Introduction

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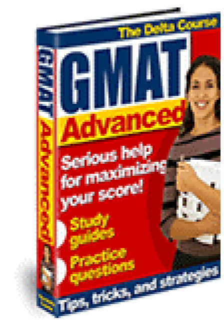

This e-book contains a small excerpt from our popular GMAT study course, GMAT Advanced. GMAT Advanced features a comprehensive review of advanced GMAT topics, including advanced algebra, statistics, probability, and advanced probability.

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The Delta Course

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Counting Methods
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Our GMAT Advanced study course covers the following counting methods topics:

This free study guide will cover the first 5 topics in our counting methods study guide.

Multiplication Principle

You may encounter questions on the GMAT that ask how many different ways that an event can occur. For instance:

- If the first digit cannot be a 0 or a 5, how many five-digit odd numbers are possible?

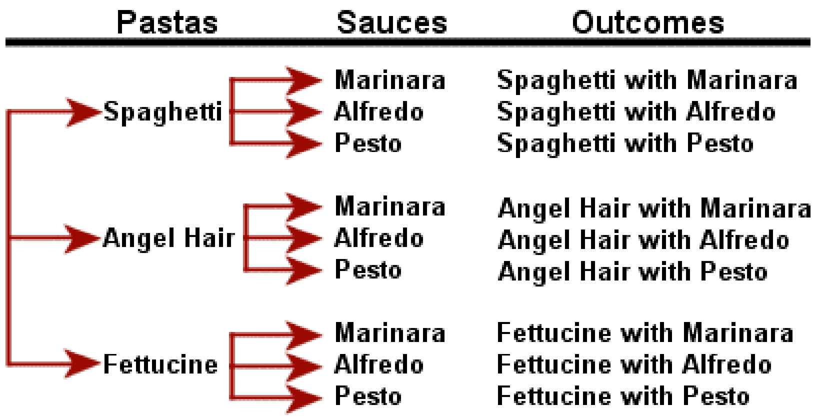
- A restaurant offers three types of pasta and four types of sauces. If a dish only has one pasta and one sauce, how many different dishes are possible?

- Sara is tired of working and wants to take a trip. She is considering five different vacation destinations: Cancun, Corfu, Moorea, Punta Cana, or Playa del Carmen. There are four airlines that fly to each location: FunAir, TripJet, IslandAir, and BankruptAir. Each airline offers four types of drinks on their flights: soft drinks, mixed drinks, beer, and wine. If Sara only has one drink on her flight, how many different ways could she choose a destination, airline, and drink?

The outcomes in all of the previous examples can be illustrated with a tree diagram. **A tree diagram shows all of the possible outcomes of an event**.

Example: A restaurant offers spaghetti, angel hair, and fettuccine pasta and marinara, alfredo, and pesto sauce. How many different ways are there to choose a pasta and a sauce?

Here is a tree diagram for this problem:



As you can see from the tree diagram, there are 9 different ways to choose a pasta and a sauce.

What if the menu choices were expanded to include 4 appetizer choices, 2 bread choices, 3 drink choices, and 4 dessert choices? The possible outcomes would be so numerous that you would find it difficult and time consuming to construct a tree diagram.

Fortunately, the **Multiplication Principle** allows us to quickly determine the number of possible outcomes.

The Multiplication Principle tells us that the number of ways independent events can occur together can be determined by multiplying together the number of possible outcomes for each event (example: each course of the meal**).**

Example: If the first digit cannot be a 0 or a 5, how many five-digit odd numbers are there?

There are 8 possibilities for the first digit (1, 2, 3, 4, 6, 7, 8, 9).

There are 10 possibilities for the second digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
There are 10 possibilities for the third digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
There are 10 possibilities for the fourth digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
There are 5 possibilities for the fifth digit (1, 3, 5, 7, 9)

Using the Multiplication Principle,

= 8 * 10 * 10 * 10 * 5
= 40,000

Example: Mary rolls a fair, six-sided die and flips a coin. How many different outcomes are possible?

There are six different outcomes that the die could produce: 1, 2, 3, 4, 5, or 6. Their are two different outcomes that the coin could produce: heads and tails.

Using the Multiplication Principle,

= 2 * 6
= 12

Remember that the Multiplication Principle is only used to solve problems involving independent events. **Independent events are events where the outcome of one event does not affect the outcome of another event**. For example, in the previous problem, rolling a six on the die does not affect the outcome of flipping the coin.

Permutations

You may encounter questions on the GMAT that ask how many ways that you can arrange a set of items. For instance:

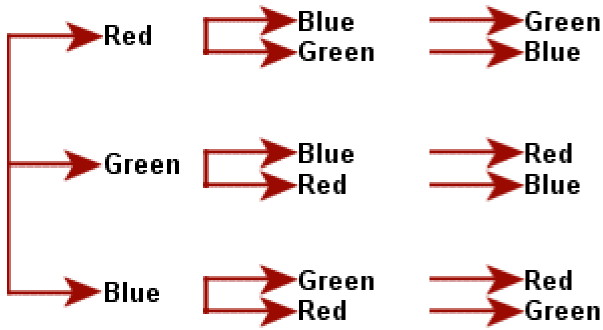
- How many different ways can you arrange three markers?
- How many different distinct ways can the letters in the word VACATION be arranged?
- A famous artist has donated 7 pieces of artwork to a certain art gallery. The gallery has raised enough money for 5 display cases to display the art throughout the gallery. How many different ways can the gallery display the 7 pieces of artwork, 5 pieces at a time, throughout the gallery?

All of these problems are permutation problems. **A permutation is a linear arrangement of elements for which the order of the elements must be taken into account**.

Tree diagrams can also be used with permutation problems. For example,

How many different ways can you arrange a red, blue, and green marker?

Here is a tree diagram that shows all of the possible ways the markers could be arranged:



The tree diagram shows there are six different ways to arrange the three markers.

What if you were asked for the number of possibilities from arranging 6 different markers? You would quickly discover that there are so many different possibilities that it would be time consuming to construct a tree diagram.

Fortunately, the multiplication principle can help us solve permutation problems.

When you are arranging n items in a set, there are n possible outcomes for the first item, n - 1 possible outcomes for the second item, n - 2 possible outcomes for the third item, and so on.

The tree diagram illustrates the number of outcomes for each position in the arrangement.

For instance, once you select the red marker for the first position, you only have n - 1 or 2 choices (blue or green) for the second position. Once you select the blue marker for the second position, there is n – 2 or 1 choice (green) for the third position.

Remember, the Multiplication Principle tells us that the number of ways independent events can occur together can be determined by multiplying together the number of possible outcomes for each event.

Therefore, we can multiply the number of possible outcomes for each position to determine the total number of possible arrangements.

In this example,

= 3 * 2 * 1
= 6

You can see that this answer agrees with our tree diagram.

Factorial

Factorial notation can be used to represent solving arrangement problems by the multiplication principle.

If you are arranging n items in a set, the number of different permutations possible is n!.

n! is pronounced **n factorial**.
n! = n(n-1)(n-2)(n-3) . . . * 2 * 1

For instance,
2! = 2 * 1
3! = 3 * 2 * 1
4! = 4 * 3 * 2 * 1
5! = 5 * 4 * 3 * 2 * 1
6! = 6 * 5 * 4 * 3 * 2 * 1

Example: How many different ways can you arrange five different markers?

The number of different ways to arrange the markers is 5!.

5! = 5 * 4 * 3 * 2 * 1
5! = 120

Indistinguishable Elements

Some arrangement problems may ask the number of different distinct ways items with indistinguishable items. For instance,

How many different distinct ways can the letters in the word TRUST be arranged?

We know the number of ways the letters can be arranged is 5!. However, the question asks how many different distinct ways the letters can be arranged. Since there are 2 T's in the word TRUST, the different ways of arranging the 2 T's are indistinguishable.

To find the number of distinct permutations of a set of items with indistinguishable items, divide the factorial of the items in the set by the product of the factorials of the number of indistinguishable items.

In this problem, the number of different distinct ways of arranging the letters in the word TRUST is 5!/2! or 60.

Let's take a look at another example:

How many different distinct ways can the letters in the word MISSION by arranged?

The word MISSION has 2 Is and 2 Ts.

Therefore, using the rule above,

= 7!/(2! * 2!)
= 5040/4
= 1260

Permutations Formula

Now suppose we don't want to arrange an entire set, but instead we want to order a subset of a larger set. For instance:

- How many different ways can five books be arranged three at a time?

- How many different ways can four markers be arranged two at a time?

- A famous artist has donated 7 pieces of artwork to a certain art gallery. The gallery has raised enough money for 5 display cases to display the art throughout the gallery. How many different ways can the gallery display the 7 pieces of artwork, 5 pieces at a time, throughout the gallery?

Fortunately, we can still use the multiplication principle to solve these problems. For example, if we are trying to fill three slots on a bookshelf with 5 books, we have 5 possibilities for the first slot, 4 possibilities for the second slot, and 3 possibilities for the third slot.

5 * 4 * 3 = 60 possible ways to arrange 5 books taken three at a time.

There is a general formula we can use for solving permutation problems. The number of permutations of n objects taken r at a time is:

P(n,r) = n!/(n-r)!

For our bookshelf problem,

P(5,3) = 5!/(5-3)!
P(5,3) = 5!/2!
P(5,3) = (5*4*3*2*1)/(2*1)
P(5,3) = 60

Permutations Shortcut

Since you now understand the multiplication principle, there is a simple shortcut you can use to solve problems involving ordering a subset of a larger set. The shortcut is to simply complete the factorial notation for the number of items in the set for as many items that are in the subset.

If there are 6 objects taken 4 at a time, complete 4 steps of 6's factorial or 6 * 5 * 4 * 3. If there are 6 objects taken 2 at a time, complete 2 steps of 6's factorial or 6 * 5.

For example,

P(7,4) = 7*6*5*4
P(7,4) = 840

This is easier than using the permutation formula. Here is this problem solved with the permutation formula:

P(7,4) = 7!/(7-4)!
P(7,4) = (7 * 6 * 5 * 4 * 3 * 2 * 1)/(3 * 2 * 1)
P(7,4) = 5040/6
P(7,4) = 840

Thanks!

We hope you have enjoyed this brief tutorial.

Here are some of the topics you'll find covered in our GMAT study course, GMAT Advanced:

Probability:

Expressing probabilities
Rule #1: Basic Probability
Rule #2: Complementary Events
Rule #3: Conditional Probability
Rule #4: The Additive Rule

Advanced Probability:

Probability and Counting Methods
Probability and Geometry
Number Properties and Probability
Weighted Coins

Trials Required
Law of Total Probability
Bayes Theorem

Counting Methods:

The Multiplication Principle
Permutations
Factorial Notation
Permutations with Indistinguishable Items
Permutations Shortcut
Combinations
Combinations Shortcut
Partitions
Circular Permutations

Statistics:

Mode
Median
Mean
Common Mean Problems
Range
Standard Deviation

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