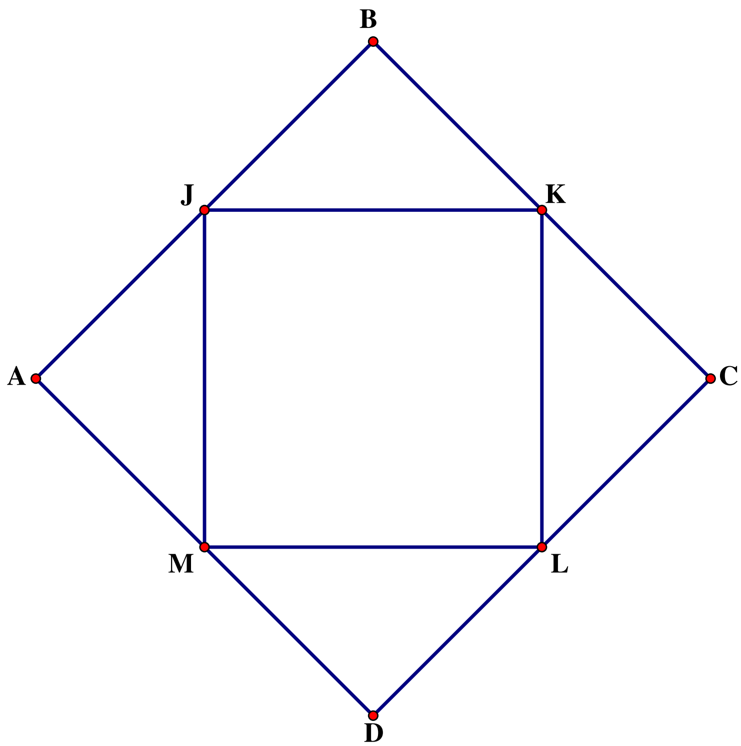
Circles are drawn with four vertices as the center and radius equal to the side of the square. If the square is formed by joining the mid points of another square of side 2*(6^0.5), find the area common to all the four circles.

First, let's consider the squares:



Square ABCD is the original square --- it's midpoints are connected to form square JKLM. This latter square, JKLM is the square with a circle at each vertex --- we'll get to that!

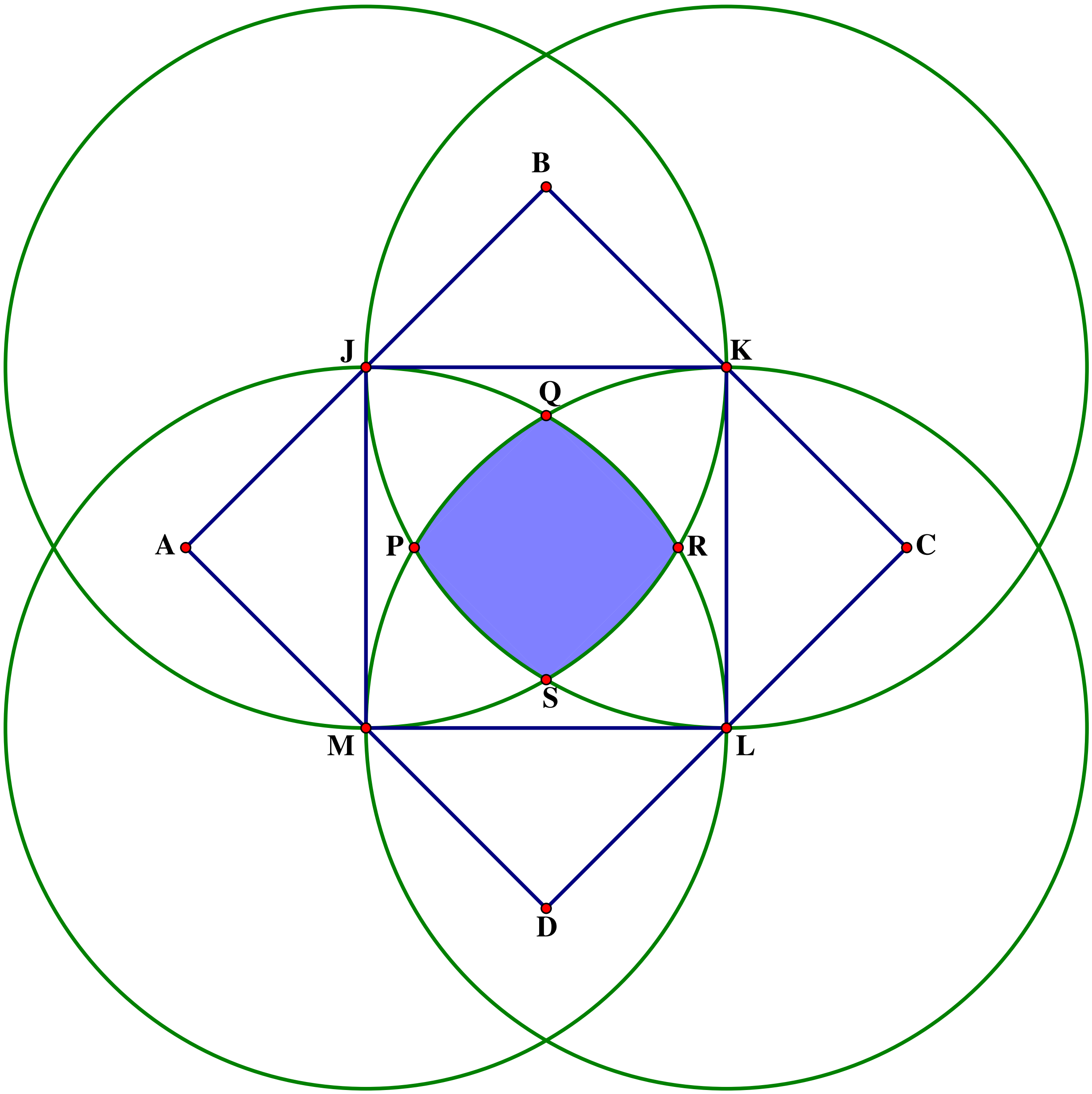
$AB = 2\sqrt{6}$

$AM = AJ = 0.5* AB = \sqrt{6}$

Triangle AJM is a 45-45-90 triangle, so $JM = \sqrt{2}\times\sqrt{6} = \sqrt{12} = 2\sqrt{3}$

Now, we know the length of the side of square JKLM.

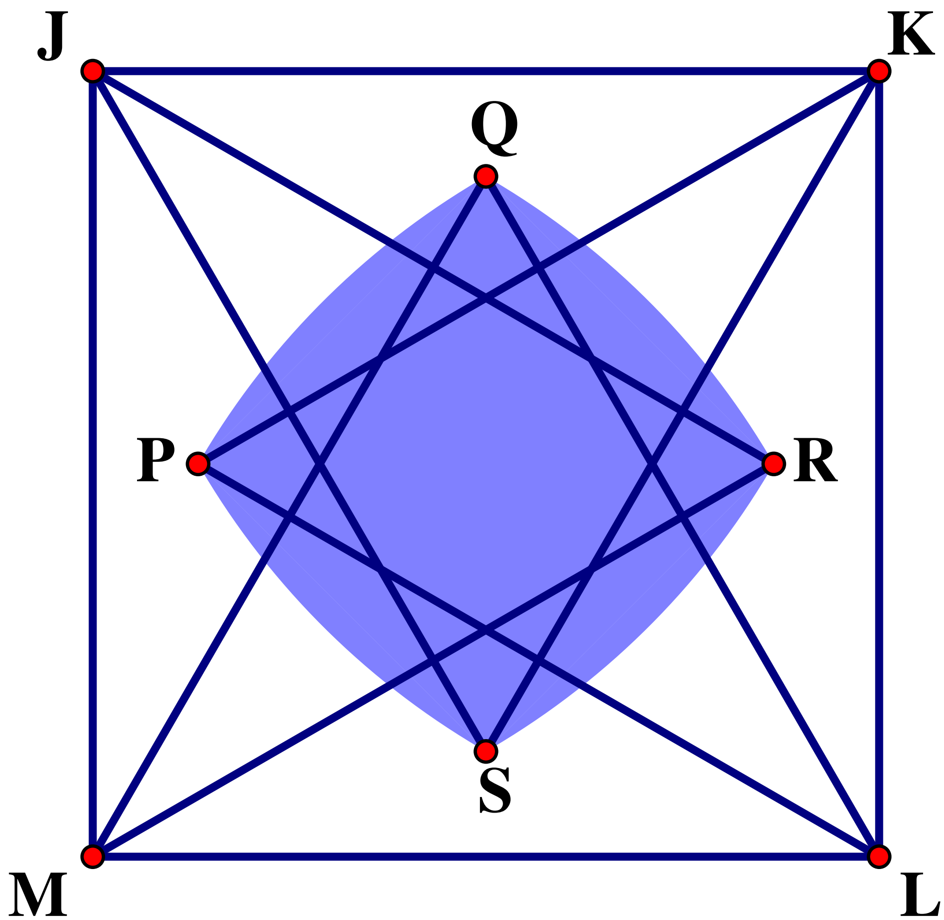
$JK = KL = LM = JM = 2\sqrt{3}$



Here’s the full diagram, and I denoted the area requested, the overlap of all four circles, in blue, region PQRS.

This is going to be a challenge!

First of all, notice just square JKLM and the points with it. Each of points, P & Q & R & S is the vertex of an equilateral triangle with the opposite side of the square as a base. Here are those four equilateral triangles:



We know ML = JK = $2\sqrt{3}$, which means the height, say from ML to Q would have to be 3. This means from Q up to JK is ($2\sqrt{3}$ - 3), which has to equal the distance from S down to ML, so the distance from Q to S is the side of the square ($2\sqrt{3}$) minus two times this extra distance ($2\sqrt{3}$ - 3):

QS = $6 - 2\sqrt{3}$

We are going to have to figure out the area of the central square PQRS, and separately calculate the area of the four circular segments.

The diagonal across square PQRS is QS = $6 - 2\sqrt{3}$. Divide this by $\sqrt{2}$ to get the side of the square:

PS = $3\sqrt{2} - \sqrt{6}$

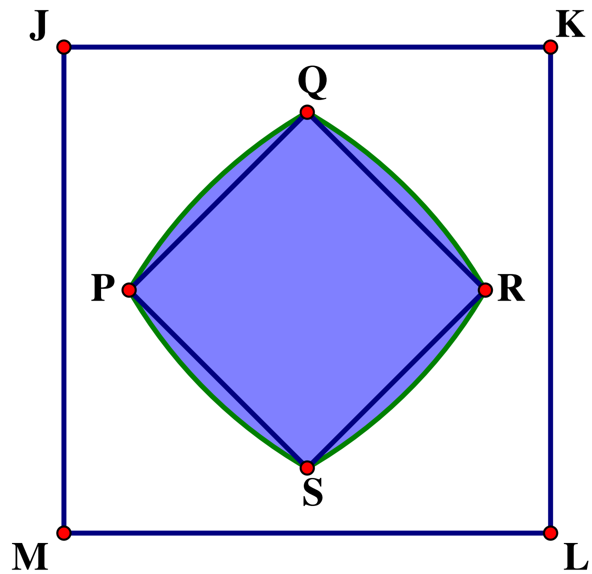
If we square that, we get the area of the central square:

area PQRS = $(3\sqrt{2} - \sqrt{6})^2$

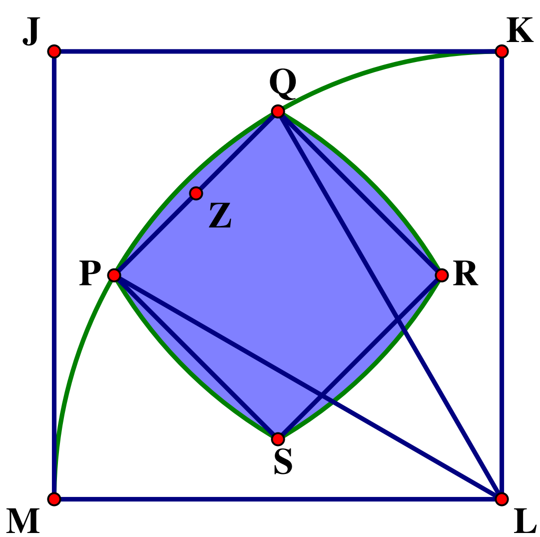
$= 18 - 6\sqrt{12} + 6$

$= 24 - 12\sqrt{3}$

That's part of our answer.



Now, consider the segment from P to Q



Angle PLK = 60°, because it's an angle of equilateral triangle PLK. This means that, if we drew JL, at the 45°, PLJ would be 15°. Similarly, QLJ would be 15°, which means PLQ = 30°. That's a huge piece of information!

The circular segment from P to Q is a segment within a 30° arc. The circle has a radius of $r = 2\sqrt{3}$. The entire circle would have an area of $A = \pi r^2 = 12\pi$

A 30° sector would be 1/12 of the circle, so this entire sector, sector PQL, has an area of π.

Now, we need the area of a triangle PQL, because the area of the circular segment = (area of sector) – (area of triangle).

We know base = PQ = $3\sqrt{2} - \sqrt{6}$. Z is the midpoint of PQ. We will divide PQ by 2, to get the distance ZQ.
We know QL = $2\sqrt{3}$ ---- this is the hypotenuse of right triangle ZQL. We need the third side, LZ, the height of the triangle, to find the area.

$(QL)^2 = (QZ)^2 + (LZ)^2 \quad \rightarrow \quad (LZ)^2 = (QL)^2 - (QZ)^2$

Without showing all the numerical detail, I'll just jump to

$(LZ)^2 = 6 + 3\sqrt{3}$

LZ = $\sqrt{\frac{3}{2}}\left(1 + \sqrt{3}\right)$

Now, the area of triangle PQL

Area = 0.5*bh = $\frac{1}{2}(3\sqrt{2} - \sqrt{6})\left(\sqrt{\frac{3}{2}}\left(1 + \sqrt{3}\right)\right)$

Without showing all the details, this simplifies enormous to Area = 3

This means

(circular segment) = (circular sector) – (triangle) = π - 3

The whole blue overlap area = square PQRS + 4*(circular segment)

$= 24 - 12\sqrt{3} + 4*(\pi - 3)$

$= 4\pi + 12(1 - \sqrt{3})$

$= 4\pi - 12(\sqrt{3} - 1)$

Answer = **D**