Quant Theory

GMAT Quantitative Section Introduction

Quant Section: 37 Q, 75 min, PS (20-22), DS (15-17), approx. 2 min / Q. Some experimental questions.

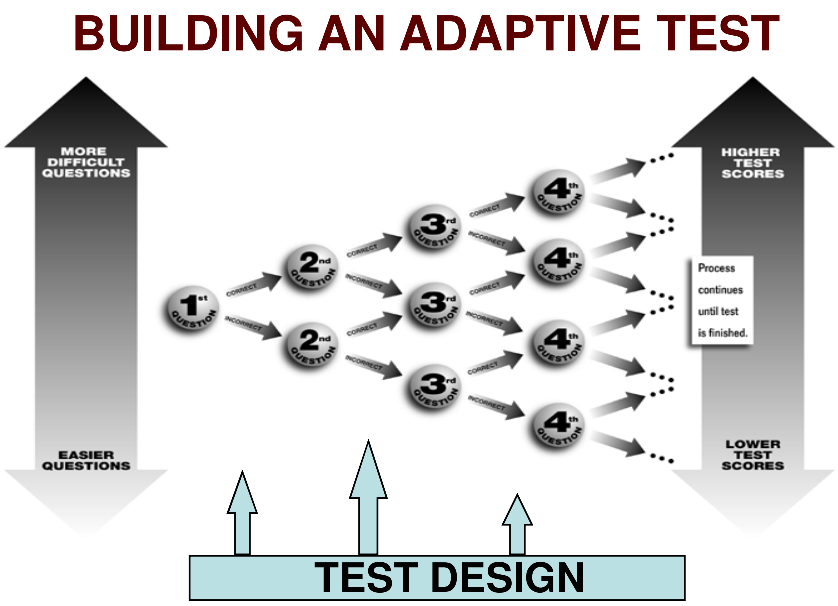
PS (Problem Solving) means simple math questions with 5 options

DS (Data Sufficiency) means Math Questions with the following directions:
The following question is followed by two statements labelled (1) and (2). Check the sufficiency of the statements and mark your answer as:
(A) Statement (1) ALONE is sufficient to answer the question, but statement (2) alone is not.
(B) Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.
(C) Statements (1) and (2) TAKEN TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question.
(E) Statements (1) and (2) TAKEN TOGETHER are NOT sufficient to answer the question.

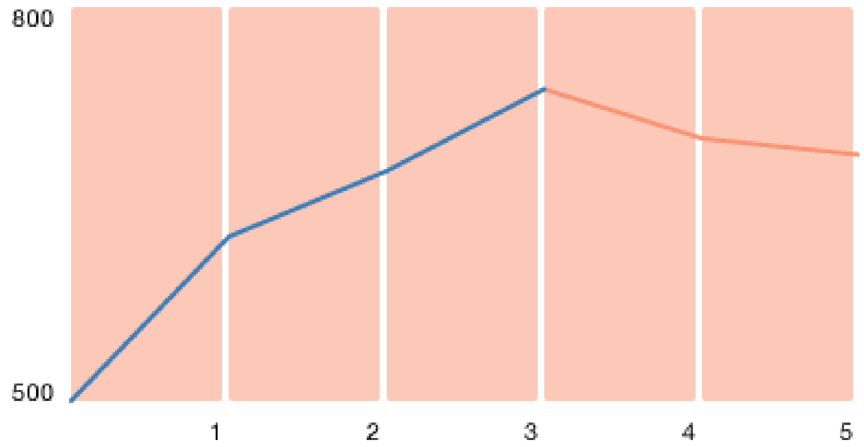
Scaled Score: Maximum 51 which is 99 percentile. Pretty much possible.

CAT - Computer Adaptive Test - Q. 1 500 level, Next question will depend on how you answer the previous question.

The nature of the test makes the **First 10 Q very important.**



HOW AN ADAPTIVE TEST PROCEEDS



GMAT is unlike any other test (a very different mindset is needed). It is not MATH. It is GMAT Quant. We are not here to do PhD in Math. What we learn here is only for the GMAT. Our objective is to improve your score on the GMAT Quant Section.

Success Mantras on GMAT Quant:

1. **Each Q is important** - Indian CAT may need you to solve only 10 etc.

2. **PACE:** You have to finish the section. Don't be on Q. 30 when you have spent 73 minutes.

3. **DS much more important than PS** (for those aspiring for 51): DS is extremely error prone and you need to
understand the intricacies/nuances of it very well.

4. **Very strong penalty (up to 50 points per section) for not finishing any section.** Worst case scenario: I am at Q. 29, I have 90 seconds left. All I should do is MARK all the answers. Don't try to solve anything then. We have had a lot of good students goofing up on this point despite being repeatedly told.

5. ***UNDERSTANDING GMAT Peculiarities:***

   a. **WORDING:** No GMAT Question is really tough because of Math / Calculations. The only way a question can be made tougher on the GMAT is by WORDING.

   b. **There is always a shorter method:** If a question looks too tough or involves a lot of calculations, there has to be an alternate method / SHORTCUT.

   c. **Pattern Recognition:** The GMAT is a standardized test. Mostly the test does not surprise you a lot. Almost types of questions can be fitted into some limited number of patterns.

   d. **Practice, Practice, Practice:** A lot of practice is needed to recognize all the GMAT patterns. You have to solve a lot of questions of the same type to develop recognition for a question type. Practice also helps you develop speed.

Quantitative Theory

Questions Based on Venn Diagrams (Overlapping Sets)

On the GMAT, you are likely to see questions with 2 or 3 variables.

In case of 2 variables, there are a maximum of four divisions possible:

Imagine that at a B-school, applicants can choose Marketing and Finance among other specializations, where dual major is allowed. In this case, there can be only four types of sets of people possible:
1. Students taking Marketing Only
2. Students taking Finance Only
3. Students taking both Marketing and Finance
4. Students taking neither Marketing nor Finance

These types of questions are best solved by making a 2-way-matrix (table). Just remember that if one row has “Marketing”, the other has to have “Not Marketing”. If one column has “Finance”, the other has to have “Not Finance”.

	Finance	**NOT FINANCE**	**TOTAL**
Marketing	**A**	**B**	**A + B**
NOT Marketing	**C**	**D**	**C + D**
TOTAL	**A + C**	**B + D**	**A + B + C + D**

The table can be completed by putting all the given values in the question and thus the unknown can be found out by simple addition / subtraction of rows / columns. In case of percentage problems (where all values are mentioned in terms of percentage), it is better to take the total as 100.

Let us solve a practical example:

50% of the apartments in a certain building have windows and hardwood floors. 25% of the apartments without windows have hardwood floors. If 40% of the apartments do not have hardwood floors, what percent of the apartments with windows have hardwood floors?

10% 16.66% 40% 50% 83.33%

On your test, you can easily solve this problem in less than 2 minutes. But we illustrate here the entire mental process.

This problem involves two sets: Set 1: Apartments with windows / Apartments without windows. Set 2: Apartments with hardwood floors / Apartments without hardwood floors. It is easiest to organize two-set problems by using a matrix as follows:

	Windows	**NO Windows**	**TOTAL**
Hardwood Floors			
NO Hardwood Floors			
TOTAL			

Let's say that there are 100 total apartments in the building. This is the first number we can put into our matrix. The absolute total is placed in the lower right hand corner of the matrix as follows:

	Windows	NO Windows	TOTAL
Hardwood Floors			
NO Hardwood Floors			
TOTAL			**100**

Next, we will attack the complex wording by reading each piece of information separately, and filling in the matrix accordingly.

50% of the apartments in a certain building have windows and hardwood floors. Thus, 50 of the 100 apartments have BOTH windows and hardwood floors. This number is now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**		
NO Hardwood Floors			
TOTAL			**100**

25% of the apartments without windows have hardwood floors. Here's where the subtlety of the wording is very important. This does NOT say that 25% of ALL the apartments have no windows and have hardwood floors. Instead it says that OF the apartments without windows, 25% have hardwood floors. Since we do not yet know the number of apartments without windows, let's call this number **x**. Thus the number of apartments without windows and with hardwood floors is **.25x**. These figures are now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood Floors			
TOTAL		**x**	**100**

40% of the apartments do not have hardwood floors. Thus, 40 of the 100 apartments do not have hardwood floors. This number is put in the Total box at the end of the "No Hardwood Floors" row of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood Floors			**40**
TOTAL		**x**	

Before answering the question, we must complete the matrix. To do this, fill in the numbers that yield the given totals. First, we see that there must be be 60 total apartments with Hardwood Floors (since 60 + 40 = 100) Using this information, we can solve for **x** by creating an equation for the first row of the matrix: **50 + 0.25X = 60 so X = 40**

Now we put these numbers in the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**10**	**60**
NO Hardwood Floors			**40**
TOTAL		**40**	**100**

Finally, we can fill in the rest of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**10**	**60**
NO Hardwood Floors	**10**	**30**	**40**
TOTAL	**60**	**40**	**100**

Note: You don't have to create 7 tables separately. You need to create only one.

We now return to the question: What percent of the apartments with windows have hardwood floors?

Again, pay very careful attention to the subtle wording. The question does NOT ask for the percentage of TOTAL apartments that have windows and hardwood floors. It asks what percent OF the apartments with windows have hardwood floors. Since there are 60 apartments with windows, and 50 of these have hardwood floors, the percentage is calculated as follows: 50/60 = 83.33%. Ans. E

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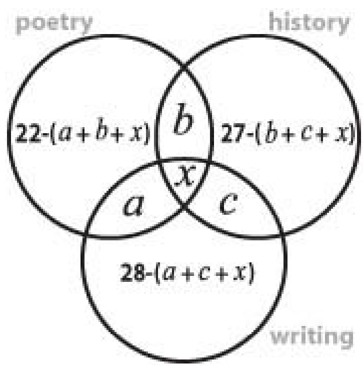
In case of 3 variables, there are a maximum of eight divisions possible. So a table will become very complicated. So we will deal with such questions by drawing 3 overlapping circles:

Each of the 59 members in a high school class is required to sign up for a minimum of one and a maximum of three academic clubs. The three clubs to choose from are the poetry club, the history club, and the writing club. A total of 22 students sign up for the poetry club, 27 students for the history club, and 28 students for the writing club. If 6 students sign up for exactly two clubs, how many students sign up for all three clubs?

2 5 6 8 9

This is a three-set overlapping sets problem. When given three sets, a Venn diagram can be used. The first step in constructing a Venn diagram is to identify the three sets given. In this case, we have students signing up for the poetry club, the history club, and the writing club. The shell of the Venn diagram will look like this:

We are told that the total number of poetry club members is 22, the total number of history club members is 27, and the total number of writing club members is 28. We can use this information to fill in the rest of the diagram:



We can now derive an expression for the total number of students by adding up all the individual segments of the diagram. The first bracketed item represents the students taking two or three courses. The second bracketed item represents the number of students in only the poetry club, since it's derived by adding in the total number of poetry students and subtracting out the poetry students in multiple clubs. The third and fourth bracketed items represent the students in only the history or writing clubs respectively.

$59 = [a + b + c + x] + [22 - (a + b + x)] + [27 - (b + c + x)] + [28 - (a + c + x)]$ OR
$59 = a + b + c + x + 22 - a - b - x + 27 - b - c - x + 28 - a - c - x$ *OR*
$59 = 77 - 2x - a - b - c$ *OR*
$59 = 77 - 2x - (a + b + c)$

By examining the diagram, we can see that $(a + b + c)$ represents the total number of students who sign up for two clubs. We are told that 6 students sign up for exactly two clubs. Consequently:

$59 = 77 - 2x - 6$ or $x = 6$.

So, the number of students who sign up for all three clubs is 6. The correct answer is C.

Percentages

The term **percentage** means **parts per 100** or "for every hundred". A fraction whose denominator is 100 is called percentage and the numerator of the fraction is called the rate percent. Thus, when we say a man made a profit of 20 percent we mean to say that he gained Rs.20 for every hundred rupees he invested in the business, i.e., 20/100 rupees for each Rupee. The abbreviation of percent is p.c. and it is generally denoted by %.

Let's start with a number X (= 1 X)
X increased by 10% would become X + 0.1 X = 1.1 X
X increased by 1% would become X + 0.01 X = 1.01 X
X increased by 0.1% would become X + 0.001 X = 1.001 X
X decreased by 10% would become X – 0.1 X = 0.9 X
X decreased by 1% would become X – 0.01 X = 0.99 X
X decreased by 0.1% would become X – 0.001 X = 0.999 X
X increased by 200% would become X + 2X = 3X
X decreased by 300% would become X – 3X = –2X

Similarly, you can work mentally with any specifically chosen number (say 500) and work out different answers.

Complete all entries in the following table.

Increased by / Number	10%	25%	50%	100%	300%
50	55				
75		93.75			
150			225		
500				1000	
600					2400
1000				2000	
2000			3000		
2500		3125			

Increased by / Number	10%	25%	50%	100%	300%
50	55	62.5	75	100	200
75	82.5	93.75	112.5	150	300
150	165	187.5	225	300	600
500	550	625	750	1000	2000
600	660	750	900	1200	2400
1000	1100	1250	1500	2000	4000
2000	2200	2500	3000	4000	8000
2500	2750	3125	3750	5000	10000

A Percentage can be expressed as a Fraction. 10% can be expressed as 10/100 or 1/10. To express a percentage as a fraction divide it by 100 **a % = a/100.**

To express a fraction as a percent multiply it by 100 $\Rightarrow \frac{a}{b} = \left[\left(\frac{a}{b}\right)\times 100\right]\%$

To express percentage as a decimal we remove the symbol % and shift the decimal point by two places to the left. For example, 10% can be expressed as 0.1. 6.5% = 0.065 etc.

To express decimal as a percentage we shift the decimal point by two places to the right and write the number obtained with the symbol % or simply we multiply the decimal with 100. Similarly 0.7 = 70%.

$$\text{Increase \%} = \frac{\text{Increase}}{\text{Original Value}}\times 100 \qquad \text{Decrease \%} = \frac{\text{Decrease}}{\text{Original Value}}\times 100$$

In increase %, the denominator is smaller, whereas in decrease %, the denominator is larger.

$$\text{Change \%} = \frac{\text{Change}}{\text{Original Value}}\times 100$$

Successive change in percentage. If a number A is increased successively by X% followed by Y%, and then by Z%, then the final value of A will be $A\left(1+\frac{X}{100}\right)\left(1+\frac{Y}{100}\right)\left(1+\frac{Z}{100}\right)$.

In a similar way, at any point or stage, if the value is decreased by any percentage, then we can replace the same by a negative sign. The same formula can be used for two or more successive changes. The final value of A in this case will be

$A\left(1-\frac{X}{100}\right)\left(1-\frac{Y}{100}\right)\left(1-\frac{Z}{100}\right)$.... etc.

Let the present population of a town be p and let there be an increase of X% per annum. Then :

(i) population after n years = p $[1 + (X/100)]^n$

(ii) population n years ago = p / $[1 + (X/100)]^n$

If the population of a town (or value of a machine) decreases at R% per annum, then :

(i) population (or value of machine) after n years = p $[1 - (R/100)]^n$

(ii) population (or value of machine) n years ago = p / $[1 - (R/100)]^n$

Also, let us remember that:

2 = 200% (or 100% increase), 3 = 300% (or 200% increase), 3.26 = 326% (means 226% increase), fourfold (4 times) = 400 % of original = 300% increase, 10 times means 1000% means 900% increase, 0.6 means 60% of the original means 40% decrease, 0.31 times means 31% of the original means 69% decrease etc.

1/2 = 50%, 3/2 = 1 + 1/2 = 100 + 50 = 150%, 5/2 = 2 + 1/2 = 200 + 50 = 250% etc.,

2/3 = 1 - 1/3 = 100 - 33.33 = 66.66%, 4/3 = 1 + 1/3 = 100 + 33.33 = 133.33%,
5/3 = 1 + 2/3 = 100 + 66.66 % = 166.66%, 7/3 = 2 + 1/3 = 200 + 33.33 = 233.33%,
8/3 = 2 + 2/3 = 200 + 66.66 = 3 - 1/3 = 300 - 33.33 = 266.66% etc.

1/4 = 25%, 3/4 = 75%, 5/4 (1 + 1/4) = 125% (= 25 increase), 7/4 (1 + 3/4 = 2 - 1/4) = 175% (= 75% increase), 9/4 (2 + 1/4) = 225% (= 125% increase), 11/4 = 275% = (175% increase).

1/5 = 20%, 2/5 = 40%, 3/5 = 60%, 4/5 = 80%, 6/5 = 120%, 7/5 (1 + 2/5) = 140% etc.

1/6 = 16.66%, 5/6 = 83.33%, 7/6 (1 + 1/6) = 116.66%, 11/6 = 183.33%

1/8 = 12.5%, 3/8 = 37.5%, 5/8 = 62.5%, 7/8 = 87.5%, 9/8 = (1 + 1/8) = 112.5%, 11/8 = (1 + 3/8) = 137.5%, 13/8 = 162.5%, 15/8 = 187.5% etc.

1/9 = 11.11%, 2/9 = 22.22%, 4/9 = 44.44%, 5/9 = 55.55%, 7/9 = 77.77%, 8/9 = 88.88%, 10/9 = 111.11%, 11/9 = (1 + 2/9) = 122.22% etc.

NOTE : In the problems on **DISCOUNT** remember the following:
1. Marked price is the price listed on the article (called list price).
2. Discount is calculated on Marked price and NOT on Cost price.

So Marked Price - Discount = Sale Price. Also Cost Price + Profit = Sale Price.

Problems

1. The production of a firm increases from 340 MT to 500 MT. What is the percent increase?

Sol. 160/340 = 8/17 = 47.05% (direct).

2. The production of a firm decreases from 500 MT to 340 MT. What is the percent decrease?

Sol. 160/500 = 32%.

Note the answers to the above 2 questions are different.

$$\%\ \text{increase} = \frac{\text{Change}}{\text{smaller value}} \times 100 \qquad \%\ \text{decrease} = \frac{\text{Change}}{\text{larger value}} \times 100$$

3. The production of a firm increases by 20%, 25% and 50% in 3 successive years over the previous year. If the production is 160 MT in the first year, find the production at the end of 3 years.

Sol. 160 × 1.2 × 1.25 × 1.5 = $160 \times \frac{6}{5} \times \frac{5}{4} \times \frac{3}{2} = 360$

4. The production of a firm decreases by 20% in the first year, then decreases by 25% in the next year and then increases by 50% the next year and then increases by 10% in the next year. All percentage changes being consecutive (over the previous year). If at the end of the changes, the value is 990 MT, what was the value initailly?

Sol. $A \times 0.8 \times 0.75 \times 1.5 \times 1.1 = 990$ or $A \times \frac{4}{5} \times \frac{3}{4} \times \frac{3}{2} \times \frac{11}{10} = 990 \Rightarrow A = 1000$

5. Which is bigger: 0.004% of 25000 or 25000% of 0.004?

Sol. Both equal. **A% of B = B % of A.**

6. If price decreases by 25%, by what % should consumption increase so that the expenditure does not increase?

Sol. 25% = 1/4, so 4 was made 3. Now 3 has to be made 4, so 1/3 = 33.33%.

7. If speed increases by 33.33%, what is the percent reduction in the time taken to travel the same distance?

Sol. 33.33% = 1/3, so 3 was made 4. Now for 4 to be made 3, drop = 1/4 = 25%.

8. The price of a product increased by 20% but the turnover increased by only 12%. What is the % drop in quantity sales?

Sol. $100 \times 100 = 10000$ **(10 × 10 Funda)**
$120 \times X = 11200$.
So we have X = 1120/12 = 280/3. So we have X = 93.33, so drop = 6.66%.

The 10 × 10 Funda. Let us take a very simple question and try to work it out by different methods.

9. The length of a rectangle increases by 25%. Find the percent drop in its width for area to remain same.

Sol. Anything that involves product of two quantities can be handled by 10 by 10 rule.

Examples where A × B may be used:

Percentage effect on expenditure. Expenditure = Price × Quantity (or Consumption)

Percentage effect on area of Rectangle / Square / Circle etc.
Area = Length × Breadth

Revenue calculation Revenue = Unit Sale × Price per unit

Distance Distance = Speed × Time

10 × 10 METHOD. In this assume length and breadth are both 10. So we have

$10 \times 10 = 100$
$12.5 \times B = 100$ or B = 8, so 20% drop.

One more Example (of 10 × 10 method). The price of petrol increases by 40% but the final expenditure increases by 20% only. By what percent does the consumption decrease?

Sol. $10 \times 10 = 100$ $14 \times X = 120$.

So we get X = 120/14 = 60/7. So the drop = 10 - (60/7) = 10/7. Now 1/7 = 14.28% Answer.

10. *When the cost of petroleum increases by 40%, a man reduces his annual consumption by 20%. Find the percentage change in his annual expenditure on petroleum.*

Sol. First expenditure : Suppose 10 litres of petroleum at 10 units of money per litre, then total expenditure = 10 x 10 units of money = 100 units of money

Second expenditure : Now, 8 litres of petroleum at 14 units of money per litre, total expenditure = 8 x 14 units of money = 112 units ⇒ Expenditure increases by 12% Ans.

Alt. Sol. : $Exp_1 = PX$, $Exp_2 = 1.4P (0.8X) = 1.12 PX$ ⇒ Directly we see, answer = 12%

11. *The number of students in a school increases at a certain rate per cent. The number at present is 1323 and the number two years ago was 1200; find the rate per cent of the increase.*

Sol. By formula, we have $1200 \times (1 + R/100)^2 = 1323 \Rightarrow R = 5\%$

12. *A hawker has a certain number of oranges, of which 2% are bad. He sells 95% of the remainder, and then has 49 oranges left. How many had he originally ?*

Sol. Suppose he has X originally, then (X – 0.02 X – 0.95 x 0.98 X) = 49 or X = 1000.

13. *A reduction of 20% in the price of apples could enable a man to get 120 more for Rs 1,440. Find the first price of one apple.*

Sol. We have 1440 = X x Y ....(1) X = no. of apples Y = price of one apple.

Now 1440 = (X + 120) x 0.8Y ......(2) From (1) and (2), X = 480 and Y = Rs 3

14. *A man's working hours a day were increased by 25%, and his wages per hour were increased by 20%. By how much percent were his daily earnings increased ?*

Sol. Let initially X be number of hours & Y = wages/hour

Later these become 1.25 X & 1.2 X respectively. Daily earnings initially = X x Y

Now Daily earnings = 1.25X x 1.2Y = 1.5 XY Hence 50% increase.

15. *A tradesman allows a discount of 15% on the marked price. How much above the C.P. must he mark his goods to make a profit of 19 % ?*

Sol. Let CP = 100, Gain = 19, SP = 100 + 19 = 119

Now Marked price should be such that Marked price reduced by 15% is equal to 119

or 85% of M.P. = 119 or MP = 119 x 100/85 = Rs 140 ⇒ Answer = 40 % above the C.P.

SIMPLE INTEREST

Principal or Sum : The money borrowed or lent out for a certain period is called the principal or the sum.

Interest : Extra money paid for using other's moeny is called interest.

Simple Interest : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called simple interest.

Formulae: Let Principal = P, Rate = R% per annum and time = T years. Then, $\text{S.I.} = \left(\frac{P \times R \times T}{100}\right)$

COMPOUND INTEREST

Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half yearly or quarterly to settle the previous account. In such case, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, the difference between the amount and the money borrowed is called the **Compound Interest** (abbreviated as C.I.) for that period.

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compounded Annually: $Amount = P\left(1+\frac{R}{100}\right)^n$

II. When interest is compounded Half-yearly : $Amount = P\left[1+\frac{(R/2)}{100}\right]^{2n}$

III. When interest is compounded Quarterly : $Amount = P\left[1+\frac{(R/4)}{100}\right]^{4n}$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years

$$Amount = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

V. When Rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively. Then, Amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

Ex 1. *A certain sum of money at C.I. amounts in 2 years to Rs 811.2 and in 3 years to Rs 843.65. Find the sum of money.*

Sol. Since $A = P(1 + R/100)^n$

$\Rightarrow$ $811.2 = P(1 + R/100)^2$ .....(1) *and* $843.65 = P(1 + R/100)^3$ ....(2)

On dividing (2) by (1), we get : $843.65/811.2 = (1 + R/100)$

$\Rightarrow$ $1.04 = (1 + R/100)$ $\Rightarrow R/100 = 0.04$ $\Rightarrow R = 4$

Now, putting $R = 4$ into (1), we get $811.2 = P(1 + 4/100)^2$

$\Rightarrow 811.2 = P(1.04)^2$ $\Rightarrow$ $P = 811.2/(1.04)^2 = 750$

$\Rightarrow$ The sum of money is Rs 750. Ans.

Ex 2. *Find the compound interest on Rs 4500 for 3 years at 6% p.a. interest being payable half yearly.*

Sol. $A = P(1 + R/100)^{2n} = 4500(1 + 6/200)^6 = 4500(1.03)^6 = 5373$

$\Rightarrow$ Compound interest = Rs (5373 – 4500) = Rs 873 Ans.

RATIOS and VARIATIONS

RATIO : The comparison between two quantities of the same kind of unit is the Ratio of one quantity to another. The ratio of a and b is usually written as a : b or a/b, where a is called the antecedent (numerator) and b the consequent (denominator).

1. $a : b = ka : kb$ where k is a constant

2. $a : b = a/k : b/k$

3. $a : b : c = X : Y : Z$ is equivalent to $a/X = b/Y = c/Z$

4. If $a/b = c/d$, then
   (i) $(a + b)/b = (c + d)/d$
   (ii) $(a - b)/b = (c - d)/d$
   (iii) $(a + b)/(a - b) = (c + d)/(c - d)$

5. If $a/b = c/d = e/f = \ldots\ldots = k$, then $\dfrac{a + c + e + \ldots.}{b + d + f + \ldots.} = k$.
   Also note that : $k = a/b = Xc / Xd = -5e / -5f$
   $\Rightarrow$ Each ratio $= (a + Xc - 5e)/(b + Xd - 5f) = k$.
   Here, we have randomly taken X and –5. You can take any factor.

6. If $a/b > 1$ or $a > b$ then $(a + X)/(b + X) < a/b$ a, b, X are natural numbers

7. If $a/b < 1$ or $a < b$ then $(a + X)/(b + X) > a/b$ a, b, X are natural numbers

VARIATION :

1. **Direct proportion :**

If two quantities X & Y are related such that any increase or decrease in 'Y' produces a proportionate increase or decrease in 'X' or vice versa, then the two quantities are said to be in direct proportion.

In other words $X : Y = X/Y = k$ (a constant)
or $X = KY$ or $Y = K'X$ (where K and K' are constants)

X is directly proportional to Y is written as $X \propto Y$ or $X = K\,Y$

2. **Inverse proportion :**

Here two quantities X & Y are related such that, any increase in X would lead to a decrease in Y or any decrease in X would lead to an increase in Y. Thus the quantities X & Y are said to be inversely related and X is inversely proportional to Y is written as

$X \propto 1/Y$ or $X = k/Y$ or $XY = k$ (constant) or the product of two quantities remains constant.

MIXTURES FORMULA

This rule enables us to find the proportion in which two or more ingredients at the given price must be mixed to produce a mixture at a given price.

The C.P. of unit quantity of the mixture is called the MEAN PRICE.

The rule says : If two ingredients are mixed in a ratio, then

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{CP of Cheaper}}$$

1. In what ratio should tea @ 35/kg be mixed with tea @ 27/kg so that mixture may cost Rs. 30/kg?
Sol. Quantity of cheaper / Quantity of dearer = (35 – 30) / (30 – 27) = 5/3
Hence the two should be mixed in the ratio 5 : 3.

2. *Find a : b : c if 6a = 9b = 10c.*
Sol. a/b = 9/6 = 3 : 2 = 15 : 10, b/c = 10/9 = 10 : 9. Hence a : b : c = 15 : 10 : 9.

3. *A's present age is to be B's as 8 : 5; and 20 years ago it was as 12 : 5. Find the present age of each.*
Sol. Let A's present age = 8X ⇒ B's present age = 5 X ⇒ (8X – 20) / (5X – 20) = 12 / 5 Solving this, we get : X = 7 ⇒ A's age = 8 X = 56 yrs, B's age = 5 X = 35 yrs

4. *An alloy contains 24% of tin by weight. How much more tin to the nearest kg must be added to 100 kg of the alloy so that the % of tin may be doubled ?*

Sol. Let X kg of tin be added to the alloy.

Tin (kg)	**Alloy (kg)**
24	100
24 + X	100 + X

$\Rightarrow$ (24 + X) / (100 + X) = 2 (24/100) $\Rightarrow$ X = 46 Hence 46 kg of tin must be added to the alloy. Ans.

5. *The expenses of a hotel consist of two parts. One part varies with the number of inmates, while the other is constant. When the number of inmates is 200 & 250, the expenses are respectively Rs. 1300 & Rs. 1600. Then find the expenses for 300 inmates.*

Sol. Let E = K_1 X + K_2, where K_1 & K_2 are constants, E stands for expenses, X for the number of inmates. When X = 200 & E = 1300 we have –> [200 K_1 + K_2 = 1300]. When X = 250 & E = 1600 we have –> [250 K_1 + K_2 = 1600]. Solving the equations we have K_1 = 6 & K_2 = 100 $\Rightarrow$ E = 6X + 100. Now when X = 300, E = 6 x 300 + 100 = Rs 1900 Ans.

6. *Two tins A and B contain mixtures of wheat and rice. In A, the weights of wheat and rice are in the ratio 2 : 3 and in B they are in the ratio 3 : 7. What quantities must be taken from A and B to form a mixture containing 5 kg of wheat and 11 kg of rice?*

Sol. Let X kg of mixture be taken from A, then (16 – X) kg is taken from B
$\Rightarrow$ 2X/5 kg of wheat from A and 3(16 – X)/10 kg of wheat from B is to be taken.
Now we have, 2X/5 + 3(16 – X)/10 = 5 or X = 2 kg $\Rightarrow$ 2 kg from A and 14 kg from B.

7. *Two vessels contain mixtures of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35–litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel ?*

Sol. Let X litres be taken from the first vessel; then (35 – X) litres are taken from the second. In the first vessel 8/11 of the mixture, and in the second vessel 5/6 of the mixture, is spirit $\Rightarrow$ The spirit in the 35–litre cask is 4/5 of the mixture $\Rightarrow$ 8/11 X + 5/6 (35 – X) = 4/5 x 35 $\Rightarrow$ X = 11 $\Rightarrow$ 11 litres are taken from the first vessel Ans.

8. *A bag contains $ 600 in the form of one–dollar, 50 cents & 25–cents coins in the ratio 3 : 4 : 12. Find the number of 25 cents coins.*

Sol. Ratio of values of coins = 3/1 : 4/2 : 12/4 = 3 : 2 : 3. Value of 25 cents coins
= Rs 600 x 3/(3 + 2 + 3) = 225. $\Rightarrow$ No. of 25 cents coins = 225 x 4 = 900 Ans.
<u>Alternate method :</u>
Assume that the number of 1 $ coins is 3X. Then the value equation would be 3 X + 4 X (0.50) + 12 X (0.25) = 600. Find X and get answer = 12 X.

9. *A mixture contains milk & water in the ratio 5 : 1. on adding 5 litres of water, the ratio of milk and water becomes 5 : 2. Find the quantity of milk in the original mixture.*

Sol. Let the quantity of milk be 5X & that of water X. Then 5X / (X + 5) = 5/2 or X = 5 $\Rightarrow$ Quantity of milk = 5X = 25 litres

10. *The ratio of the number of boys to the number of girls in a school of 546 is 4 : 3. If the number of girls increases by 6, what must be the increase in the number of boys to make the new ratio of boys to girls 3 : 2 ?*

Sol. Original no. of boys = 546 x 4/7 = 312. Original no. of girls = 78 x 3 = 234. Final no. of girls = 234 + 6 = 240 $\Rightarrow$ No. of boys reqd. to make the new ratio = 240 x 3/2 = 360 $\Rightarrow$ The reqd. increase in the no. of boys = 360 – 312 = 48 Ans.

11. *In what ratio should 30% milk be mixed with 50% milk to get a 35% milk strength?*
Sol. From mixture rule we have :
Qty. of 30% / Qty of 50% = (50 – 35) / (35 – 30) = 3 : 1 Ans.

12. *Two numbers are in the ratio of 3 : 4. If 5 is subtracted from each, the resulting numbers are in the ratio 2 : 3. Find the numbers.*
Sol. Let 3X and 4X be the numbers $\Rightarrow$ (3X – 5) / (4X – 5) = 2/3 $\Rightarrow$ 9X – 15 = 8X – 10 $\Rightarrow$ X = 5 $\Rightarrow$ The reqd numbers are 15 and 20 Ans.

Work / Rate

FUNDAMENTAL CONCEPTS :

1. If a man can do a piece of work in N days (or hours or any other unit of time), then the work done by him in one day will be 1/N of the total work.

2. If A is twice as good a workman as B, then A will take half the time B takes to finish a piece of work.

3. If A and B can do a piece of work in X & Y days respectively while working alone, they will together take
   XY / (X + Y) days to complete it.

4. If A, B, C can do a piece of work in X, Y, Z days respectively while working alone, they will together take XYZ / [XY + YZ + ZX] days to finish it.

5. If an inlet pipe can fill a cistern in X hours, the part filled in 1 hour = 1/X

6. If an inlet pipe can fill a tank in X hours and an outlet pipe empties the full tank in Y hours, then the net part filled in 1 hour when both the pipes are opened = (1/X) – (1/Y) $\Rightarrow$ In 1 hour, the part filled (or emptied) = 1/X – 1/Y
   $\Rightarrow$ Time required to fill or empty the tank = XY / (X ~ Y) hours. X ~ Y indicates [X – Y] or [Y – X], whichever is positive).

1. *If A and B together finish a piece of work in 10 days & B alone can finish it in 20 days. In how many days can A alone finish the work ?*
Sol. Let X and Y be the number of days required by A and B respectively.
$\Rightarrow$ By the standard formula, XY / (X + Y) = 10 & Y = 20
$\Rightarrow$ X x 20 / (X + 20) = 10 or X = 20 days Ans.

2. *Four men working together all day, can finish a piece of work in 11 days; but two of them having other engagements can work only one half–time and quarter time respectively. How long will it take them to complete the work ?*
Sol. Each man will take 11 x 4 = 44 days to complete the work. If one man works half day/day he will take 44 x 2 = 88 days to finish the work. Similarly, a man working quarter day/day will take 44 x 4 = 176 days to finish the work. When these work together they will require
1 / [(1/44) + (1/44) + (1/88) + (1/176)] = 16 days.

3. *20 men can complete a piece of work in 10 days, but after every 4 days 5 men are called off, in what time will the work be finished ?*

Total work = 20 x 10	=	200 monday
1. First 4 days' work	=	20 x 4 = 80 md
2. Next 4 days' work	=	15 x 4 = 60 md
3. Next 4 days' work	=	10 x 4 = 40 md
4. Next 4 days' work	=	5 x 4 = 20
		Σ = 200 md
Hence, days reqd	=	4 + 4 + 4 + 4 = 16

4. *A vessel can be filled by one pipe A in 10 minutes, by a second pipe B in 15 minutes. It can be emptied by a waste pipe C in 9 minutes. In what time will the vessel be filled if all the three were turned on at once ?*

Sol. We will follow exactly the same method as in time & work. The part of vessel filled in 1 minute when all three are on = 1/10 + 1/15 – 1/9 = 1/18 $\Rightarrow$ Total vessel will be filled in 18 minutes Ans.

5. *Three pipes A, B and C can fill a cistern in 15, 20 and 30 min resp. They were all turned on at the same time. After 5 minutes the first two pipes were turned off. In what time will the cistern be filled ?*

Sol. A, B and C can fill (1/15 + 1/20 + 1/30) or 3/20 of the cistern in 1 minute $\Rightarrow$ A, B and C filled (3/20 x 5) or 3/4 of the cistern in 5 min. Now A and B are turned off $\Rightarrow$ (1 – 3/4) or 1/4 of the cistern wil be filled by C $\Rightarrow$ C will fill 1/4 of the cistern in (30 x 1/4) or 7.5 minutes $\Rightarrow$ The cistern will be filled in 7.5 + 5 or 12.5 min. Ans.

6. *A cistern can be filled by two taps A and B in 12 minutes and 14 minutes respectively and can be emptied by a third in 8 minutes. If all the taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes ?*

Sol. We have (7/12) + (7/14) – 7/8 = 5/24 part filled in 7 minutes. Hence 1 – 5/24 = 19/24 th of the tank is unfilled.

Time, Speed and Distance

1. **Speed:** The rate at which anything covers a particular distance is called its speed
   $\Rightarrow$ Speed = Distance Travelled / Time Taken
   Generally speed is expressed in the following units : miles / hr, km/hr, m/sec, m/min, etc.

2. If A goes from X to Y at U km/hr and comes back from Y to X at V km/hr, then Average speed during the whole journey = 2UV / (U + V) km/hr.

3. If a man changes his speed in the ratio m : n then the ratio of times taken becomes n : m.

4. If three men cover the same distance with speeds in the ratio a : b : c, the times taken by these three will be resp. in the ratio 1/a : 1/b : 1/c.

Relative Speed:

When two objects travel in the same direction, relative speed = difference of speeds

When two objects travel in opposite directions, relative speed = sum of speeds

5. When two objects travel in the same direction,

   relative speed = difference of speeds

   time to catch / overtake = lead distance / difference of speeds

6. When two objects travel in the Opposite directions,

   relative speed = sum of speeds

   time to meet = lead distance / sum of speeds.

7. If the speed of a boat (or man) in still water be X km/hr, and the speed of the stream (or current) be Y km/hr, then
   (a) Speed of boat with the stream (or Downstream or D/S) = (X + Y) km/hr
   (b) Speed of boat against the stream (or upstream or U/S) = (X – Y) km/hr
   We have X = [(X + Y) + (X – Y)] / 2 and Y = [(X + Y) – (X – Y)] / 2
   ⇒ Boat's speed in still water = [Speed downstream + Speed upstream] / 2
   Speed of current = [Speed downstream – Speed upstream] / 2

1. *A policeman goes after a thief who is 176 m ahead of him. When and where will the policeman catch the thief when they run at the rates of 11440 and 10560 meters per hour respectively?*

Sol. Time to catch / overtake = lead distance / difference of speeds = 176 / (11440 – 10560)
= 176 / 880 = 1/5 hours = 12 minutes ⇒ The time required to overtake the thief = 12 min. Ans.
(b) The distance from the starting point = 11440 x 12/60 kms = 2288 meters

2. *If I walk at the rate of 4 kms an hour, I reach my destination 30 min too late; If I walk at the rate of 5 kms an hour I reach 30 minutes too soon. How far is my destination ?*

Sol. Let time taken be T hrs for the distance to be covered at the normal speed (neither fast nor slow). Then we have 4 (T + 0.5) = 5 (T – 0.5) {Note : 0.5 here is 30 min} ⇒ T = 4.5 hours ⇒ Distance = 4 (T + 0.5) = 4 x 5 = 20 kms.

3. *A man rows 18 kms down a river in 4 hours with the stream and returns in 12 hours; find his speed and also the velocity of the stream.*

Sol. Speed with the stream = 18/4 = 4.5 kms an hour. ⇒ Speed against the stream = 18/12
= 1.5 kms an hour. ⇒ Speed of the stream = 1/2 (4.5 – 1.5) = 1.5 kms an hour and his speed
= 4.5 – 1.5 = 3 kms an hour Ans.

4. *A, B and C can walk at the rates of 3, 4, 5 kms an hour. They start from X at 1, 2, 3 o'clock respectively; when B catches up with A, B sends him back with a message to C; when will C get the message ?*

Sol. In one hour A covers 3 kms Now (B – A) = 1 kmph hence B catches up A after 3 hours i.e. at 5 o' clock. Now upto 5 o' clock A has covered 12 kms and C has covered 10 kms. Hence Distance between A & C = 2 kms and their relative speed (3 + 5) = 8 kmph. To cover 2 kms at 8 kmph, Time = 2/8 hours = 15 min. Hence C gets the message at 15 minutes past 5 o'clock. Ans.

5. *A student walks to school at the rate of 2.5 kms an hour and reaches 6 minutes too late. Next day he increases his speed by 2 kms an hour and then reaches there 10 minutes too soon. Find the distance of the school from his home.*

Sol. Let t be the usual time We have $\Rightarrow$ 2.5 x (t + 1/10) = 4.5 (t – 1/6), or t = 1/2 hours.
Hence distance = 2.5 (1/2 + 1/10) = 2.5 x 6 /10 = 1.5 kms.

6. *A man can row in still water a distance of 4 kms in 20 minutes and 4 kms with the current in 16 min. How long will it take him to row the same distance against the current ?*

Sol. X = 4 / (20/60) = 12, X + Y = 4 x 60 /16 = 15 or Y = 3.
or Time = 4 / (X – Y) = 4 x 60 /9 = 80/3 minutes Ans.

PROBLEMS ON DIGITS

In these problems the student should carefully understand the difference between the DIGIT VALUE (or FACE VALUE or ABSOLUTE VALUE) and LOCAL VALUE (or PLACE VALUE) of the digits forming a number.

Thus the face value of the digits of the number 789 are 7, 8 and 9 resp. But the local values are 700, 80 and 9 respectively.

The following table will make the difference clear :

100's digit	10's digit	Unit's digit	Number Formed
	5	4	5 x 10 + 4 = 54
9	7	8	9 x 100 + 7 x 10 + 8 = 978
	X	Y	X x 10 + Y = 10X + Y
	Y	X	Y x 10 + X = 10Y + X
X	Y	Z	100 X + 10Y + Z

Translating Word Problems into equations

1. *Find two consecutive odd numbers the difference of whose squares is 296.*

Sol. Let the numbers be 2X + 1 and 2X + 3 Then $(2X + 3)^2 – (2X + 1)^2 = 296 \Rightarrow X = 36$ Hence 2X + 1 = 2 x 36 + 1 = 73 and 2X + 3 = 75 $\Rightarrow$ The required numbers are 73 and 75.
[Verification. $(75)^2 – (73)^2 = 5625 – 5329 = 296$]

2. *A is 29 years older than B, B is 3 years older than C and D is 2 years younger than C. Two years hence A's age will be twice the united ages of B, C and D. Find their present ages.*

Sol. Let D's age be = X years Then C's age = (X + 2) years, B's age = (X + 5) years and A's age = (X + 34) years. Two yrs hence A's, B's, C's and D's ages will be X + 36, X + 7, X + 4 and X + 2 years respectively. $\Rightarrow$ 2 (X + 2 + X + 4 + X + 7) = X + 36 $\Rightarrow$ X = 2 $\Rightarrow$ A's age = 36 yrs; B's age = 7 yrs, C's age = 4 yrs; D's age = 2 yrs.

3. *A number consists of three consecutive digits, that in the unit's place being the greatest of the three. The number formed by reversing the digits exceeds the original number by 22 times the sum of the digit. Find the number.*

Sol. Let the hundred's digit be X. Then the ten's digit = X + 1 and the unit's digit = X + 2
$\Rightarrow$ The number = 100 x X + 10(X + 1) + X + 2 = 111X + 12.
The number formed by reversing the digits = 100(X + 2) + 10(X + 1) + X
= 111X + 210 $\Rightarrow$ 111X + 210 – 111X – 12 = 22 (X + 2 + X + 1 + X) $\Rightarrow$ X = 2. Hence the required number = 234.

4. *The crew of a boat can row at the rate of 5 miles an hour in still water. If to row 24 miles, they take 4 times as long as to row the same distance down the river, find the speed at which the river flows.*

Sol. Let X miles per hour be the speed of the river. Hence, on equating the times, we get:
24/(5 – X) = 4 x 24/(X + 5) $\Rightarrow$ X = 3 Thus, the river flows at the rate of 3 miles an hour.

5. *The area of a rectangle remains the same if the length is increased by 7 metre and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 m and breadth is increased by 5 m. Find the dimensions of the rectangle.*

Sol. Let the length of the rectangle be X m and breadth of the rectangle = Y m. Area = XY sq. m.
I Case : Length = (X + 7) m and breadth = (Y – 3) m
$\Rightarrow$ Area = (X + 7) (Y – 3) sq. m. $\Rightarrow$ (X + 7) (Y – 3) = XY or – 3X + 7Y – 21 = 0 .... (1)
II Case : Length = (X – 7) m and breadth = (Y + 5) m $\Rightarrow$ (X – 7) (Y + 5) = XY
or 5X – 7Y – 35 = 0 .... (2) $\Rightarrow$ Y = 15 and X = 28
Hence length = 28 m and breadth = 15 m Answer.

Note : Assume L = 28 and B = 15 as an option and try checking the conditions given in the problem. You will see that working backwards is exceptionally fast in such cases.

6. *The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 200 per month, find their monthly incomes.*

Sol. Let the monthly income of first person be Rs 9X and the monthly income of second person be Rs 7X. Let the expenditure of first person be 4Y and the expenditure of second person be 3Y. $\Rightarrow$ Saving of the first person = Rs (9X – 4Y) and solving of second person = Rs (7X – 3Y). Using the given informations, we have : 9X – 4Y = 200 .... (1) and 7X – 3Y = 200 .... (2)
X = 200. Hence, the monthly income of first person = Rs. 9 x 200 = Rs. 1800 and the monthly income of second person = Rs. 7 x 200 = Rs. 1400 Ans.

7. *Find two consecutive even numbers such that 1/6th of the greater exceeds 1/10th of the smaller by 29.*

Sol. Let the numbers be 2X and 2X + 2 Then (2X + 2)/6 – 2X/10 = 29.
$\Rightarrow$ X = 21 Hence 2X = 430 and 2X + 2 = 432. $\Rightarrow$ The required numbers are 430 and 432.
[Verification. 432/6 – 430/10 = 72 – 43 = 29]

8. *A number consists of two digits whose sum is 12. The ten's digit is three times the unit's digit. What is the number?*

Sol. Let the unit's digit be X, Then the ten's digit is 12 – X. ⇒ 3X = 12 – X ⇒ X = 3
Hence the number is 93. [Verification. 9 = 3 x 3; and 9 = 3 = 12]

9. *A train travelled a certain distance at a uniform rate. Had the speed been 6 miles an hour more, the journey would have occupied 4 hours less; and had the speed been 6 miles an hour less, the journey would have occupied 6 hours more. Find the distance.*

Sol. Let us suppose that X miles per hour is the speed of the train and Y hours is the time taken for the journey.
⇒ Distance travelled = XY = (X + 6) (Y – 4) = (X – 6) (Y + 6) This gives two simultaneous equations. Solving, we get : X = 30, Y = 24 ⇒ Distance = XY = 720 miles Ans.

10. *A sum of money was divided equally among a certain number of persons; had there been six more, each would have received a rupee less, and had there been four fewer, each would have received a rupee more than he did; find the sum of money and the number of men.*

Sol. Let X be the number of persons and Rs Y be the share of each. Then by the conditions of the problem, we have (X + 6) (Y – 1) = XY ....(1) (X – 4) (Y + 1) = XY ....(2). Thus the number of person is X = 24 and the share of each is Y = Rs 5. The sum of money = 5 x Rs 24 = Rs 120.

STATISTICS

MEAN

1. Avg or mean or AM = Sum of n quantities (or numbers) / number of them (n) OR
2. Arithmetic Mean (A.M) is given by $\bar{\mathbf{X}} = \frac{\Sigma \mathbf{x}}{\mathbf{N}}$
3. Mean of the Combined Series If n_1 and n_2 are the sizes and $\bar{X}_1, \bar{X}_1$ are the respective means of two series then the mean $\bar{X}$ of the combined series of size $n_1 + n_2$ given by $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_2 + n_2}$
4. If each one of the given numbers is increased (or decreased) by K, their average is increased (or decreased) by k.
5. If each one of some given numbers is multiplied by K, their average is multiplied by K.
6. If a man (or train or boat or bus) covers some journey from A to B at X km/hr (or m/sec) and returns to A at a uniform speed for Y km/hr, then the average speed during the whole journey is [2XY / (X + Y)] km/hr.

   TIP: The average speed in such a case will be a bit less than the simple average.
7. If there are r series of observations $N_1, N_2, ..., N_r$, the mean M of the whole series is related to the mean $M_1, M_2, ...$ of the component series by the equation $\mathbf{NM = N_1M_1 + N_2M_2 + ... + N_rM_r}$.
8. Weighted Arithmetic Average is given as $\bar{\mathbf{X}}_{\mathbf{w}} = \frac{\Sigma \mathbf{wx}}{\Sigma \mathbf{w}}$
9. The sum of first "n" natural numbers is given by n(n + 1)/2.

10. **For consecutive integers or for equally spaced numbers (AP), Mean = (First term + Last term) / 2.**

11. Count of consecutive numbers inclusive = last term - first term + 1, Example 9 to 15, total = 7.

12. Count of consecutive numbers exclusive (terms greater than x but less than y) = last term - first term -1. Example: 9 to 15, total = 5

13. If the average of a few consecutive integers is 0, then either all numbbers are zero or there will be an odd number of integers.

14. **The average of an odd number of consecutive integers is an integer and the average of an even number of consecutive integers is a non-integer.**

15. If in a set of numbers, the average = any other number, all the numbers will have to be equal.

MEDIAN

1. Median is the middle value or the average of two middle values when the **values are arranged in an order**, either ascending or descending.

2. If there are odd number of observations, median is directly the middle number.

3. If there are even number of observations, median is the average of the two middle numbers.

4. **For consecutive integers or for equally spaced numbers (AP), Median = (First term + Last term) / 2.**

   So, Median = Mean in this case.

Ex. (i) The following are the marks of 9 students in a class. Find the median
34, 32, 48, 38, 24, 30, 27, 21, 35
(ii) Find the median of the daily wages of ten workers.
Rs. 20, 25, 17, 18, 8, 15, 22, 11, 9, 14

Sol.(i) Arranging the data in ascending order of magnitude, we have 21, 24, 27, 30, 32, 34, 35, 38, 48. Since, there are 9, an odd number of items, therefore median is 32.
(ii) Arranging the wages in ascending order of magnitude, we have 8, 9, 11, 14, 15, 17, 18, 20, 22, 25. Since, there are 10 observations, therefore median is the arithmetic mean of 15 and 17. So median = (15 + 17)/ 2 = 16.

Ex. The number of students absent in a school was recorded every day for 147 days and the raw data was presented in the form of the following frequency table.

No. of students absent :	5	6	7	8	9	10	11	12	13	15	18	20
No. of days :	1	5	11	14	16	13	10	70	4	1	1	1

Obtain the median and describe what information it conveys.

Sol. Calculation of median

x_i	f_i	CF
5	1	1
6	5	6
7	11	17
8	14	31
9	16	47
10	13	60
11	10	70
12	70	140
13	4	144
15	1	145
18	1	146
20	1	147

We have N = 147. So N/2 = 147/2 = 73.5. The cumulative frequency just greater than N/2 is 140 and the corresponding value of x is 12. Hence, median = 12. This means that for about half the number of days, more than 12 students were absent.

Mode is the value which occurs most frequently in a set of observations

In case more than one values occurs most frequently, all those values are called the modes.

Range. It is defined as the difference between the two extreme observations of the distribution.

$$\textbf{Range} = X_{max} - X_{min}$$

where X_{max} is the greatest observation and X_{min} is the smallest observation of the variable value.

If Range = 0, all the observations are equal.

Standard deviation. It is defined as positive square root of the A.M. of the squares of the deviations of the given observations from their A.M. If $X_1, X_2, ...X_N$ is a set of N observations then its standard deviation is given by Standard Deviation $\sigma = \sqrt{\frac{\Sigma d^2}{N}}$ or $\sigma = \sqrt{\frac{1}{N}\sum\left(X - \overline{X}\right)^2}$. It is a measure of how much each value varies from the mean of all the values.

Less SD implies more consistency, less variation, less spread, more compactness AND vice versa.

If SD = 0, all the observations are equal.

Range is always greater than SD, except when all observations are equal, when both are equal to 0.

Change in respective statistical parameters:

	Addition	**Subtraction**	**Sign Change**	**Multiplication**	**Division**
Mean	Change	Change	Change	Change	Change
Median	Change	Change	Change	Change	Change
Mode	Change	Change	Change	Change	Change
Range	NO Change	NO Change	NO Change	Change	Change
SD	NO Change	NO Change	NO Change	Change	Change

Ex. Find the S. D. of the set of the numbers 3, 4, 9, 11, 13, 6, 8 and 10.

Sol. Here we have

Numbers (x)	$x - \bar{x}$	$(x - \bar{x})^2$
3	–5	25
4	–4	16
9	1	1
11	3	9
13	5	25
6	–2	4
8	0	0
10	2	4
$\Sigma x = 64$		$\sum (x - \bar{x})^2 = 84$

Here N = number of items = 0. $\therefore \bar{x}$ = arithmetic mean = Σx / N = 64/8 = 8.

$$\therefore \sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}} = \sqrt{\frac{84}{8}} = \sqrt{10.5} = 3.24$$

INEQUALITIES / Absolute Value / Modulus / Positive / Negative

Let a and b be real numbers. If a – b is negative we say that a is less than b and write a < b. If a – b is positive then a is greater than b, i.e., a > b.

Properties of Inequalities.

1. For any two real numbers a and b, we have a > b or a = b or a < b.

2. If a > b and b > c, then a > c. If a > b then (a + c) > (b + c) and (a - c) > (b - c), however, ac > bc and (a/c) > (b/c) (not sure) (is true only when c is positive)

3. If a > b, then a + m > b + m, for any real number m.

4. If $a \neq 0$, $b \neq 0$ and a > b, then 1/a < 1/b.

5. If a > b, then am > bm for m > 0 and am < bm for m < 0, **that is, when we multiply both sides of inequality by a negative quantity, the sign of inequality is reversed.**

6. If a > X, b > Y, c > Z then
(1) a + b + c + ... > X + Y + Z + ... (2) abc .... > XYZ .... (Provided none is negative)

7. If x > 0 and a > b > 0, then $a^x > b^x$.

8. If a > 1 and x > y > 0, then $a^x > a^y$.

9. If 0 < a < 1 and x > y > 0, then $a^x < a^y$.

10. Do not cancel anything from both sides of an inequality unless you are sure that the cancelled quantity is positive, so ax > ay does not necessarily mean x > y, etc.

11. The concept of number line is very useful in checking inequalities. The common values to check are x = 0, 1, -1, >1 (preferred value = 2), between 0 and 1 (preferred value = 1/2), between - 1 and 0 (preferred value = -1/2), and less than -1 (preferred value = -2). So in short, there are 7 points: -2, -1, -1/2, 0, 1/2, 1, 2.

12. |x| is defined as the non-negative value of x and hence is never negative. On the GMAT, $\sqrt{x^2} = |x|$, that means, the square root of any quantity is defined to be non-negative, so $\sqrt{36} = 6 \text{ and not } -6$ on the GMAT. BUT if $x^2 = 36 \Rightarrow x = 6 \, or -6$ both. So $\sqrt{x^2} = x \; or -x$ both are possible. If, x is negative, then $\sqrt{x^2} = -x$ as it has to be +ve eventually. In this case x is negative and -x is positive.

13. |5| = 5, |-5| = 5, so |x| = x, if x is positive or 0 and |x| = -x if x is negative.

14. If |x| > x, then x is negative.

15. If |x| = a, then x = a or x = -a.

16. If $|x| > a$, then $x > a$ or $x < -a$.

17. If $|x| < a$, then $x < a$ or $x > -a$.

18. If $|x - a| > b$, then either $x - a > b$ or $x - a < -b$

19. If $|x - a| < b$, then either $x - a < b$ or $x - a > -b$.

20. If $|x| = x$, then x is either positive or 0.

21. $|a + b| \leq |a| + |b|$, $|a - b| \geq ||a| - |b||$, $|ab| = |a|\,|b|$, $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$, $|a^2| = a^2$.

22. If $(x - a)(x - b) < 0$, then x lies between a and b. OR $a < x < b$.

23. If $(x - a)(x - b) > 0$, then x lies outside a and b. OR $x < a$, $x > b$.

24. If $x^2 > x$, then either $x > 1$ or x is negative ($x < 0$)

25. If $x^2 < x$, then x lies between 0 and 1. ($0 < x < 1$)

26. If $x^2 = x$, then $x = 0$ or $x = 1$.

27. If $x^3 > x$, then either $x > 1$ or x is between -1 and 0(either $x > 1$ or $-1 < x < 0$).

28. If $x^3 < x$, then either x lies between 0 and 1 or x is less than -1. (either $0 < x < 1$ or $x < -1$)

29. If $x^3 = x$, then $x = 0$ or $x = 1$ or $x = -1$.

30. If $x^3 = x$, then $x = 0$ or $x = 1$ or $x = -1$.

31. If $x > y$, it is not necessary that $x^2 > y^2$ or $\sqrt{x} > \sqrt{y}$ etc. So even powers can't be predicted.

32. If $x > y$, it is necessarily true that $x^3 > y^3$ or $\sqrt[3]{x} > \sqrt[3]{y}$ etc. So odd powers and roots dont change sign.

33. $ab > 0$ means $a/b > 0$ and vice versa. The two are of the same sign.

34. $ab < 0$ means $a/b < 0$ and vice versa. The two are of the opposite sign.

35. If x is positive, $x + \frac{1}{x} \geq 2$.

36. If X is positive, then
    (1) $(a + X) / (b + X) > a/b$ if $a < b$
    (2) $(a + X) / (b + X) < a/b$ if $a > b$

37. If X is negative, then
    (1) $(a + X) / (b + X) > a/b$ if $a > b$
    (2) $(a + X) / (b + X) < a/b$ if $a < b$

38. $(a + c + e + ...) / (b + d + f + ...)$ is less than the greatest and greater than the least of the fractions a/b, c/d, e/f, ...

NUMBERS

<u>*Numbers are basically of two types:*</u> 1. Real and 2. Imaginary

For the purpose of the GMAT, all numbers are real.

REAL numbers are basically of two types :

1. **Rational numbers :** A rational number can always be represented by a fraction of the form p/q where p and q are integers and $q \neq 0$. Examples : finite decimal numbers, infinite repeating decimals, whole numbers, integers, fractions i.e.
   3/5, 16/9, 0.666 ..... ∞ = 2/3 etc.

2. **Irrational numbers :** Any number which can not be represented in the form p/q where p and q are integers and $q \neq 0$ is an irrational number. AN INFINITE NON–RECURRING DECIMAL IS AN IRRATIONAL NUMBER. Examples – $\sqrt{2}$, π, $\sqrt{5}$, $\sqrt{7}$.

3. **Prime numbers :** A natural number which has no other factors besides itself and unity is a prime number. Examples – 2,3,5,7,11,13,17,19 ......
   If a number has no factor equal to or less than its square root, then the number is prime. This is a test to judge whether a number is prime or not.

NOTE :

1. The only even prime number is 2
2. 1 is neither prime nor composite (by definition)

Composite numbers : A composite number has other factors besides itself and unity. e.g. 8, 72, 39 etc. Alternatively, we might say that a natural natural number that is not prime is a composite number.

NATURAL NUMBERS : The numbers 1,2,3,4,5 ..... are known as natural numbers. The set of natural numbers is denoted by N. Hence N = {1, 2, 3, 4 ..... }

WHOLE NUMBERS : The numbers 0,1,2, 3,4, ... W = {0, 1, 2, 3, ... }

INTEGERS : The set of Integers I = {0, ±1, ±2, ±3, ... }

EVEN NUMBERS : The numbers divisible by 2 are even numbers. Eg. 0, 2, 4, 6, 8, 10 .... Even numbers are expressible in the form 2n where n is an integer other than zero. **Thus –2, –6 etc. are also even numbers.**

For the purpose of the GMAT, 0 is an even number.

ODD NUMBERS : The numbers not divisible by 2 are odd numbers e.g. 1, 3, 5, 7, 9 ..... Odd numbers are expressible in the form (2n + 1) where n is an integer other than zero (not necessarily prime). Thus, –1, –3, –9 etc. are all odd numbers.

<u>TESTS FOR DIVISIBILITY:</u>

1. A number is divisible by 2 if its unit's digit is even or zero e.g. 128, 146, 34 etc.

2. A number is divisible by 3 if the sum of its digits is divisible by 3 e.g. 102, 192, 99 etc.

3. A number is divisible by 4 when the number formed by last two right hand digits is divisible by '4' e.g. 576, 328, 144 etc.

4. A number is divisible by 5 when its unit's digit is either five or zero : e.g. 1111535, 3970, 145 etc.

5. A number is divisible by 6 when it's divisible by 2 and 3 both. e.g. 714, 509796, 1728 etc.

6. A number is divisible by 8 when the number formed by the last three right hand digits is divisible by '8'. e.g. 512, 4096, 1304 etc.

7. A number is divisible by 9 when the sum of its digits is divisible by 9 e.g. 1287, 11583, 2304 etc.

8. A number is divisible by 10 when its units digit is zero. e.g. 100, 170, 10590 etc.

9. A number is divisible by 11 when the difference between the sums of digits in the odd and even places is either zero or a multiple of 11. e.g. 17259, 62468252, 12221 etc. For the number 17259 : Sum of digits in even places = 7 + 5 = 12, Sum of digits in the odd places = 1 + 2 + 9 = 12 Hence 12 – 12 = 0.

10. A number is divisible by 12 when it is divisible by 3 & 4 both. e.g. 672, 8064 etc.

11. A number is divisible by 25 when the number formed by the last two Right hand digits is divisible by 25. e.g. 1025, 3475, 55550 etc.

12. A number is divisible by 125, when the number formed by last three right hand digits is divisible by 125. e.g. 2125, 4250, 6375 etc.

NOTE :

1. When any number with even number of digits is added to its reverse, the sum is always divisible by 11. e.g. 2341 + 1432 = 3773 which is divisible by 11.

2. If X is a prime number then for any whole number "a" ($a^X - a$) is divisible by X e.g.
   Let X = 3 and a = 5. Then according to our rule $5^3 - 5$ should be divisible by 3.
   Now ($5^3 - 5$) = 120 which is divisible by 3.

<u>HCF (GCD / GCF) & LCM OF NUMBERS</u>

<u>HCF :</u> It is the greatest factor common to two or more given numbers. It is also called GCF OR GCD (greatest common factor or greatest common divisor). e.g. HCF of 10 & 15 = 5, HCF of 55 & 200 = 5, HCF of 64 & 36 = 4

To find the HCF of given numbers, resolve the numbers into their prime factors and then pick the common term(s) from them and multiply them if more than one. This is the required HCF.

<u>LCM :</u> Lowest common multiple of two or more numbers is the smallest number which is exactly divisible by all of them.
e.g. LCM of 5, 7, 10 = 70, LCM of 2, 4, 5 = 20, LCM of 11, 10, 3 = 330

To find the LCM resolve all the numbers into their prime factors and then pick all the quantities (prime factors) but not more than once and multiply them. This is the LCM.

NOTE:

1. LCM x HCF = Product of two numbers (valid only for “two”)

2. HCF of fractions = HCF of numerators ÷ LCM of denominators

3. LCM of fractions = LCM of numerators ÷ HCF of denominators

<u>Calculating LCM :</u>

Method 1 : Factorization Method

Rule – After expressing the numbers in terms of prime factors, the LCM is the product of highest powers of all factors.

Q. Find the LCM of 40, 120, 380.

A. $40 = 4 \times 10 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$,

$120 = 4 \times 30 = 2 \times 2 \times 2 \times 5 \times 3 = 2^3 \times 5^1 \times 3^1$

$380 = 2 \times 190 = 2 \times 2 \times 95 = 2 \times 2 \times 5 \times 19 = 2^2 \times 5^1 \times 19^1$

$\Rightarrow$ Required LCM $= 2^3 \times 5^1 \times 3^1 \times 19^1 = 2280$.

Method 3 : Division method

Q. Find the LCM of 6, 10, 15, 24, 39

A.

2	6	10	15	24	39
2	3	5	15	12	39
3	3	5	15	6	39
5	1	5	5	2	13
2	1	1	1	2	13
13	1	1	1	1	13
	1	1	1	1	1

Explanation : Write these numbers separately. Then divide by 2 and write the result below the numbers divisible by 2. Leave the others untouched. Do the same for all steps till you get 1 as the remainder in each column.

Ans. LCM $= 2 \times 2 \times 3 \times 5 \times 2 \times 13 = 1560$.

<u>Calculating HCF:</u>

Factorization

After expressing the numbers in term of the prime factors, the HCF is product of COMMON factors.

Ex. Find HCF of 88, 24, 124

$88 = 2 \times 44 = 2 \times 2 \times 22 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11^1$

$24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$124 = 2 \times 62 = 2 \times 2^1 \times 31^1 = 2^2 \times 31^1$ $\Rightarrow$ HCF $= 2^2$

DECIMAL FRACTIONS

Recurring Decimals (Conversion to a Rational Number): If in a decimal fractions a figure or a set of figures is repeated continually, then such a number is called a recurring decimal.

If a single figure is repeated, it is shown by putting a dot on it. Also, if a set of figures is repeated, we express it by putting on dot at the starting digit and one dot at the last digit of the repeated set.

Rule : Write the recurring figures only one in the numerator and take as many nines in the denominator as the number of repeating figures.

Ex. (1) $0.6^{\text{—}} = 6/9 = 2/3$ (2) $0.2^{\cdot}34^{\cdot} = 234/999$ (3) $3.5^{\cdot} = 3\,{}^{5}/_{9}$ (4) $2.0^{\cdot}3^{\cdot}5 = 2\,{}^{35}/_{999}$

ROUNDING OFF:

Examples of Rounding Decimals:

Number	Nearest tenth	Nearest hundredth	Nearest thousandth
1.2346	1.2	1.23	1.235
31.6479	31.6	31.65	31.648
9.7462	9.7	9.75	9.746

Whether a fraction will result in a terminating decimal or not? To determine this, express the fraction in the lowest form and then express the denominator in terms of Prime Factors. If the denominator contains powers of only 2 and 5, it is finite (terminating). If the denominator contains any power of any other prime number, it is infinite (non-terminating).

Division : Division is the method of finding how many times one number is contained in another. The former of these numbers is called the divisor, the latter the dividend and the number of times found the quotient. That which is left after the operation is finished is the remainder.

Hence, (Divisor x Quotient) + Remainder = Dividend

SOME FORMULAS

The following formulae are very handy tools while solving any type of mathematical problem. Memorise each by heart.

1. $(a \pm b)^2 = a^2 \pm 2ab + b^2$
2. $(a + b)^2 - (a - b)^2 = 4ab$
3. $(a + b)^2 + (a - b)^2 = 2\,(a^2 + b^2)$
4. $a^2 - b^2 = (a + b)\,(a - b)$
5. $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$
   $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$
6. (1) $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$ (2) $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
7. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$
8. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - ac - bc)$
   $\Rightarrow$ if $(a + b + c) = 0$ then $a^3 + b^3 + c^3 = 3abc$.

<u>**SURDS**</u> (COMPLICATED ROOTS)

Surds are irrational roots of a rational number.

e.g. $\sqrt{6}$ = a surd $\Rightarrow$ it can't be exactly found. Similarly – $\sqrt{7}$, $\sqrt{8}$, $\sqrt[3]{9}$, $\sqrt[4]{27}$ etc. are all surds.

Pure Surd : The surds which are made up of only an irrational number e.g. $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ etc.

Mixed Surd : Surds which are made up of partly rational and partly irrational numbers e.g. $3\sqrt{3}$, $6\sqrt[4]{27}$ etc.

Q. Convert $\sqrt{27}$ to a mixed surd A. $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$

Q. Convert $2\sqrt{8}$ to a pure surd A. $2\sqrt{8} = \sqrt{8 \times 4} = \sqrt{32}$

<u>Rationalisation of Surds:</u> In order to rationalize a given surd, multiply and divide by the conjugate of denominator [conjugate of $(a + \sqrt{b})$ is $(a - \sqrt{b})$ and vice versa].

e.g. $$\frac{(6+\sqrt{2})}{(1-\sqrt{3})} = \frac{(6+\sqrt{2})\,(1+\sqrt{3})}{(1-\sqrt{3})\,(1+\sqrt{3})} = \frac{(6+6\sqrt{3}+\sqrt{2}+\sqrt{6})}{(1-3)} = \frac{(6+6\sqrt{3}+\sqrt{2}+\sqrt{6})}{-2}$$

<u>**Prime factors:**</u>

A composite number can be uniquely expressed as a product of prime factors.

Ex. $12 = 2 \times 6 = 2 \times 2 \times 3 = 2^2 \times 3^1$ $20 = 4 \times 5 = 2 \times 2 \times 5 = 2^2 \times 5^1$

$124 = 2 \times 62 = 2 \times 2 \times 31 = 2^2 \times 31$ etc.

NOTE :

- The number of divisors (factors) of a given number N (including one and the number itself) where $N = a^m \times b^n \times c^p$ ..... where a, b, c are prime numbers are $(1 + m)(1 + n)(1 + p)$ ......

  e.g. (1) $90 = 2 \times 3 \times 3 \times 5 = 2^1 \times 3^2 \times 5^1$
  hence here a = 2 b = 3 c = 5, m = 1 n = 2 p = 1
  then number of divisors = $(1 + m)(1 + n)(1 + p)$ .... = $2 \times 3 \times 2 = 12$
  Now factors of 90 = 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90 = 12.

- **The number of factors of a perfect square number will always be odd.**
- **Or if a number has odd number of factors, the number has to be a perfect square.**
- Every non zero integer is a factor of integer 0.
- 0 is the multiple of all non-zero integers but not a factor of any integer
- The product of r consecutive integers is divisible by r !
- If p is a prime number, then 1 + (p – 1) ! is divisible by p.

  Example. *16 ! + 1 i.e.,(17–1) ! + 1 is divisible by 17.*

- The fifth power of any single digit number has the same right hand digit as the number itself.

  SO EVERY DIGIT HAS A CYCLICITY OF 4.

- For any integer n, $n^3 - n$ is divisible by 3, $n^5 - n$ is divisible by 5, $n^{11} - n$ is divisible by 11, $n^{13} - n$ is divisible by 13.

  In general, if p is a prime number then for any whole number a, $a^p - a$ is divisible by p.

- When any number with even number of digits is added to its reverse, the sum is always divisible by 11. e.g. 2341 + 1432 = 3773 which is divisible by 11.

- The product of r consecutive integers is divisible by r !

- If p is a prime number and n is prime to p, then $n^{p-1} - 1$ isdivisible by p.
  Example. *($99^6 - 1$) i.e., ($99^{7-1} - 1$) is divisible by 7.*

- If p is a prime number, then 1 + (p – 1) ! is divisible by p.
  Example. *16 ! + 1 i.e.,(17–1) ! + 1 is divisible by 17.*

- Whenever an even power of (n – 1) is divided by n the remainder is always 1 and whenever an odd power of (n – 1) is divided by n the remainder is always n – 1.

Factor Theorem : If f(x) is completely divisible by (x – a), then f(a) = 0. So, (x – a) **is a factor of** f(x), f(a) = 0

Check whether (x + 1) is a factor of f(x) = $4x^2 + 3x - 1$.

Putting x + 1 = 0, i.e., x = –1 in the given expression we get f(–1) = 0. So, (x + 1) is a factor of f(x).

Remainder Theorem : If an expression f(x) is divided by (x – a), then the remainder is f(a).

Let f(x) = $x^3 + 3x^2 - 5x + 4$ be divided by (x – 1). Find the remainder.

Remainder = f(1) = $1^3 + 3 \times 1^2 - 5 \times 1 + 4 = 3$.

Some properties of square numbers:

- A square number cannot end with 2, 3, 7, 8 or an odd number of zeroes.
- The square of an odd number is odd.
- The square of an even number is even.
- Every square number is a multiple of 3, or exceeds a multiple of 3 by unity.
- Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.
- If a square number ends in 9, the preceding digit is even.
- Square numbers have odd number of factors.

Power of a Prime Number in a Factorial. If we have to find the power of a prime number p in n!, it is found using a general rule, which is

$\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots\ldots$, where $\left[\frac{n}{p}\right]$ denotes the greatest integer less than or equal to $\left[\frac{n}{p}\right]$ etc.

For example power of 3 in 100 ! $=\left[\frac{100}{3}\right]+\left[\frac{100}{3^2}\right]+\left[\frac{100}{3^3}\right]+\left[\frac{100}{3^4}\right]+\left[\frac{100}{3^5}\right]+\ldots.$

$= 33 + 11 + 3 + 1 + 0 = 48.$

For example power of 5 in 200 ! $=\left[\frac{200}{5}\right]+\left[\frac{200}{5^2}\right]+\left[\frac{200}{5^3}\right]+\ldots$

$= 40 + 8 + 1 + 0 = 49.$

Number of Zeroes at the end of a Factorial. It is given by the power of 5 in the number. Actually, the number of zeroes will be decided by the power of 10, but 10 is not a prime number, we have 10 = 5 × 2, and hence we check power of 5.

For example, the number of zeroes at the end of 100 ! $=\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]+\ldots = 20+4=24.$

The number of zeroes at the end of 500 ! $=\left[\frac{500}{5}\right]+\left[\frac{500}{5^2}\right]+\left[\frac{500}{5^3}\right]+\ldots = 100+20+4=124.$

The number of zeroes at the end of 1000 ! $=\left[\frac{1000}{5}\right]+\left[\frac{1000}{5^2}\right]+\left[\frac{1000}{5^3}\right]+\left[\frac{1000}{5^4}\right]\ldots = 200+40+8+1$

$= 249.$

Indices

- We define,

$$a^m = a.a.a....a \text{ (m times)}$$

Here a is called the base and m the exponent, index or power.

Laws of Indices

- If a, b are two positive real numbers and m, n are two real numbers then
  (i) $a^m . a^n = a^{m+n}$
  (ii) $a^m / a^n = a^{m-n}$
  (iii) $(a^m)^n = a^{mn}$
  (iv) $(ab)^m = a^m.b^m$
  (v) $(a/b)^m = a^m/b^m$
  (vi) $a^0 = 1$, $a \neq 0$
  (vii) $a^{-m} = 1/a^m$, $a \neq 0$
  (viii) $a^m = a^n \Rightarrow a = 0$ or 1 or -1 or $m = n$
  (ix) $a^m = b^m \Rightarrow m = 0$ or $a = b$
  (x) $\sqrt{(ab)} = \sqrt{a}\,\sqrt{b}$
  (xi) $\sqrt[m]{a} = a^{1/m}$
  (xii) $a^{p/q} = \sqrt[q]{a^p}$

Ex 1. *What will be the unit's digit in 128^{96} ?*

Sol. ***In all such questions, divide the power by 4 and check the remainder.***
If the remainder is 1, 2 or 3, then convert the question to LAST DIGIT RAISED TO REMAINDER.
If the remainder is 0, convert the question to LAST DIGIT RAISED TO FOUR.

In this question, 96/4 = 0, so the question converts to $8^4 = 8^2 \times 8^2 = 6\underline{4} \times 6\underline{4} = 4 \times 4 = 1\underline{6} = 6$
$\Rightarrow$ Required answer = 6

Ex 2. *A number X when divided by 289 leaves 18 as the remainder. The same number when divided by 17 leaves Y as the remainder. Find Y.*

Sol. We know that : DIVIDEND = (DIVISOR X QUOTIENT) + REMAINDER
$\Rightarrow X = (289 \times K) + 18 = (17 \times 17 \times K) + 18$
$= (17 \times 17 \times K) + (17 \times 1) + 1 = 17\,(17 \times K + 1) + 1$
= 17 (New Quotient) + 1 $\Rightarrow$ Y = 1 **Answer.**

Ex 3. *Find the largest number in $\sqrt[4]{4}$, $\sqrt[3]{3}$ and $\sqrt{2}$*

Sol. In this we can compare by taking the LCM of roots and we see that LCM = 4 x 3 = 12
Hence we have $\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$, $\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$
$\sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$ $\Rightarrow$ Obviously $\sqrt[3]{3}$ is the largest.

Ex 4. *An orderly has a number of pegs to peg in a row. At first he tried to peg 5 in each row, then 6, then 8 and then 12, but had always 1 left. On trying 13 he had none left. What is the smallest number of pegs that he could have had ?*

Sol. LCM of 5, 6, 8, and 12 = 120 $\Rightarrow$ Let the number be 120 P + 1 (because 1 was always left)
$\Rightarrow$ Number = (13 x 9 + 3) P + 1 = (13 x 9) P + 3 P + 1 which is divisible by 13
$\Rightarrow$ 3 P + 1 has to be divisible by 13. The least value of P that makes it divisible by 13 is P = 4
$\Rightarrow$ Number = 120 P + 1 = 120 x 4 + 1 = 481

Ex 5. *A number 1568X35Y is divisible by 88. Find X & Y.*

Sol. As the no. is divisible by 8 x 11, the last 3 digits must be divisible by 8 or 35Y should be 352 or Y = 2. To be divisible by 11, the differences of the sum of its digits in the even and odd places must be either zero or a multiple of 11 $\Rightarrow$ Adding the digits of even and odd places, we get 2 + 3 + 8 + 5 = 18 & 5 + X + 6 + 1 = 12 + X. To make the difference zero X + 12 = 18 or X = 6. Hence X = 6, Y = 2 Ans.

Ex 6. *A heap of stones can be made up exactly into groups of 25. But when made into groups of 18, 27 and 32, there are always 11 left. Find the least number of stones that may be contained is such a heap.*

Sol. The LCM of 18, 27, 32 = 864. Hence the required number must be of the form (864 K + 11) such that it is divisible by 25. The least such value of K = 1. Hence the required number = 875.

Ex 7. *Solve for X : $|\,3X - 6\,| < 9$*

Sol. Since the absolute value of a number denotes both +ve & –ve values, then either
(a) $3X - 6 < 9$ or (b) $-(3X - 6) < 9$ or $-3X + 6 < 9$ or $-3X < 3$
$\Rightarrow$ (a) $3X < 15 \Rightarrow X < 5$
(b) $-3X < 3 \Rightarrow X > -1 \Rightarrow$ Combining, we get : $-1 < X < 5$ Ans.

Ex 8. *Find the greatest fraction out of 2/5, 5/6, 11/12, 7/8*

Sol. We have LCM of 5, 6, 12, 8 = 120
Hence the numbers are 48/120, 100/120, 110/120, 105/120
Hence 110/120 or 11/12 is the largest Ans.

Ex 9. *Find the value of :*
$[1 / (\sqrt{9} - \sqrt{8})] - [1 / (\sqrt{8} - \sqrt{7})] + [1 / (\sqrt{7} - \sqrt{6})] - [1 / (\sqrt{6} - \sqrt{5})] + [1 / (\sqrt{5} - \sqrt{4})]$

Sol. By rationalization we have :
$1/(\sqrt{9} - \sqrt{8}) = [1/(\sqrt{9} - \sqrt{8})] \times [(\sqrt{9} + \sqrt{8}) / (\sqrt{9} + \sqrt{8})] = (\sqrt{9} + \sqrt{8})/(9 - 8) = \sqrt{9} + \sqrt{8}$
Similarly $1/(\sqrt{8} - \sqrt{7}) = (\sqrt{8} + \sqrt{7})$ etc. Hence the given expression
$= (\sqrt{9} + \sqrt{8}) - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6}) - (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + \sqrt{4}) = \sqrt{9} + \sqrt{4} = 3 + 2 = 5$ Ans

Progressions

"A progression or sequence or series is simply a function whose variable stands only for natural numbers."

e.g.if $f(n) = n^2$ then $f(1) = (1)^2 = 1$
$f(2) = 2^2 = 4$, $f(3) = 3^2 = 9$
$f(10) = 10^2 = 100$ and so on. The nth term of a sequence is usually denoted by T_n.

Thus T_1 = first term, T_2 = second term, T_{10} = tenth term and so on.

ARITHMETICAL PROGRESSION (A.P.)

It is a series of numbers in which every term after the first can be derived from the term immediately preceding it by adding to it a fixed quantity called COMMON DIFFERENCE.

Ex. 1, 5, 9, 13 ....;   1, –2, –5, –8, ...;

a, a + d, a + 2d, a + 3d, are in A.P.
If in an A.P. a = the first term, d = common difference, T_n = the nth term, *l* = the last term, S_n = Sum of n terms, We have $\mathbf{T_n = a + (n - 1)\,d}$ $\mathbf{S_n = n/2\,(a + \mathit{l})}$ $\mathbf{S_n = n/2\,[2a + (n - 1)\,d]}$

Arithmetic Mean :

If a, m, b are in A.P., m is called the Arithmetic mean between a and b and is given by **m = (a + b)/2**

Some General formulae : (to be crammed thoroughly)

1. Sum of first n natural numbers = $\frac{n(n+1)}{2}$
2. Sum of first n odd natural numbers = n^2
3. Sum of first n even natural numbers = n (n + 1)
4. Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$
5. Sum of cubes of first n natural numbers = $\frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2$

GEOMETRIC PROGRESSION (G.P.)

A series in which each term is formed from the preceding by multiplying it by a constant factor is called a Geometric Progression or G.P. The constant factor is called the common ratio and is formed by dividing any term by the term which precedes it. Ex.: 1,2,4,8,16.....; 3,9,27,81,243.... etc.

The General form of a G.P. with n terms is a, ar, ar^2, ... ar^{n-1}
Thus if a = the first term, r = the common ratio,
T_n = the nth term and S_n = the sum of n terms,

We have : $\mathbf{T_n = ar^{n-1}}$ $S_n = \dfrac{a(1-r^n)}{1-r}$ where r < 1 $S_n = \dfrac{a(r^n-1)}{r-1}$ where r > 1

If a G.P. has infinite terms and –1 < r < 1, the sum to infinity (S_∞) is : $S_\infty = \dfrac{a}{1-r}$

NOTE

1. Three numbers in an A.P. should be taken as a – d, a, a + d
2. Four numbers in an A.P. should be taken as a – 3d, a – d, a + d, a + 3d
3. Three numbers in a G.P. should be taken as a/r, a, ar
4. Four numbers in a G.P. should be taken as a/r^3, a/r, ar, ar^3

HARMONIC PROGRESSION (H.P.)

A series of quantities is said to be in harmonic progression when their reciprocals are in A.P.
Ex. 1/3, 1/5, 1/7, ... 1/a, 1/(a + d), 1/(a + 2d) .....
1/2, –1/3, –1/8, –1/13 are in H.P. as their reciprocals are in an AP.

Ex 1. *Find the sum of all even numbers from 10 to 200 inclusive, excluding those which are multiples of 6.*
Sol. a = 10, l = 200, n = 96 for even numbers a = 12, l = 198, n = 32 for multiples of 6
{ 96/2 (10 + 200) } – { 32/2 (12 + 198) } = 10080 – 3360 = 6720
⇒ The sum is 6720 Ans.

Ex 2. *A person saves each year Rs 100 more than he saved in the preceding year, and he saves Rs 200 the first year. How many years would it take for his savings, not including interest, to amount to Rs 23000 ?*
Sol. Let n be the number of years required ⇒ a = 200, d = 100
n/2 x [2(200) + (n – 1) 100] = 23000 ⇒ $n^2 + 3n - 460 = 0$
⇒ (n – 20)(n + 23) = 0 ⇒ n = 20 or n = –23 (rejected)
⇒ The number of years is 20 Ans.

Ex 3. *A man receives Rs 60 the first week and Rs 3 more each week than the preceding. What does he get in the 20th week ? And what is the total amount received in 20 weeks ?*
Sol. a = 60, d = 3, n = 20 ⇒ $T_n = a + (n-1)d$
⇒ $T_{20} = 60 + (20-1)(3) = 117$ ⇒ $S_n = n/2\,(a + l)$
⇒ $S_{20} = 20/2\,(60 + 117) = 1770$
⇒ He receives Rs 117 in the 20th week and the total amount is Rs 1770.

Ex 4. *A and B set out to meet each other from two places 165 km apart. A travels 15 km the first day, 14 km the second, 13 km the third and so on. The other travels 10 km the first day, 12 km the second, 14 km the third and so on. When will they meet ?*

Sol. Suppose they meet in n days.

Dist travelled by A in n days = $n/2\,[2(15) + (n-1)(-1)] = n/2\,(31 - n)$

Dist travelled by B in n days = $n/2\,[2(10) + (n-1)(2)] = n/2\,(18 + 2n)$

$\Rightarrow n/2\,(31 - n) + n/2\,(18 + 2n) = 165 \quad \Rightarrow (n + 55)(n - 6) = 0$

$\Rightarrow n = -55$ (rejected) or $n = 6$ $\Rightarrow$ They will meet in 6 days Ans.

Ex 5. *A man's salary is Rs 800 per month in the first year and is promised to have an increment of Rs 40 per month at the end of each year's service. How many years does it take him to earn a total of Rs 64800 ?*

Sol. Let n be the time in years

$\Rightarrow n/2\,[2(800 \times 12) + (n-1)(40 \times 12)] = 64800$

$\Rightarrow n^2 + 39n - 270 = 0 \quad \Rightarrow (n + 45)(n - 6) = 0 \quad \Rightarrow n = 6 \Rightarrow$ The time is 6 years Ans.

Ex 15. *In a certain colony of cancerous cells, each cell divides into two every hour. How many will be produced from a single cell if the rate of division continues for 10 hours ?*

Sol. 1, 2, 4, ... $a = 1, r = 2, n = 10$

$S_{10} = 1\,(2^{10} - 1)/(2 - 1) = 2^{10} - 1 = 1023$

There are 1023 cells. Ans.

QUADRATIC EQUATIONS

A Quadratic Equation in X is one in which the highest power of X is 2. The equation is generally satisfied by two values of X, but these values may be equal to each other.

The quadratic form is generally represented by $aX^2 + bX + C = 0$ where $a \neq 0$, and a, b, c are constants.

e.g. $X^2 + 4X - 12 = 0$

$3X^2 - 3X + 2 = 0$

The values of the unknown quantity (X) for which the equation is satisfied are called its roots and the process of finding the roots is called solving the equation.

SOLUTION OF QUADRATIC EQUATIONS :

1. **Factorization method :**

Ex. Solve $X^2 - 4X + 3 = 0$

Sol. We have $X^2 - 4X + 3 = (X - 1)(X - 3) = 0$

$\Rightarrow$ either $(X - 1) = 0$ or $(X - 3) = 0$

$\Rightarrow X = 1$ and $X = 3$ Ans.

2. **General Method :**

The general solution to $AX^2 + BX + C = 0$ is given as $X = \dfrac{-B \pm \sqrt{(B^2 - 4AC)}}{2A}$

Ex. Solve $2X^2 - 7X + 6 = 0$

Sol. Here we have $A = 2$, $B = -7$, $C = 6 \Rightarrow X = [-(-7)^2 \pm (4 \times 2 \times 6)] / (2 \times 2)$

$\Rightarrow X = [7 \pm \sqrt{(49 - 48)}] / (2 \times 2) = (7 \pm 1)/4$

Now $X = (7 + 1)/4 = 8/4 = 2$ [taking + sign]

AND $X = (7 - 1)/4 = 3/2$ [taking – sign]

Hence $X = 2, 3/2$ Ans.

If X_1 and X_2 are the two roots of $aX^2 + bX + C = 0$ then (sum of roots) $= X_1 + X_2 = -b/a$

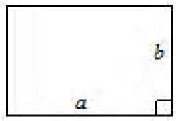
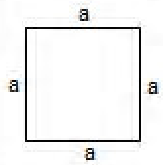
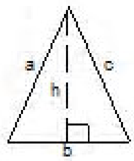
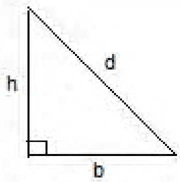
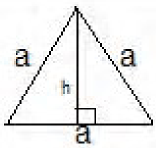
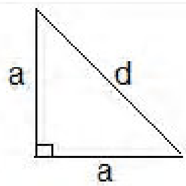
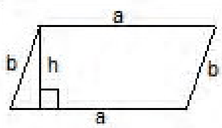
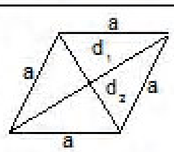
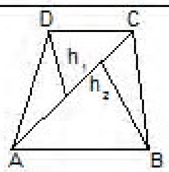
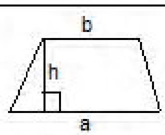
and product of roots $= X_1 X_2 = c/a$

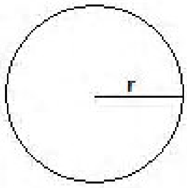
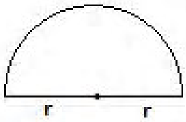
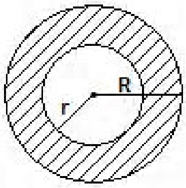
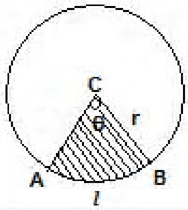
1. If X_1 and X_2 are the two roots then $(X - X_1)(X - X_2) = 0$ is the required equation

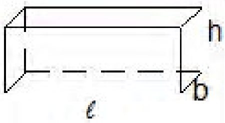
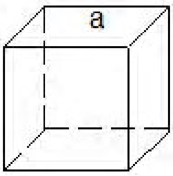
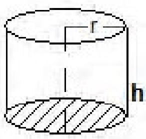
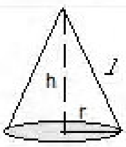
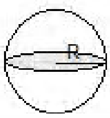
2. If $(X_1 + X_2)$ and $X_1 . X_2$ are given then equation is $X^2 - (X_1 + X_2) X + X_1 X_2 = 0$

$\Rightarrow X^2 - SX + P = 0$ where S = sum of roots, P = product of roots.

GEOMETRY (FROMULA SHEET)

Name	Figure	Perimeter	Area	Nomenclature
Rectangle		2 (a + b)	ab	a = length b = breadth
Square		4a	a^2	a = side
Triangle		a + b + c = 2s	½ b x h $\sqrt{s(s-a)(s-b)(s-c)}$	b is the base and h is the altitude. a, b, c are three sides of Δ. s is the semiperimeter
Right triangle		b + h + d	½ bh	d (hypotenuse) = $\sqrt{b^2+h^2}$
Equilateral triangle		3a	½ ah $\frac{\sqrt{3}}{4}a^2$	a = side h = altitude = $\frac{\sqrt{3}}{2}$ a.
Isosceles right triangle		2a + d	½ a^2	d (hypotenuse) = a√2, a = Each of equal sides.
Parallelogram		2 (a + b)	ah	a = side b = side adjacent to a h = distance between the parallel sides (a) and (b)
Rhombus		4a	½ d_1 d_2	a = side of rhombus d_1, d_2 are the two diagonals.
Quadrilateral		Sum of its four sides	½ (AC) (h_1 +h_2)	AC is one of its diagonals and h_1,h_2 are the altitutdes on AC from D, B respectively.
Trapezium		Sum of its four sides	½ h (a + b)	a, b are parallel sides and h is the distance between parallel sides.

Circle		$2\pi r$	πr^2	r = radius of the circle π = 22/7 or 3.1416
Semicircle		$\pi r + 2r$	$\frac{1}{2} \pi r^2$	r = radius of the circle
Ring (shaded region)		----	$\pi (R^2 - r^2)$	R = outer radius r = inner radius
Sector of a circle		$l + 2r$ where $l = \theta/360 \times 2\pi r$	$\theta/360 \times \pi r^2$	$\theta°$ = central angle of the sector r = radius of the sector l = length of the arc

S. No.	Name	Figure	Lateral/curved surface area	Total surface area	Volume	Nomenclature
1.	Cuboid		$2h(l + b)$	$2(lb + bh + lh)$	lbh	l=length b=breadth h=height
2.	Cube		$4a^2$	$6a^2$	a^3	a = edge
3.	Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$	r-radius of base h=height of the cylinder
	Right circular cone		$\pi r l$	$\pi r(l + r)$	$1/3.\pi r^2 h$	h = height r=radius l=slant height
	Sphere		$4\pi r^2$	$4\pi r^2$	$4/3.\pi r^3$	r= radius

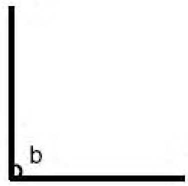
General Notation for Geometry:

A = area, P = perimeter, C = circumference
V = volume, H = Height, hyp = Hypotenuse

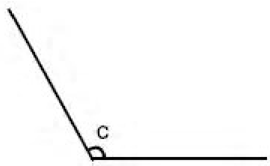
ANGLES and PARALLELS

(a) Two straight lines which meet at a point form an angle between them.

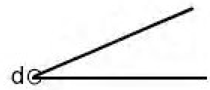
Acute angle : 0° < a < 90°



Right angle : b = 90°



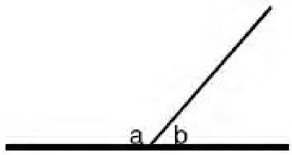
Obtuse angle : 90° < c < 180°



Reflex angle : 180° < d < 360°

(b) Theorems :

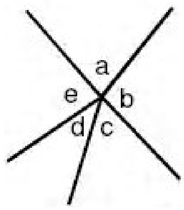
1. If AOB is a straight line, then



$a + b =$ **180°**

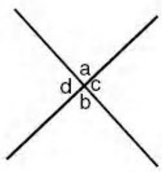
(Adjacent angles on a straight line)

2. The sum of all the angle at a point, each being adjacent to the next, is 4 right angles.



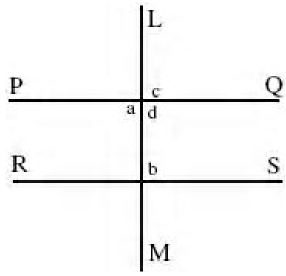
$\Rightarrow a + b + c + d + e =$ **360°** ***(Angles at a point)***

3. If two straight lines intersect, the vertically opposite angles are equal.



$a = b$, $c = d$ ***(Vertically Opposite angles)***

4. Parallel lines PQ and RS are cut by a transversal LM, then we have :



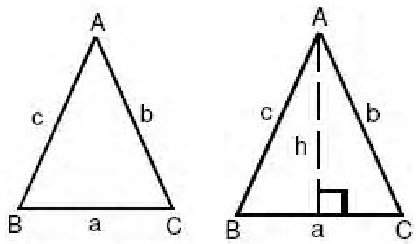
The corresponding angles are equal c = b ***(Corresponding angles, PQ || RS)***

The alternate angles are equal a = b ***(Alternate angles, PQ || RS)***

The interior angles are supplementary b + d = 2 right angles ***(Interior angles, PQ || RS)***

5. (i) Two angles whose sum is 90°, are complementary. Each one is the complement of the other.
(ii) Two angles whose sum is 180º, are supplementary. Each one is the supplement of the other.

TRIANGLES :



A closed figure enclosed by 3 sides is called a Triangle. ABC is a triangle. The sides AB, BC, AC are respectively denoted by c, a, b. Please carefully note the capital and small letters.

In any triangle ABC

(1) A = ½ a x h = ½ base x perpendicular to base from opposite vertex

(2) A = $\sqrt{s(s-a)(s-b)(s-c)}$, s = (a + b + c)/2 = semi–perimeter

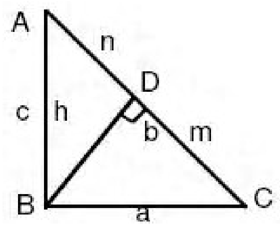
(3) P = (a + b + c) = 2 s

TRIANGLES

PROPERTIES :

1. Sum of the three interior angles is 180°
2. When one side is extended in any direction, an angle is formed with another side. This is called the exterior angle.
3. There are six exterior angles of a triangle.
4. Interior angle + corresponding exterior angle = 180°.
5. An exterior angle = Sum of the other two interior angles not adjacent to it
6. Sum of any two sides is greater than the third side.
7. Difference of any two sides is less than the third side.
8. Side opposite to the greatest angle will be the greatest and vice versa.
9. A triangle must have at least two acute angles.
10. Triangles on equal bases and between the same parallels have equal areas.
11. If a, b, c denote the sides of a triangle then
    (i) if $c^2 < a^2 + b^2$, Triangle is acute angled
    (ii) if $c^2 = a^2 + b^2$, Triangle is right angled
    (iii) if $c^2 > a^2 + b^2$, Triangle is obtuse angled

Right Angled Triangle :



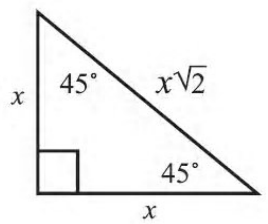
A triangle whose one angle is 90° is called a right (angled) Triangle. In the figure, b is the hypotenuse,
and a & c the legs, called base and height resp.

(1) $h^2 = m\,n$
(2) $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)
(3) $h = ac/b$
(4) Area = $ac / 2$

NOTE : *You should remember some of the Pythagorean triplets (e.g. 3,4,5 because $5^2 = 3^2 + 4^2$). Some others are (5, 12, 13), (7, 24, 25) etc.*

Exercise : Try to find out at least three other sets of Pythagorean Triplets. But be clear that we are talking about distinct triplets. Thus if 3,4,5 is a Pythagorean triplet, then it does not mean that (3 x 10), (4 x 10), (5 x 10) is a distinct triplet.

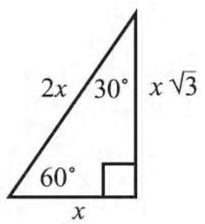
Isosceles Right Triangle :



A right triangle whose two legs are equal is an isosceles right triangle. The ratio of sides is $1 : 1 : \sqrt{2}$

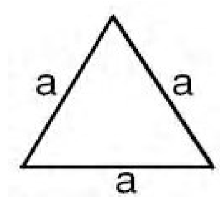
30, 60, 90 triangle :

This is a special case of a right triangle whose angles are 30°, 60°, 90°.



In this triangle side opposite to angle 30° = HYP/2, Side opposite to Angle 60° = $\sqrt{3}/2$ x HYP. The ratio of sides is $1 : \sqrt{3} : 2$.

Equilateral Triangle :



A triangle whose all sides are equal is called an equilateral triangle. If a be the side of an equilateral triangle, then

1. Area $= \frac{\sqrt{3}}{4}a^2$ Altitude = h = $\frac{\sqrt{3}}{2}a$

2. Given the perimeter, equilateral triangle has the maximum area.

3. Of all the triangles that can be inscribed in a circle, the equilateral triangle has the greatest area.

4. **Area of Outer Circle is 4 times the area of Inner Circle**

5. The triangle formed by joining the mid-points of the sides will be half in perimeter and one-fourth in area.

6. For similar figures and solids, $\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$ and for similar solids $\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

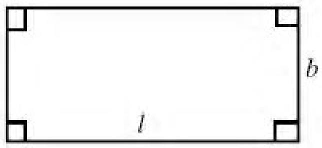
Isosceles Triangle : A triangle with 2 sides (and two angles) equal is called isosceles.

QUADRILATERALS :

A closed figure (plane) bounded by four sides is called a quadrilateral. In any quadrilateral :
A = ½ x one diagonal x (sum of perpendiculars to it from opposite vertices).

(a) Sum of the four interior angles = 360°
(b) If a quadrilateral can be inscribed in a circle, it is called a cyclic quadrilateral. Here opposite angles are supplementary.
If one side is produced, then the exterior angle = Remote interior angle.

Rectangle :

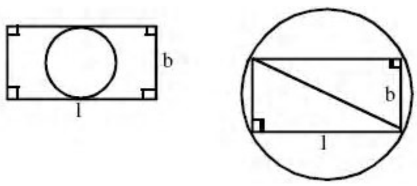


A quadrilateral whose opposite sides are equal and each internal angle equal to 90°, is called a rectangle.

l = length b = breadth A = l x b P = 2 (l + b)

Diagonal = $\sqrt{(l^2 + b^2)}$, diagonals are equal and bisect each other.

(a) Opposite sides equal, each angle = 90°
(b) Diagonals bisect each other (not at 90°).
(c) Of all rectangles of given perimeter, a square has max. area
(d) When inscribed in a circle, it will have maximum area when it's a square.
(e) Figure formed by joining the midpoints of a rectangle is a rhombus.
(f) The biggest circle that can be inscribed in a rectangle will have the diameter equal to the breadth of the rectangle.



(g) When a rectangle is inscribed in a circle, the diameter of the circle is equal to the diagonal of the rectangle.

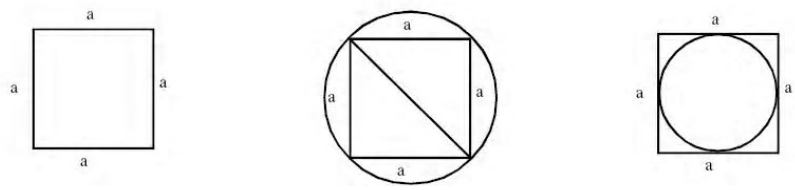
Parallelogram

A quadrilateral in which opposite sides are equal and parallel is called a parallelogram.

$A = bh$ $P = 2(a + b)$, Diagonals bisect each other.

(a) Opposite angles are equal
(b) Diagonals bisect each other (not at 90°)
(c) Sum of any two adjacent angles = 180°
(d) Diagonals are unequal in lengths and do not bisect angles at vertices.

Square



A quadrilateral whose all sides are equal and all angles 90° is called a square.

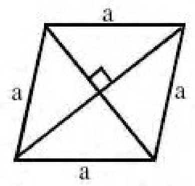
Area = a^2 Diagonal = $a\sqrt{2}$ Area = $\frac{\text{Diagonal}^2}{2}$ P = 4 x side

(a) Diagonals bisect each other at 90° and are equal
(b) When inscribed in a circle, diagonal = diameter of circle
(c) When circumscribed about a circle, Side of square = Diameter of circle.
(d) The figure formed by joining the mid-points of the sides of a square is also a square. In this case the side will become $\frac{1}{\sqrt{2}}$ times the side of the original square, perimeter will become $\frac{1}{\sqrt{2}}$ times the perimeter of the original square and area will become $\frac{1}{2}$ times the area of the original square.

(e) If a square (biggest possible) is inscribed in a circle of radius r, then Diameter of circle = Diagonal of Square. If the area of the circle is A then $A = \pi r^2 = \frac{\pi D^2}{4}$ or $\frac{D^2}{2} = \frac{2A}{\pi}$ or Area of Square $= \frac{2A}{\pi}$ or $2r^2$.
(f) If a circle is inscribed in a square of side a, then **Side of square = Diameter of circle.** If the area of the square is S, then the area of circle $= \frac{\pi S}{4} = \frac{\pi a^2}{4}$.

Rhombus

A parallelogram whose all sides are equal is called a rhombus.

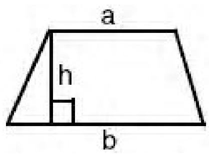


$A = \frac{1}{2} \times D_1 \times D_2$ (D_1, D_2 are diagonals) $P = 4a$

Diagonals are at right angles $D_1^2 + D_2^2 = 4a^2$

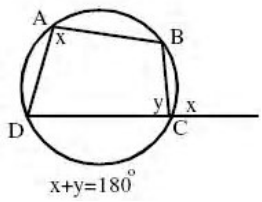
(a) Opposite angles are equal
(b) Diagonals bisect each other at 90°.
(c) Sum of any two adjacent angles = 180°.
(d) Diagonals are unequal.

Trapezium (Trapeziod)



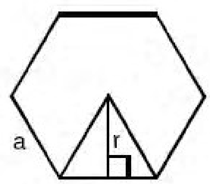
A quadrilateral in which one pair of opposite sides is paralel is a trapezium. $A = \frac{1}{2} \times (a + b) \times h$

CYCLIC QUADRILATERAL :



1. The four vertices lie on a circle.
2. Opposite angles are supplementary.
3. If any one side is produced, Exterior angle = Remote interior angle

REGULAR POLYGONS :
A many sided closed figure is called a polygon. If all the sides of a polygon are equal, it is called a regular polygon.

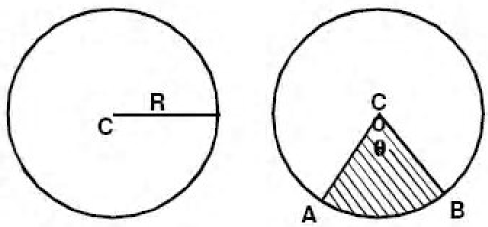


1. Interior Angle + Exterior angle = 180°
2. P = n x a (n = number of sides, a = side)
3. Sum of exterior angles = 360°
4. Sum of Interior angles = (2n – 4) 90°
5. Each Interior angle = [(2n – 4)/n] x 90°
6. Each exterior angle = 360/n
7. A = ½ P r = ½ n a r
   P = perimeter (r = ⊥ from centre to any one side = radius of incircle)

Regular Hexagon :

1. Each interior angle = 120°
2. Sum of Interior angles = 720°
3. Each exterior angle = 60°
4. Area = $(3\sqrt{3}/2) \times a^2$ (a = side)
5. P = 6a

Circle :
A circle is the path traversed by a point which moves in such a way that its distance from a fixed point always remains constant.



C is called centre and R the radius of the circle. Circumference = $2\pi R = \pi D$

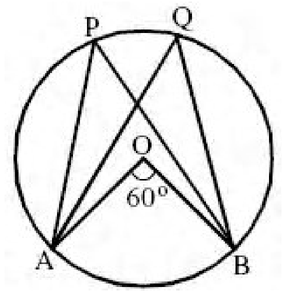
1. Area = $A = \pi R^2 = \pi D^2/4$ (D = Diameter = 2R) $\pi = 22/7$ or 3.14

2. Length of arc of a circle is given by $\ell(AB) = (\theta°/360°) \times 2\pi R$

3. Area of Sector ABC = $(\theta/360) \times \pi R^2 = \frac{1}{2}\, l\,(AB) \times R$

4. Distance travelled by a wheel in n revolutions = n x circumference

5. Angle at the centre made by an arc = twice the angle made by the arc at any point on the remaining part of the circumference.

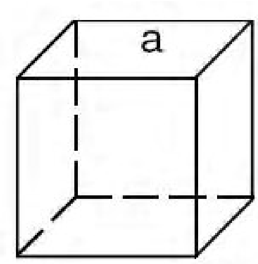


We have $\angle APB = \frac{1}{2} \angle AOB = 30° = \angle AQB$

6. Angle in a semicircle is a right angle.

SOLIDS

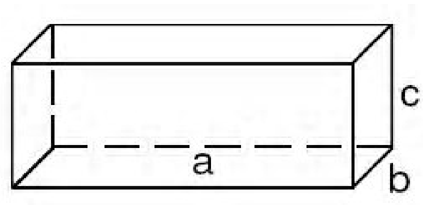
CUBE :
A six–faced solid figure with all faces equal and adjacent faces mutually perpendicular is a cube.



If “a” be the edge of a cube,

1. The longest diagonal = $a\sqrt{3}$ The face diagonal = $a\sqrt{2}$
2. Volume = a^3
3. Total surface area = $6\ a^2$

CUBOID or RECTANGULAR BOX :

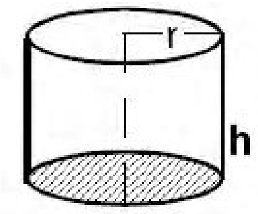


If a,b,c are the edges of a box,

1. The longest diagonal = $\sqrt{(a^2 + b^2 + c^2)}$
2. Surface area = 2 (ab + bc + ac)
3. Volume = abc

RIGHT CIRCULAR CYLINDER :

If r is the radius of base and h is the height, then

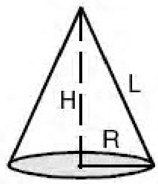


1. Volume = $\pi r^2 h$
2. Curved surface area = $2\pi rh$
3. Total surface area = $2\pi r(r + h)$

4. If a rectangle of length L and breadth B is rotated about its length to form a cylinder, then $L = 2\pi R$ and $B = h$.

5. If a rectangle of length L and breadth B is rotated about its breadth to form a cylinder, then $B = 2\pi R$ and $L = h$.

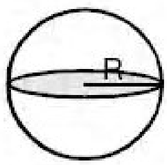
RIGHT CIRCULAR CONE :



R = radius of base   H = Height   L = slant height = $\sqrt{(H^2 + R^2)}$

1. Volume = $1/3 \times (\pi R^2 H)$
2. Curved surface Area = $\pi R L$
3. Total Surface Area = $\pi R (R + L)$

SPHERE :



R = Radius

1. Volume = $4/3 \times \pi R^3$
2. Surface Area = $4 \pi R^2$

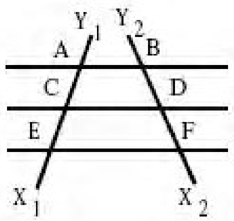
SIMILAR PLANE FIGURES & SOLIDS

1. If A_1 & A_2 denote the areas of two similar figures, and l_1 & l_2 denote their corresponding linear measures, then $A_1 / A_2 = (l_1 / l_2)^2$

2. If V_1 & V_2 denote the volumes of two similar solids, and l_1, l_2 denote their corresponding linear measures, then $V_1 / V_2 = (l_1 / l_2)^3$

GENERAL THEOREMS ON SIMILARITY

1. Proportionality Theorem :

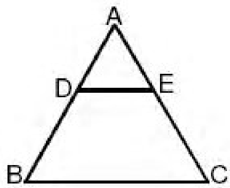


Intercepts made by two transversal lines (cutting lines) on three or more parallel lines are proportional. In the figure, lines X_1Y_1 & X_2Y_2 are transversals cutting the three parallel lines AB, CD, EF. Then AC, CE, BD, DF are intercepts

Also, AC/BD = CE/DF

2. Midpoint Theorem :
A triangle, the line joining the mid points of two sides is parallel to the third side and half of it.

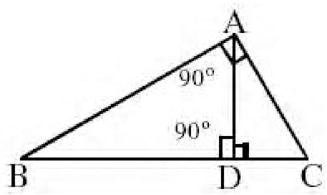
3. Basic Proportionality Theorem :



A line parallel to any one side of a triangle divides the other two sides proportionally. If DE is parallel to BC, then
(a) AD/BD = AE/EC,
(b) AB/AD = AC/AE,
(c) AD/DE = AB/BC and so on.

PROPERTIES OF SIMILAR TRIANGLES :

(1) Ratio of areas = Ratio of squares of corresponding sides
(2) RIGHT TRIANGLE :



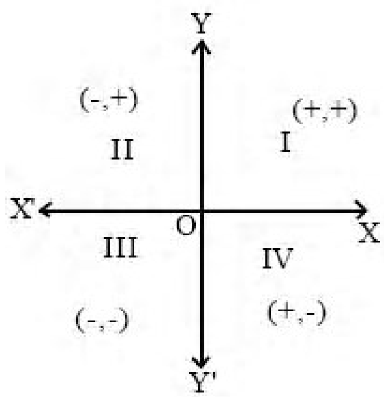
ABC is a Right Triangle with A as the Right angle.
AD is perpendicular to BC then
(a) Triangle ABD ~ Triangle CBA & $BA^2 = BC \times BD$
(b) Triangle ACD ~ Triangle BCA & $CA^2 = CB \times CD$
(c) Triangle ABD ~ Triangle CAD & $DA^2 = DB \times DC$.

Some More Important Points:

1. If with a given perimeter, different figures are formed like equilateral triangle, square, regular hexagon, regular octagon ...... and eventually a circle (a regular polygon of infinite sides), then the triangle will have the minimum area and circle will have the maximum area.
2. If different triangles are inscribed in a circle, then the equilateral triangle will have the maximum area.
3. If the perimeter of a triangle is fixed, then the equilateral triangle will have the maximum area.
4. If the sum of two sides of a triangle is constant, then the isosceles right angled triangle will have the maximum area.

Co-ordinate Geometry Basics

Co-ordinate System: Sign Convention and Quadrants:



1. DISTANCE BETWEEN TWO POINTS : If there are two points A (X_1, Y_1) and B (X_2, Y_2) on the XY plane, the distance between them is given by AB = d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
   Distance of the point P(x, y) from the origin O is **OP = √(x^2 + y^2)**
2. The co–ordinates of P(X,Y) such that P divides the line joining A (X_1, Y_1) and B (X_2, Y_2) iternally in the ratio m : n will be : $X = \frac{mX_2 + nX_1}{m+n}, Y = \frac{mY_2 + nY_1}{m+n}$
3. If P (X,Y) is the midpoint between A & B then X = (X_1 + X_2) /2 , Y = (Y_1 + Y_2) /2

EQUATION OF A CURVE :

An equation in two variables X and Y (with or without a constant term) is called the equation of a particular curve if the graph of that equation plotted on the XY cartesian plane give that particular curve. e.g. $X^2 + Y^2 = 36$. If we plot this curve, by taking different values of Y (and thereby different values of X), we get a circle. Hence $X^2 + Y^2 = 36$ is the equation of a circle.

STRAIGHT LINE :

An equation of the form AX + BY + C = 0 is called the general equation of a straight line, where X and Y are variables and A, B, C are constants.

Any point lying on this line will satisfy the equation of the line. i.e. the coordinates of the point when substituted by X & Y resp. in the above equation will make the LHS vanish.

GENERAL CONCEPTS

If AB is a straight line on the XY plane, then the ratio of y intercept to x intercept (with signs) is called its slope and is denoted by 'm'. The lengths OP and OQ are respectively called the intercepts on X and Y axes, made by the line.

So slope = RISE / RUN

Also, slope = DIFFERENCE OF Y COORDINATES / DIFERENCE OF X COORDINATES

For convenience, same straight line can be represented by many different forms of the same equation. The different forms are given below one by one.

1. If 'm' is the slope of the line and 'c' the intercept made by the line on Y axis, the equation is $Y = mX + c$

2. If the slope of the line is m and it passes through (X_1, Y_1), the equation is $(Y - Y_1) = m(X - X_1)$

3. If the line passes through two points (X_1, Y_1) and (X_2, Y_2), the equation is

$$Y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(X - X_1)$$

Hence the slope of the line through (X_1, Y_1), (X_2, Y_2) is given by $m = (Y_2 - Y_1)/(X_2 - X_1)$ or $(Y_1 - Y_2)/(X_1 - X_2)$

4. If the X intercept of the line is a and Y intercept is b, the equation is : $X/a + Y/b = 1$

5. General form : $AX + BY + C = 0$ In this slope = $m = -A/B$
X intercept = $-C/A$ Y intecept = $-C/B$
(a) If two lines are parallel then their slopes are equal ($m_1 = m_2$). If two lines do not intersect, they are parallel.
(b) If two lines are perpendicular to each other, the product of their slopes is -1. ($m_1 m_2 = -1$).
(c) $a_1X + b_1Y + c_1 = 0$ and $a_2X + b_2Y + c_2 = 0$ will represent the same straight lines if $a_1/a_2 = b_1/b_2 = c_1/c_2$. In this case, the lines are coincident and theoretically intersect at infinite points.

7. The point of intersection of two lines (X,Y) is obtained by simultaneously solving both the equations.

8. The equation of a line parallel to a given line $AX + BY + C = 0$, will be $AX + BY + K = 0$ where K is a constant which can be found by any additional given condition.

9. The equation of a line perpendicular to a given line $AX + BY + C = 0$ will be $BX - AY + K = 0$, where K is a constant which can be found by an additional given conditions.

10. The length of perpendicular (p) from (X_1, Y_1) on the line $AX + BY + C = 0$ is :

$$P = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

11. Equation of a line parallel to X axis is $Y = b$ (b is a constant)

12. Equation of a line parallel to Y axis is $X = a$ (a is a constant)

13. Equation of X and Y axes are $Y = 0$ and $X = 0$ respectively

14. Any point on the X axis can be taken as (a,0)

15. Any point on the Y axis can be taken as (0,b)

16. In order to find the X–intercept of a line, put Y = 0 in the equation of the line and find X

17. In order to find the Y intercept of a line, put X = 0 and find Y

18. The image of the point (a, b) in x axis is (a, -b)

19. The image of the point (a, b) in y axis is (-a, b)

20. The image of the point (a, b) in the line y = x is (b, a)

21. To plot a line, first put y = 0, find the point on x axis; then put x = 0, fins the point on y axis. Join the two points to get the desired graph.

<u>Note :</u>

The equation of a circle with centre (h, k) and radius r is $(x – h)^2 + (y – k)^2 = r^2$.

The equation of a circle with centre (0, 0) and radius r is $x^2 + y^2 = r^2$.

The equation $y = Ax^2 + Bx + C$ is the equation of the quadratic graph which is a parabola with axis parallel to y axis. If A > 0, the parabola opens upwards. If A < 0, the parabola opens downwards.

If $B^2 > 4AC$, the parabola cuts the x-axis at 2 different points.

If $B^2 = 4AC$, the parabola touches the x-axis at one point (the two points become co-incident).

If $B^2 < 4AC$, the parabola does not cut the x-axis at all.

Examples:

1. *Find the ratio in which the line segment joining the points P(–2,3) and Q (3,5) is divided by the Y–axis. Find also the Y–coordinate of the point of division.*

Sol. Let the line segment cut at the point R(0,Y), then 0 = (–2 + 3r)/(1 + r) r = 2/3
Also Y = (3 + 2/3 (5)) /(1 + 2/3) = $3^4/_5$ ⇒ The required ratio is 2 : 3. Ans.
The Y–coordinate of the point of division is $3^4/_5$ Ans.

2. *Given three points A(–1,0), B(2,–1), C(3,2), show that AB is perpendicular to BC.*

Sol. Slope of AB = (–1 – 0) /(2 – (–1)) = –1/3 Slope of BC = (2 – (–1)) /(3 – 2) 3
⇒ Product of slope = (–1/3) (3) = –1 ⇒ AB ⊥ BC Ans.

3. *If the points A(–1,1), B(5/2,3/2), C(2,5) are the three vertices of a parallelogram :*
(a) find the fourth one. *(b) prove that ABCD is a square.*

Sol. (a) Let D(X,Y) be the required point.
Coordinates of the mid–point M(X_1, Y_1) of AC is given by
X_1 = (–1 + 2) /2 = 1/2 Y_1 = (1 + 5)/2 = 3. Thus the mid–point of BD = M (X_1, Y_1)
⇒ X_1 = 1/2 = (5/2 + X)/2 ⇒ X = –2.5. Y_1 = 3 = (3/2 + Y)/2 ⇒ Y = 4.5
Thus the fourth point is (–2.5, 4.5)
(b) For square just check that 1. Sides are equal and 2. Diagonals are equal Ans.

4. *The equation of the straight line is $-2X + 5Y + 7 = 0$. Find :*

(a) the slope of the line, *(b) the X–intercept of the line and*

(c) the y–intercept of the line.

Sol. By general form $AX + BY + C = 0$ where $A = -2$, $B = 5$, $C = 7$

(a) The slope $m = -A/B = -(-2/5) = 2/5$ Ans.

(b) The X–intercept $= -C/A = -(7/-2) = 7/2$ Ans.

(c) The Y–intercept $= -C/B = -(7/5) = -7/5$ Ans.

5. *(a) Find the equation of the line L passing through the point P_1 (3,–2) and parallel to the line $L_1 : 2X - 3Y - 4 = 0$.*

(b) If (2,p) is on the line L, find the value of p.

Sol. (a) Convert L_1 to slope–intercept form $\Rightarrow 2X - 3Y - 4 = 0$, $3Y = 2X - 4$, $Y = 2/3\ X - 4/3$

$\Rightarrow$ Slope of $L_1 = 2/3$ Slope of L = slope of $L_1 = 2/3$

By point–slope form, equation of L is $Y + 2 = 2/3\ (X - 3) \Rightarrow 2X - 3Y - 12 = 0$ Ans.

(b) Since (2,p) is one the line L, (2,p) satisfies the equation of L.

$\Rightarrow 2(2) - 3p - 12 = 0 - 3p = 8\ p = -8/3$ Ans.

6. *Find the equation of the line L passing through the point P_1 (–1,3) and perpendicular to the line $L_1 : 5X - 2Y + 3 = 0$*

Sol. $L_1 : 5X - 2Y + 3 = 0\ Y = 5/2\ X - 3/2 \Rightarrow$ slope of $L_1 = m_1 = 5/2$

$\Rightarrow$ Slope of L . $m_1 = -1$ ($L \perp L_1$) $\Rightarrow$ Slope of L $= -2/5 \Rightarrow P_1$ (–1,3) is on the line L.

$\Rightarrow$ By point–slope form, equation of L is $(Y - 3) = -2/5\ (X + 1)$. i.e. $2X + 5Y - 13 = 0$ Ans.

7. *Find the equation of the line L perpendicular to the line $L_1 : X - 3Y - 1 = 0$ and having X–intercept same as L_1.*

Sol. $L_1 : X - 3Y - 1 = 0 \Rightarrow Y = 1/3\ X - 1/3 \Rightarrow$ slope of $L_1 = m_1 = 1/3$

$\Rightarrow$ slope of L . $m_1 = -1$ ($L \perp L_1$) $\Rightarrow$ slope of L $= -3$ Put Y = 0 into L_1,

$X - 3(0) - 1 = 0\ X = 1 \Rightarrow$ X–intercept of $L_1 = 1$

Hence, eqn of L is $Y - 0 = -3(X - 1) \Rightarrow 3X + Y - 3 = 0$ Ans.

Permutations and Combinations, Probability

Definitions :

1. Each of the different orders of arrangements, obtained by taking some, or all, of a number of things, is called a **Permutation.**

2. Each of the different groups, or collections, that can be formed by taking some, or all, of a number of things, irrespective of the order in which the things appear in the group, is called a **Combination.**

Example : Suppose, there are four quantities A,B,C,D. The different orders of arrangements of these four quantities by taking three at a time, are :

ABC, ACB, BAC, BCA, CAB, CBA, ... (1)
ABD, ADB, BAD, BDA, DAB, DBA, ... (2)
ACD, ADC, CAD, CDA, DAC, DCA, ... (3)
BCD, BDC, CDB, CBD, DBC, DCB. ... (4)

Thus, each of the 24 arrangements, of the four quantities A,B,C,D by taking three at a time, are each called a permutation. Hence, it is clear that the number of permutations of four things taken three at a time is 24.

Again, it may be easily seen, from the above that out of these 24 permutations, the six, given in (1), are all formed of the same three quantities A,B,C in different orders; hence, they all belong to the same group. Similarly, the permutations, given in (2), all belong to a second group; those given in (3), belong to a third and those in (4), belong to a fourth. Hence, we see that **there are only four different groups** that can be formed of four quantities A,B,C,D by taking three at a time.
Thus, the number of combinations of four things taken three at a time is only four.

Note:
It may be observed that the total number of permutations, as given by (1), (2), (3), (4), may be supposed to have been obtained by either :
(1) forming all possible different groups and then re–arranging the constituents of each of these groups in different orders in all possible ways; or

(2) filling up three places by means of the four quantities A,B,C,D in all possible ways.

If there are **m** ways of doing a thing and **n** ways of doing a second thing and **p** ways of doing a third thing, then the total number of "distinct" ways of doing all these together is **m x n x p.**

Meaning of factorial :
Factorial of a number (whole number only) is equal to the product of all the natural numbers upto that number.

Factorial of n is written as $\lfloor n$ or n! and is read as factorial n.

Hence $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
$n! = n\,(n-1)\,(n-2) \times \ldots\ldots 3 \times 2 \times 1$

NOTE : 1. $0! = 1$ (by definition)
2. ${}^{n}P_{r} = n!/(n-r)!$ where $r \le n$
3. ${}^{n}C_{r} = n!/[(n-r)!\,r!]$ where $r \le n$

PERMUTATIONS

1. Permutations of n different things taken 'r' at a time is denoted by ${}^{n}P_{r}$ and is given by $\mathbf{{}^{n}P_{r} = n!/(n-r)!}$

2. The total number of arrangements of n things taken r at a time, in which a particular things always occurs = $\mathbf{r \times {}^{n-1}P_{r-1}}$

3. The total number of permutations of n different things taken r at a time in which a particular thing never occurs = $\mathbf{{}^{n-1}P_{r}}$

4. The total number of permutations of n dissimilar things taken r at a time with repetitions = n^{r}

5. (a) No. of circular permutations of n things taken all at a time = $(n-1)!$
   (b) No. of circular permutations of n different things taken r at a time = ${}^{n}P_{r}/r$

7. The number of permutations when things are not all different : If there be n things, p of them of one kind, q of another kind, r of still another kind and so on, then the total number of permutations is given by $n!/(p!\,q!\,r!...)$

COMBINATIONS

1. Number of combinations of n dissimilar things taken 'r' at a time is denoted by ${}^{n}C_{r}$ & is given by ${}^{n}C_{r} = n!/[(n-r)!\,r!]$

2. Number of combinations of n different things taken r at a time in which p particular things will always occur is ${}^{n-p}C_{r-p}$

3. No. of combinations of n dissimilar things taken 'r' at a time in which 'p' particular things will never occur is ${}^{n-p}C_{r}$

4. ${}^{n}C_{r} = {}^{n}C_{n-r}$

PROBABILITY

THE MEANING OF PROBABILITY :

In our daily life, we often come across events which are concerned with the idea of the likelihood or the chance of occurrence of future events.

Probability is the measure of the chance of occurrence of a future event. It tells us how likely we expect the event to happen.

Now, we may define the probability of an event occurring as follows :

$$\text{Probability of an event occurring} = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

Note :

1. If an event E is sure to occur, we say that the probability of the event E is equal to 1 and we write P (E) = 1.
2. If an event E is sure not to occur, we say that the probability of the event E is equal to 0 and we write P (E) = 0.

   Therefore for any event E, $0 \leq P(E) \leq 1$

Mathematical definition of probability :

(A) If the outcome of an operation can occur in n equally like ways, and if m of these ways are favourable to an event E, the probability of E, denoted by P (E) is given by P (E) = m/n

(B) As $0 \leq m \leq n$, therefore for any event E, we have $0 \leq P(E) \leq 1$

(C) The probability of E not occuring, denoted by P(not E), is given by P(not E) or $P(\bar{E}) = 1 - P(E)$

(D) Odds in favour = No. of favourable cases / No. of unfavourable cases

(E) Odds against = No. of unfavourable cases / No. of favourable cases

Mutually Exclusive Events and Addition Law

(A) Mutually Exclusive Events :

Two events are mutually exclusive if one happens, the other can't happen & vice versa. In other words, the events have no common outcomes. For example

1. In rolling a die
   - E :– The event that the no. is odd
   - F :– The event that the no. is even
   - G :– The event that the no. is a multiple of three
2. In drawing a card from a deck of 52 cards
   - E :– The event that it is a spade
   - F :– The event that it is a club
   - G :– The event that it is a king

In the above 2 cases events E & F are mutually exclusive but the events E & G are not mutually exclusive or disjoint since they may have common outcomes.

(B) ADDITION LAW OF PROBABILITY :

If E & F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by : **P(E or F) = P(E) + P(F)**

If the events are not mutually exclusive, then **P(E or F) = P(E) + P(F) – P(E & F together).**

Note : Compare this with $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ of set theory.

Similarly **P (neither E nor F) = 1 – P(E or F).**

Independent Events And Multiplication Law :

(A) Two events are independent if the happening of one has no effect on the happening of the other.

For ex :

1. On rolling a die & tossing a coin together
   E :– The event that no. 6 turns up.
   F :– The event that head turns up.
2. In shooting a target
   E :– Event that the first trial is missed.
   F :– Event that the second trial is missed.

In both these cases events E & F are independent.

BUT 3. In drawing a card from a well shuffled pack
E :– Event that first card is drawn
F :– Event that second card is drawn without replacing the first
G :– Event that second card is drawn after replacing the first
In this case E & F are not Independent but E & G are independent.

(B) MULTIPLICATION LAW OF PROBABILITY :

If the events E & F are independent then **P(E & F) = P (E) x P (F)**
& **P (not E & F) = 1 – P (E & F together).**

SOLVED EXAMPLES

Ex 1. *Find the number of ways in which the letters of the word "machine" can be arranged such that the vowels may occupy only odd positions ?*

Ans. "Machine" consists of seven letters : four of them are consonants and three vowels. Let us mark out the position to be filled up as follows :

1	2	3	4	5	6	7
(a)	( )	(i)	( )	(e)	( )	( )

Since the vowels can be placed only in three out of the four positions marked 1,3,5,7, the total number of ways in which they can be made to occupy odd positions = ${}^4P_3$ = 4.3.2 = 24. ... (1)
Suppose one arrangement of the vowels is as shown in the diagram; then for this particular arrangement of the vowel, the number of ways in which the 4 consonants can be made to occupy the remaining positions (marked 2,4,6,7) = ${}^4P_4$ = 4.3.2.1 = 24.
Hence, for each way of placing the vowels in odd positions there are 24 arrangements of the whole set. Consequently the total number of arrangements of the given letters under the given condition = 24 x 24 = 576 Ans.

Ex 2. *Sixteen jobs are vacant; how many different batches of men can be chosen out of twenty candidates ? How often may any particular candidate be selected ?*

Sol. We have only to find out the number of different groups of 16 men that can be formed out of 20 without any reference to the appointment to be given to each.
Hence, the required number of ways = ${}^{20}C_{16} = {}^{20}C_4$
= 20 x 19 x 18 x 17 / (1 x 2 x 3 x 4) = 5 x 19 x 3 x 17 = 4845.
Let us now find out how many times a particular candidate may be chosen.
Every time that a particular candidate is selected the other 15 candidates will have to be chosen from the remaining 19 candidates.
Hence a particular man may be selected as many times as we can select a group of 15 men out of the remaining 19. Hence, the required number of times = ${}^{19}C_{15} = {}^{19}C_4$
= 19 x 18 x 17 x 16 / (1 x 2 x 3 x 4) = 19 x 3 x 17 x 4 = 3876 Ans.

Ex 3. *How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?*

Sol. Since, each number is to consist of not less than 7 digits, we shall have to use all the digits in forming the numbers. Now, among these 7 digits there are 2 two's and 3 three's; hence the total number of ways of arranging the digits = 7! / (2! 3!) = 420. But out of these arrangements we have to reject those that begin with zero, for they are six–digit numbers. Now, evidently there are as many such arrangements as there are ways of arranging the remaining 6 digits among themselves, and $\Rightarrow$ Their no. = 6!/2! 3! = 60 Hence, the reqd number = 420 – 60 = 360.

Ex 4. *In how many ways can 3 letters be posted in four letter boxes in a village ? If all the three letters are not posted in the same letter box, find the corresponding number of ways of posting.*

Sol. We can post the first letter in 4 ways. Similarly the second and third can be posted in 4 ways each. So the total number of ways = 4 x 4 x 4 = 64. Now all the three letters together can be posted in any letterbox. In this case there will be four ways and when all the letters are not posted together, the number of ways = 64 – 4 = 60.

Ex 5. *In rolling two dice, find the probability that*

(1) there is at least one '6' *(2) the sum is 5.*

Sol. The total possible outcomes are 36 as shown below.

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) ; (2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6) ; (4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6) ; (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

The outcomes with at least one '6' are (1,6), (2,6), ... (6,6). There are 11 such pairs.

(1) $\Rightarrow$ P (at least one '6') = 11/36 Ans.

(2) The pairs with a sum of 5 are (1,4), (2,3), (3,2), (4,1).
$\Rightarrow$ P (the sum is 5) = 4/36 = 1/9 Ans.

Ex 6. *A single card is selected from a deck of 52 bridge cards. What is the probability that*

(1) it is not a heart,

(2) it is an ace or a spade ?

Sol. A deck of bridge cards has 4 suits – spade, heart, diamond and club. Each suit has 13 cards.– Ace, two, three, ... , ten, jack, Queen, King.

(1) P (not a heart) = 1 – P (a heart) = 1 – 13/52 = 39/52 = 3/4 Ans.

(2) There are 4 aces and 12 spades besides the ace of spades
$\Rightarrow$ P (an ace or a spade) = 16/52 = 4/13 Ans.

Ex 7. *A box contains 2 red, 3 yellow and 4 blue balls. Three balls are drawn in succession with replacement. Find the probability that*

(1) all are yellow,

(2) the first is red, the second is yellow, the third is blue,

(3) none are yellow,

(4) all three are of the same colour.

Sol. (1) In a draw, P (red) = 2/9, P (yellow) = 3/9, P (blue) = 4/9.
In 3 draws, $\Rightarrow$ Prob of all are yellow = (3/9) . (3/9) . (3/9) = 1/27 Ans.

(2) Required probability = P (1st red) . P (2nd yellow) . P (3rd blue)
= 2/9 . 3/9 . 4/9 = 8/243 Ans.

(3) Probability that none are yellow = P (1st not yellow) . P (2nd not yellow) . P (3rd not yellow)
= (1 – 3/9) x (1 – 3/9) x (1 – 3/9) = 8/27 Ans.

(4) Probability that all three are of the same colour
= P (all red) + P (all yellow) + P (all blue) {mutually exclusive}
= $(2/9)^3 + (3/9)^3 + (4/9)^3$ = 11/81 Ans. A standard doubt : If we have only 2 red balls, how can we get 3 in succession ? Answer : Because we're replacing !

Ex 8. *With the data in Example 7, answer those questions when the balls are drawn in succession without replacement.*

Sol. (1) Prob of all yellow = P(1st yellow) . P (2nd yellow) . P (3rd yellow)
= 3/9 . 2/8 . 1/7 = 1/84 Ans.
since when the first yellow ball has been drawn, there are 8 balls remaining in the bag of which 2 are yellow.

(2) Required probability = P (1st red) . P (2nd yellow) . P (3rd blue)
= 2/9 . 3/8 . 4/7 = 1/21 Ans.

(3) Probability that none are yellow
= P (1st not yellow) . P (2nd not yellow) . P (3rd not yellow)
= (1 – 3/9) (1 – 3/8) (1 – 3/7) = 6/9 . 5/8 . 4/7 = 5/21 Ans.

(4) Probability that all three are of the same colour
= P (all red) + (all yellow) + P (all blue)
= 2/9 . 1/8 . 0/7 + 3/9 . 2/8 . 1/7 + 4/9 . 3/8 . 2/7 = 5/84 Ans.

Note : The probability that all are red = 0.

Ex 9. *There are 7 Physics and 1 Chemistry book in shelf A. There are 5 Physics books in shelf B. One book is moved from shelf A to shelf B. A student picks up a book from shelf B. Find the probability that the Chemistry book :*

(1) is still in shelf A,
(2) is in shelf B,
(3) is taken by the student.

Sol. (1) The probability that it is in shelf A = 7/8 Ans.
(this means that Physics book was picked up)

(2) The probability that it is in shelf B =
P (it is moved from A to B) . P (it is not taken by the student) = 1/8 . 5/6 = 5/48 Ans.

(3) The probability that is it taken by the student
= P (it is moved from A to B) . P (it is taken by the student) = 1/8 . 1/6 = 1/48 Ans.

Ex 10. *The ratios of number of boys and girls in X A and X B are 3 : 1 & 2 : 5 respectively. A student is selected to be the chairman of the students' association. The chance that the student is selected from X A is 2/3. Find the probability that the chairman will be a boy.*

Sol. Probability that the boy comes from X A = 2/3 . 3/4 = 1/2
Probability that the boy comes from X B = 1/3 . 2/7 = 2/21
$\Rightarrow$ The required probability = 1/2 + 2/21 = 25/42 Ans.

Ex 11. *The probability that a man will be alive in 25 years is 3/5 and the probability that his wife will be alive in 25 years is 2/3. Find the probability that :*

(1) both will be alive,
(2) only the man will be alive,
(3) only the wife will be alive,
(4) at least one will be alive.

Sol. (1) P (both alive) = P (man alive) x P (wife alive) = 3/5 x 2/3 = 2/5
(2) P (only man alive) = P (man alive) x P (wife dead) = 3/5 x 1/3 = 1/5
(3) P (only wife alive) = P (man dead) x P (wife alive) = 2/5 x 2/3 = 4/15 Ans.
(4) P (at least one will be alive) = 1 – P (both dead) = 1 – (2/5 x 1/3) = 13/15 Ans.