around a number neutralize, or do away with, the sign of the number? The answer is that absolute value bars are conceptually a measure of the distance of the expression inside the bars from zero. -3 and 3 are both three units away from zero. We will use these two ways of thinking absolute values-- (1) "the guarantee of positive" and (2) the measure of distance from zero--as the basis for delving deeper into absolute values.

Absolute value expressions: start to become difficult when variable expressions are placed inside the bars. For example, /x/. Upon a cursory examination, the expression /x/ seems like it should be equal to x. Since there is no sign in front of the x, the absolute value bars should be able to be removed without jeopardizing the "guarantee of positive." What this line of reasoning fails to account for, however, is that x itself could be negative! When dealing with absolute value expressions that contain variables, two scenarios must be considered:

(1) the scenario whereby the expression inside the bars is positive and

(2) the scenario whereby the expression inside the bars is negative.

In this example, for scenario (1) if x > 0, the expression /x/ can simply be represented as x; for scenario (2) if x < 0, the expression /x/ must be represented as (-x). Notice that in the negative scenario, we don't simply remove the absolute value bars. We remove the absolute value bars and negate the entire expression within.

Let's look at a more complicated example: the expression /x - 3/. As always, we must consider both the positive and negative scenarios. When is the expression inside the absolute value bars positive? Not simply when x > 0, but when x - 3 > 0 or when x > 3. Likewise the expression will be negative when x < 3.

To recap, the two scenarios are:

(1) /x - 3/ can be rewritten as x - 3 when x > 3
(2) /x - 3/ can be rewritten as -(x - 3) or 3 - x when x < 3
One more for the road: /3x + y/.
(1) /3x + y/ can be rewritten as 3x + y when 3x + y > 0
(2) /3x + y/ can be rewritten as -(3x + y) when 3x + y < 0

To finish our discussion of the definition of absolute value expressions, let's return to the concept of the **"measure of distance from zero."** In our first example with an algebraic expression, /x/, the absolute value expression rather straightforwardly represents the distance of x from zero. But what about in the expression /x - 3/? We might think of this as the distance of "x - 3" from zero; however, this will rarely be useful on the GMAT. A more sophisticated way of looking at this is "the distance of x from three."

Let's see how this conceptualization is put into practice. If x is to the right of the number 3 on the number line (let's say x is four units to the right of 3, which would make x equal to 7), the expression x - 3 will be positive and the expression /x - 3/ (in this case 7 - 3 = 4 units) represents the distance between x and 3. If x is to the left of 3 on the number line (keeping x four units away from 3, would make x equal to -1), the expression x - 3 will be negative and the expression /x - 3/ (in this case 3 - (-1) = 4 units) again represents the distance between x and 3.

On 700+ level GMAT questions, it is sometimes useful to be able to think of the general expression /x - y/ as the distance of x from y. In data sufficiency questions, this line of thinking is sometimes faster than running through the different algebraic scenarios

- **Absolute Value and Inequality - 2**

The first step to solving an equation containing an absolute value expression is to remove the absolute value bars. To do so, however, we must consider both the positive and negative scenarios

Consider the following example: /x/ + 8 = 12.

SCENARIO 1: If x is positive, then the absolute value bars can simply be dropped and the equation can be rewritten as: x + 8 = 12, so x = 4. Therefore, if x > 0, x = 4.

SCENARIO 2: If, however, x is negative, then the absolute value bars are dropped and the expression is negated, so the equation can be rewritten as (-x) + 8 = 12, so x = -4. Therefore, if x < 0, x = -4.

Now consider a slightly more difficult example: /x + 1/ = 8

SCENARIO 1: If (x + 1) is positive, x + 1 = 8 so x = 7.
Notice that the condition here is that x + 1 > 0 or x > -1
Therefore, if x > -1, x = 7

SCENARIO 2: If (x + 1) is negative, -(x + 1) = 8, so x = -9 |
Therefore, if x < -1, x = -9

Notice that in both of these examples, there were two solutions to the absolute value equation. That's because a variable absolute value expression behaves differently depending on the sign of the variable expression it contains within. Each variable absolute value expression has a breaking point, known as the critical point, where the behavior changes. This critical point is the "zeroing point" of the expression in the absolute value bars. For the simple expression /x/, the critical point is 0. All x's to the right of 0 on the number line reduce the expression /x/ to x; all x's to the left of 0 on the number line reduce the expression /x/ to (-x). For the expression /x - 2/, the critical point is 2; for the expression /2x + 3/ the critical point is -3/2. The critical point is important because it defines the range whereby each equation is true.

Now we can look at a more complicated example: /x + 1/ + /x - 3/ = 6.

Notice that this equation has two absolute value expressions, both in terms of the variable x. With two scenarios per absolute value expression, solving this equation could potentially involve solving four different equations! Because the absolute value expressions contain the same variable x, however, it turns out there are only three possible equations.

The easiest way to see the various scenarios is to track the critical points of the two absolute value expressions on a number line.

The critical point of the expression $/x + 1/$ is -1 and the critical point of the expression $/x - 3/$ is 3. How many distinct regions are there on the number line with the points -1 and/or 3 as boundaries? The answer is three: $x < -1$, $-1 < x < 3$, and $x > 3$. (The theoretical "fourth" scenario here is not a possibility, i.e. $3 < x < -1$.)

So what does the equation look like for each one of these three scenarios?

SCENARIO 1: when $x < -1$ $(x + 1)$ is negative in this region and $(x - 3)$ is also negative, so the equation becomes:
$-(x + 1) - (x - 3) = 6$, which simplifies to $x = -2$

With complex absolute value questions, the solution ($x = -2$) must be checked against the range for which the equation holds true ($x < -1$). In this case there is no conflict (since -2 is less than -1) so this is an actual solution.

SCENARIO 2: when $-1 < x < 3$ $(x + 1)$ is positive in this region and $(x - 3)$ is negative, so the equation becomes:
$(x + 1) - (x - 3) = 6$, which simplifies to $4 = 6$, i.e. mathematical jibberish!

This means that there is no solution for the equation in this range.

SCENARIO 3: when $x > 3$ $(x + 1)$ is positive in this region and $(x - 3)$ is also positive, so the equation becomes:
$(x + 1) + (x - 3) = 6$, which simplifies to $x = 4$

This solution ($x = 4$) check out since it is in the range for which the equation hold true ($x > 3$).

Therefore, there are two potential solutions to this absolute value equation, $x = -2$ and 4. Conceptually, it is important to understand that because this equation had two absolute value expressions for the same variable x, there were two critical points and therefore three regions of "different behavior." This created the need for the investigation of three different scenarios/equations.

- **Absolute Value and Inequality - 3**

In general, inequalities are not that different from equations. In fact, most of the algebraic manipulations that can be used to solve equations can be applied to inequalities. There is however, one notable exception: when you multiply or divide an inequality by a negative value, you must change the direction of the inequality symbol. This may seem trivial, but the ramifications are numerous:

(1) You cannot multiply/divide an inequality by a variable unless you know the sign of that variable because you need to know whether or not to switch the direction of the inequality symbol. The inequality $p/q > 1$ does not necessarily imply that $p > q$ (unless q is positive).

(2) You cannot take the square root of both sides of an inequality expression. $x2 < 4$ is not simply equivalent to $x < 2$ (it's actually equivalent to $x < -2$ OR $x > 2$).

(3) You can't square both sides of an inequality. $x > y$ does not necessarily imply that $x2 > y2$.

(4) Finally, absolute value expressions must be treated with special care, since as we learned in last article, absolute value expressions have two possible scenarios. Special care must be taken when considering the negative scenario.

This last point is of primary interest to us in our discussion of absolute values. Let's look at an example of a simple inequality containing an absolute value: $/x/ < 2$. Just as we did when working with equations, we must consider two scenarios for the absolute value expression.

Scenario 1: when $x > 0$, the inequality simply becomes $x < 2$.

Scenario 2: when $x < 0$, the inequality becomes $-x < 2$, which can be simplified to $x > -2$. Notice that the direction of the inequality symbol switches when we multiply by a negative.
The solution is $-2 < x < 2$

Now let's take a slightly more difficult example. $/x - 4 / > 8$

Scenario 1: when $x - 4 > 0$ (i.e. when $x > 4$), $x - 4 > 8$, which simplifies to $x > 12$

Scenario 2: when $x - 4 < 0$ (i.e. when $x < 4$), $-(x - 4) > 8$ which simplifies to $x < -4$

The solution is $x > 12$ OR $x < -4$.

More often than not, absolute value and inequalities show up as data sufficiency questions on the GMAT.

Now that we have covered the basics of inequalities and absolute values, let's take a look at a sample GMAT data sufficiency question.

Is $x > 0$?
(1) $/x - 3/ < 5$
(2) $/x + 2/ > 5$

The first statement can be solved using the method described above.

Scenario 1: When $x > 3$, $x - 3 < 5$, which can be simplified to $x < 8$
Scenario 2: When $x < 3$, $-(x - 3) < 5$ or $x > -2$

Statement (1) can be simplified as $-2 < x < 8$, which is NOT sufficient to answer the question "is $x > 0$?"

The second statement can be solved in a similar manner.

Scenario 1: When $x > -2$, $x + 2 > 5$ or $x > 3$.
Scenario 2: When $x < -2$, $-(x + 2) > 5$ or $x < -7$.

Statement (2) can be simplified as $x > 3$ or $x < -7$, which is NOT sufficient to answer the question "is $x > 0$?"

Together, the two statements ARE SUFFICIENT to answer the question "is $x > 0$?" Since both statements are true (remember this is always the case in Data Sufficiency), x must be greater than 3 and less than 8. This is the only overlapping region between the ranges from statements (1) and (2). All of the values between 3 and 8 are positive so TOGETHER the statements are SUFFICIENT and the answer is C.

- **Absolute Value and Inequality - 4**

We can conclude our discussion of absolute values on the GMAT with a more conceptual way of solving these expressions. In more difficult questions involving absolute value expressions on the GMAT, considering the positive and negative scenarios for each absolute value expression can be either overly tedious or altogether not useful. In these cases, it is preferable to take a more conceptual approach.

Consider the following sample data sufficiency question.

If **y = /x + 7/ + /2 - x/**, is y = 9?

(1) $x < 2$
(2) $x > -7$

This question could be solved using a systematic algebraic approach. It would start like this: the two absolute value expressions each have two scenarios that must be considered. If we consider the intersection of these four scenarios, we will come up with three different equations (Theoretically there should be four different equations given the two scenarios for each expression, however, one of the equations would never hold since x cannot be both less than -7 and greater than 2). The process of considering these three different equations would be quite tedious and time consuming.

If we apply a conceptual understanding of absolute values, we can greatly simplify the solution to this problem. As we learned earlier in the series, each absolute value expression has two scenarios, a positive and a negative one. Recall that these scenarios are true for a given range of values.

For example, the first expression $/x + 7/$ has two scenarios: one when x is greater than -7 and one when x is less than -7. -7 is called the critical point of this absolute value expression. What is the critical point of the second absolute value expression in the above equation? 2. Notice that the critical point is the value of x that would cause the absolute value expression to be zero.

Now that we have the critical points of the two absolute value expressions in this equation, we are poised for a strategic, conceptual solution to this question. Because there are two critical points, the equation $y = /x + 7/ + /2 - x/$ will have three different forms, one for each of three critical regions: $x < -7$, $-7 < x < 2$, and $x > 2$.

The question asks us if this equation can be simplified to $y = 9$. Let's look at the statements.

Statement (1) says that $x < 2$. Is $y = 9$ for all x's less than 2? If we plugged a few values for x less than 2, i.e. $x = 1, 0, -1$, we might come to the hasty decision that in fact y is always equal to 9 when $x < 2$. The problem with this set of numbers, however, is that it doesn't do justice to the critical regions in the problem.

For statement (1) we must check values not only less than 2 but also less than -7. Otherwise we are artificially restricting the statement to one of the three critical ranges for the problem. In fact when we check $x = -10$, we see that y is not equal to 9, so statement (1) is not sufficient.

The same process can be applied when looking at statement (2) on its own. We must try values of x that fall in the two critical regions which meet the criteria in the statement. Thus, we must test values in the region $-7 < x < 2$ and values in the region $x > 2$. When we do, we find that y is not always equal to 9.

When we look at the two statements together, the only critical region to be considered is $-7 < x < 2$. Now the two statements are sufficient (y is always 9 in this region) and the correct answer is C, statements (1) and (2) TAKEN TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient.

Let's take a look at another example.

If **/x/ + /y/ = /x + y/** then which of the following must be true?
(A) $x + y > 0$
(B) $x + y < 0$
(C) $x - y > 0$
(D) $xy > 0$
(E) $xy < 0$

At first glance, one might consider writing out the different scenarios for the absolute value expressions in this equation, however, that would be a mistake. A conceptual approach is much better.

Let's use the "guarantee of positive" that we spoke about in the first week to think about the equation /x/ + /y/ = /x + y/. The left side of the equation takes the variables x and y and makes them positive before they are added. The right side of the equation adds the variables first and then makes the result positive.

What must be true about x and y for the two sides of the equation to be equal? If x and y are both positive (x = 2, y = 3), both sides of the equation equal 5. If x and y are both negative (x = -2, y = -3), both sides of the equation still equal 5.

If, however, one value is positive and the other is negative (x = -2, y = 3), the left side of the equation is 5, but the right side of the equation is 1. We see that for the two sides of the equation to be equal, x and y must have the same sign. This means that the product of x and y must be greater than zero, so the correct answer is D. Notice that the solution to this question did not involve breaking the absolute value expressions up into two scenarios, but instead involved a conceptual understanding of absolute value expressions.

In this series we have covered a variety of approaches to solving absolute value expressions on the GMAT. From the algebraic to the conceptual, these methods should aid you in tackling even the most difficult absolute value questions.

- **Inequality Tip→gurpreetsingh→Gmatclub.com**

I learnt this trick while I was in school and yesterday while solving one question I recalled.
Its good if you guys use it 1-2 times to get used to it.

Suppose you have the inequality

$f(x) = (x-a)(x-b)(x-c)(x-d) < 0$

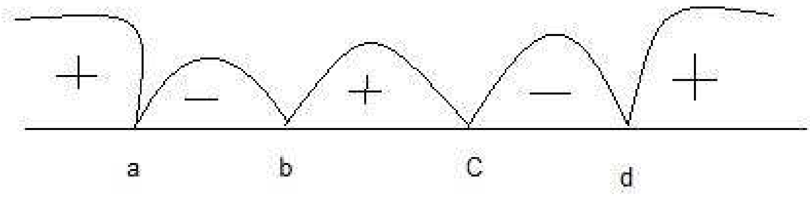
Just arrange them in order as shown in the picture and draw curve starting from + from left.

now if $f(x) < 0$ consider curve having "-" inside and if $f(x) > 0$ consider curve having "+" and combined solution will be the final solution. I m sure I have recalled it fully but if you guys find any issue on that do let me know, this is very helpful.

Dont forget to arrange then in ascending order from left to right. $a<b<c<d$

So for $f(x) < 0$ consider "-" curves and the ans is : $(a < x < b)$, $(c < x < d)$
and for $f(x) < 0$ consider "+" curves and the ans is : $(x < a)$, $(b < x < c)$, $(d < x)$

Attachments:



1.jpg [6.73 KiB | Viewed 191 times]

- **<u>Number System</u>**

GMAT number system is a critical chapter for all aspiring gmatters. On an average about 5-7 questions on number system will be asked during the GMAT CAT.So please go through the definitions before cracking the problems

Whole Numbers: Whole Number includes all the numbers in the positive coordinates **including 0**.Let us represent the whole number by W,then

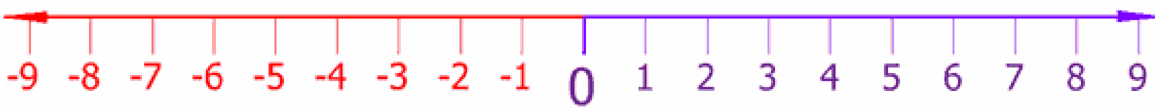
W = {0,1,2,3....}

Positive Numbers/Natural Numbers:All numbers in the positive coordinate greater than 0 represents Positive numbers or are termed as natural numbers.If positive number is represented by P then

P = {1,2,3,4,5}

Note that 0 is neither positive nor negative.

Integers : Set of whole numbers and negative numbers constitute Integers and can be represented on the number line as follows



The distance between any two points say between -9 and -4 is calculated by subtracting the lower value from higher value.

Distance(-9,-4) = -4-(-9)=5

Absolute Value : The distance of the number from 0 (Zero) is represented by the absolute value(|X|)

Absolute value of -4 (|-4|) = 0-(-4) = 4

Rational Numbers : A number of the form p/q where q#0 is called a rational number.

For example 5/6 is a rational number . 5/0 is undefined and doesn't satisfy the rationality condition mention above.

A rational number can result in an **integer (6/2=3) or a decimal fraction (5/6=0.833333...)**

Decimal Fractions : There are two types of decimal fractions :

a) **Terminating Decimal Fraction** : Like 6/5 = 1.2 Note that the decimal terminates after 2

b) **Non-Terminating Decimal Fraction** : Like 5/6 = 0.833333... The decimal does not terminate

Non-Terminating Decimal Fraction : There are two types of non-terminating decimal fractions

a) **Non-Terminating Recurring** : For example 5/6=0.833333.. is a non-terminating recurring decimal fraction

Note that 3 is recurring to infinity

b) **Non-Terminating Non-Recurring** : For example 3.316624790355 is a non-terminating non-recurring decimal

fraction

Irrational Numbers : Any number that cannot be represented in the form p/q where q # 0 is called an irrational number

Most common irrational numbes are square roots, cube roots and logarithms ex sqrt(2) and log(8) etc

Real Numbers : Set of rational and irrational numbers are called Real numbers

Real Number

|

Rational Number + Irrational Number

|

Integer + Fractions

| |

Negative + Positive Decimal

|

Terminating + Non-Terminating

|

Non-terminating recurring + Non-terminating Non-recurring

- **GMAT Divisibility Rule Shortcuts**

GMAT Divisibility Rule to cram

Dividing by 2

1. All even numbers are divisible by 2. E.g., all numbers ending in 0,2,4,6 or 8.

Dividing by 3

1. Add up all the digits in the number.
2. Find out what the sum is. If the sum is divisible by 3, so is the number
3. For example: 12123 (1+2+1+2+3=9) 9 is divisible by 3, therefore 12123 is too!

Dividing by 4

1. Are the last two digits in your number divisible by 4?
2. If so, the number is too!
3. For example: 358912 ends in 12 which is divisible by 4, thus so is 358912.

Dividing by 5

1. Numbers ending in a 5 or a 0 are always divisible by 5.

Dividing by 6

1. If the Number is divisible by 2 and 3 it is divisible by 6 also.

Dividing by 7 (2 Tests)

Test 1

1.Take the last digit in a number.

2. Double and subtract the last digit in your number from the rest of the digits.

3. Repeat the process for larger numbers.

Example: 357 (Double the 7 to get 14. Subtract 14 from 35 to get 21 which is divisible by 7 and we can now say that 357 is divisible by 7.

Test 2

1. Take the number and multiply each digit beginning on the right hand side (ones) by 1, 3, 2, 6, 4, 5.

Repeat this sequence as necessary

2. Add the products.

If the sum is divisible by 7 - so is your number.

Example: Is 2016 divisible by 7?
6(1) + 1(3) + 0(2) + 2(6) = 21
21 is divisible by 7 and we can now say that 2016 is also divisible by 7.

Dividing by 8

1. This one's not as easy, if the last 3 digits are divisible by 8, so is the entire number.
2. Example: 6008 - The last 3 digits are divisible by 8, therefore, so is 6008.

Dividing by 9

1. Add up all the digits in the number.
2. Find out what the sum is. If the sum is divisible by 9, so is the number.
3. For example: 43785 (4+3+7+8+5=27) 27 is divisible by 9, therefore 43785 is too!

Dividing by 10

1. If the number ends in a 0, it is divisible by 10.

Divisibility by 13

1. Remember this series(1, 10, -4, -1, -10, 4 repeat the pattern.)
2. Multiply the right most digit of the number with the left most number in the sequence shown above and the second right most digit to the second left most digit of the number in the sequence

and so on.

Ex: 321 = 1 x 1 + 2 x 10 - 3 x 4 = Check whether the number is divisible by 13
321 not divisible by 13
1234567 = 7 x 1 + 6 x 10 - 5 x 4 - 4 x 1 - 3 x 10 + 2 x 4 + 1 x 1 = Check whether the number is divisible by 13
1234567 not divisible by 13

- **Set Terminologies**

A Set is a collection of unique objects.

There are two methods to represent sets

1) Rooster Method : In this method all the elements are written within braces separated by commas

Example X = {a, b, c, d, e}

2)Set Builder Method/Property Method : In this method elements are represented by stating properties that are satisfied by all the elements in the set

If A is a set of all positive factors of 12
Example A = { x: x is a factor of 12 and x >0 }

Set Types

1) Null Set : A Set with no elements represented by "{}," "∅", and "∅".

Example: Set of months with 34 days

2) Singleton Set: A Set with one element

Example: Set of even prime number {2}

3) Finite Set : A Set whose elements can be counted.

Example: Set of positive even numbers above 2 and below 12
A= {4, 6, 8, 10)
n(A) = 4

4) Infinite Set : A Set whose elements cannot be counted

Example: Set of irrational numbers between 1 and 100

5) Equal Sets : Two sets are equal if all the elements in one Set are also presents in the other Set

Example: A = {a,c,b}   C= {a,d,c}   B = {b,c,a}

A = B   A#C

6) Subsets : Set A is the subset of Set B if all the elements in Set A is contained in Set B

A = { a,b,c }   B = { a,d,c,b,e}

$A \subseteq B$

Points to Remember

1) Every set is a subset of itself
2) Every set has an empty set as its subset
3) Total subsets of a Set with n elements = 2^n

7) Universal Set: If a Set 'A' represents all the elements in multiple sets then 'A' is a Universal set

A = { 1,2}   B={1,2,5} C = {5,1}

Universal Set U = AUBUC = {1,2,5}

8) Power Set: A set containing all the subsets of a Set is a Power Set

A = {4,5}
Subsets of A = { }, {4}, {5}, {4,5}
Power Set P(A) = { { }, {4}, {5}, {4,5}}

- **Back solving in GMAT**

Back-solving is a technique of trying out the answer choices in the question. Each choice is selected one at a time and is assumed to be the right choice. Analyze the value or conditions by injecting the choices into the question. Back-solving is incredibly time saving in algebraic equations and number properties problems.

Case 1: <u>To find the value of an equation</u>
Example: Which number gives the lowest value for the equation?
1/X + (1/X) ^2 + 5X

Approach 1: Start from the middle value in the answer choice.
For example, if the five answer choices are 14,5,19,8,21

Plug 14 into the equation and check the value. If the value is too high, find out an answer choice with a much smaller value, like 5.

Approach 2: If you inspect the equation, the first two terms will result in small numbers, smaller than 1, according to the given answer choices. However, the 3rd term of the equation is the leading one. A small value in 5X will make the equation's net value smaller, and a big value in 5X will make the equation's net value bigger. So, the answer is almost immediately x = 5.

Case 2: To satisfy a condition
Example: If y # 3 and 2x/y is a prime integer greater than 2, which of the following must be true?

I. x = y
II. y = 1
III. x and y are prime integers.

(A) None

(B) I only

(C) II only

(D) III only

(E) I and II

For 1: Let us plug-in x=y=1
2x/y = 2, which is not greater than 2

For 2: y=1
Therefore 2x/y = 2x
Plug-in value for x:
x = 1, 2x = 2 which is equal to 2 and not greater than 2

For 3: x and y are prime integers

x = 13 y= 2
2x/y = 13, which is a prime integer greater than 2
x=2 y=13
2x/y = 0.3, which is less than 2 and not a prime integer

Note: If a certain value satisfies the condition, always try with other values before confirming the answer.

Correct Answer : A

For 3: x=13, y=2 satisfied the condition (Where x is a large prime number and y, a small one), but when we plugged in opposite numbers, x=2 and y =13 (Where x is a small prime number and y, a large prime number) the condition failed. Always test with opposite numbers to validate the condition.

- **Fast Math Multiplication**

Multiplication by 5

It's often more convenient **instead of multiplying by 5 to multiply first by 10 and then divide by 2**.
For example, 237x5=2370/2=1185

Same way you can also multiply by 50

Multiplication by 4

Replace either with a **repeated operation by 2**.
For example 124x4=248·2=476

Multiplication by 25

Multiply first by 100 and then divide by 4
For example 37x25=3700/4=1850/2=925.

Multiplication by 8

Replace either with a repeated operation by 2.
For example: 124x8= 248x4= 496x2= 992.

Multiplication by 45

Multiply first by 90 and then divide by 2.
For example37x45=37x90/2=3330/2=1665

Multiplication by 55

Multiply first by 110 and then divide by 2.
For example37x55=37x110/2=4070/2=2035

Multiplication by 75

Multiply first by 300 and then divide by 4 .
For example37x75=37x300/4=11100/4=2775

Multiplication by 125

Multiply first by 500 and then divide by 4
For example 37x125=37x500/4=18500/4=4625

Multiplication by 150

Multiply first by 300 and then divide by 2
For example37x150=37x300/2=11100/2=5550

Multiplication by 175

Multiply first by 700 and then divide by 4
For example37x175=37x700/4=25900/4=6475

Multiplication by 225

Multiply first by 900 and then divide by 4
For example37x225=37x900/4=33300/4=8325

Multiplication by 275

Multiply first by 1100 and then divide by 4

For example37x275=37x1100/4=40700/4=10175

A Great Time Saver

Q A music shop has a sound system marked at Rs. 14000 on which a sales tax of 12.5% is chargeable. Find the total value a customer would have to pay for the sound system.

$$\text{sales tax} = \frac{14000 \times 12.5}{100} = \frac{14000 \times 125}{1000} = \frac{14 \times 500}{4} = \frac{7000}{4} = 1750$$

Rs 14000+1750 =Rs15750

Multiplication by 11

To multiply any figure number by 11 we just put

the total of the two figures between the 2 figures.

<u>32X11</u>

Here first we write 2 of 32 to extreme right , now add up 3+2 =5 , put 5 before 2 and write 3 to extreme left .

Required answer is 352

<u>322x11</u>

Take the 2(Extreme right) from 322 and add up 2&2 of 322 , 2+2=4 , write 4 before 2 Now our partial answer is 42 Again add up 3&2 of 322 .3+2 =5 write before 42 .Now our partial answer becomes ..542 .Put 3 before 5 . Now our required answer is 3542

77 x 11 = <u>847</u> This involves a carry figure because 7 + 7 = 14

we get 77 x 11 = $7_1 47$ = 847

You can do calculations in Mensuration very fast by this tool

As you know value of pi is 22/7 .You can break it as 2x11/7

Ex Find the area of circle of radius of 14 cm

(22/7)x14x14=11x2x2x14=11x56=616square cm

All calculation mentally, no rough work , a great time saver.

Method for multiplying numbers where the first figures are the same and the last figures add up to 10.

42 x 48 = Both numbers here start with 4 and the last figures (2 and 8) add up to 10.

Just multiply 4 by 5 (the next number up) to get **20** for the first part of the answer.

And we multiply the last figures: 2 x 8 = **16** to get the last part of the answer
42 x 48=2016

Method for multiplying numbers where the first numbers add up10 and

the second number's digits are same 46X55

Here first number's add up is 10 and second number's digits are common i.e 5

Just multiplying last figures of both numbers 6x5 =30 put it in right hand side

Again multiplying first figures of both numbers and add common digit of second number

(4x5)+5 =20+5 =25 put it left hand side

Required answer is 2530 (If multiplication is in unit's place in first step add zero before it)

Multiplying numbers just over 100.

108 x 109 = <u>11772</u>

The answer is in two parts: 117 and 72,

117 is 108 + 9 (or 109 + 8),

and the 72 is just an 8 x 9.

Similarly **107 x 106 = <u>11342</u>**

- **Prime Factorization**

The purpose of this exercise is to express the number with which you are presented as a product of prime numbers. This exercise will include all numbers less than 1000 that can be expressed as a product of the prime numbers less than 30. You may find it useful to familiarize yourself with the following common prime factorizations.

For example, the number 350 can be expressed as 2 X 5^2 X 7.

$1024=2^{10}$	$729=3^6$	$625=5^4$	$343=7^3$
$512=2^9$	$243=3^5$	$125=5^3$	$49=7^2$
$256=2^8$	$81=3^4$	$25=5^2$	$7=7^1$
$128=2^7$	$27=3^3$	$5=5^1$	
$64=2^6$	$9=3^1$		
$32=2^5$	$3=3^1$		
$16=2^4$			

Prime factorization is useful in conjunction with the Fundamental Theorem of Arithmetic, which states that "All integers greater than 1 can be expressed as the unique product of prime numbers."

e.g.

If a, b, and c are prime numbers and ab^2c^3= 3500, what is the value a + b - c?

To solve this problem, we can prime factor 3500 - $3500=2^2 5^3 7^1$

By substitution we get $ab^2c^3=2^2 5^3 7^1$

So a=7, b=2, and c=5, and a + b – c =7 + 2 – 5, which equals 4.

Prime factorization can not only be useful on GMAT questions that test number property concepts such as factors, multiples, and prime numbers, but also on question that test arithmetic concepts such as multiplication, division, and powers.

- **The Importance of Prime Factorization on the GMAT**

By Josh Anish on March 3rd, 2010 (Knewton guy) →**on *beatthegmat.com***

Prime Factorization: My single favorite topic on the GMAT. Hands down.

My passionate (some would say evangelical!) advocacy of prime factorization results not only from my finding prime numbers so intrinsically fascinating in and of themselves, but also from the plain and simple truth that prime factorization proves surprisingly useful on questions on which prime numbers aren't even mentioned.

For example, any time you're given a question asking about multiples and factors, you can bet that prime factorization will help you get to the answer quicker.

Check out this Data Sufficiency question from the Official GMAT Guide:

If positive integer *x* is a multiple of 6 and positive integer *y* is a multiple of 14, is *xy* a multiple of 105?

(1) *x* is a multiple of 9
(2) *x* is a multiple of 25

Note that there is no mention of prime numbers at all. But take any other approach to this problem, and you're likely to get pretty frustrated and lost rather quickly. You could certainly test numbers, but good luck taking only two minutes finding values that work for every case!

Now, I'm going to re-write the question and statements using only prime factorizations:

If positive integer *x* is a multiple of 2*3 and positive integer *y* is a multiple of 2*7, is *xy* a multiple of 3*5*7?

(1) *x* is a multiple of 3*3
(2) *x* is a multiple of 5*5

All of a sudden, the question becomes much more manageable. We know that *x* carries at least one 2 and one 3 as factors. We also know that *y* carries at least one 2 and one 7 as factors. Therefore, the product *xy* must carry at least two 2s, one 3, and one 7.

We are asked if *xy* carries at least one 3, one 5, and one 7 as factors. So far, we know we *xy* has one 3 and one 7, so all that's missing is the one 5. What we've just done is show that in order to establish sufficiency, all we need to do is determine whether there's a factor of 5 somewhere in *x* or *y* (or both).

Statement 1 lets us know that *x* has two 3s and mentions nothing of 5s. But that doesn't necessarily mean there isn't a 5 there. There also might be a factor of 5 in *y*. Because we cannot determine the presence or absence of factors of 5, this statement is insufficient.

Statement 2, on the other hand, lets us know that *x* definitely has a factor of 5. And again, we already know from the prompt that *x* has a factor of 3 and *y* has a factor of 7. Therefore, the product *xy* has at least one 3, one 5, and one 7 as factors, and we can conclude unequivocally that *xy* is a multiple of 3*5*7 = 105. Sufficient.

Final answer: **B**

Even on questions that do explicitly mention prime numbers, things can get really ugly really quickly if you don't use prime factorization.

For example, take this Problem Solving question, also from the Official Guide (answer choices not included):

In a certain game, a large container is filled with red, yellow, green, and blue beads worth, respectively, 7, 5, 3, and 2 points each. A number of beads are then removed from the container. If the product of the point values of the removed beads is 147,000, how many red beads were removed?

The use of 2, 3, 5, and 7 is a prime clue (pun indeed intended). You might look at 147,000 and panic because the number is so large. But let's break down 147,000 into its prime factorization:

147,000
= 147 * 1000
= (7 * 21) * 10 * 10 * 10
= (7 * 7 * 3) * (2*5) * (2*5) * (2*5)

Now, the question asks us how many red beads were removed. Red beads are associated with a point value of 7.

We know that the final point total was 147,000, and when we broke that number down, we found that there were only two factors of 7. Therefore, the only way we could get that score is if we removed 2 red beads. That's it! 2 is our final answer!

These are just two examples of a large number of questions made easier by prime-factor prowess. Practice making those factor trees! And notice how prime numbers help you answer questions about other topics like Greatest Common Factor and Least Common Multiple.
--
Rich Zwelling is a stellar teacher in Knewton's GMAT course. And it's not just shtick: He really does love prime factorization.

<u>William Wallace's blog→Quant tips:</u>

Quant - tips 1

While i am recovering from my back injury and while my prep has taken a back seat, i just wanted to share a few quant tips which might be useful. Feel free to add your own tips to these and i will also add many more as soon as i refresh new fundae and find something useful.

1. **Keep a note of basics and imp formulae** --eg area of a triangle can be calculated as .5*base*altitude, or
0.5*a*b*sin(q) {product of adjacent sides and sine of INCLUDED angle}, or
{s(s-a)(s-b)(s-c)}^(1/2) where "s" is semi-perimeter. These variations can help you in DS section. Consider the following:

Calculate the area of paralellogm ABCD with diagonal AC=14 units ;

1.) AB = 10units

2.) BC = 12 units {the measurements might not be of a true paralellogm, i have just taken imaginary measurements}

here the third formula will come handy.

2. Whenever you solve an equation and get 2 values for dependent variable then ask yourself whether it is possible for the dep var to have 2 values. Consider the following ques:

Solve the problem in 2 minutes and chances are you will mark the choice C as answer, but here's the catch. Y can't be negative {see the question}, the answer should be D.

3. **Don't panic** -- if you are not getting the answer right even after applying the logic ...then check your logic again ... due to stress during the exam it is quite possible that you are using the formula wrongly or using the wrong logic. I faced this problem while solving a progressions problem. I was using the sum of series of AP as {a/2(2a + (n-1)d)}; whereas right formula is {n/2(2a + (n-1)d)};

4. See {actually just glance} the answer choices before you solve the problem, No need to find the answer in decimal if answer choices are in fractions. This technique will help when GMAT cleverly hides the right answer in a different form {and you will not be baffled to see a different set of answers after solving the problem}.

5. Always put down what you have to find and the units in which you have to give the answer and most importantly place a question mark in front of it. After finding the answer check whether you have found the right variable in right units or not. If question is asking about $ then

don't work the problem in cents. Use the scratch paper well.

<u>To take square of any number by AtifS (me) →gmatclub.com:</u>

Ok guys! I couldn't find how to take square of any number especially 2 digit numbers and I think instead of cramming squares of numbers up to 25 why not find a fast and easy way to get the square (power 2) of any number. By the way I'll update this in GMAT Quant Notes file posted in first post. And yea I'll prove this method as well :)

Let's start from easy one, (P.S. The beauty of math is that you can solve it in several ways)

The formula is as follows

(a-d)^2= a^2 + d^2 -2ad

This can also be written as

(a-d) ^2= [u]a^2 - 2ad [/u] + d^2

Now by taking "a" as common in underlined portion of the equation, we get => [b]a*(a-2d) + d^2[/b]

Now remember this one and let's take Square of 7 (start from easy one). To take square of & we'll use above formula and the rule is always to get to nearest number multiple of 10 meaning number with "0" as unit digit or number with the base 10 . Here in case it is "10" so to get to 10 we have to add "3" to "7", which means

7= 10-3 --> same as (a-d), so "10" (the base number) is as "a" and "3" is as "d", the difference between "7" and nearest number that is multiple of 10.

Now let's solve it,

7^2= (10-3) ^2= [10*{10-2*(3)}] + (3) ^2= {10*(10-6)} + 9 = (10*4) + 9=40+9=49 proved

Whenever the square of any number comes in your mind like in this case "7", quickly add some number(or integer) to round it to a nearest number that is multiple of 10(the one with base 10) like in this it would be "10" so the difference is "3" now similarly subtract the difference from "7" which would be 7-3=[b]4[/b], Now multiple 10 (base 10 number) and 4 (after subtracting the difference from actual number) and then add the [i]square of difference[/i] which would yield as 3^2=9. Now let's look at this

To take the square of 7 let's first get the difference between base 10 number and 7, which in this case is 10-7=3=d (difference), Suppose a=7 (actual number whose square is to be taken) and d=3 (difference between actual number and base 10 number), Now let's get the formula and solve it

[b]a^2[/b] = [u](a-d)*(a+d)[/u] + d^2=

By solving underlined portion to (a^2 - d^2) and the put it in above equation we get

=> (a^2 - d^2) + d^2= a^2 - [u]d^2 + d^2[/u]

Now the terms in underlined portion will cancel each other and we'll get only

=> [b]a^2[/b] Hence proved :) (See above the bold term a^2 which is equal to this term)

Now solving for 7

7^2= (7+3)*(7-3) + 3^2=10*4 + 9=40+9=49

We can also use [b] (a+d) ^2[/b], this is used when the number to be squared is just above Base number (multiple of 10 or 100 and so on)

And (a-d) ^2 is used for the number just below the base number (multiple of 10 or 100 and so on)

Now for (a+d) ^= a^2 + d^2 + 2ad= a^2 + 2ad + d^2= a*(a+2d) + d^2

Let's take 11

11^2= (10+1) ^2 = 10 (10 + 2*1) + 1^2= 10 (10+2) +1= 10*(12) + 1= 120 +1=121 proved

Another way to solve it is (2nd method mentioned above already)

11^2= (11-1)*(11+1) + 1^2= (10*12) + 1= 120 +1=121 it's fast and easy once you get a grasp over it, multiplying any number with a number with base 10 makes it easy as you have to just put "0"s on right side of the resultant number and then add the square of difference gives the square of the number.

Now let's practice with different numbers,

68^2= 70*(70- 2*2) + 2^2= [b] 70*(70-4) [/b] + 4 = [u] 70*66[/u] + 4= 4620 + 4= 4624

By the way multiplying underlined portion is easy as (by breaking down the numbers)

70*66=70*(60+6) =(70*60) + (70*6)=4200 + 420 = 4200+400+20=4600+20=4620 and then add 4 gives 4624=68^2

Similarly bold portion of above equation can also be solved as

70*(70-4) = (70*70) - (70*4) =4900 - 280=4900 -200-80=4700-80=4620 and then adding 4 gives 4624=68^2

Now with other way

68^2= (68+2)*(68-2) + 2^2=70*66 + 2^2=4620+4=4624

Now let's take 46

46^= (46+4)*(46-4) + 4^2=50*42 + 16=50*(40+2) + 16=2000+100 +16=2100+16=2116

Also can be solved as

46^2= (50-4) ^2= 50*(50 - 2*4) + 4^2=50*(50-8) +16 = (2500-400) + 16=2100+16=2116

116^2 = (116+4)*(116-4) + 4^2= (120*112) + 16= 120*(100+12) +16= (120*100- 120*12) + 16 = (12000+1440) + 16= (12000+1000+440) + 16= (13000+440) + 16= 13440 + 16= 13456

116^2= (120-4)^2= 120*(120 – 2*4) + 4^2= 120*(120 - 8) + 16= (120*120 – 120*8) + 16= (14400 – 960) + 16= (14400 -900-60) +16= (14000-900+400-60) + 16= (13100+400-60) + 16= (13500-60) +16= 13440+16=13456

Whew! I think that's enough you can try other numbers you wish to, that's easy instead of just cramming squares of numbers till 25 (after you practice and then try to solve it in your mind, you'll find it easy). I wrote it in detail step by step so that you can understand the basic concept behind it otherwise it’s fast to do especially for 2-digit numbers.

Sriharimurthy’s Remainder Tips & Tricks→gmatclub.com:

Hi guys,
this is in conjunction with another post which has questions dealing with remainders (collection-of-

remainder-problems-in-gmat-74776.html). I'm just trying to put together a list of tips and tricks which we can use to solve these kinds of problems with greater accuracy and speed. Please feel free to comment and make suggestions. Hopefully we can add onto this list and cover all sorts of strategies that would help us deal with remainders!
Cheers.

Please read this first :
1) Take your time with these points. Some of them might be a little difficult to follow in the first reading, but don't give up. The concepts are fairly simple.
2) These tips if mastered will be extremely valuable in the GMAT to help solve a variety of questions not limited specifically to remainders. I have been using them for quite a while now and they have not only helped me improve my accuracy but also my speed.
3) If you have any doubts, please do not hesitate to ask (no matter how stupid you might think them to be!). If you do not ask, you will never learn.
4) Lastly, have fun while trying to understand these tips and tricks as that, according to me, is the best possible way to learn.

All the best!

-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-
NOTE: Where ever you see R of 'x' it just stands for Remainder of x.

1) The possible remainders when a number is divided by 'n' can range from 0 to (n-1).
Eg. If n=10, possible remainders are 0,1,2,3,4,5,6,7,8 and 9.

2) If a number is divided by 10, its remainder is the last digit of that number. If it is divided by 100 then the remainder is the last two digits and so on.
This is good for questions such as : ' What is the last digit of.....' or ' What are the last two digits of.....' .

3) If a number leaves a remainder 'r' (the number is the divisor), all its factors will have the same remainder 'r' provided the value of 'r' is less than the value of the factor.
Eg. If remainder of a number when divided by 21 is 5, then the remainder of that same number when divided by 7 (which is a factor of 21) will also be 5.

If the value of 'r' is greater than the value of the factor, then we have to take the remainder of 'r' divided by the factor to get the remainder.
Eg. If remainder of a number when divided by 21 is 5, then the remainder of that same number when divided by 3 (which is a factor of 21) will be remainder of 5/3, which is 2.

4) Cycle of powers : This is used to find the remainder of , when divided by 10, as it helps us in figuring out the last digit of .

The cycle of powers for numbers from 2 to 10 is given below:

2: 2, 4, 8, 6 → all will have the same last digit.

3: 3, 9, 7, 1 → all will have the same last digit.

4: 4, 6 → all will have the same last digit.

5: 5 → all will have the same last digit.

6: 6 → all will have the same last digit.

7: 7, 9, 3, 1 → all will have the same last digit.

8: 8, 4, 2, 6 → all will have the same last digit.

9: 9, 1 → all will have the same last digit.

10: 0 → all will have the same last digit.

5) Many seemingly difficult remainder problems can be simplified using the following formula:

Eg.

Eg.

6) , can sometimes be easier calculated if we take it as
Especially when x and y are both just slightly less than n. This can be easier understood with an example:

Eg.

NOTE: Incase the answer comes negative, (if x is less than n but y is greater than n) then we have to simply add the remainder to n.

Eg. Now, since it is negative, we have to add it to 25.

[Note: Go here to practice two good problems where you can use some of these concepts explained : numbers-86325.html]

7) If you take the decimal portion of the resulting number when you divide by "n", and multiply it to "n", you will get the remainder. *[Special thanks to h2polo for this one]*

Note: Converse is also true. If you take the remainder of a number when divided by 'n', and divide it by 'n', it will give us the remainder in decimal format.

Eg.

In this case,

Therefore, the remainder is 3.

This is important to understand for problems like the one below:

If s and t are positive integer such that s/t=64.12, which of the following could be the remainder when s is divided by t?

(A) 2
(B) 4
(C) 8
(D) 20
(E) 45

OA :

OA=E