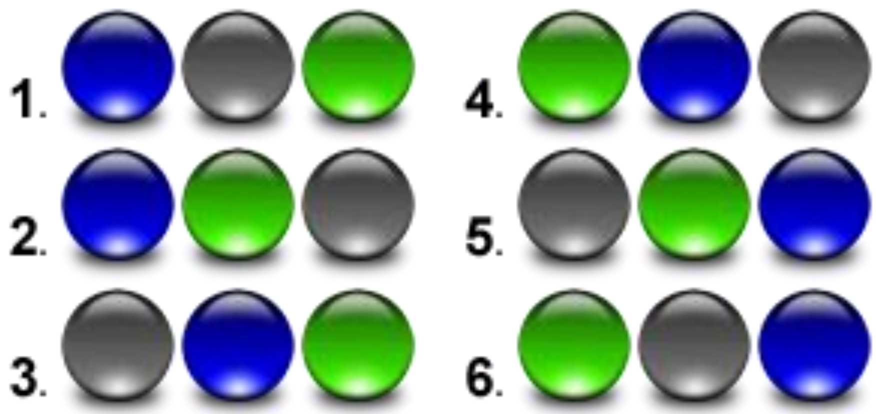
Enumeration

- There are three marbles: 1 blue, 1 gray and 1 green. In how many ways is it possible to arrange marbles in a row?

Enumeration | Answers

- **Solution:** Let's write out all possible ways
- Total: 6



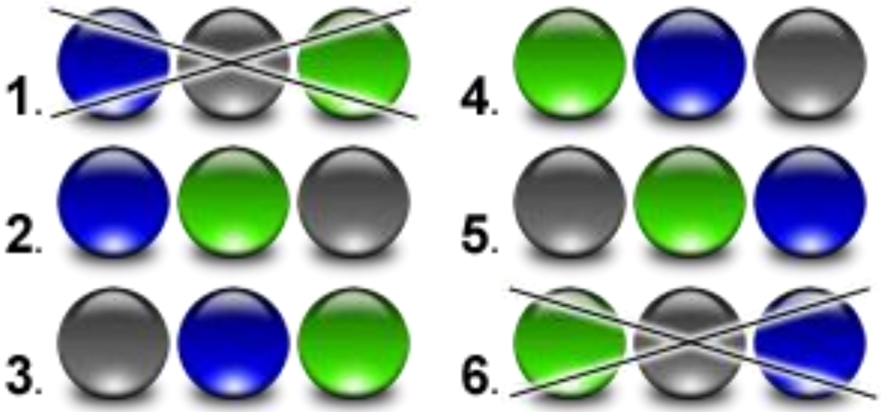
- Enumeration is a method of counting all possible ways to arrange elements. Although it is the simplest method, it is often the fastest method to solve hard GMAT problems and is a pivotal principle for any other combinatorial method. In fact, combination and permutation is shortcuts for enumeration. The main idea of enumeration is writing down all possible ways and then count them.

Enumeration 2

- There are three marbles: 1 blue, 1 gray and 1 green. In how many ways is it possible to arrange marbles in a row if blue and green marbles have to be next to each other?

Enumeration 2 | Answers

- **Solution:** Let's write out all possible ways to arrange marbles in a raw and then find only arrangements that satisfy question's condition:



- Answer: 4

Enumeration 3

- In how many ways can 5 dresses be arranged in a store display?

Enumeration 3 | Answers

- 1. How many objects we can put at 1st place? **5.**
- 2. How many objects we can put at 2nd place? **4** and so on: 3, 2, 1
  Therefore, the total number of arrangements of n different objects in a row is
  $$N = n \times (n-1) \times (n-2) \ldots 2 * 1 = n!$$
- 5! = 5*4*3*2*1 = 20 * 6 = 120
- N! is called a factorial. Factorial equals to the product of numbers from N to 1.

Enumeration 4 (Ultra Hard)

- If N is a positive integer, what is the last digit of *1! + 2! + …. N! ?*
  - *N is divisible by 4*
  - $\frac{N^2+1}{5}$ *is an odd integer*

Enumeration 4 | Answers

- This is a very hard question that requires a non-traditional approach (as some of the hardest official GMAT questions often do)
- Analyzing factorials, you will notice that the sum of factorials will have 3 as the last digit if $N>3$, (starting with $5!$, each sum ends with a zero since $5!=120$, $6!=720$, and so on.)
- S1 is sufficient since we know $N>3$ and thus we can say with certainty that last digit equals to 3
- S2 tells us that N is not 1 or 3 and is either 2 or greater than 3. In either case, the last digit will be 3
- **The correct answer is D**

Combinations 1

- What is the number of possible arrangements of objects k from a collection of distinct objects n?

Combinations 1 | Answers

- A combination is an unordered collection of k objects taken from a set of n distinct objects. The number of ways how we can choose k objects out of n distinct objects is denoted as: C_k^n
- Total number of arrangements of *n* distinct objects is *n!*
- Now we have to exclude all arrangements of k objects *(k!)* and remaining *(n-k)* objects *((n-k)!)* as the order of chosen *k* objects and remained *(n-k)* objects doesn't matter.
- $C_k^n = \frac{n!}{k!(n-k)!}$

Combinations 2

- What is the number of possible arrangements of objects k in a certain order from a collection of distinct objects n?

Combinations 2 | Answers

- A permutation is an <u>ordered</u> collection of *k* objects taken from a set of *n* distinct objects. The number of ways how we can choose *k* objects out of *n* distinct objects is denoted as: P_k^n
  1. The total number of arrangements of n distinct objects is *n!*
  2. Now we have to exclude all arrangements of remaining *(n-k)! Objects*
- $P_k^n = \dfrac{n!}{(n-k)!}$

Combinations 3

- What is the difference between combinations and permutations?
- When to use which formula?

Combinations 3 | Answers

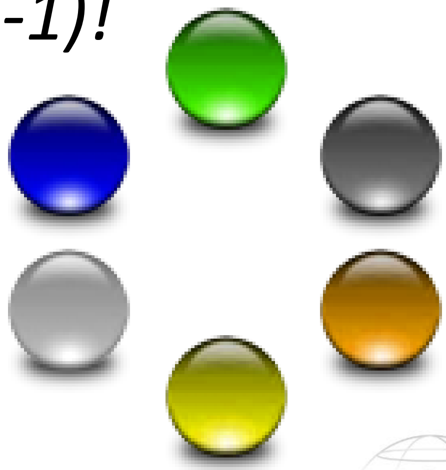
- Permutations formula $P = \frac{n!}{(n-k)!}$ is used when sequence of choice matters (meaning a group ABC is different from BAC or CBA). Classic example is choosing nominees for 3 specific positions from a pool of 10 candidates
- Combinations formula C = $\frac{n!}{k!(n-k)!}$ is used when order of selection has no impact and once a small group is formed, it does not matter how they arrived there. Classic example is picking 3 marbles from a bag of 10

Combinations 4

- If six business partners are having a dinner at a round table, how many seating arrangements are possible?

Combinations 4 | Answers

- The difference between placement in a row and that in a circle is following: if we shift all object by one position, we will get different arrangement in a row but the same relative arrangement in a circle. So, for the number of circular arrangements of *n* objects, instead of *n!*, we have *(n-1)!*
- *Thus, the answer is 5! or 120*



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Combinations 5

- If there are 5 chairs in a room and Bob and Rachel want to sit so that Bob is always left of Rachel, in how many ways this seating arrangement be achieved?

Combinations 5 | Answers

- Note that left of Rachel, does not mean immediately next to Rachel, just left of her.
- This condition is called symmetry because it eliminates half of the possibilities (Rachel can sit only left or right of Bob).
- Therefore, the number of ways that Bob is left of Rachel is exactly $\frac{1}{2}$ of all possible ways or
- $N = \frac{1}{2} \times P_5^2 = 10$

Combinations 6 (Ultra Hard)

- In how many different ways can a group of 8 people be divided into 4 teams of 2 people each?
  - 90
  - 105
  - 168
  - 420
  - 2520

Combinations 6 | Answers

- The solution to this problem is the number of combinations. First we get one team out of 8 . The number of ways to do this would be C_8^2. The next combination is 2 out of 6 or C_4^2, and so on. Having all four combinations multiplied, we need to divide the total number by the number of ways the teams can be chosen , since we are not interested if the team with two certain people is chosen first, second or third. Therefore, the answer is found by the following formula: $\frac{C_8^2 \times C_6^2 \times C_4^2 \times C_2^2}{4!} = 105$
- The correct answer is B.

Probability 1

- What is the probability that an event *n* will occur?
- What is the probability that an event *n* will <u>not</u> occur?

Probability 1 | Answers

- The probability that an even *n* will take place is $\frac{n}{N}$ where *N* is the total number of possible occurrences

- The probability that an even *n* will not occur is the opposite of it occurring, so $1-\frac{n}{N}$ or *1- p*

Probability 2

- What is the probability of getting *Tails* when flipping a coin?

Head Tail

- What is the probability of getting a 4 when rolling a die?

Probability 2 | Answers

- The probability of getting Tails when flipping a coin is $\frac{1}{2}$ or 50% since there are 2 total possibilities and only one outcome each time the coin is flipped

- The probability of getting a 4 when casting a die is $\frac{1}{6}$; there are a total of 6 potential possibilities (1,2,3,4,5,6) and only one chance to roll one of them.

Probability 3

- A bucket contains 10 green and 90 white marbles. If Adam randomly chooses a marble, what is the probability that it will be green?

Probability 3 | Answers

- The number of green marbles: n = 10
- The number of all marbles: N = 10 + 90 =100
- Probability: $\frac{10}{100} = \frac{1}{10} = 10\%$
  There is one important concept in problems with marbles/cards/balls. When the first marble is removed from a jar and not replaced, the probability for the second marble differs ($\frac{10}{100}$ vs. $\frac{10}{99}$). Whereas in case of a coin or dice the probabilities are always the same ($\frac{1}{6}$ and $\frac{1}{2}$). Usually, a problem explicitly states: it is a problem <u>with replacement or without replacement</u>.

Probability 4

- If there is a coin and a die, what is the probability of getting heads and a "4“ after one flip and one toss?

Probability 4 | Answers

- Tossing a coin and rolling a die are <u>independent events</u> (occurrence of one event does not influence occurrence of other events). For n independent events the probability is the product of all probabilities of independent event.
- So, the probability of getting heads is $\frac{1}{2}$ and probability of getting a "4" is $\frac{1}{6}$. Therefore, the probability of getting heads and a "4" is: $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

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Probability 5

- If there is a 20% chance of rain on an average day, what is the probability that it will rain on the first day and will be sunny on the second?

Probability 5 | Answers

- The probability of rain is 0.2; therefore probability of sunshine is q = 1 - 0.2 = 0.8. This yields that the probability of rain on the first day and sunshine on the second day is:
  $P = 0.2 * 0.8 = 0.16$
- **Note:** when working with percents, it is important to convert them into a decimal format (such as 0.2 for 20% or a fraction format such as $\frac{1}{5}$ for 20%)

Probability 6

- There are two sets of cards with numbers: {1,3,6,7,8} and {3,5,2}. If Robert chooses randomly one card from the first set and one card from the second set, what is the probability of getting two odd numbers?

Probability 6 | Answers

- There is a total of 5 cards in the first set and 3 of them are odd: {1, 3, 7}. Therefore, the probability of getting odd card out of the first set is $\frac{3}{5}$.
- There are 3 cards in the second set and 2 of them are odd: {3, 5}. Therefore, the probability of getting an odd card out of the second set is $\frac{2}{3}$. Finally, the probability of getting two odd integers is: $\frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$ or 40%

Probability 7

- If Jessica rolls a die, what is the probability of getting at least a "3"?

Probability 7 | Answers

- Two events are mutually exclusive if they cannot occur at the same time. For *n* mutually exclusive events the probability is the sum of all probabilities of events:
  P(A or B) = P(A) + P(B)
- There are 4 outcomes that satisfy our condition (to roll at least 3): {3, 4, 5, 6}. The probability of each outcome is $\frac{1}{6}$.
- The probability of getting at least a "3" is:
  $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

Probability 8

- There are 8 employees including Bob and Rachel. If 2 employees are to be randomly chosen to form a committee, what is the probability that the committee includes both Bob and Rachel?

Probability 8 | Answers

- **Combinatorial approach**:
  - The total number of possible committees is C_2^8=28
  - The number of possible committee that includes both Bob and Rachel is 1
  - P = $\frac{1}{28}$
- **Probability approach**:
  - The probability of choosing Bob or Rachel as a first person in committee is $\frac{2}{8}$. The probability of choosing Rachel or Bob as a second person when first person is already chosen is $\frac{1}{7}$. The probability that the committee includes both Bob and Rachel is $\frac{1}{28}$

Probability 8 – Part 2

- There are 8 employees including Bob and Rachel. If 2 employees are to be randomly chosen to form a committee, what is the probability that the committee includes both Bob and Rachel?

Probability 8 – Part 2 | Answers

- **Reverse Combinatorial Approach**:
  - Instead of counting probability of occurrence of certain event, sometimes it is better to calculate the probability of the opposite and then use formula *p = 1 - q.*
  - The total number of possible committees is C_2^8=28
  - The number of possible committee that does not includes both Bob and Rachel is: $m = C_2^6 + 2 \times C_1^6$ where, C_2^6 is the number of committees formed from 6 remaining people
    $2 \times C_1^6$ is the number of committees formed from Rob or Rachel and one out of 6 other people
  - $P = 1 - \frac{m}{N} = 1 - \frac{C_2^6+2\times C_1^6}{C_2^8} = 1 - \frac{15+2\times 6}{28} = 1 - \frac{27}{28} = \frac{1}{28}$

Probability 8 – Part 3

- There are 8 employees including Bob and Rachel. If 2 employees are to be randomly chosen to form a committee, what is the probability that the committee includes both Bob and Rachel?

Probability 8 – Part 3 | Answers

- **Reverse probability approach**:
  - We can choose any first person.
  - Then, if we have Rachel or Bob as the first choice, we can choose any other person out of the 6 remaining people.
  - If we have neither Rachel nor Bob as first choice, we can choose any person out of the remaining 7 people.
  - The probability that the committee includes both Bob and Rachel is: $P = 1 - \left(\frac{2}{8} \times \frac{6}{7} + \frac{6}{8} \times 1\right) = 1 - \frac{27}{28} = \frac{1}{28}$

Probability 9

- Julia and Brain play a game in which Julia takes a ball and if it is green, she wins. If the first ball is not green, she takes the second ball (without replacing first) and she wins if the two balls are white or if the first ball is gray and the second ball is white. What is the probability of Julia winning if the jar contains 1 gray, 2 white and 4 green balls?

Probability 9 | Answers

- Sometimes, at 700+ level you may see complex probability problems that include conditions or restrictions. For such problems it could be helpful to draw a probability tree that include all possible outcomes and their probabilities.
- Now, It is pretty obvious:
- $P = \frac{4}{7} + \frac{2}{7} \times \frac{1}{6} + \frac{1}{7} \times \frac{2}{6} = \frac{2}{3}$