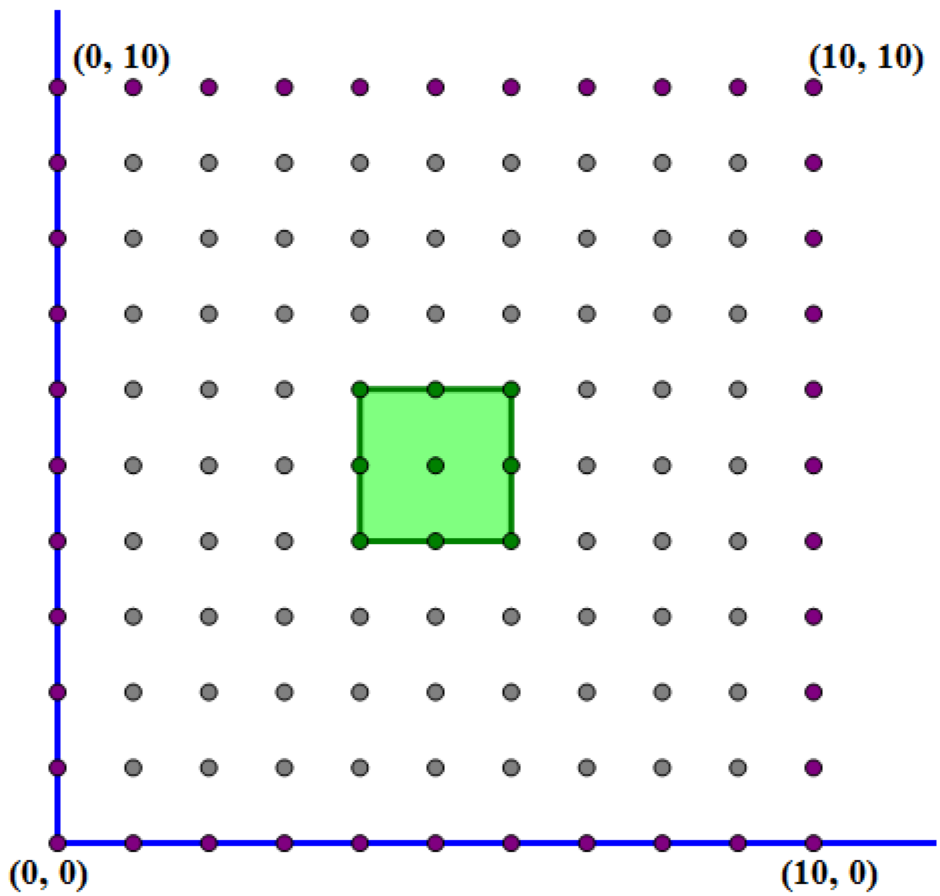
In the x-y plane, the square region bound by (0,0), (10, 0), (10, 10) and (0, 10) is isolated. A **boundary point** is any of the 40 points on the edge of this region for which both coordinates are integers; boundary points are indicated as purple in the diagram. Square J, bound by the points (4, 4), (4, 6), (6, 6), and (6, 4), are shown in green. If two boundary points are selected at random, and the line segment connecting these two is drawn, what is the probability that this line segment touches or passes through Square J?

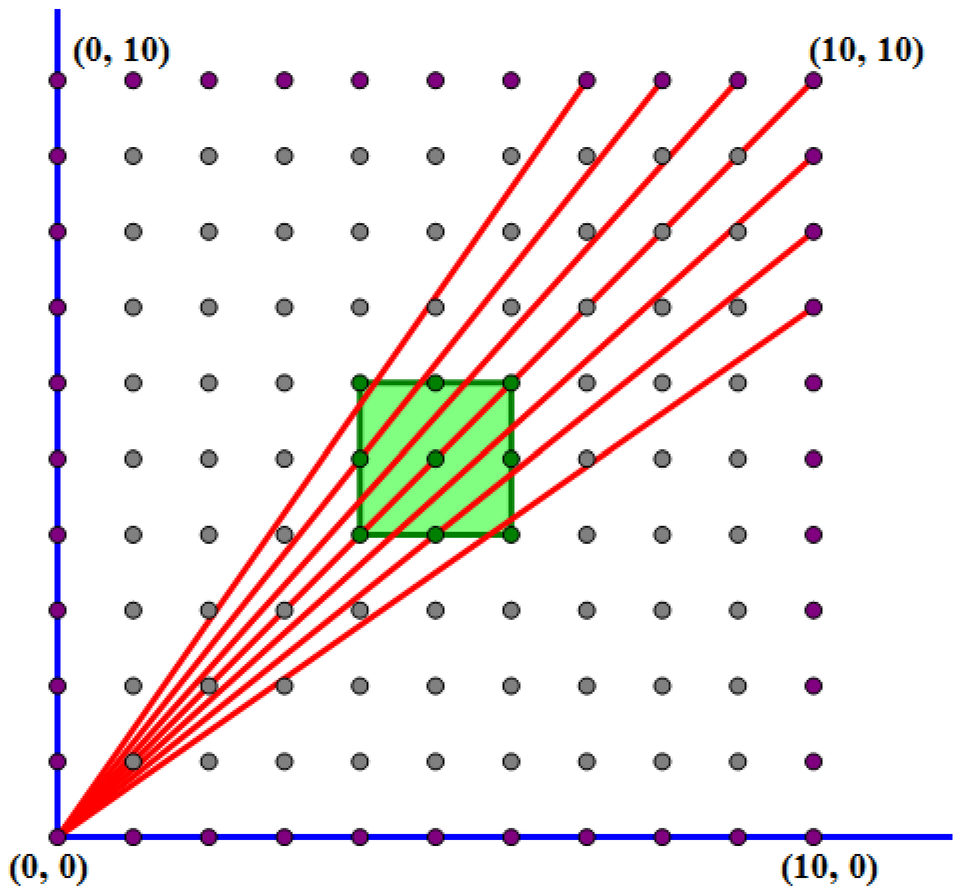


First of all, the total number of pairs of points is

$$40C2 = \frac{40\times39}{2} = 780$$

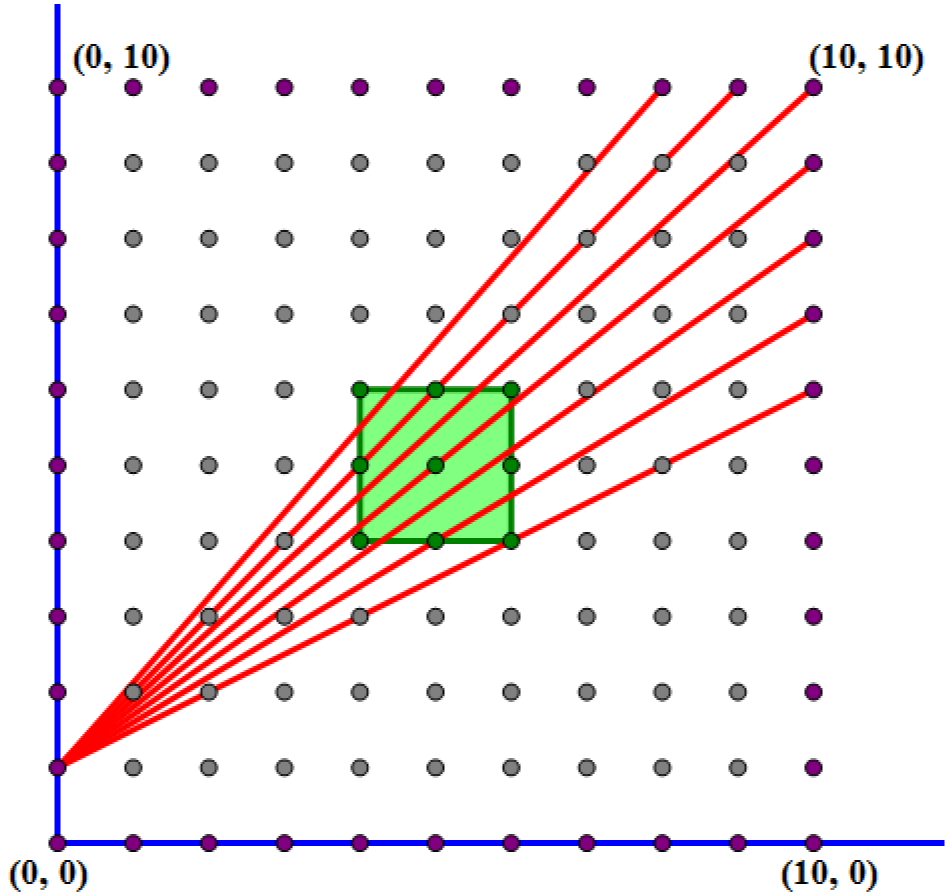
For counting, every segment will be counted twice, once from each endpoint, so we will count the total number, and then divide by two.

From a corner point, there are **seven** choices that work.



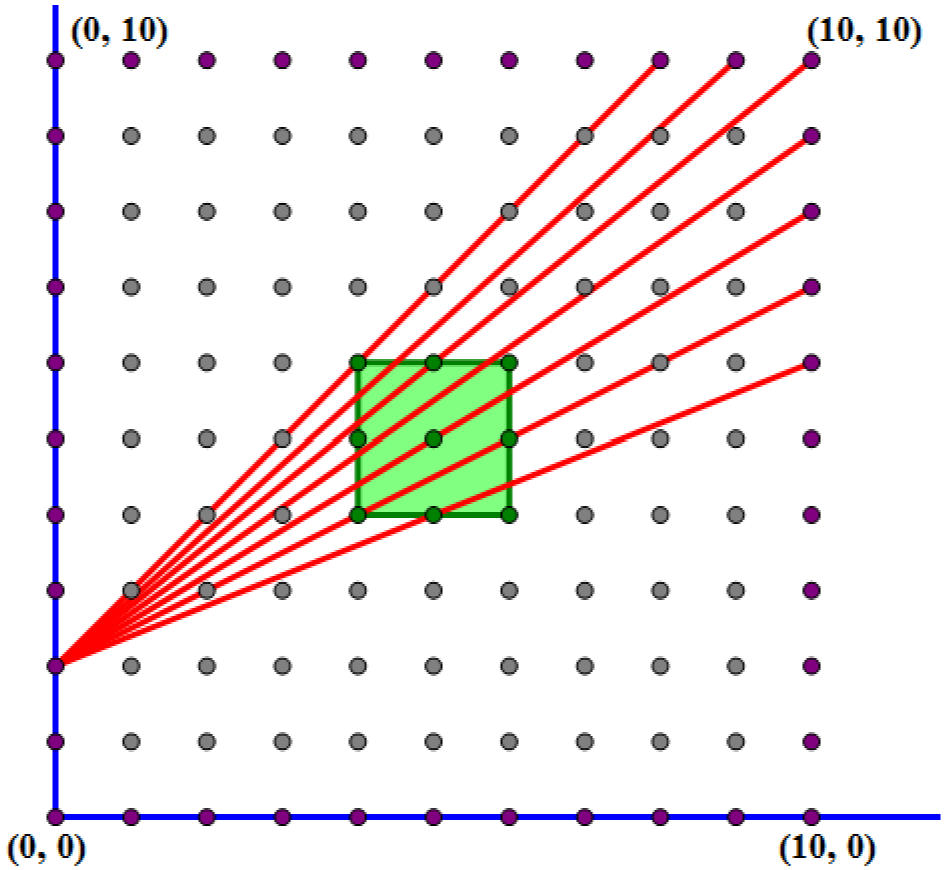
These can be drawn from any corner, so there are **28** such possibilities.

Consider a point one step from a corner. From one such point, there are **seven** possibilities.



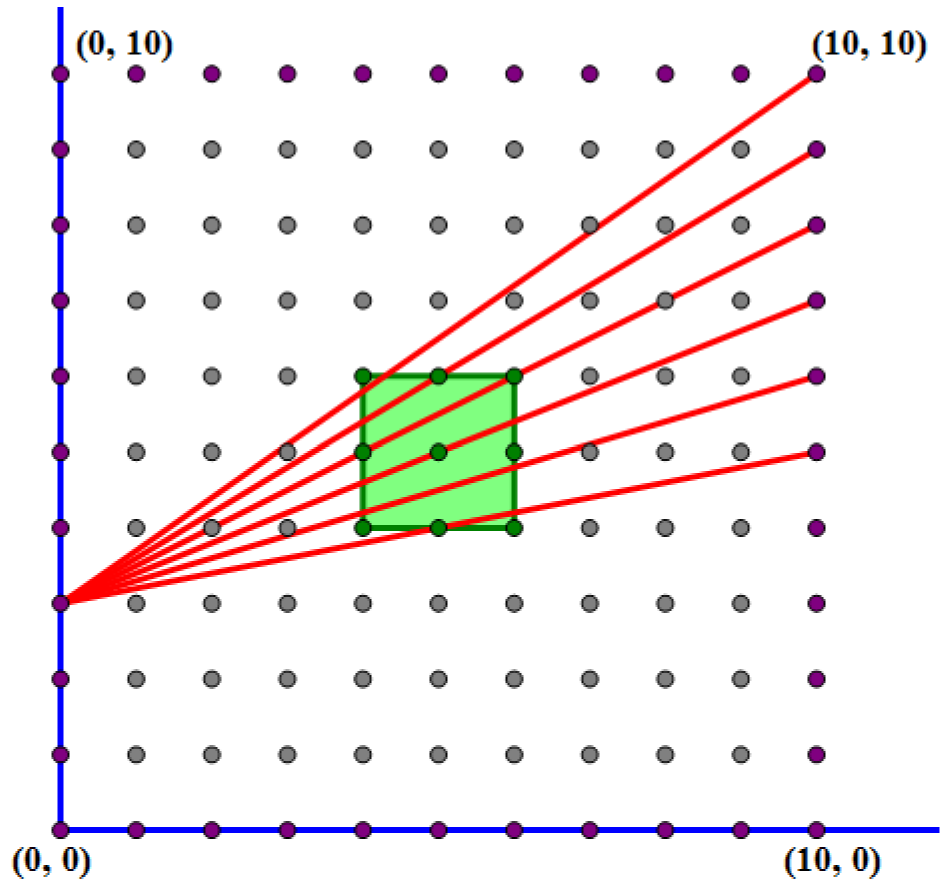
There are eight boundary points that are exactly one unit away from a corner, so the total number of these lines is 7 x 8 = **56**.

Consider a point two steps from a corner. From one such point, there are **seven** possibilities.



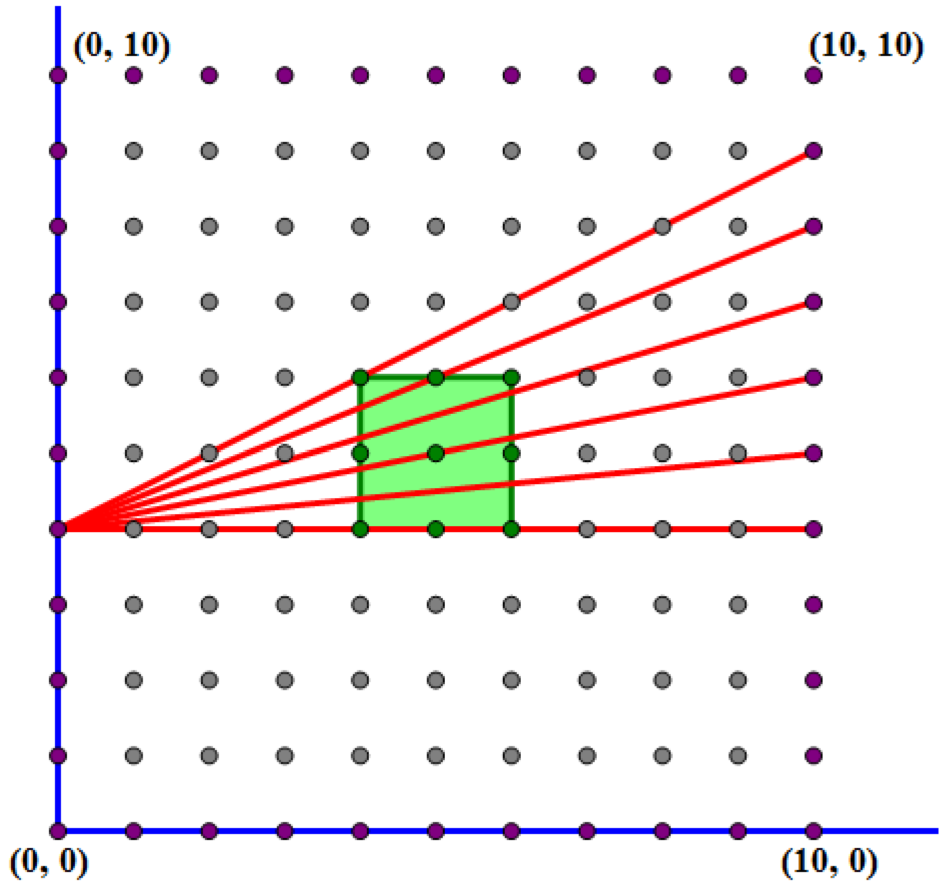
There are eight boundary points that are exactly two units away from a corner, so the total number of these lines is 7 x 8 = **56**.

Consider a point three steps from a corner. From one such point, there are **six** possibilities.



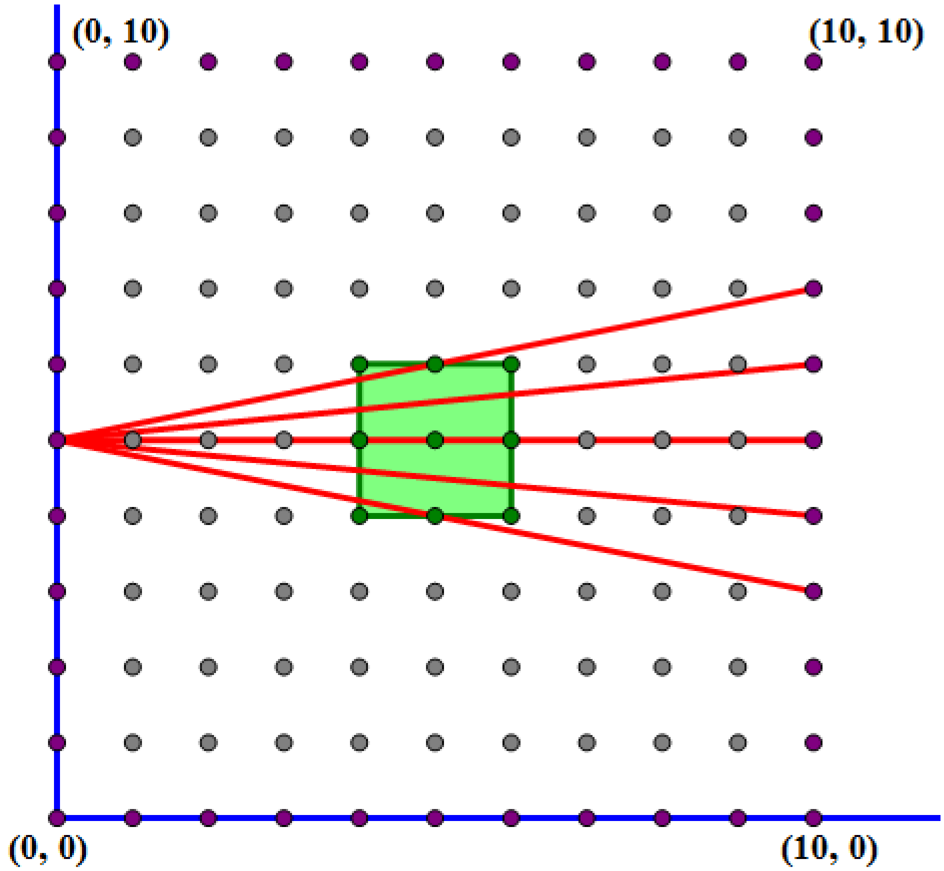
There are eight boundary points that are exactly three units away from a corner, so the total number of these lines is 6 x 8 = **48**.

Consider a point four steps from a corner, a boundary point adjacent to the midpoint of a side. From one such point, there are **six** possibilities.



There are eight boundary points that are exactly three units away from a corner, so the total number of these lines is 6 x 8 = **48**.

Consider to the midpoint of a side. From one such point, there are **five** possibilities.



There are four midpoints, so the total number of these lines is 5 x 4 = **20**.

Add these numbers:

28 + 56 + 56 + 48 + 48 + 20 = 256

Because every line is counted twice, once for each endpoint, we divide this by 2.

128 lines, each determined by a unique pair of boundary points.

This is out of 780 pairs of boundary points.

Probability = $\frac{128}{780} = \frac{32}{195}$