Cyclicity:

Cyclicity of Digits at Unit's place:

In this chapter, we are going to learn about cyclicity. Cycility as the name suggests means a cycle or a pattern or a period in the unit's place, ten's place, hundred's place and so on of a number.

We will learn this logic but let's start with a very simple example.

Assume a series is given as a, b, c, d, e, a, b, c, d, e, a, b, ...

And we have to find out the 27th term of this series.

On observing the series, we will notice that the terms are repeating after 5th term. So, we can conclude that the series has a cyclicity of 5.

Now, we can find out the 27th term easily. We can divide 27 by 5, we will get 5 cycles complete and 2 terms are remaining. So, we can say that 'b' will be the 27th term.

If we had to find 40th term, then again we can divide 40 by 5(since the cycle is of 5), we will get 8 cycles completed and no terms remaining. We are getting no terms as the remainder that means we have reached the end of the cycle. Thus, we will take the last term of the cycle. So the 40th term of the series will be 'e'.

Same logic is used for the unit's place of a digit. Any operation on a Unit place of a digit is only dependent on the unit digit of that particular number.

For example if we want to find the unit digit of 347×243.

Rather than multiplying the entire number, we can say that the unit place will be just the multiplication of unit digits of the two numbers. So, the answer will be $7 \times 3 = 1$.

The unit digit of successive powers of any number (2^1, 2^2, 2^3, 2^4, 2^5, 2^6,.................... or 7^1, 7^2, 7^3, 7^4, 7^5, 7^6,................... or any digit$^{\text{raised to a power}}$) show a pattern or a cycle.

Let's say, we want to find the cyclicity of unit's place of digit 2.

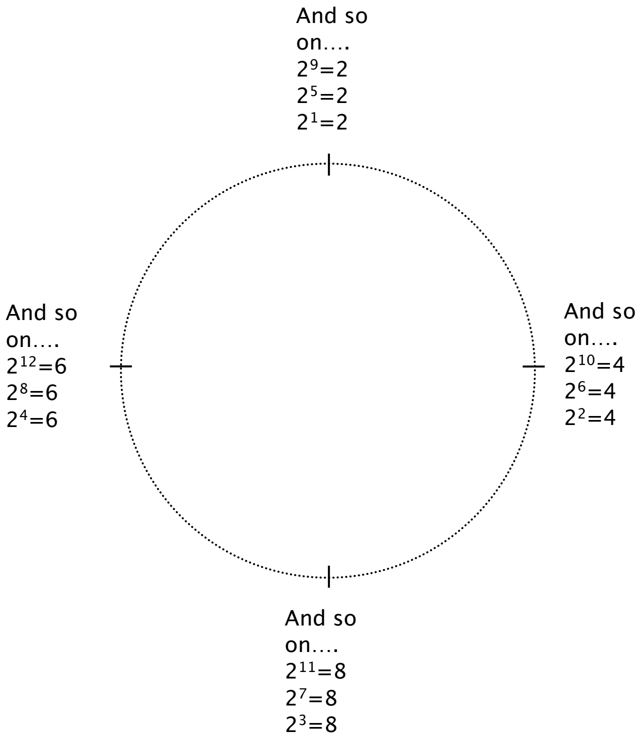
$2^1 = 2$ $\quad$ $2^5 = 2$ (We are just writing the unit' place of $2^5 = 32$.)

$2^2 = 4$ $\quad$ $2^6 = 4$

$2^3 = 8$ $\quad$ $2^7 = 8$

$2^4 = 6$ $2^8 = 6$

The diagram below also depicts the same thing. So, the unit's place of 2^1, 2^5, 2^9, 2^{13},.........and so on will be 2. The unit's place of 2^2, 2^6, 2^{10}, 2^{14},........ and so on will be 4. The unit's place of 2^3, 2^7, 2^{11}, 2^{15},........... and so on will be 8. The unit's place of 2^4, 2^8, 2^{12}, 2^{16},........ and so on will be 6.



We can see that the unit's place of 2 starts repeating after 2^4 in cycles of 2, 4, 8, 6, so we can conclude that cyclicity of digit 2 is 4.

Once we know the cycle or the pattern, we can easily find out any term in that sequence.

E.g. To find the unit's place of 2^{65}.

Unit's place of 2^x will repeat like 2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6,.............................. as x takes values starting from 1, 2, 3, 4, 5, 6, and so on. We will try to reach as close to 65 using the multiple of 4. We can reach uptill 64 and conclude that the 64th term will be the last term of the cycle.(Any multiple of 4 will be the last term of the cycle, any term which is a multiple of 4y + 1 will be the first term of the cycle and so on.) Already we have reached uptill 2^{64}, so the unit place of 2^{65} will be the the first term of the cycle which happens to be 2.

Similarly, the cyclicity of all the digits from 1 to 9 can be found.

We will learn the cycle of other digits in clusters.

Cyclicity of 4 and 9:

The cycle of unit digit of 4^x as x takes values of 1, 2, 3, 4, 5, 6........... is 4, 6, 4, 6, 4, 6,.......

The cycle of unit digit of 9^x as x takes values of 1, 2, 3, 4, 5, 6,.......... is 9, 1, 9, 1, 9, 1,........

In these two cases, rather than memorising that 4 and 9 have a cyclicity of 2, if we observe carefully we will find that whenever the power/index of 4^x is odd, the unit place is 4 and when the power is even, the unit place is 6.

Same thing can also be observed for 9^x.

So, we just have to keep in mind that odd/even funda is associated with the cyclicity of 4 and 9.

Cyclicity of 2, 3, 7, 8:

The cycle of unit digit of 2^x as x takes values of 1, 2, 3, 4, 5, 6........... is 2, 4, 8, 6, 2, 4,.......

The cycle of unit digit of 3^x as x takes values of 1, 2, 3, 4, 5, 6........... is 3, 9, 7, 1, 3, 9,.......

The cycle of unit digit of 7^x as x takes values of 1, 2, 3, 4, 5, 6........... is 7, 9, 3, 1, 7, 9,.......

The cycle of unit digit of 8^x as x takes values of 1, 2, 3, 4, 5, 6........... is 8, 4, 2, 6, 8, 4,.......

Cyclicity of 1, 5, 6:

We don't need to worry about the numbers ending with 1, 5 and 6 at the unit place as 1^x, 5^x, 6^x will always end with 1, 5 and 6 respectively for any values of x.

For e.g: We want to find the unit's place of 6^{10}.

6^1 has unit's place of 6, 6^2 has unit's place of 6, 6^3 has also unit's place of 6, so 6^{anything} will always have a unit digit of 6.

Same logic can be applied for cyclicity of 1 and 5. 1^{anything} results in 1 at the unit's place and 5^{anything} results in 5 at the unit's place.

Summary:

So, if we know the cycle or pattern of all the digits from 1 to 9, cyclicity of all the numbers are covered. If we have to find the unit's place of 1234^x, we will find out the unit's place of 4^x as the unit's place of a number is just dependent on the unit's place.

Also, we don't need to remember the cycles of the digits. We can find out the cycle at that instant by quick calculation in our mind. For example, if we want to construct the cycle of 8^x:

First in the cycle will be 8 itself; next will be the unit's digit of $8 \times 8 = 4$;

Next in the cycle will be the unit's digit of $8 \times 8 \times 8$. We have already found out the unit of 8×8 which is 4. So, $8 \times 8 \times 8$ can be written as $4 \times 8 = 2$.

Next in the cycle will be the unit's digit of $8 \times 8 \times 8 \times 8$. We have already found out unit's place of $8 \times 8 \times 8$ which was 2, so unit's place of $8 \times 8 \times 8 \times 8$ will be $2 \times 8 = 6$.

Examples:

E.g. 1: Find the unit's place of 2^{14}.

We already the cycle of digit 2 is 4, so we divide $\frac{14}{4}$, we get 3 cycles and 2 remainder. And 3 cycles means that we have reached uptill 2^{12} and two remainder means that we will take the unit's place of the 2^{nd} term of the cycle, which is $2^2 = 4$.

E.g. 2: Find the unit's place of 2^{96}.

We will divide the power by 4, we get $\frac{96}{4}$ = 24 cycles complete and 0 remainder. And a remainder of 0 and all cycles means that we have reached the end of the cycle. Whenever the remainder is 0, it means all the cycles are complete; so we will take the unit digit of the last term of the cycle which is $2^4 = 6$.

Let's try out a different example.

E.g. 3: Find the unit's place of 12^{147}.

The unit's place of 12 will be same as that of 2 since the unit place of a number is dependent on the unit's place and not the ten's place.

So, applying the logic which we have learnt, we will divide the power of 12(which is 147) by 4.

$\frac{147}{4}$ =some cycles and 3 remainder. We are not interested in the actual number of cycles, we know that if the remainder is 3; we will take the 3rd term of the cycle from beginning which is 8.($2^3 = 12^3 = 8$)

E.g. 4: Find the unit's place of $142^{49} \times 177^{35} \times 166^{54} \times 244^{37}$.

Practically, this question boils down to finding out the unit's place of $2^{49} \times 7^{35} \times 6^{54} \times 4^{37}$. We know the cycle of digit 2 is 4, so we divide the power 49 by 4, the unit's place of 2^{49} will be that of 2^1. Similarly for 7^{35}, we divide the power 35 by 4, since the digit 7 has a cycle of 4, we get the remainder as 3. So, the unit place of 7^{35} will be same as that of 7^3 which is 3. We know that $6^{\text{any power}}$ results in 6 at the unit's place. Finally, we know for any power of 4, the odd/even funda is related. $4^{\text{odd power}}$ has a unit's place of 4 and $4^{\text{even power}}$ has a unit's place of 6.

So, if we combine all these, we can find the answer comfortably.

Thus, the answer will be $2 \times 3 \times 6 \times 4 = 4$.

All these questions are based on the fundamental logic, so if the logic is well-understood, these questions can be solved orally also.

E.g. 5: Find the unit's place of $29^{39^{45}}$.

We know the unit place of digit 9 has odd/even funda. We just need to know whether the power is odd or even.

In this case, the power is 39^{45}. 39^{45} means that 39 is multiplied 45 times. 39 is an odd number and an odd number multiplied with another odd number any number of times will always be an odd number.

Thus, 39^{45} is an odd number. So, the unit's place of $29^{\text{odd power}}$ will be 9.

E.g. 6: Find the unit's place of $47^{73^{80}}$.

This is a slightly tricky question, but still the same logic will be followed. For all the digits which have a cycle of 4, we divide the power by 4 and find out the remainder. Depending on what is the remainder, we can reach the solution.

In this question we will divide the power 73^{80} by 4.

$\frac{73^{80}}{4}$ will give us a remainder of $\frac{1^{80}}{4}$ which is equal to 1. If remainder is 1, that means we will take the first term of the cycle of 7^x.

So, the unit's place of $47^{73^{80}}$ will be 7^1 which is 7.

Exercise:

1. Find the unit's place of the following:

a. $345^{99}+12^{87}+144^{87}$ b. $34^{101}\times18^{100}\times49^{67}$ c. $3333^{222}\times2222^{333}$

2. Find the unit's place of $49^{36^{45}}$.

3. Find the unit's place of $99^{129^{134}}$.

4. Find the unit's place of $28^{43^{20}}$.

5. Find the digit in the ten's place of 5×2^{50}.

6. Find the digit in the hundred's place of $5^{2}\times2^{100}$.

7. Find the digit in the hundred's place of $5^{2}\times2^{100}\times33^{99}\times77^{88}\times88^{66}$.

8. For how many two digit values of n would the unit's place of 2^{n} be same as 4^{n}? (n is a natural number)

9. For how many two digit values of n would the unit's place of $3^{n}+7^{n}$ end with zero?

10. Find the largest three digit value which n can take if the unit's place of 17^{n} is 9.

11. Find the rightmost non-zero digit of 10!.

12. Find the unit's digit of the expression $\left(66^{234}\times53^{512}\times22^{181}\right)-\left(54^{29}\times24^{35}\times56^{15}\right)$

13. Find the greatest three-digit value which n can take if the unit's place of $2^{n}+8^{n}$ is 0.

14. The digit in the unit place of $\left(7^{3}\right)^{4^{N}}+1$, where N is any natural number is

Answer Key & Explanations:

1. a) 7 b) 6 c) 8

2. 1 3. 9 4. 8 5. 2 6. 4 7. 2

8. 22 9. 45 10. 998 11. 8 12. 6 13. 999

14. 2

Explanations:

2. Unit's place of $49^{36^{45}}$ can be found easily as we know that unit's digit of $9^{\text{any power}}$ involves even/odd power. We just need to know whether the power is odd/even.

36^{45} means 36 is multiplied 45 times and since 36 is an even number; so, 36^{45} will always be an even number.

So, the unit's place of $49^{36^{45}} = 9^{\text{even power}} = 1$

3. Same logic as used in the previous question. Only difference is that in this question the power 129^{134} is odd. So, the unit's place of $99^{129^{134}} = 9^{\text{odd power}} = 9$.

4. Unit's place of $28^{43^{20}}$ can be found out by dividing the power by 4, since a number ending with 8 at the unit digit's place has a cycle of 4. And when dividing the power by 4, our objective should be to find the remainder which will help us in identifying which term of the cycle to take.

$\frac{43^{20}}{4} = \frac{3^{20}}{4} = \frac{9^{10}}{4} = \frac{1^{10}}{4} = 1$ is the remainder which means that we will take the first term of the cycle which is 8^1 i.e. the unit's place will be 8.

5. For finding out the ten's place of 5×2^{50}, we need to know a simple logic.

Concept of Zero at Unit's place or ten's place and so on:

If a particular number is multiplied with another number having one zero, then the unit's place will be occupied by zero and the digit at the ten's place will be the unit's place of the remaining number. For example: If we want to find the digit which will occupy the ten's place in the multiplication of 979 × 130; We can rewrite this question as 979 × 13 × 10.

We know that 0 will be shifted to unit's place and the ten's place will be occupied by the unit's place of 979 × 13 which is 7.

Similarly, if a number is multiplied with another number having two zeros, then the unit's and ten's place will be occupied by two zeros and the digit at the hundred's place will be the unit's place of the remaining number.

For example: Find the hundred's place of $864 \times 230 \times 380$.

We can rewrite it as $864 \times 23 \times 38 \times 100$.

Last two places i.e. unit's and ten's place will be occupied by zeros and we have to find the digit at hundred's place.

Digit which will come at hundred's place will be the unit's place of $864 \times 23 \times 38$ and that will be 6.

So, coming back to the main question of finding the ten's place of 5×2^{50} ,we should be anticipating that there must be one zero which will occupy the unit's place and ten's place will depend on the remaining number. If there is one 5 and one 2, one zero will be formed and if we multiply any number with a number ending with a zero, zero will always be at the unit's place.

So, 5×2^{50} can be written as = $(5^1\times2^1)\times2^{49}$. One 5 and one 2 will make 0 at the unit's place and ten's place will be the unit's place of 2^{49} which is $2^1 = 2$ ($\frac{49}{4}$ = remainder of 1, since cyclicity of digit 2 is 4).

6. To find the hundred's place of $5^2\times2^{100}$, we will separate two 2's and two 5's because they will make two zeros which will be placed at the unit's place and ten's place.

So, it can be simplified and written as $(5^2\times2^2)\times2^{98}$. So, the digit at the hundred's place will be the unit's place of 2^{98} which is 4(as $\frac{98}{4}=$ remainder of 2).

Even after all this, there is some confusion, then think of real life examples.

For e.g., If there are 6 males and 4 females, then how many couples can be formed? Obviously, the answer will be 4 couples as two females are less.

Similarly, if there 5 vadas and 10 pavs, then how many vada pavs can be made? Again the answer will depend on the variable which has lesser number. 5 vada pavs can be formed.

Now, relate it with the power of 2 and 5.

$(2^1\times5^1)=1\times(2^1\times5^1)=$ 10. One zero will be formed because of presence of one 2 and one 5. Zero will occupy the unit's place and then remaining is 1 which will go at ten's place.

$\left(2^2 \times 5^1\right) = 2^1 \times \left(2^1 \times 5^1\right) =$ 20. Again only one zero will be formed as even if two 2's are present, only one 5 is there, so one zero will be formed and then remaining number is 2^1 and that will go at ten's place.

$\left(2^2 \times 5^2\right) = 1 \times \left(2^2 \times 5^2\right) =$ 100. Two zeros will be formed as two 5's and two 2's are present. These two zeros will occupy the unit's and ten's place. Remaining number is 1 and that will go at hundred's place.

$\left(2^3 \times 5^1\right) = 2^2 \times \left(2^1 \times 5^1\right) =$ 40. Only one zero will be formed as only one 5 is present is there and it will combine with one of 2's to form a zero at the unit's place. Left number is 2^2, so the digit at the ten's place will be the unit's place of 2^2 which is 4.

So, we can find out any such pattern. We just have to keep in mind that if there are 2's and 5's, they will multiply with each other and form zeros and those zeros will occupy unit's place, ten's place, hundred's place and so on. First non-zero digit will depend on the number which is left after excluding the power of 2 and 5 making zeroes.

7. We have to find hundred's place of $5^2 \times 2^{100} \times 33^{99} \times 77^{88} \times 88^{66}$.

All of us would have got an idea that there are two 5's and it will combine with two 2's to form two zeros which will occupy the unit's and ten's place. That's why the question is asking to find the hundred's place.

Hundred's place will be the unit's place of $\left(2^{98} \times 33^{99} \times 77^{88} \times 88^{66}\right)$ $as(2^2 \times 5^2)$ has been excluded forming two zeros. We can easily find out the unit's place of $\left(2^{98} \times 33^{99} \times 77^{88} \times 88^{66}\right)$.

Unit's place of the number in the bracket will be 4 × 7 × 1 × 4 = 2

So, the answer is 2.

8. For solving this question, just write the powers of 2 and 4, we will get the answer.

$2^1 = 2$ $4^1 = 4$

$2^2 = 4$ $4^2 = 6$

$2^3 = 8$ $4^3 = 4$

$2^4 = 6$ $4^4 = 6$

By observing the sequence, we can say that the unit's place of 2^4 and 4^4 is same. We don't need to go further and write the power. Again, when the power of 2 and 4 will be 8, their unit's place will be same.

We can conclude that whenever n(the power) is a multiple of 4, the unit's place of 2^n will be equal to 4^n. But, asking for the two digit values of n. So, our series will start from 12, 16, 20, 24, 28,............,92, 96.

Here another problem arises. How to count the terms in this sequence?

Whenever there is a difference between terms (in this case the common difference is 4) which is constant, we can find out the no. of terms efficiently by using a simple formula.

No. of terms = $\frac{Firstterm - Lastterm}{commondifference} + 1$.

So, no. of terms = $\frac{96-12}{4} + 1 = 21 + 1 = 22$.

So, n can take 22 values.

9. To find out for which value of n, unit's place of $3^n + 7^n$ will be zero; we just need to write the powers of 3 and 7 uptill 4.

$3^1 = 3$ $7^1 = 7$

$3^2 = 9$ $7^2 = 9$

$3^3 = 7$ $7^3 = 3$

$3^4 = 1$ $7^4 = 1$

By observing the sequence, we can say that whenever the power is odd, the sum of $3^n + 7^n$ will end in a 0 at the unit's place.

So, n can take all the odd values starting from 11, 13, 15,............., 97, 99.(since asking for the two digit values of n).

We just learnt in the previous question how to find number of terms in a sequence.

So, n can take = $\frac{99-11}{2} + 1 = 44 + 1 = 45$ values

10. To find the largest three-digit value of n, for which the unit's place of 17^n is 9. We just need to write the first 4 powers of 7.

$7^1 = 7$ $7^2 = 9$ $7^3 = 3$ $7^4 = 1$

We know that the cycle is of 4, so if 7^2 has unit's place of 9. Then 7^6, 7^{10} and so on will also end with 9. Since, we have to find out largest three-digit value of n, n can take all the even values starting from 100, 102, 104, 106,............................, 996, 998.

Answer is n = 998.

11. We have to find the right-most non-zero digit of 10!. We will solve it, but first look at a example. If we want to know the right-most non-zero digit of 14576000, answer will be 6. We will ignore the number of zeros and look for the first non-zero digit from right which happens to be 6 in this case.

Coming back to the original question of finding right-most non-zero digit of 10!, we know that $10! = 1\times2\times3\times4\times5\times6\times7\times8\times9\times10$.

Two zeros will be formed as one 5 will combine with one 2 and 10 is already there. These two zeros will occupy the unit's and the ten's place. So, the right-most non-zero digit will be the unit's place of the remaining number.

So, we can write it like $(1\times 3 \times 4 \times 6 \times 7 \times 8 \times 9) \times 10 \times 10$ = It will be a large number ending with zeros at the unit's place and ten's place and the right-most non-zero digit will be the unit's place of $(1\times 3 \times 4 \times 6 \times 7 \times 8 \times 9)$ which is 8.

12. To find the unit's digit of the expression $\left(66^{234}\times53^{512}\times22^{181}\right)-\left(54^{29}\times24^{35}\times56^{15}\right)$

Answer can be found out easily. We can find out the unit's place of both the brackets and subtract them.

(a number ending with 2 at the unit's place) – (a number ending with 6 at the unit's place).

Answer should be 6. But many of us make mistake here of writing the answer as – 6. The number in the first bracket is the greater as compared to the number in the second bracket.

13. We have to find the greatest three-digit value of 'n' such that the unit's place of $2^n + 8^n = 0$.

Exactly the same logic as $3^n + 7^n$ = resulting in a 0 at the unit's place.

When the power is odd, the sum will result into a zero at the unit's place.

$2^1 = 2$	$8^1 = 8$
$2^2 = 4$	$8^2 = 4$
$2^3 = 8$	$8^3 = 2$
$2^4 = 6$	$8^4 = 6$

It can be observed that the whenever the power is odd, the sum of $2^n + 8^n = 0$.

So, the answer is n = 999.

14. The digit in the unit place of $\left(7^3\right)^{4N}+1$, (where N is any natural number) can be calculated easily. We know the cyclicity of digit 7 is of 4, and if the power is a multiple of 4, then dividing that power by 4 will result in a remainder of zero.

Zero remainder means that we have reached the end of the cycle and the 4th term in that cycle will be taken as the unit's digit.

So, here $\left(7^3\right)^{4N}$: The power 12N when divided by 4 leaves a remainder of zero, the unit's place of $\left(7^3\right)^{4N}$ will be same as 7^4 which is 1.

Answer is 1 + 1 = 2