Triangles

□ Sum of angles of ANY triangle equals ?

□ What can we say about sides of a triangle?

□ What is the right triangle?

Triangles 2

□ List all methods for finding an area of a triangle.

Triangles | Answers

□ Sum of all angles in any triangle is always 180°

□ One side is always smaller than the sum of the other two and is always greater than the difference of the other two

□ A right triangle is the one that has a 90 degree angle (it has the right angle). A triangle can only have one angle at 90 egrees since sum of the 3 angles is 180

Triangles 2 | Answers

□ 1. Area = $\frac{1}{2} \ base \ \times height$

□ 2. Hero's formula: $\sqrt{s(s-a)(s-b)(s-c)}$ where a,b,c are sides of a triangle and **s** is semi-perimeter $s = \frac{a+b+c}{2}$

□ 3. If you know 2 sides of a triangle but not its height, you can add an equally sized triangle to create a square/rectangle/rhombus and find its area (may be easier). Remember to divide your result by 2.

Triangles 3

☐ These are 2 sides of a right triangle, find the third side:
  ☐ 3, 4, x
  ☐ 6, 8, x
  ☐ 5, 12, x
  ☐ 12, 16, x
  ☐ 7, 24, x

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Triangles 4

☐ What is the relationship between sides in a right isosceles triangle?
☐ What is the relationship between angles?
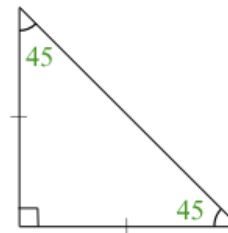
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Triangles 3 | Answers

☐ GMAT relies on these easy triangles. If you memorize these combinations, it will save you time on the Geometry section
  ☐ 3, 4, 5
  ☐ 6, 8, 10
  ☐ 5, 12, 13
  ☐ 12, 16, 20
  ☐ 7, 24, 25

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Triangles 4 | Answers

☐ A right isosceles triangle will have angles that are 90, 45, 45 degrees
☐ It will have sides that are x, x, and the hypotenuse of $x\sqrt{2}$

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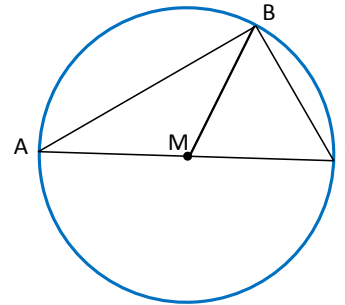
Triangles 5

☐ Name as many properties, relationships, and formulas you know about and equilateral triangle

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Triangles 6

☐ What can you derive from this figure? (M is the center of a circle)


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Triangles 5 | Answers

☐ All sides are equal

☐ All angles are equal

☐ Area = $a^2 \times \frac{\sqrt{3}}{4}$ where as is a side of a triangle

☐ A height is = $a\frac{3}{2}$

☐ For a given area, an equilateral triangle has the smallest perimeter – it is the most "efficient" triangle

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Triangles 6 | Answers

☐ AC is the diameter of a circle

☐ $R = \frac{AC}{2}$ where R is radius

☐ AM = MC = MB

☐ Angle ABC is a right angle

☐ If one of the sides on an inscribed triangle is the circle's diameter, then this is a right triangle regardless of the position of point B
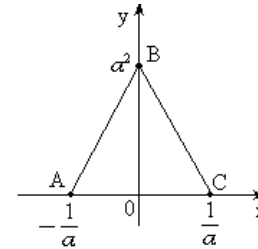
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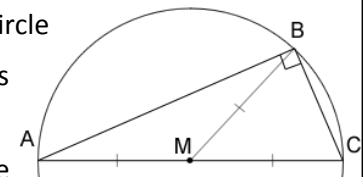
Triangles 7

□ What is the value of sides in a 30-60-90 triangle?
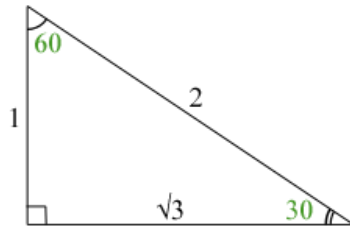

Triangles 8 (Ultra Hard)

□ Is the area of the triangle ABC less than 1?

   ◼ ABC < 90 degrees

   ◼ Perimeter of triangle ABC is greater than $\frac{a}{4}$

Triangles 7 | Answers

□ In 30-60-90 triangle, the sides are x, $x\sqrt{3}$, and the hypotenuse is 2x (double the size of the smallest side)

Triangles 8 | Answers

□ The area of the triangle ABC is $\frac{1}{2}\left(\frac{1}{a} + \frac{1}{a}\right) a^2 = a$

□ Statement (1) by itself is sufficient. In the extreme case when Angle ABC is right, the triangle BOC is isosceles and thus $a^2 = \frac{1}{a}$ and the area of the triangle ABC is a = 1. If angle ABC is smaller than 90 degrees, then the area exceeds 1 due to the increase of the height a^2.

□ Statement (2) by itself is insufficient. As long as a>0, the perimeter of the triangle ABC is always greater than $\frac{4}{a}$

□ The correct answer is A.

Circles

Please define the following:

- Center - ?
- Radius - ?
- Diameter - ?
- Circumference - ?
- Area - ?
- Chord - ?
- Tangent - ?
- Secant - ?

MATH: GEOMETRY

Circles 2

- Area of a circle = ?
- Length of a circle = ?
- $\pi = $?

MATH: GEOMETRY

Circles | Answers

- Center – A point inside the circle. All points on the circle are equidistant from the center
- Radius – distance between the center and any point on the circle. It is half the diameter
- Diameter – a chord passing through the center
- Circumference – distance around the circle
- Area – a region enclosed by the circle
- Chord – a line segment linking any two points on a circle
- Tangent – line touching the circle at one point only; tangent lines are always at 90 degrees to the radius
- Secant – a line that intersects a circle in 2 points

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Circles 2 | Answers

- Area of a circle = πR^2
- Length of a circle = $2\pi R$
- $\pi = 3.14 \approx 3 \approx \frac{22}{7}$

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Coordinate Geometry

□ What is the equation of the slope of a line?

Coordinate Geometry 2

□ If a line has a negative slope less than 1 – what does it say about the line?

Coordinate Geometry | Answers

□ Slope of a line equation:

$$\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Where x and y are coordinates of point 1and point 2 on that line.
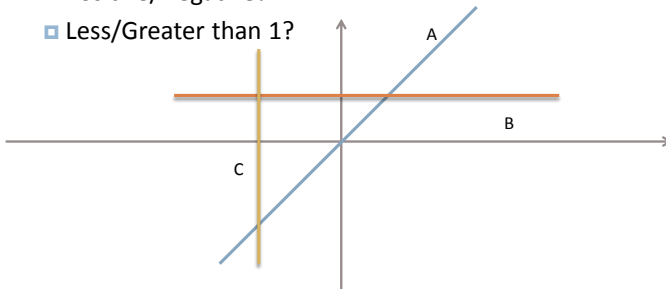
Coordinate Geometry 2 | Answers

□ Negative slope – means line moves from the upper left hand quarant (Q2) to the bottom right hand quadrant (Q4) or in simple terms, it is a decreasing line. Positive slope means the opposite (duh)

□ Since the slope is less than 1, it is a flat line (as opposed to steep). Since slope is rise over run, in this case, there is less rise than run

Coordinate Geometry 3

- What slopes do Lines A, B, and C have?
  - Positive/Negative?
  - Less/Greater than 1?

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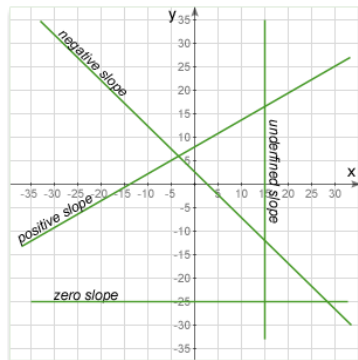## Coordinate Geometry 4

- How to find the X and Y intercepts of a line?

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Coordinate Geometry 3 | Answers

- **Line A**
  - Positive Slope
  - Slope greater than 1
- **Line B**
  - Slope is neither positive or negative
  - Slope is Zero
- **Line C**
  - Slope is undefined

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Coordinate Geometry 4 | Answers

- Best option is to plug in the values into the equation of the line
- For example, a line is $y = mx + b$
- To find the Y intercept (this is when the line crosses the Y axis and thus X is zero) solve: y = b
- To find X intercept (this is when the line crosses the X axis and Y is zero) solve: $0 = mx + b$
- The trick is to use Y = 0 when looking for X intercept and X = 0 when looking for Y intercept

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Coordinate Geometry 5

□ If line *M* with a slope of $\frac{5}{9}$ goes through points A(-5, -2) and B(4, 3), what is the length of the segment AB?
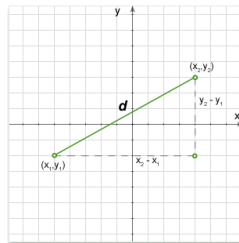
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Coordinate Geometry 6

□ Find the equation of a line passing through the points **A (5,4)** and **B (2,3)**

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Coordinate Geometry 5 | Answers

□ The slope information in this irrelevant
□ To find distance between A and B is calculated using the Pythagorean Theorem by drawing a triangle
□ $9^2 + 5^2 = d^2$
□ $\sqrt{81 + 25} = d$
□ $d = \sqrt{106}$

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Coordinate Geometry 6 | Answers

□ **A (5,4)** and **B (6,3)**
□ To find an equation of a line based on two points, use this formula: $\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$
□ $\frac{y - 4}{x - 5} = \frac{4 - 3}{5 - 6}; \frac{y - 4}{x - 5} = \frac{1}{-1}$
□ $-y + 4 = x - 5$
□ $y = -x + 9$
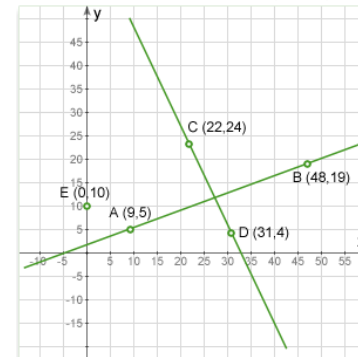
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Coordinate Geometry 7

- □ If lines A and B are perpendicular to each other, what is the relationship between their slopes?
  - A. Inverse
  - B. Opposite
  - C. Positive
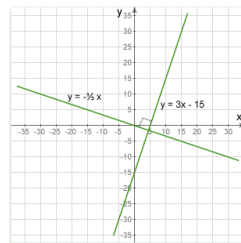 - D. Reciprocal
  - E. Reciprocal and Negative

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Coordinate Geometry 8

- □ Are the two lines below perpendicular?

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Coordinate Geometry 7 | Answers

- □ The relationship between slopes of 2 perpendicular lines is negative reciprocal $-\frac{1}{m}$. In other words, the two lines are perpendicular if and only if the product of their slopes is -1.

- □ E.g. $3 \times -\frac{1}{3}$ =-1

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Coordinate Geometry 8 | Answers

- □ To answer, find the slope of each line and then check to see if one slope is the negative reciprocal of the other or if their product equals to -1.
- □ Slope AB = $\frac{5-19}{9-48} = \frac{-14}{-39} = \frac{14}{39}$
- □ Slope CD = $\frac{24-4}{22-31} = \frac{20}{-9} = -\frac{20}{9}$
- □ Multiply the slopes: $\frac{14}{39} \times \frac{20}{9} \neq -1$;
- □ **Not Perpendicular**

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Coordinate Geometry 9

- What is the point of intersection of two lines that have the following equations: y=3x-3 and y=2.3x+4?

Coordinate Geometry 10 (Hard)

- Does the curve $(x - a)^2 + (y - b)^2 = 16$ intersect the Y axis?
  1) $a^2 + b^2 > 16$
  2) $a = |b| + 5$

Coordinate Geometry 9 | Answers

- The key to solving the intersection questions is that at the point of intersection, both lines will have the same X and Y coordinates.
- Thus, if Y coordinates are the same, then we can put the two equations together: 3x-3 = 2.3x+4
- 0.7x = 7; x = 10
- Now we still need to find the Y intercept. Plug 10 into one of the equations: 3*10-3 = 27
- **Intersection point: (10, 27)**

Coordinate Geometry 10 | Answers

- $(x - a)^2 + (y - b)^2 = 16$ is the equation of a circle centered at with radius 4.
- Statement (1) by itself is insufficient. S1 says that the center of the circle is further than 4 units away from the origin but it doesn't specify whether the circle is far enough from the axis not to intersect it.
- Statement (2) by itself is sufficient. From S2 it follows that and thus the center of the circle is at least 5 units away from the axis. As the radius of the circle is only 4 units, we can conclude that the circle does not intersect the axis.
- **The correct answer is B. Statement 2 is sufficient.**