1.

OA: A

the word "parity", as used in the following discussion, means "whether something is even or odd", in the same way in which "sign" means whether a number is positive or negative.

the best way to approach things like this is to **break down the compound statements into statements about the parity of the individual variables**.
to do that, you'll almost certainly have to **split the statements into cases**.

"xy + z is odd"
two numbers can add to give an odd sum only if they have opposite parity. hence:
case 1: xy is odd, z is even
there's only one way this can happen:
x = odd, y = odd, z = even. **(1)**
case 2: xy is even, z is odd
there are 3 ways in which this can happen:
x = even, y = even, z = odd (2a)
x = odd, y = even, z = odd (2b)
x = even, y = odd, z = odd (2c)

this is a bit awkward, but, once you've divided the question prompt up into cases, all you have to do is look at your results, check the cases, and you'll have an answer.

—

statement (1)
the easiest way to handle expressions like this is to factor out common terms. you can handle the statement without doing so, but it's more work that way.
pull out x:
x(y + z) is even.
this means that *at least one* of x and (y + z) is even.
* if x is even, regardless of the parity of (y + z), then the answer to the prompt question is "yes" and we're done.
* the other possibility would be x = odd and (y + z) = even. this is impossible, though, as it doesn't satisfy any of the cases above.
therefore, the answer must be "yes".
sufficient.

—

statement (2)
this means that y and xz have opposite parity.
* y = even, xz = odd —> this means x = odd, y = even, z = odd. that's case (2b), which gives "no" to the question.

at this point you're done, because STATEMENTS CAN'T CONTRADICT EACH OTHER, se you know that "yes" MUST be a possibility with this statement (as statement #1 gives exclusively "yes" answers).
if you use this statement first, you'll have to keep going through the cases.
insufficient.

ans = a

2.

this problem involves two fractions that are added together. for no other reason than that 'it's the normal thing to do with two fractions added together', let's find the common denominator:
w/x + y/z = wz/xz + xy/xz = (wz + xy)/xz

therefore
the question can be rearranged to:
is (wz + xy)/xz – which is the same thing as w/x + y/z – odd?

— (2) alone —

if wz + xy is an odd integer, then all of its factors are odd. this means that (wz + xy)/xz, which is guaranteed to be an integer**, must also be odd – because it's a factor of an odd number.

sufficient

**we know this is an integer because it's equal to w/x + y/z, which, according to the information given in the problem statement, is integer + integer.

— (1) alone —

try to come up with contradictory examples**:

w=2, x=1, y=3, z=1 (so that wx + yz = 5 = odd, per the requirement):
w/x + y/z = 2 + 3 = 5 = odd

w=2, x=2, y=3, z=1 (so that wx + yz = 7 = odd, per the requirement):
w/x + y/z = 1 + 3 = 4 = even

insufficient

**of course, if you're at a loss for the theory, you should try this for statement (1) too ... but you'll find that all the examples you get are odd.

—

answer = b

3.

JUST PLUG IN NUMBERS.

statement (1)

let's just PICK A WHOLE BUNCH OF NUMBERS WHOSE GCF IS 2 and watch what happens. let's try to make the numbers diverse.
say,
4 and 6
6 and 8
8 and 10
10 and 12
...
4 and 10
6 and 14
6 and 16
8 and 18
8 and 22
...
in all nine of these examples, the remainders are greater than 1. in fact, there is an obvious pattern, which is that **they're all even**, since the numbers in question must be even.

in fact, i just thought of this, which is a much nicer, more ground-level approach to statement one:
in statement 1, both m and p are even. therefore, the remainder is even, so it's greater than 1.

done.

sufficient.

--

statement (2)
just pick various numbers whose lcm is 30.
notice the numbers selected above:
5 and 6 --> remainder = 1
10 and 15 --> remainder = 5 > 1
insufficient.

ans (a)

4.

Answer: B

(1)
this is a disguised way of saying 'n is prime'
therefore, insufficient

(2)
this says *any* two factors. that means **any** two factors – i.e., ALL pairs of factors have an odd difference.
there's only one way to do this: one odd factor and one even factor. (as soon as you get 2 odd factors or 2 even factors, you get an even difference by subtracting them.)
2 is the only # with only 1 odd factor and only 1 even factor.
therefore, sufficient

5.

Take a prime number and figure out a specific soln for that prime number.
Let p = 5. So, excluding 1, the other numbers that have no factors common with 5 are 2,3,4.
Let p = 7. So, excluding 1, the other numbers that have no factors common with 7 are 2,3,4,5,6
Do you see the pattern? For any prime number, all the numbers less than it will have no factors in common with it except 1.
So f(p) = p – 2 Answer is B. **NOOOOOOOOOO**

We need to include 1 and hence the correct answer is p–1 (A).

Pick a prime number for p. Let's say p=5.

The positive integers less than 5 are 4, 3, 2, and 1.

5 and 4 share only 1 as a factor
5 and 3 share only 1 as a factor
5 and 2 share only 1 as a factor
5 and 1 share only 1 as a factor

There are four positive integers, therefore, that are both less than 5 and share only 1 as a factor. In other words, we include 1 in this set of integers.

6.

Let's first consider the prime factors of h(100). According to the given function,
h(100) = 2*4*6*8*...*100

By factoring a 2 from each term of our function, h(100) can be rewritten as
2^50*(1*2*3*...*50).

Thus, all integers up to 50 - **including all prime numbers up to 50** - are factors of h(100).

Therefore, h(100) **+ 1** *cannot* have any prime factors 50 or below, since dividing this value by any of these prime numbers will yield a remainder of 1.

Since the smallest prime number that can be a factor of h(100) + 1 has to be greater than 50, The correct answer is E.

7.

n is an integer
is n odd?
yes/no question, so I will try to prove it wrong (that is, get a yes and a no based upon the statements)
(1) n/3
n could be 6 (that is divisible by 3). Is n odd? No
n could be 9 (that is divisible by 3). Is n odd? Yes
Elim A and D
(2) 2n has twice as many factors as n
n could be 1, which has one factor; 2n would be 2, which has two factors; is n odd? Yes
n could be 2, which has two factors; 2n would be 4, which has three factors. Oops, can't use this combo of numbers (has to make statement 2 true, and this combo doesn't)

What's going on here?
general rule: 2n will be divisible by 2 and also by whatever number 2n is.
If I make n an even number, even numbers are already divisible by 2. So 2n will only be divisible by one new number, equal to 2n. That is, I add only one new factor for 2n. **[editor: there's a mistake in this explanation - see below for a correction]**
Any even number, by definition, has at least two factors - 1 and 2. So I would need to add at least two more factors to double the number of factors. But I can't - the setup of statement 2 only allows me to add one new factor if n is even. So I can never make statement 2 true using an even number for n.

Sufficient. Answer is B.
let's say a number has "n" different factors.
when you multiply this number by 2, you POTENTIALLY create "n" MORE factors - by doubling each factor.

HOWEVER,
the only way that ALL of these factors can be NEW (i.e., not already listed in the original *n* factors) is if they are ALL ODD.
if there are ANY even factors to start with, then those factors will be repeated in the original list. (for instance, note that 2, 4, 26, and 52 all appear in both lists above.) therefore, **if the number is even, then the number of factors will be less than doubled** because of the repeat factors.

thus if statement (2) is true, then the number must be odd.

8.

Start with statement 2. This doesn't tell us one value of d, so elim. B and D.

Statement 1: 10^d is a factor of f. This isn't going to be sufficient. If you're not sure why try the easiest possible positive integers. Is 10^1 = 10 a factor of f? Yes, so 1 is a possible value for d. Is 10^2 = 100 a possible factor of f? Yes, so 2 is a possible value for d. I just found 2 possible values for d. Elim. A. Only C and E are left.

d is a pos int (given in stem) and is greater than 6 (statement 2). Smallest possibility, then, is 7. If d is anything greater than 7, then 7 will work too (eg, if d actually is 8, then 7 would also satisfy both statements and we wouldn't be able to tell, just from the statements, whether d is 7 or 8). So it's either 7, exactly, which is sufficient, or it's something greater than 7, which is not sufficient.

So how many 10's are in f?
write down the numbers that contain 2s and 5s (only those)
30*28*26*25*24*22*20*18*16*15*14*12*10*8*6*5*4*2

Now ask yourself Is my limiting factor going to be 5 or is it going to be 2?

It's going to be 5 because there are many more 2's up there. So circle the numbers that contain 5's:
30, 25, 20, 15, 10, 5
How many 5's do you have? Seven 5's (don't forget – 25 has two 5's!), so you can make seven 10's. That's it. Answer is C.

"limiting factor" means "which is least common or likely." Think of it this way: there are many more multiples of 2 than there are multiples of 5. In probability terms, a number is more likely to be even than to be a multiple of 5. In divisibility terms, take some large number that is divisible by both 2 and 5, and it is likely to have more factors of 2 than 5.

For example: 400 = 4*10*10 = (2*2)(2*5)(2*5) = (2^4)(5^2).

I know, numbers with more factors of 5 than factors of 2 exist...this is just a bet we make to ease the computation.

In general, the larger the factor, the less likely it is to divide evenly into a number. The larger the factor, the more of a "limiting factor" it is.

here's all you have to do:
forget entirely about 10, 20, and 30, and **ONLY THINK ABOUT PRIME FACTORIZATIONS**.
(TAKEAWAY: this is the way to go in general – when you break something down into primes, you should not think in hybrid terms like this. instead, just translate *everything* into the language of primes.)

each PAIR OF A '5' AND A '2' in the prime factorization translates into a '10'.

there are **seven 5's**: one each from 5, 10, 15, 20, and 30, and two from 25.

there are waaaaaaayyyyy more than seven 2's.

therefore, **30! can accommodate as many as seven 10's** before you run out of fives.

—

statement 2 is clearly insufficient.

statement 1, by itself, means that d can be anything from 1 to 7 inclusive.

together, d must be 7.

ans (c)

9.

96 =

6*8*2 or 2*8*6 or 826 or 628.....
3*8*4 or 483 or 843 or ....
According to 1: the number is odd; We have only one odd digit: 3. – correct
while II says: hundreds digit of m is 8; There are many combination: 682 or 483 or.... incorrect.

96 = 2*2*2*2*2*3,

statement 1: m is odd, so unit's digit could be 1,3,5,7,9.
But we have only one odd factor in 96(product of digits of m) i.e. 3. Therefore, unit's digit of m is 3. – sufficient

statement 2: hundred's digit is 8, so we are left with 2*2*3. Therefore, m could be 826, 843, 834, 862. So no unique unit's digit. Insufficient

10.

the correct answer: B

if we are told that four different prime numbers are factors of 2n then can't i further assume that one of those four prime numbers is 2 (since it's 2n)

it's possible that 2 is already a factor of n to start with, in which case n itself would still have 4 different prime factors (because, in that case, the additional 2 would not change the total number of prime factors).

for instance, if n = 3x5x7 = 105 (which has three prime factors), then 2n = 2x3x5x7 = 210 has four prime factors.
if n = 2x3x5x7 = 210, which has four prime factors, then 2n = 2x2x3x5x7 = 420, which still has two prime factors.
therefore, #1 is not sufficient.

11.
the question is asking whether *k* has a factor that is *greater than 1, but less than itself.*
if you're good at these number property rephrasings, then you can realize that this question is equivalent to "is k non–prime?", which, in turn, because it's a data sufficiency problem (and therefore we don't care whether the answer is "yes" or "no", as long as there's an answer), is equivalent to "is k prime?".
but let's stick to the first question – "does k have a factor that's between 1 and k itself?" – because that's easier to interpret, and, ironically, is easier to think about (on this particular problem) than the prime issue.

—

key realization:
every one of the numbers 2, 3, 4, 5, ..., 12, 13 is a factor of 13!.

this should be clear when you think about the definition of a factorial: it's just the product of all the integers from 1 through 13. because all of those numbers are in the product, they're all factors (some of them several times over).

—

consider the lowest number allowed by statement 2: 13! + 2.
note that 2 goes into 13! (as shown above), and 2 also goes into 2. therefore, 2 is a factor of this sum (answer to question prompt = "yes").

consider the next number allowed by statement 2: 13! + 3.
note that 3 goes into 13! (as shown above), and 3 also goes into 3. therefore, 3 is a factor of this sum (answer to question prompt = "yes").

etc.
all the way to 13! + 13.
works the same way each time.
so the answer is "yes" every time —> sufficient.

—

in this problem, the prompt asks, "**Is there a factor p** such that...?"
this means that, *if you can show that there is even one such factor*, then it's "sufficient" and you are DONE.
we have ascertained that every one of the "k"s in that range has *at least one such factor*.
to wit, 13! + 2 has the factor 2; 13! + 3 has the factor 3; ...; 13! + 13 has the factor 13.
that's all we need to know.
sufficient.

you are right that it's difficult to ascertain whether numbers greater than 13 are factors of these "k"s. luckily, we don't have to care about that.

12.

OA: D

Since it has only 2 prime factors but 6 factors (4 of which are 1, 3, 7, k) this means that the prime factors must be combined to generate the other 2 factors – the other 2 can only be either 3 which means 3x3=9 and 3.7=21 is a factor OR 7 which means the other 2 factors are 21 and 49.

SHORTCUT METHOD:
if you know the following useful fact, then you can solve this problem much more quickly.
USEFUL FACT: if a, b, ... are the EXPONENTS in the prime factorization of a number, then the total number of factors of that number is the product of (a + 1), (b + 1), ...
example:
540 = (2^2)(3^3)(5^1), in which the exponents are 2, 3, and 1. therefore, 540 has (2 + 1)(3 + 1)(1 + 1) = 3 x 4 x 2 = 24 different factors.

with this shortcut method, realize that 6 (the total number of factors) is 3 x 2. therefore, the exponents in the prime factorization must be 2 and 1, in some order.
therefore, **there are only two possibilities: k = (3^2)(7^1) = 63, or k = (3^1)(7^2) = 147.**

statement (1) includes 63 but rules out 149, so, sufficient.
statement (2) includes 63 but rules out 149, so, sufficient.
answer = (d).

—

IF YOU DON'T KNOW THE SHORTCUT:

statement (1)
if 3^2 is a factor of k, then so is 3^1.
therefore, we already have four factors: 1, 3^1, 3^2, and 7.
but we also know that (3^1)(7) and (3^2)(7) must be factors, since 3^2 and 7 are both part of the prime factorization of k.
that's already six factors, so we're done: k must be (3^2)(7). if it were any bigger, then there would be more than these six factors.
sufficient.

statement (2)
if 7 is a factor of k, but 7^2 isn't, then the prime factorization of k contains EXACTLY one 7.
therefore, we need to find out how many 3's will produce six factors when paired with exactly one 7.
in fact, it's data sufficiency, so we don't even have to find this number; all we have to do is realize that adding more 3's will always increase the number of factors, so, there must be exactly one number of 3's that will produce the correct number of factors. (as already noted above, that's two 3's, or 3^2.)
sufficient.

13.

when you take the product of two numbers, all you're doing, in terms of primes, is throwing *all* the prime factors of both numbers together into one big pool.
therefore, the original question – 'what's the greatest prime factor of the product?' – can be rephrased as,
what's the greatest prime that's a factor of *either* t or n?

(1)
because the gcf only tells us which primes are in BOTH t and n. there could be great big fat primes that are factors of only one of them, and they wouldn't show up in the gcf.
insufficient.

(2)
the lcm of two numbers contains EVERY prime that appears in either one of the two numbers (because it's a multiple of both numbers). therefore, whatever is the largest prime factor of the lcm is also the largest prime that goes evenly into either t or n.
sufficient.

—

if you don't realize why the relationships between lcm/gcf and primes, stated above, are what they are, you can just try a few cases and watch the results for yourself. for instance, consider the two numbers 30 (= 2 x 3 x 5) and 70 (= 2 x 5 x 7).
the gcf of these 2 numbers is 10 (= 2 x 5), which doesn't show anything about the presence of the

prime factor 7 in one of the numbers.
the lcm of these 2 numbers is 210 (= 2 x 3 x 5 x 7), which contains all of the primes found in either number.
Ans. B

OR

(1) The first statement does not tell us about factors that are not common to n and t. One of those factors might be greater or less than 5. Tis statement alone is NOT SUFFICIENT

(2) The LCM is 105 which can be factored as 3*5*7. Since LCM incorporates all the factors of n and t we know the greatest factor for the two numbers is 7. Hence this statement is sufficient.

OR

consider the 'prime box' approach (= an imaginary box that contains all the numbers in the prime factorization of a number, for those of you who are uninitiated into our curriculum).

you're looking for the greatest prime # that would be in the 'box' obtained from dumping all the factors of n and all the factors of t, including all repetitions, into a bigger box. (this is what multiplication does: it multiplies the complete factorization of one number by that of another. for instance, 12 = 2x2x3 and 20 = 2x2x5, so 12 x 20 = 2x2x2x2x3x5.) therefore, the question can be rephrased as follows: what is the greatest prime # that is a factor **of either** t or n ?

(1) this only tells is that the greatest number that is in **both** factorizations – those of n and t – is 5. but there could be a larger factor that is part of only one of the factorizations. for instance:
– it's possible that n = t = 5. then the greatest prime factor of nt is 5.
– it's possible that n = 5 and t = 35. then the greatest prime factor of nt is 7.
insufficient.

(2) the least common multiple contains every factor of t or n at least once. (it has to; if, say, t had a factor that wasn't contained in it, then it would fail to be a multiple of t.) so, the biggest prime factor of this # will also be the biggest prime factor of the product nt.
sufficient.
try a few combinations of n and t if you aren't convinced.

answer = b

14.
Answer is A.

From (1), I could figure (t+3)(t+2) will always have a remainder 2, hence SUFFICIENT

I had trouble with (2) as I could not come up with an algebraic approach. I understand I can plug numbers to see that t^2 = 36 and t^2 = 64 fit the criterion BUT yield different remainders when the corresponding values of t are plugged into t^2 + 5t +6 and hence INSUFFICIENT.

OR

one fact that's pretty cool, and which happens to apply to this problem, is that *you can do normal arithmetic with remainders, as long as all the remainders come from division by the same number.* the only difference is that, if/when you get numbers that are too big to be authentic remainders (i.e., they're equal to or greater than the number you're dividing by), you have to take out as many multiples of the divisor as necessary to convert them back into "legitimate" remainders again. you can think of the remainders as on an odometer that rolls back to 0 whenever you reach the number you're dividing by.

so with statement (1), all the remainders are upon division by 7, so we can do normal arithmetic with them:
if t gives a remainder of 6, then t^2 = t x t gives a remainder of 6 x 6 = 36 —> this is more than 7, so we take out as many 7's as possible: 36 – 35 = 1.
if t gives a remainder of 6, then 5t gives a remainder of 5(6) = 30 —> this is more than 7, so we take out as many 7's as possible: 30 – 28 = 2.
and finally, 6 itself gives a remainder of 6.
therefore, the grand remainder when t^2 + 5t + 6 is divided by 7 should be 1 + 2 + 6 = 9 —> take out one more seven —> remainder will be 2.
sufficient.

by the way, much more generally (and therefore perhaps more importantly), **the patterns in remainder problems will always emerge fairly early when you plug in numbers.** therefore, if you don't IMMEDIATELY realize a good theoretical way to do a remainder problem, you should get on the number plugging RIGHT AWAY.

with statement (1), generate the first 3 numbers for which the statement is true: 6, 13, 20.
try 6: 36 + 30 + 6 = 72, which yields a remainder of 2 upon division by 7.
try 13: 169 + 65 + 6 = 240, which yields a remainder of 2 upon division by 7.
try 20: 400 + 100 + 6 = 506, which yields a remainder of 2 upon division by 7.
i'm convinced. (again, remember that PATTERNS EMERGE EARLY in remainder problems. 3 examples may not be enough for other types of pattern recognition, but that's usually pretty good in a remainder problem.)

with statement (2), as a poster has already mentioned above, find the first two t^2's that actually do this, which are 1^2 = 1 and 6^2 = 36.
if t = 1, then 1 + 5 + 6 = 12, which yields a remainder of 5 upon division by 7.
if t = 6, then 36 + 30 + 6 = 72, which yields a remainder of 2 upon division by 7.
insufficient.

OR
remainder problems usually show patterns after a very, very small number of plug–ins.

statement (1):
it's easy to generate t's that do this: 6, 13, 20, 27, ... (note that 6 is a member of this list, and an

awfully valuable one at that; it's quite easy to plug in)

try 6: 36 + 30 + 6 = 72; divide by 7 —> remainder 2
try 13: 169 + 65 + 6 = 240; divide by 7 —> remainder 2
try 20: 400 + 100 + 6 = 506; divide by 7 —> remainder 2

by this point i'd be convinced.
note that 3 plug–ins is NOT good enough for a great many problems, esp. number properties problems. however, as i said above, remainder problems don't keep secrets for long.

sufficient.

statement (2):
it's harder to find t's that do this. however, **the gmat is nice to you. if examples are harder to find, then the results will usually come VERY quickly once you find those examples.**
just take perfect squares, examine them, and see whether they give the requisite remainder upon division by 7.

the first two perfect squares that do so are 1^2 = 1 and 6^2 = 36.
if you don't recognize that 1 ÷ 7 gives remainder 1, then you'll have to dig up 6^2 = 36 and 8^2 = 64. that's not *that* much more work.

in any case, you'll have
1 + 5 + 6 = 12 —> divide by 7; remainder = 5
36 + 30 + 6 = 72 —> divide by 7, remainder = 2 (the work for this was already done above; you should NOT do it twice. i'm reproducing it here only for the sake of quick understanding.)

or
36 + 30 + 6 = 72 —> divide by 7, remainder = 2 (the work for this was already done above; you should NOT do it twice. i'm reproducing it here only for the sake of quick understanding.)
64 +40 + 6 = 110 —> divide by 7, remainder = 5

either way, insufficient within the first two plug–ins!

answer (a)

15.

A) (8*n + 5)/4 –> sufficient

B) p is odd. So in the sum one is odd and one is even.

p = e^2 + o^2

e^2 will be divisible by 4.
o^2 = (2n+1)(2n+1) / 4 = (4n^2 + 4n + 1)/4 –> sufficient

you can always plug in a bunch of numbers until you've satisfied yourself that the statements are sufficient.
for (1), just find the first few numbers that give remainder 5 upon division by 8: 5, 13, 21, 29, 37, etc. all of these give remainders of 1 upon division by 4, so that's convincing enough. sufficient.
(note: the gmat WILL NOT give problems on which a spurious pattern appears, only to be broken after the 40th or 50th number; if you see a pattern persist for 4–5 cases, you can take it on faith that the pattern persists indefinitely.)

for (2), you should make the same realization you made above: one of the numbers has to be odd and the other even. then just try a bunch of possibilities:
1^2 + 2^2 = 5
2^2 + 3^2 = 13
3^2 + 4^2 = 25, etc
1^2 + 4^2 = 17
2^2 + 5^2 = 29
3^2 + 6^2 = 45, etc
all these give a remainder of 1 upon division by 4. sufficient.

16.
1. p can be represented as p = 8n+5
so p can take values , 13, 21, 29,37, 45.....just plugging in various values of integer n.

Now the next task is to represent all these odd numbers as sum of 2 perfect squares.

13 = 4+9 = 2^2+3^2 this implies x=2, y=3 . As question already told us that y is odd.
21 = cant represnt as sum of two +ve integers
29 = 4+ 25 = 2^2+ 5^2 ....implies x=2, y=5.
37 = 1+36 ....implies y=1 , x=6
45 = 9+36....implies y=3, x=6

So it tells us that x = 2, 6...not divisble by 4.

Hence SUFFICIENT

2. Condition B tells us
x–y= 3 and y is odd

so y=1 , x=4 Div by 4
y= 3 , x= 6 Not Div by 4
y= 5, x= 8 Div by 4
y= 7, x=10, Not DIv by 4
So INSUFFICIENT

17.

TAKEAWAY:

in REMAINDER PROBLEMS:
if you don't INSTANTLY see the algebraic solution, then IMMEDIATELY start LOOKING FOR A PATTERN.

there's also a fact that you should know concerning this problem statement:

fact:
REMAINDERS UPON DIVISION BY 10 are simply UNITS DIGITS.

for instance, when 352 is divided by 10, the remainder is 2.

since remainders are fundamentally based on stuff repeating over and over and over again, it shouldn't be a surprise that patterns emerge early and often among remainders.

this solution isn't necessarily "easier" – that judgment depends upon how comfortable you are with the algebra and theory – but it can be quite efficient.

using (1)
9*3^4n + m becomes 9*3^8 + m
considering only units digit , 9*1 + m
INSUFFICENT

instead, you can just realize that, since *m* can be anything at all, you can have any units digit you want.

using (2)
9*3^4n + 1 , as shown above, for all values of n, units digit 3^4n remains the same. (UD of 3^4=1,UD of 3^8 =1)
Now , considering only units digit
9*1 + 1 = 10 ,Hence B SUFFICIENT

yeah.

technically, you should also throw away the "1" in your sum of 10, reducing to a final units digit of 0.

The answer is 'B', but I don't get it!!! if m *is one, you still don't know what 3^(4n+2) is... right? all we know is it's a power of 3... so it's units digit could be any number between 0–9.... thus, we still don't know what the remainder would be if divided by 10.... please help!!!!*

Rephrase the expression: –
3^(4n+2) +m = (9)*3^(4n) + m

statement (1) n =2 so the expression = (9)*3^8 + m but we do not know what m is – so cannot predict the value of the expression. INSUFFICIENT

statemtn (2) m = 1 which makes the expression:

(9)*3^(4n) + 1 Since we know n is +ve integer, now it gets tricky: –

for n = 1,2,3,4 the exponential component of the expression will be

3^4, 3^8, 3^12 or
9^2, 9^4, 9^6 or

81, 81^2, 81^3 ans so on... the unit digit of all these values will be 1, now this value will be multiplied by 9 and '1' will be added to the result. It will make the unit digit of the result – 0. It means the result will be prefectly divided by 10. So the remainder will be 0

SUFFICIENT, So the answer is (B)

18.

(1)
if n = 3, then (n – 1)(n + 1) = 8, so the remainder is 8
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
insufficient

(2)
if n = 2, then (n – 1)(n + 1) = 3, so the remainder is 3
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
insufficient

(together)
the best approach, unless you're really good at number properties, is to try the first few numbers that satisfy both statements, and watch what happens.
if n = 1, then (n – 1)(n + 1) = 0, so the remainder is 0
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
if n = 7, then (n – 1)(n + 1) = 48, so the remainder is 0
if n = 11, then (n – 1)(n + 1) = 120, so the remainder is 0
...you can see where this is headed.

here's the theory:
– if n is not divisible by 2, then n is odd, so both (n – 1) and (n + 1) are even. moreover, since every other even number is a multiple of 4, one of those two factors is a multiple of 4. so the product (n – 1)(n + 1) contains one multiple of 2 and one multiple of 4, so it contains at least 2 x

2 x 2 = three 2's in its prime factorization.
– if n is not divisible by 3, then exactly one of (n – 1) and (n + 1) is divisible by 3, because every third integer is divisible by 3. therefore, the product (n – 1)(n + 1) contains a 3 in its prime factorization.
– thus, the overall prime factorization of (n – 1)(n + 1) contains three 2's and a 3.
– therefore, it is a multiple of 24.
– sufficient

answer = c

takeaway:
once you're established "insufficient", do not bother testing additional cases!
the fact that n = 2 and n = 5 are both of the form (3k + 2) is random coincidence.

two:
if you look at the treatment of the 2 statements together, i have included both (3k + 1) and (3k + 2)–type cases in that treatment. unlike statement (2) alone, the combination of the 2 statements turns out to be sufficient, so this time i *must* consider all of the possibilities.
therefore, i do.

if n is not divisible by 3, then exactly one of (n – 1) and (n + 1) is divisible by 3

if n – 1 is divisible by 3, then n has the form 3k + 1.
if n + 1 is divisible by 3, then n has the form 3k + 2.
both have been considered.

19.

We can rephrase the statement as such:

Is: n(n^2 – 1) divisible by 4?

Is N(N–1)(N+1) divisible by 4?

Is the product of three consecutive integers divisible by 4?

Final rephrasing:

Is N an odd integer or is N a multiple of 4?

Evaluate the statements:

1) n = 2k + 1, where K is an integer.

2K + 1 will give us an odd integer for N. (YES)

The problem I had was with plugging in 0 for K.
2(0) + 1 = 1 0x1x2 = 0 (OA: A **0 is divisible by every positive integer.**)

note the following:
the *only* way you will encounter this sort of query is if you *plug in your own numbers*. in other words, the official problems WILL NOT require you to decide the issue of whether 0 is divisible by *n* (for whatever *n*); they restrict the scope of divisibility problems strictly to positive divisors and positive dividends.

however, you should still know this fact, because, as you have seen here, you will often encounter "extra" questions like this as artifacts of plugging in your own numbers. therefore, even though the gmat won't test the concept directly, you may still have to rely on it to solve the problem because of your number plugging.

—

as long as we're at it, if you encounter "negative multiples" in your number plugging adventures, then yes, those are divisible too. for instance, –4 is divisible by 4, as are –8, –12, and the whole lot.

20.

Method 1: Visual/Number Line approach.
(1) r is 3 times farther away from 0 than m is. But we have no "distances" given, nor any info about sign (i.e. is m left or right of 0?)

(2) On a number line, put a dot at 12. Put two dots on either side of it for m and r. What can vary? The distance between m and r--they can be very close to 12, or both very far away. Also, we don't know whether m is the dot to the left or to the right of 12.

(1)&(2) together: We still don't know distances (from 12 or 0), or whether m is left or right of r. We can either have (case A) r = 18 and m = 6 or (case B) r = 36 and m = -12.

Method 2: Algebra approach
(1) r = +/-3m
(2) r-12 = 12-m, or r+m = 24.
(1)&(2) together: r+m= (+/-3m)+m = 24. Either 4m = 24 (i.e. m=6) or -2m = 24 (i.e. m = -12).

since the natural instinct is to try only *positive* values for m and r, this is a very tricky problem.

Statement (1) tells us that r = 3m or r = –3m (as either case would result in an r with an absolute value that is three times that of m). Insufficient. Eliminate AD from AD/BCE Grid.

Statement (2) tells us that (r+m)/2 = 12. Insufficient. Eliminate B from remaining BCE Grid.

By substituting each equation from Statement (1) into the equation from Statement (2), the statements together tell us that $3m + m = 24$, so $m = 6$ and $r = 18$, or that $-3m + m = 24$, so $m = -12$ and $r = 36$. As there are still two possible values for r, the correct answer is E.

OR

if you'd rather conceptualize it (which is always a good idea for number–line problems like this one), you can think of it this way:
r is 3 times as far away from 0 as is m, *but we don't know in which direction*.
that's the big thing.

since 12 is halfway between m and r, imagine m and r both starting out at 12, and 'sliding' equally in opposite directions, with r moving to the right and m moving to the left. (you can't slide r to the left and m to the right, because, if you do so, then r will be closer to 0 than is m.)
when the numbers have 'slid' a certain distance – specifically, 6 units each, so that $m = 6$ and $r = 18$ – they'll arrive at a point where the distance between m and 0 is 1/3 of the distance between r and 0. that's the first point that satisfies both criteria.
now keep sliding the points away from 12.
eventually, m will pass through 0 itself, and will come out on the negative side. if you keep sliding, you'll reach*another* point at which the distance from 0 to m is 1/3 of the distance from 0 to r, only this time m is negative. (specifically, this will happen when $m = -12$ and $r = 36$.)

21.

the OA is C

Consider Data 2 independently

We have two possibilities:
1) Keep S on the right side of zero and satisfy the condition
2)Keep S on the left side and again satisfy the condition

In both ways the data is sufficient, but our quest is whether zero is halfway ...this is where u seem to have miss out...

If u dont consider the Data 1 then u may or may not get zero halfway...

Thats y the answer is C

<——————————R——————S———T———————>

OR

Choice (B) does not eliminate the possibility that R & S are zero. Combining the two statements eliminates zero as an answer and gives us a definite "yes" as an answer.

watch those assumptions.

the distance between t and (–s) must be a positive number, but the problem is that we don't know which way to subtract to get that positive number. if t > –s, then the distance is t – (–s), as you've written here. however, if –s > t, then the distance is actually (–s – t) instead.
if s is to the left of zero, then –s will be to the right of zero – which could well place –s to the right of t. if that happens, then the distance will become (–s – t), rendering your calculation inaccurate. try drawing out this possibility – put zero WAY to the right of both s and t on the number line, then find –s, and watch what happens).

if s lies to the right of zero, then –s must lie even further to the left than does s itself. since s is already to the left of t, it then follows that –s is also to the left of t. therefore, in that case, you can definitively write the distance as t – (–s), and your calculation is valid. therefore, (c).

—

ironically, the presence of statement (1) should make it *easier* to see that statement (2) is insufficient. specifically, statement (1) calls your attention to the fact that s *could* lie to the *left* of zero, in which case you could get the alternative outcome referenced above. that's something you might not think about if statement (1) weren't there.

plug in numbers to the number line here:

Statement 1)
if the line reads: r=–1, zero, s=1, t=3, then zero is halfway between r and s.
if the line reads: zero, r=1, s=2, t=3, then zero is not between r and s.
Insufficient.

Statement 2)
by definition zero is halfway between s and –s.

a) if the line reads: –s=r=–2, zero, s=2, and t=4, then (t to r)=(t to –s)=6.
zero is halfway in between r and s.

b) if the line reads: r=–4, s=–2, t=–1, zero, and –s=2, then (t to r) = (t to –s) = 3.
zero is between t and –s.
Insufficient.

Together)
Forces the case 2a). Sufficient.

OR

the particular trap you may have fallen into in your interpretation of (2) is that of assuming "–s" is to the LEFT of "t". there is no good reason whatsoever to make this assumption, and, what's more, at least one good reason (viz., "the gmat loves to test *exactly* these sorts of assumptions) *not* to make it.

of course, you don't need reasons to be very careful about your assumptions; that should be your default state.

if "–s" is to the right of "t", then you have
<—r————s—–t——————(–s)—>
in which case 0 is in no–man's–land between "t" and "–s".
in this case, note that "s" is negative. also note that **(–s) is positive** in this case, a situation that is difficult to digest for most students.

taking statements (1) and (2) together eliminates the above possibility, leaving only the case that you have outlined.

—

incidentally, the fault in the algebraic approach lies in writing the distance between t and (–s) as t – (–s). this writing is correct only if t is greater than (–s), an assumption that, as we've seen, is unjustified.
the correct way to write the distance is $|t - (-s)| = |t + s|$, an expression that is thoroughly unhelpful in solving this problem.

22.

The answer is A.

S and t are different numbers on the line segment, Is s+t=0?
We need to know where s and t are in the line segment

Using BDACE Grid ,

2 says 0 is between s and t
In a line segment s and t are two points and 0 is between them. Let says s at –7 in the coordinate, t could be in 3 and 0 is between them. It does not give a statement that s+T=0. Insuff

1 says distance between s and o is = d(betwen t and o)
Clearly, 0 is between S and t because distance from s to 0 is equal to distance from t to 0.
This gives a way to solve for s+t=0. Hence A is sufficient

Statement 2 is insufficient because 0 is between s and t. But that means s can equal –5 and t can equal +3. In such a case, 0 is still between s and t but that does not make them equidistant from 0. Or, s and t can be –4 and +4 respectively in which case they are equidistant from 0. Therefore, this statement doesn't necessarily answer the question because it can have different results.

The question states that s and t are different numbers, so they cannot both be –5. Therefore, they must be opposites of each other.

just as in normal parlance, "between" only means "between", and carries no connotations of equidistance from the two points.
for instance, it's quite true that 1 is between 0 and 100, but obviously false that 1 is the midpoint between 0 and 100.

same thing with statement two. if 0 is between s and t, then all this means is that one of s and t is positive and the other is negative. that is all; there's nothing barring possibilities such as s = –1,000,000 and t = 1.

23.

well, first, think about the qualitative aspects of the sequence: if the sequence consisted entirely of 7's, then there would be fifty terms in the sequence. these answer choices are reasonably close to fifty, so it stands to reason that by far the majority of the terms will be 7's. therefore, try as few 77's as possible.

try only one 77:
remaining terms = 350 - 77 = 273
this would be 273 / 7 = 39 sevens
so ... you'd have one '77' and thirty-nine '7's

this works!

answer = c

OR

Since the units digit of 350 is zero, you know that the number of terms in the equation must be such that:

n*7 = number with units digit of zero

The only time this is true is if n is 10 or a multiple thereof, and 40 is the only answer that satisfies that.

24.

after the first two terms i.e (2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8) is a GP series with first term as 2^2 and ratio as 2.
Using the formula for sum of GP series, for this part, the original equation becomes
2^2 + 2^2(1–2^7)/(1–2)
= 2^2 + 2^2(127)
= 2^2(1 + 127)
= 2^2 * 2^7
=2^9

OR

there are several ways.

(1) **PATTERN RECOGNITION**
it should be clear that there's nothing special about 2^8 as an ending point; in other words, they just cut the sequence off at a random point. therefore, if we **investigate smaller "versions" of the sequence, we should be able to detect a pattern**.
let's look:
first term = 2
sum of first 2 terms = 4
sum of first 3 terms = 8
sum of first 4 terms = 16
ok, it's clear what's going on: each new term doubles the sum. **if you see a pattern this clear, it doesn't matter whether you understand WHY the pattern exists; just continue it.**
so, i want the sum of nine terms, so i'll just double the sum five more times:
32, 64, 128, 256, 512.
this is choice (a).
this is a general rule, by the way: IF SOMETHING CONTAINS MORE THAN 4–5 IDENTICAL STEPS, YOU SHOULD BE ABLE TO EXTRACT A PATTERN FROM LOOKING AT SIMILAR EXAMPLES WITH FEWER STEPS.

(2) **ALGEBRA WITH EXPONENTS** ("textbook method")
the first two terms are 2 + 2. this is 2(2), or 2^2.
now, using this combined term as the "first term", the first two terms are 2^2 + 2^2. this is 2(2^2), or (2^1)(2^2), or 2^3.
now, using *this* combined term as the "first term", the first two terms are 2^3 + 2^3. this is 2(2^3), or (2^1)(2^3), or 2^4.
you can see that this will keep happening, so it will continue all the way up to 2^8 + 2^8, which is 2(2^8) = (2^1)(2^8) = 2^9.

(3) ESTIMATE

these **answer choices are ridiculously far apart**, so you should be able to estimate the answer.
memorize some select powers of 2. notably, 2^10 = 1024, which is "about 1000". 2^9 = 512, which is "about 500". and of course you should know all the smaller ones (2^6 and below) by heart.
thus we have 2^8 is about 250, and the other terms are 128, 64, 32, 16, 8, 4, 2, 2.
looking at these numbers, i'd make a ROUGH ESTIMATE WITHIN A FEW SECONDS:
250 is 250.
128 is ~130.
64 and 32 together are ~100.
the others look like thirty or so together.
so, 250 + 130 + 100 + 30 = 510.
the only answer choice within shouting range is (a); the others are absurdly huge.

—

even if you have no idea how to do anything else, **you should still be able to do out the arithmetic within the two–minute time limit.**
it won't be fun, but you should be able to do it. if you can't, then the reason is probably "you stared at the problem for too long, and didn't get started when you should have".

yes ,the shortest method on this planet to solve the above question.

This formula may be of use:2^1+2^2+........+2^n =[2^(n+1)] – 2, where n equals to number of terms.

question is: 2+2+(2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8)

this can be written as : 2+(2^1)+(2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8)

Therefore, 2+[2^(8+1)] – 2 = 2^9 (answer)

25.

OA is C

statement (1) means that the smallest and largest elements of the list have the same sign, i.e., are both positive or both negative.
but, since those are *the smallest and largest elements of the list*, that means that all the elements *between* have to have that same sign, too.
or:
you can't have 0 between two positive numbers, or between two negative numbers.
either *everything* in the list is positive, or *everything* in the list is negative.

From statement (1), we know the product of the highest and the lowest integer is + ve, it means either both of them are +ve or –ve

For ex: –2,–1,1,2 in this list the product of –2&2 is –4. It proves both the highest and the lowest terms have to be of same sign.

(+ve or –ve) the other factor that needs to be considered is the no. of terms in the list,

If the no. of terms is odd and all the integers are –ve, the product of all the integers will be –ve this information is given by statement (2)

no. of terms in the list are even, hence the product of all the integers in the list will always be +ve.

So if you combine (1) & (2), they are sufficient.

You have to multiply the smallest and the largest to satisfy case I. For exmple {+,–,–,+} does not satisfy case I. You have taken both negative numbers in the middle. But the smallest number will be one of the negative numbers and the largest one of the positive ones, giving a negative product of as opposed to positive. Same holds for the third example you have used.

if you have numbers arranged from least to greatest, then any '–' numbers must show up to the left of *all* '+' numbers. otherwise, you've created an impossible situation in which a negative number is somehow bigger than a positive number.

26.

(1) imagine 0, 0, 0, 0 OR 0, 0, 0, 2 NS
(2) in order for the sum of "any 2 numbers" to = 0, all the numbers must equal 0
SUFFICIENT

statement 2 gives: the sum of ANY TWO...
So, since there are "more than 2" numbers in the set, the set contains (in your example): (–2, 2, x). because the set contains at least that x, the sum of "any 2" numbers, for instance 2 + x, does not have to equal zero.
so, INSUFFICIENT

you need to have more than 2 numbers in the set. the problem is that ANY two numbers have to sum to zero – which means that if you pair the mystery third number with EITHER of the existing two numbers, you must get a sum of zero.

if your first two numbers are 2 and –2, that's impossible: there's no number that will add to 2 to give zero, and will ALSO add to –2 to give zero.

in general, if your first two numbers are –x and x, then your third number must be x (so that it adds to –x to give zero), but it must ALSO be –x (so that it adds to x to give zero). the only way that x can equal –x is if x is zero – which means that all three numbers are zero.

therefore, everything must be zero.

27.

The answer is A
With statement 1:
this function can only be addition or multiplication
with either of these two operations the left side does indeed equal the right...sufficient
With statement 2
this function can be either multiplication or division
with multiplication the left and right side equal one another
with division it doesn't...
hence 2 is insuffcient.

note the general takeaway here:
if you have a problem like this, in which a mystery symbol stands for *one or more* of a collection of operations, then your #1 goal is to figure out ANY AND ALL operations for which that symbol can stand.

28.

Let's consider (1) N+1 >0. This clearly tells you nothing about p, so is insuffienct by itself, ruling out A&D.

Let's look at (2). np >0. This tells us that n,p and either both positive or both negative. Therefore it is insufficient to answer whether p >0, so we can eliminate B.

Now let's consider (1) and (2) together. (1) combined with the fact that N and P are integers tells us that N>= 0. (2) tells us that N and P are either both positive or both negative and that neither are equal to 0. Combined with (1) we therefore know what N is positive, and from (2) P must be positive too. So (1) and (2) together are sufficient and the answer is C.

29.

(2)

Takeaway #1: when you plug numbers on a DS problem, YOUR GOAL IS TO PROVE THAT THE STATEMENT IS INSUFFICIENT.

Therefore, as soon as you get a 'yes' answer, you should be TRYING to get a 'no' answer to go along with it; and, as soon as you get a 'no' answer, you should be TRYING to get a 'yes' answer to go along with it.

Statement (2)

you need to pick numbers such that $x + y > z$, per this statement.

First, pick a completely random set of numbers that does this: how about $x = 1$, $y = 1$, $z = 0$. These numbers give a YES answer to the prompt question, since $1^4 + 1^4$ is indeed greater than 0^4. Now remember: **your goal is to prove that the statement is INSUFFICIENT.** This means that we have to try for a 'no' answer. This means that *we have to make z^4 as big as possible, while still obeying the criterion $x + y > z$.* Fortunately, this is somewhat simple to do: just make z a big negative number. Try $x = 1$, $y = 1$, $z = -100$. In this case, $x + y > z$ (satisfying statement two), but $x^4 + y^4$ is clearly less than z^4, so, NO to the prompt question. Insufficient.

Statement (2)

you need to pick numbers such that $x^2 + y^2 > z^2$, per this statement. First, pick a completely random set of numbers that does this: how about $x = 1$, $y = 1$, $z = 0$ (the same set of numbers we picked last time). These numbers give a YES answer to the prompt question, since $1^4 + 1^4$ is indeed greater than 0^4. Now remember: **your goal is to prove that the statement is INSUFFICIENT.** This means that we have to try for a 'no' answer. This means that *we have to make z^4 as big as possible, while still obeying the criterion $x^2 + y^2 > z^2$.* Unfortunately, this isn't as easy to do as it was last time; we can't just make z a huge negative number, because z^2 would then still be a giant positive number (thwarting our efforts at obeying the criterion). So, we have to finesse this one a bit, but the deal is still to *make z as big as possible while still obeying the criterion*. Let's let x and y randomly be 3 and 3. Then $x^2 + y^2 = 18$; we need z^2 to be less than this, but still as big as possible. So let's let $z = 4$ (so that $z^2 = 16$, which is pretty close). With these numbers, $x^4 + y^4 = 162$, which is much less than $z^4 = 256$. Therefore, NO to the prompt question, so, insufficient. Answer = e.

Takeaway #2: if a statement is sufficient, then you WILL be able to PROVE that it is, algebraically or with some other form of theory.

In other words, you'll never get a statement that's sufficient, but for which you can only figure that out by number plugging.

It's obvious that you can get a YES answer to the question; all you have to do is take ridiculously big numbers for x and y, and a small number for z. for instance, $x = y = 100$, $z = 0$, satisfy both statements, and clearly give a YES answer. So, you're trying for a NO answer. Try to make Z as big as possible while still satisfying the criteria (i.e., less than $x^2 + y^2$). Let's let $x = y = 3$ then to satisfy both statements, we need z^2 less than 18, and z less than 6. We'll take $z = 4$, which is pushing the limit of the first one. In this case, then, x^4 + y^4 = 81 + 81 = 162, but z^4 = 256, giving a NO answer. Insufficient   Answer = e

30.

Ans. A

(1).. add 1 to both sides… you get r > w.

(2) gives contradictory answers… NS

31.

We're told x and y are positive but not whether they are greater than 1, so I have to consider fractional possibilities. How do I know what to try?

When I take a square root:
Anything greater than 1 will get smaller (but remain larger than 1)
1 will stay the same
Anything between 0 and 1 will get bigger (but remain a fraction between 0 and 1)

When I take a reciprocal in each of the above cases:
1/something larger than 1 = something smaller than 1 (but still positive)
1/1 = 1
1/something smaller than 1 = something larger than 1

If I want to try numbers now, then I know I need to try a number from each set. Or I can continue with logic and the algebraic representations. Do whichever you are most comfortable with.

For trying numbers, first try something greater than 1:
x=2, y=2 (I'm trying the same numbers b/c I'm trying to see if I can prove things false and funny things happen when you use the same number for different variables). 1/(4)^.5 = 1/2.
Roman Numeral 1 (RN1): (4)^.5 / 2(2) = 2/4 = 1/2. Same, not greater, so elim RN1.
RN2: (2^.5 + 2^.5) / (4) = 2(2^.5) / 4. Well, 2^.5 is about 1.7. 2*1.7 = 3.4 / 4 = more than 1/2. So RN2 is okay, at least with this instance.
RN3: (2^.5 – 2^.5) / 4 = 0/4 = 0. Elim RN3.

At this point, I don't know whether I have to try more numbers b/c the answer choices haven't been listed. If I have both "none" and "II only" as options, then I have to try more numbers. If "none" is not an option, then I'm done.

You will notice that equation 2 will always be more .
THE FASTEST WAY IS TO EXPRESS EACH EQUATION AS A FUNCTION OF $1/(X+Y)^{0.5}$. This aproach takes less than a minute.

there's little sense in dealing with #3 algebraically: because of the subtraction, it can clearly equal 0 (if *x* and *y* are the same number). since $1/\sqrt{x + y}$ is a positive number, the possibility of 0 rules out roman numeral III. (in fact, that expression can even be negative, as nothing prohibits *x* from being smaller than *y*.)

if you want to **compare two fractions**, you can use the technique of **cross products** to perform the comparison.
to use this technique, you take the two 'cross products' (one of the numerators, times the denominator of the *other* fraction), and associate each of the cross products with whichever fraction donated the *numerator*.
for instance, if you're comparing 2/3 vs. 11/17, then the cross products are $2 \times 17 = 34$ (associated with 2/3) and $3 \times 11 = 33$ (associated with 11/17). because 34 is greater than 33, it follows that 2/3 is greater than 11/17.

notice that this technique only applies to **positive** fractions... but that's all you really need: if the fractions have opposite signs, then the comparison is trivial (the positive one is bigger!), and if the fractions are both negative, then the comparison is the opposite of whatever it would be if they were positive.

find cross products in #(i):
$\sqrt{x + y}/2x$ vs. $1/\sqrt{x + y}$
cross products are $(x + y)$ vs. $2x$
subtract one x from both sides —> this comparison is the same as y vs. x
we don't know which is bigger.

find cross products in #(ii):
$(\sqrt{x} + \sqrt{y})/(x + y)$ vs. $1/\sqrt{x + y}$
cross products are $(\sqrt{x} + \sqrt{y})\sqrt{x + y}$ vs. $(x + y)$
divide both sides by $\sqrt{x + y}$ to give $(\sqrt{x} + \sqrt{y})$ vs. $\sqrt{x + y}$ —– remember that (quantity) divided by √(quantity) is √(quantity) — that's the definition of what a square root is.
since both of these quantities are positive, we can square them and compare the squares:
$(\sqrt{x} + \sqrt{y})^2$ vs. $(\sqrt{x + y})^2$
$x + 2\sqrt{xy} + y$ vs. $x + y$
left hand side is bigger
so the original fraction is bigger than $1/\sqrt{x + y}$

ans = ii only

32.

OA: B

1. $|x - 3| \geq y$
Taking numbers:
x: –2, 1 both can satisfy the above equation. Insufficient.

2. $|x - 3| \leq -y$
Since $|x-3|$ is an absolute value, the smallest it can go is 0.
And since y is given to be >0, thus – y will give a negative value which will cause the equation to fall apart unless it is 0.

so $|x-3| = 0$.
x = 3.

33.

OA: C

we can rephrase this to x = z – y.
there are thus 3 possibilities for the absolute value |x| :
(a) if z – y is positive, then |x| = z – y, and will NOT equal y – z (which is a negative quantity).
(b) if z – y is negative, then |x| = y – z (the opposite of z – y).
(c) if z – y = 0, then |x| equals both y – z and z – y, since each is equal to 0.
TAKEAWAY: when you consider absolute value equations, you'll often do well by considering the different CASES that result from different combinations of signs.
notice that (a) and (b), or (a) and (c), taken together prove that statement 1 is insufficient.

statement 2:
we don't know anything about y or z, so this statement is insufficient.**
if you must, find cases: say y = 2 and z = 1. if x = –1, then the answer is YES; if x is any negative number other than –1, then the answer is NO.

together:
if x < 0, then this is case (b) listed above under statement 1.
therefore, the answer to the prompt question is YES.
sufficient.

—

WE could craft a statement that doesn't mention all three of x, y, z and yet IS STILL SUFFICIENT.
here's one way we could do that:
(2) y < z
in this case, y – z is negative and therefore CAN'T equal |x| — *no matter what x is* — since |x| must be nonnegative.

so, this statement is a definitive NO, and is thus sufficient even though it doesn't mention x at all.

34.

Ans. C… standard property.

35.

If $x = 0.1$, then $x^2 < 2x < 1/x$ (so 1 is possible)

If $x = 0.9$, then $x^2 < 1/x < 2x$ (so 2 is possible)

(1) $x^2<2x<1/x$

This means that $x^2<2x$ so divide by x to get $x<2$. The second one tells you that $2x<1/x$ which simplifies to $x < 1/\sqrt{2}$. These can obviously both be satisfied at the same time, so (1) works.

(2) $x^2<1/x<2x$

This means that $x^2<1/x$ which gives $x^3<1$, or $x<1$. The second half gives you $1/x<2x$ or $1<2(x^2)$ or $x>1/\sqrt{2}$. So any number that satisfies $1/\sqrt{2}<x<1$ will work.

(3) $2x<x^2<1/x$. The first part gives $2x<x^2$ or $x>2$. The second half gives $x^2<1/x$ or $x^3<1$ or $x<1$. Since the regions $x>2$ and $x<1$ do not overlap, (3) can not be satisfied.

The Answer choice is (4), 1 and 2 only

> if $x = 1/2$, then:
> $x^2 = 1/4$
> $1/x = 2$
> $2x = 1$
> so the order would be $x^2 < 2x < 1/x$. So (a) is possible. Eliminate i and iii. (Looks like you got this far)
>
> if $x = 3/4$, then:
> $x^2 = 9/16$
> $1/x = 4/3$
> $2x = 3/2$
> so the order would be $x^2 < 1/x < 2x$. So (b) is possible. Eliminate ii. (Looks like this is where you had trouble.)

36.

Notice that we're dealing with a *fractional inequality*, which, worse yet, CAN'T be multiplied by the common denominator (since we don't know the sign of that denominator).
Therefore, **pick numbers**.
Just be careful to pick APPROPRIATE numbers. i.e., this problem contains **sums and differences**, as well as **sign considerations**, so i would pick:
* POSITIVES AND NEGATIVES (as allowed by the statements)
* DIFFERENT RELATIVE SIZES (i.e., "bigger" and "smaller" numbers) — this is important because of *addition and subtraction*.
so, for statement (1), i would pick:
1, 2
2, 1
1, –2
2, –1
1, 0
for statement (2), i would pick:
1, –2
2, –1
–1, –2
–2, –1
0, –1
for "together" i would look at the two common elements, which are (1, –2) and (2, –1).
note that this is a lot of plug–ins, but you don't wind up trying them all – you STOP as soon as you get "insufficient".

GMAT's answer is (E)

this is a difficult problem, because it resists simple algebra. **you CANNOT multiply through by the denominator (x + y), because the sign of that denominator is unknown.**

therefore, you have to leave the problem as written (ugly as it may be).

since there's no simple algebraic solution, a fallback is to **recognize the types of numbers that are important in the problem, and try numbers across those categories.**

there are two things that matter in this problem (as may be deduced from an inspection of the problem + experience with these sorts of things):
1. positive vs. negative
2. the relative magnitudes of x and y

let's try numbers across both of these categories.

statement (1)
x must be positive, but y could be positive or negative, and smaller or bigger (or the same) in

magnitude.
if x = 1 and y = 2 —> answer = NO
if x = 2 and y = 1 —> answer = NO
if x = 1 and y = –2 —> answer = NO
if x = 2 and y = –1 —> answer = YES
insufficient
[at this point you could notice that the last two examples also satisfy statement 2, and therefore satisfy statements 1 and 2 together. this fact proves that the answer is E right now, and you're done. if you don't notice this (most students won't), then go on.]

statement (2)
y must be negative, but x could be positive or negative, and smaller or bigger (or the same) in magnitude.
if x = –1 and y = –2 —> answer = NO
if x = –2 and y = –1 —> answer = NO
if x = 1 and y = –2 —> answer = NO
if x = 2 and y = –1 —> answer = YES
insufficient

together
x must be positive and y must be negative, but the relative magnitudes can go any way (bigger/smaller/same)
if x = 1 and y = –2 —> answer = NO
if x = 2 and y = –1 —> answer = YES
insufficient

ans (e)

37.

Let us take statement I – In this, it is given that 1/(k–1) >0. This implies that k must be postive and k must be greater than 1. Hence, 1/k is definitely greater than zero. For example, k's value is 2, then 1/(2–1) = 1 which is > 0. This implies that 1/2 = 0.5 which si still greater than '0'. Hence, this is sufficient.

Let us take II – It says 1/ (k+1) > 0 which means that this will satisfy for both positive and negative values of k which are > than –1. for example, if k is 2, 1/(k+1) is >0 and 1/k wll be >0. But if k's value is –0.5, it will satisfy the second equation but 1/k will be –2 which is <0 and hence, INSUFFICIENT.

Hence, A alone is sufficient to answer this question.

38.

The sign of a variable has nothing to do with addition and subtraction.
It is only MULTIPLICATION AND DIVISION that are affected by the sign of the quantity.
There's a much worse problem here: you CAN'T SUBTRACT INEQUALITIES that face the same way. You can add them, but you can't subtract them.

The answer is B

if you don't have an approach, then you should immediately start plugging in. you should do ANYTHING to ensure that you're not just sitting there staring at a problem.

—

Statement (2):
All you need is $y < w$, which is EXACTLY equivalent to $w - y > 0$. There are two ways you could figure this out:
So #2 is sufficient. they're just including x in there to try to get you to waste your time.

You cannot subtract two inequalities that face the same way.

Think about this:
$x < 10$
$y < 10$
if you try to subtract these, then you'll get $x - y$ (?) 0.
but that clearly doesn't work, since you could create possibilities for "<" (e.g. $x = 7$, $y = 8$); "=" (e.g., $x = y = 8$); or ">" (e.g., $x = 8$, $y = 7$).

Incidentally, if two inequalities **face in OPPOSITE ways**, then you can subtract them. But if that's the case, it's easier to just **multiply one of them by –1 and then add them**.

The question is asking is $w-y>0$

And After reading the the St.1 , things you know are
$w+x<0$ from Question Stem &
$x+y<0$ from St.1
Nothing more is given, don't use the inequality used in the Question.
So the above statements can be true
$w=1$, $y=2$ and $x=-10$
$w+x = -9<0$
$w+y = -8 <0$

or
$w=2$, $y=1$, $x=-10$
$w+x = -8<0$
$w+y = -9<0$

And as you can see we cant say whether $w>y$ or $w<y$ and hence insufficient.

39.

OA: A
The quick way to approach will be pick a number $x<0$
Lets pick –5
so we know $x=-5$
$\text{sqrt}(-x|x|) = \text{sqrt}(-(-5)|-5|)$
$= \text{sqrt}(5*5) = \text{sqrt}(25) = 5 = -(-5) = -x$
So Answer A.

40.

If statement 2 contains statement 1, then ELIMINATE (b) and (c).
If statement 1 contains statement 2, then ELIMINATE (a) and (c).

Ans– D (BOTH SUFFICIENT)

SQRT ($(x-5)^2 = |x-5|$
squaring a quantity, and then square–rooting, is equivalent to taking the absolute value.

we can make this more clear:
$|x - 5|$ can be either $(x - 5)$, the actual quantity within the absolute–value bars, or $(5 - x)$, the opposite of that quantity.
if it's to be the original quantity $(x - 5)$, then that quantity must be at least 0: $x \geq 5$.
if it's to be the opposite $(5 - x)$, then that opposite quantity must be at least 0. for that to happen, $x \leq 5$.
(notice that, if x is actually 5, then $|x - 5|$ equals *both* $(x - 5)$ and $(5 - x)$, since both of them are zero.)

therefore, we can rephrase the question:
is $x \leq 5$?
when you see this statement, it may bewilder you at first, but you should look at it and think: "ok, just absolute–value bars and negative signs. no other numbers; no other operations; this could only possibly have to do with the sign of x."
then just test it to see whether it works for PNZ (positive, negative, zero).
turns out that it only works for negative numbers.
therefore, rephrase:

1. $-x|x| > 0$

the above is possible only for $x < 0$, therefore $(x-5) < 0$

|x–5| = 5–x this becomes – – – > –(x–5) = 5–x and hence sufficient

2. Clearly states that (x–5) < 0 so this is sufficient and the answer is D

(1) x < 0

this is sufficient, since x is definitely less than 5 if it's negative.

41.

the absolute value will do one of two things to a quantity:
(a) LEAVE THE QUANTITY ALONE, if the quantity is POSITIVE;
(b) REVERSE THE SIGN of the quantity, if the quantity is NEGATIVE.
if the quantity is exactly 0, then both of these result in the same number, so it doesn't matter which of them you call it.

therefore:
the expression |x – 3| will equal one of two expressions:
LEFT ALONE as (x – 3), if x – 3 is POSITIVE — i.e., if x is greater than 3;
REVERSED to (3 – x) (which is the same as –x + 3), if x – 3 is NEGATIVE — i.e., if x is less than 3;
EITHER of these (since both equal 0) if x is exactly 3.

therefore, we now have a rephrase of the question.
REPHRASE:
is x ≤ 3 ?

so, the answer is (b).

you can also solve this problem, perhaps more easily, by **PLUGGING IN NUMBERS**.

statement (1):
since 3 is clearly a pivotal number in this problem, try numbers that are greater than 3 and numbers that are less than 3.

try x = 0:
√((x – 3)^2) = 3
3 – x = 3
answer to prompt question = YES

try x = 5:
√((x – 3)^2) = 2
3 – x = –2
answer to prompt question = NO

insufficient.

—

statement (2)
this statement is an obnoxious way of stating that x is a negative number. (you still have to figure *that* out — no way around it)

if you try plugging in a vast array of negative numbers – big, small, even, odd, etc. – you'll find that the equality in the prompt question holds for ALL of them.

sufficient.

42.

A
When multiplying or dividing an inequality by a negative, we have to switch the sign. We're not told whether the quantity (a–b) is pos or neg, so we have to check this both ways.
IF a–b is pos, then 1 < (a–b)(b–a) (and you can simplify further from there)
IF a–b is neg, then 1 > (a–b)(b–a) (and again you can simplify further from here)
But the point is that I have to follow both possibilities through and any particular statement is only sufficient if BOTH equations give me the SAME definitive answer, yes or no.
I'll stop there – try this again and come back if you have more questions.
Is 1/(a–b) < b – a?

Rephrase: 1< (a–b) (b–a)

1) a < b

If a, b are both +ve, Both –ve, or –ve & +ve

1 > (a–b) (b–a)
Statement (1) is sufficient.

2) 1 < |a–b|

This statement mentions |a–b| and nothing about (b–a)

(b–a) can be +ve or –ve.

Therefore, statement (2) is not sufficient.

43.

if you have
| QUANTITY 1 | = | QUANTITY 2 |
(with NOTHING ADDED to, or SUBTRACTED from, the abs. values)
then
just SOLVE TWO EQUATIONS:
*** QUANTITY 1 = QUANTITY 2**
*** QUANTITY 1 = –(QUANTITY 2)**
this is all you need. (there are also the possibilities with a negative sign in front of quantity 1, but those will just be equivalent the two already written here.)

statement (1)
for EQUATIONS involving absolute value, like this one, the key realization is that the absolute value of a quantity can signify either the quantity itself or the *opposite* of the quantity. therefore, if you try each of the sign combinations (pos/neg) of the absolute values in the problem, you'll be guaranteed to find all of the solutions.
(note: in what follows, "+" means leaving the expression within the absolute value bars alone; "–" means reversing the sign of that expression)
in this equation, there are ostensibly four sign combinations, +/+, +/–, –/+, –/–, but it's only necessary to try two of them:
** first, either +/+ or –/–, in which *both* or *neither* of the absolute value expressions are flipped. may as well go with +/+ (i.e., leaving both of the absolute value expressions alone while removing the bars): x + 1 = 2(x – 1), or x = 3. plugging this back into the original equation shows that it works.
** second, either +/– or –/+, in which *one* of the absolute value expressions is flipped. let's go (at random) with flipping the first one: –x – 1 = 2(x – 1), or x = 1/3. plugging this into the original equation also shows that it works.

therefore, **statement 1 means that x = 3 or x = 1/3**.

statement (2)

two ways to interpret absolute value inequalities like this one:
** *memorize the template of the solution* (preferred for efficiency's sake): you should just know that |*expression*| > *a* means "either *expression* > *a* or *expression* < –*a*".
** *conceptualize absolute value as distance*: in this case, |*x* – 3| means the distance between *x* and 3. therefore, this statement means that the distance between x and 3 is greater than 0 (in either direction).
either of these interpretations means that *x* < 3 or *x* > 3, or, equivalently, ***x* is not equal to 3.**

statement 1 is insufficient, because 1/3 gives a "yes" and 3 gives a "no". statement 2 is also insufficient, because *every* number except 3 is possible. taken together, though, the two statements are sufficient because they yield a unique value, 1/3, for x.

notice that there's no reason even to figure out whether 1/3 gives "yes" or "no" at this point; it's *one value*, meaning that it is guaranteed to be sufficient no matter what the answer.

answer = c

44.

I is definitely true.

For II. z > y > x. Consider, x=1/4, y=1/3, z=1/2. This satisfies given primary inequality. Hence, II also holds true.

For III. Consider x=1, y=1/3,z=1/2. This satisfies given primary inequality. Hence, III also holds true.

Hence, E.

basically, the idea is that fractions (i.e., numbers between 0 and 1) "act funky" when they're raised to powers.
so do negatives.
therefore, when you pick numbers, you MUST consider these sorts of numbers!

you can take 3 cases,
1. When X,Y,Z are positive integers in that case:
I. X > Y> Z holds true

2. When X, Y, Z are fractions:
Z > Y > X
1/2 > 1/3 > 1/4
X>Y^2>Z^4
1/4 > 1/9 > 1/16

3. X > Z > Y (Negative, positive)
20 > –3 > –4
X>Y^2>Z^4
20 > 16 > 9

Therefore answer is E.

45.

Take –4,–3,–2–,–1 as the 4 consecutive integers. The answer is A the numbers can be –4,–3,–2,–1 as well but do we know that those are the numbers?

–2,–1,0,1.. then $0 > -1$, however, if the numbers were 1,2,3,4 then $3<8$.

If p, q, r, and s are consecutive integers, with $p < q < r < s$ and $pq < rs$;then $r>0.5$ and cannot be 0. In fact $pr<qs$ making statement 1 sufficient.
Statement 2 give no information other than the universal truth $2>0$.
The correct answer is indeed A.

46.

Statement 1 is sufficient.
Stmt 2 is $y < 1$, since y is an integer , it can be 0 only if it is less than 1.
So Stmt 2 is sufficient too.
Both statements are sufficient enough to deduce if $y = 0$.
Answer is D

47.

$M - 3Z > 0$
$-M + 4Z > 0$

if you add the two equations, M cancels off and you are left with only one Z which is practical :

$M-M -3Z +4Z > 0$
gives :
$Z>0$

NOTE : obviously you can do this only if the two inequality signs face in the same direction!

So we now Z is positive. Now consider "$M - 3Z > 0$". We can conclude M is positive

so M + Z ===> positive + positive so is positive

Thus C.

48.

OA is D.
believe it or not, it turns out that **JUST THE PROBLEM STATEMENT IS ALREADY SUFFICIENT** on this problem!

in other words, this problem is already "sufficient", ***even without EITHER of the two statements!***

yes, you read that correctly.

proof:

* "$zy < xy < 0$" means that z and y have opposite signs, and x and y have opposite signs.
therefore, x and z have the same sign.
furthermore, z must be farther away from zero than x (because the magnitude of zy is greater than the magnitude of xy).

therefore, there are only 2 possibilities (shown on number line):

y————0————x—————z

or

z————x————0————y

now let's turn to the problem statement.

$|x - z|$ is the distance between x and z.
$|x|$ is the distance between 0 and x.
$|z|$ is the distance between 0 and z.

using these interpretations, it's plain that $|x - z| + |x| = |z|$ is ALREADY true for both of these statements.

neither of statements (1) and (2) is necessary.

technically, there's no answer choice that does this ("the problem statement is already sufficient"), although it's clear that you should pick (d) should this situation ever arise on the real exam.

There are different breeds of absolute value problems, so (as usual) there's no one neat, solid answer to a question like that. however:
* if a problem contains the symbols "> 0" or "< 0" at **any point**, you can rest assured that the crux of the problem involves the **signs** of quantities. (the problem in this thread is a perfect example.)
if you encounter such a problem, you should immediately devote all of your energy to rephrasing the question prompt and/or statements to equivalent formulations involving 'positive'/'negative'.
for instance, if you see
$zy < xy < 0$
you should think:

* z and x have the same sign
* y must have the opposite of whatever sign those two have
* therefore, (x y z) is either (+ – +) or (– + –)

that sort of reasoning will be an excellent start. from there, there's no telling which way the wind will blow – just study your number properties, and you should be able to figure out the rest.
oh yeah, you should avoid 'solving' if at all possible: you should try to think in the abstract about the signs of the numbers, and about the situation resulting from each possible combination of signs. if that sort of reasoning gets you nowhere, *then* try plugging in numbers and solving as plan b.

49.

Well Z could be –1 and n could be any even integer. Then the result is 1. So from (1) we can say that Z is either 1 or –1. So the correct answer should be C.
For z^n = 1, and n being a non zero integer, there are 3 possible ways.
a. 1^1 = 1
b. 1^– 1 = 1
c. – 1^2 = 1
Statement 1 not conclusive. Z could be 1 or –1.
Statement 2: z > 0.

==> we need both statements to solve for z.
Answer: C.

50.

Answer is D

the first thing you should do here is rephrase the question.
big takeaway:
if you see the ABSOLUTE VALUE OF A DIFFERENCE, you should recast it as the DISTANCE BETWEEN THE TWO THINGS on the number line.

therefore, |y – a| is the distance between y and a, and so on.

hence:

QUESTION: is y closer to a than to b ?
(1) z is closer to a than to b
(2) y is closer to a than z is to b

statement (1):
note that the distance y–a is less than the distance z–a, because y is placed between a and z.
also, note that the distance y–b is greater than the distance z–b, since z lies between y and b.
therefore:
distance y–a < distance z–a < distance z–b < distance y–b (note that these are color–coded to the

statements above)
so, distance y–a < distance y–b
"yes"
SUFFICIENT

statement (2):
same thing as statement (1), except for the second term of the inequality above isn't there anymore.
i.e., distance y–a < distance z–b < distance y–b.
SUFFICIENT

ans = (d)

OR

First, let's try to clear the absolute values.

Because we know that $a < y < z < b$, we know
$abs(y - a) = y - a$
$abs(y - b) = b - y$ (since $y - b$ is negative)

We can rephrase the question:
Is $y - a < b - y$?
or
Is $2y < a + b$?

Statement (1) can be rephrased: $z - a < b - z$, so $2z < a + b$. We also know that since $y < z$, then $2y < 2z$. So $2y < 2z < a + b$. (1) is sufficient.

Statement (2) can be rephrased: $y - a < b - z$, so $y + z < a + b$. Since $y < z$, we can add y to both sides: $2y < y + z$. So $2y < y + z < a + b$, so (2) is sufficient as well.

51.

Answer: E.
Specifically, rounding rounds ALL 5's *up*, ALL the time, even if they're followed by nothing at all.
This is why the problem doesn't contain the word "round": according to traditional rounding, 4.5 rounds to 5

the wording in the actual problem, though, is completely unambiguous: "4 is *the* integer that is

closest to x + y".
this statement actually rules out BOTH 3.5 and 4.5, because each of those numbers is *equidistant* from two integers: the former from 3 and 4, and the latter from 4 and 5.

therefore, here are the CORRECT rephrases:
(1) $3.5 < x + y < 4.5$
(2) $0.5 < x - y < 1.5$

all four of those signs are strict inequalities. there are no ≤'s or ≥'s in this problem.

you can add all 3 corresponding parts of the inequalities directly:
$3.5 < x + y < 4.5$
$0.5 < x - y < 1.5$

$4 < 2x < 6$

therefore
$2 < x < 3$

notice that all this discussion of <'s, ≤'s, >'s, and ≥'s is immaterial in the final analysis, because there are still numbers greater than 2.5 (which are closest to 3) and numbers less than 2.5 (which are closest to 2). therefore, insufficient even if you misinterpret the question prompt as referring to "rounding".
but they *could*, easily, write a problem that would turn on the inclusion/exclusion of a number such as 4.5.

here's an example:
what number results if the number x is rounded to the nearest hundred?
(1) the multiple of 20 that is closest to x is 140.
(2) x is within ten units of 140.
here, statement (1) means that $130 < x < 150$. that's a strict inequality, which doesn't apply to 130 and 150 themselves (since 130 is just as close to 120 as to 140, and 150 is just as close to 160 as to 140).
all of these numbers give 100 when rounded to the nearest hundred, so this statement is sufficient.
statement (2), on the other hand, means that $130 \leq x \leq 150$. this inequality includes 130 and 150.
since 150 rounds to 200, this statement is insufficient.
in this problem, the inclusion vs. exclusion of 150 makes all the difference.

52.

you can ADD TWO INEQUALITIES TOGETHER if the inequalities are BOTH "<" OR BOTH ">".

(you can't add them if one is "<" and the other is ">". if that's the situation, then you should multiply one of the inequalities by –1, or just turn it around, so that both of them are either "<" or ">".)

$-q>n-p$
+
$q>p$
____
$0>n$

therefore, C.

53.

statement (1)

all we know is that z^3 is AN INTEGER. in particular, we can't deduce that z^3 is a perfect cube.

if z^3 is a PERFECT CUBE, such as 1, 8, or 27, then z will be an integer.
if z^3 is NOT a perfect cube, such as 2, 3, 4, etc., then z will NOT be an integer.

therefore, INSUFFICIENT.

(notice that you can easily find this by PLUGGING IN NUMBERS. in fact, the very first two positive integers, 1 and 2, give "yes" and "no" respectively, so that's a clear "insufficient".)

if we assume that z^3 is a perfect cube, then we're *assuming* that z is an integer. if we make that (totally unfounded) assumption, then we shouldn't be surprised when we find a specious answer of "yes".

—

statement (2) is insufficient

—

together is actually SUFFICIENT.

* consider all the numbers that satisfy statement (2):
1/3, 2/3, 1, 4/3, 5/3, 2, etc.

* of these, the only ones that satisfy statement (1) as well are 1, 2, 3, ...
(all the fractional ones will still be fractions when you cube them)

* since these – the numbers that satisfy BOTH statements – are all integers, we have TOGETHER = SUFFICIENT.

answer = (c)

54.

(2) tells us that the magnitude of x is more than the magnitude of y and x and y are either both positive or both negative.
If x and y are both positive, x has to be more than y… if x and y are both negative, x has to be less than y (-2 is less than -1).
(1) tells us that x – y = ½… so x has to be more than y…
Combining, x and y are both +ve. Ans. C

55.

if you know that x > y, then you know that x – y is positive, and vice versa.
if you know that x < y, then you know that x – y is negative, and vice versa.

it's not hard to manipulate to get these statements; for instance, merely subtracting y from both sides of x > y will give x – y > 0.
but that's not the point; the point is to *recognize, INSTANTLY*, that knowing the status of the *inequality*involving x and y (i.e., whether x > y or x < y) is equivalent to knowing the sign of x – y.

well, the question prompt is:
is (m – k)(x – y) > 0?

based on the considerations above, statement #1 gives us the sign of the expression (m – k), and statement #2 gives us the sign of the expression (x – y).
if we have both of these signs, then we can figure out the sign of their product, so both statements together are sufficient.
(note that we don't even have to figure out the actual signs; it's good enough to *realize that we can find them*)

so, should be (c)

OR

mx + ky > kx + my
mx – kx > my – ky

(m–k)x > (m – k)y
(m–k)x – (m–k)y > 0
(m–k)(x–y) > 0
ie m > k and x > y

Answer C

56.

"500 is the multiple of 100 that is closest to X"

this means that, of all multiples of 100, 500 comes closest to x.
in other words, 500 is closer to x than is 100, 200, 300, 400, or 600, 700, 800, ...
if you think about this for a sec, you'll realize that it means x has to be strictly between 450 and 550.

Since the numbers don't have to be integers, you have
1. 450 < x < 550 (excluding BOTH endpoints) – note that x could be 450.00001 or 549.99999
2. 350 < y < 450 (again excluding both endpoints)

also, watch your <'s and ≤'s.

Range of X: 450<X<550
Range of Y: 350<X<450

By 1: Say X=499
Now if Y = 449 then nearest multiple of 100 to X+Y will be: 900
Say X=449 Now if Y = 350 then nearest multiple of 100 to X+Y will be: 800
Similarly you can prove that it is E

57.

Here are the TWO DEADLY ASSUMPTIONS:
1. NEVER assume that numbers are integers, unless you're told, or can infer, that they are.
2. NEVER assume that numbers are positive, unless you're told, or can infer, that they are.

There are numbers between –2 and –1, and those numbers are precisely the reason why the answer to this problem is (e).

58.

IS 1 / p > [r / (r^2 + 2)]

St 1. case 1 P=r=2

a=1/p = 0.5 and b=r/(r^2+2) = 1/3 = 0.33

a>b

case 2 p=r=-2

a=1/p = 1/-2 = -0.5 and b= r/(r^2+2) = -1/3 = -0.33

a < b

not suff

St 2 don't know anything about p not sufficient

combine both, as shown in statement 1 whenever p or r > 0, a > b. sufficient

answer C

59.

8/9 + 1/8 = 73/72, which is greater than 1.
therefore, knowing that x + y < 73/72 is insufficient to address the issue of whether x + y < 1, because x + y could be, say, 1/2 ("yes") or any value *between* 1 and 73/72 ("no").

therefore, (e).

60.

statement (1):

(a + b)/(a – b) < 0
therefore, a + b and a – b have opposite signs. we can divide this statement into 2 cases.

CASE 1: a + b is positive and a – b is negative
a – b is negative —> immediately know **a < b**
also, in this case, a + b > a – b, so therefore b > –b, so therefore **b is positive**.
that's all we know, though; we know nothing about the sign of a. (note that this case works for (a, b) = (2, 4) but also (–2, 4))

CASE 2: a + b is negative and a – b is positive
a – b is positive —> immediately know **a > b**
in this case, a + b < a – b, so therefore b < –b, so therefore **b is negative**.
again, that's all we know. (this case works for (a, b) = (2, –4) but also (–2, –4))

this is insufficient, because there's a case in which a < b and a case in which a > b.

statement 2:
obviously insufficient

together:
this has to be CASE 2 above, so therefore a > b.
sufficient.

61.

The first step is rephrase the question. IS my=rx?
Statement 1 states that m/y = x/r
This does not help us to calculate as to whether m/r is equal to x/y. So, MAYBE! (INSUFFICIENT)
Statement 2 states m+x/r+y = x/y
You can cross multiply rewriting the equation as y(m+x) = x(r+y) ———> my+yx = rx+yx
Subtracting yx from both sides, the equation then becomes my=rx, which is the rephrase of the question itself. SUFFICIENT.
B is the answer.

* **if you don't know what else to do with a proportion, cross–multiply it.**
(1) the reason this is valuable is because there are all sorts of versions of the same proportion that LOOK different as proportions, but which are shown to be the same when cross–multiplied. for instance, ALL of the following proportions
a/b = c/d
a/c = b/d
d/b = c/a
d/c = b/a
are equivalent, as all of them multiply to give ad = bc (as do countless others, such as (a + c)/(b + d) = c/d, after cancellation).
(2) **this applies only to proportions with EQUALS SIGNS** in them, **NOT to inequalities**. if you have an inequality such as a/b < c/d, then you can't cross–multiply it unless you know the sign of the product of the two denominators, bd (because that's all cross multiplication is: multiplying by both denominators at once on both sides). if bd is positive, then the sign won't flip; if bd is negative, then the sign must flip.

62.

(1) is surely enough.

if you have a simultaneous equation and inequality, then solve the equation and then substitute it into the inequality.

Correct answer is D

2x+5y= 20

Or, $y = (20-2x)/5$

$-2x > 3y$
Substiture for y
$-10x > 60-6x$
$-4x > 60$

The only way this would be possible is if x is negetive
Hence 2 is sufficient

63.

note that the expression $c + d$ appears in the question prompt. therefore, solve for this expression in statement 2: $c + d = 300 - b$.
now, substitute this into the question prompt, and also substitute $a + b = 200$:
is $200 > 300 - b$?
rephrase by solving —> **is b > 100?**

thus, it still comes down to observation that b must be more than 100, because it's the larger one of two numbers that add to 200 and therefore must be greater than half of 200.
but if you rephrase the question in this way, it's much more clear that you actually have to *think* about whether $b > 100$.

TAKEAWAY:
takeaway: if 2 numbers add up to n, then the larger number is more than n/2, and the smaller number is less than n/2.

it is given $a+b = 200$, $a < b$, is $a+b > c+d$
rephrase the question
we know $a+b = 200$, so
is $200 > c+d$ or is $c+d < 200$? This is a YES/NO question

1. $c+d < 200$ (Sufficient)
2. $b + c + d = 300$ – eq 1
add a to both the sides
$a + b + c + d = 300 + a$
we know $a + b = 200$, so
$200 + c + d = 300 + a$
$c + d = a + 100$
Now we know that $a < b$, a was equal to b a would be 100 and b would be 100, thus since $a < b$, $a < 100$
therefor $a + 100 < 200$ and $c + d < 200$ — > Sufficient

Answer D

64.

when you consider a problem like this, in which you are GIVEN INEQUALITIES, you should always CONSIDER THE EXTREMES of the given inequalities.

this technique is very simplistic, yet very powerful: *consider the extremes to find the extremes*. therefore, it's sufficient to think about, say, 0.1 and 0.9 for r, and 1.1 and 1.9 for s.

statement (i): 0.1/1.1, 0.9/1.1, 0.1/1.9, and 0.9/1.9 are all less than 1, so you're good.
statement (ii): works for (0.1)(1.1), (0.9)(1.1), and (0.1)(1.9), but NOT (0.9)(1.9).
statement (iii): only works for 1.1 – 0.9, doesn't work for any of the other pairs.

notice that this method is *systematic*: you don't just generate numbers at random, you generate numbers at the extremes of the intervals dictated by the inequality/ies.

I. Any number between 0 & 1 divided by any number between 1 & 2, will always be < 1
II. 2 cases: Consider r = 0.9 and s = 1.5, rs = 1.35. Consider r = 0.1 and s = 1.1, then rs = 0.11, so not true
III. 2 cases: 1.9 – 0.1 = 1.8 (this is > 1), 1.1 – 0.9 = 0.2 (< 1)

hence only I

65.

statement (1):
there's a statement called the pigeonhole principle, which basically says the following two things:
* if the AVERAGE of a set of integers is an INTEGER n, then at least one element of the set is ≥ n.
* if the AVERAGE of a set of integers is a NON–INTEGER n, then at least one element of the set is ≥ the next integer above n.
this principle is easy to prove: if you assume the contrary, then you get the absurd situation in which every element of a set is below the average of the set. that is of course impossible.

specifically, statement (1) is a case of the first part of the principle: the average of the set is 6/3 = 2, so at least one element of the set must be 2 or more.
again, you can prove this by reductio ad absurdum: if *no one* had sold 2 or more tickets, then you'd have a set in which everyone sold either 0 or 1 ticket, but the average is somehow still 2. that's untenable.

—

statement (2):
there are only two ways *not* to sell at least 2 tickets: sell 0 tickets, and sell 1 ticket.
if everyone sells a different # of tickets, then you can't fit three people into these two categories.
therefore, *someone* must have sold at least 2 tickets.

(1) if they sold 6 together, the possibilities (2,2,2), (1,2,3), (0,3,3) (different variations of these). In all cases, there is at least one with 2 or more.

(2). This I think is real cool.. if one of them is 0, the other is 1, the third one has to be 2 or more, hence sufficient.

Hence the answer is D.

66.

we have the equation zt < –3, and it wants to know whether z < 4.

(1) alone: If z < 9, then we still don't know whether z < 4. (For instance, z could be 3 [yes] or 5 [no].)
(2) alone: If t < –4, then we know that z is a positive number (because the product is negative and t is negative). you can't really divide two inequalities in any simple way, so just try plugging numbers. let t be, say, –10, and try z's that are greater than 4 and less than 4 (remember, the point here, as on all DS problems, is to prove 'MAYBE'). make sure that you don't violate the condition zt < –3.

if t = –10 and z = 1, then the condition zt < –3 is satisfied, and z is not < 4.
if t = –10 and z = 5, then the condition zt < –3 is satisfied, and z < 4.
these two results show that (2) alone is insufficient.
in fact, the same two results show that statements (1) and (2) TOGETHER are insufficient (since both of the z's selected here happen to be < 9).

answer = e

OR

The answer is E.

It is a YES/NO question whether z < 4

Looking at it 1st statement, for the 1st inequality to be true either z is –ve or t –ve, however both cannot be negative.

1. z < 9 – Insufficient as z = 8 , t = –9 then zt < –3 , similarly z = 1 and t = –5, zt < –3 thus it is insufficient to state that z <4

2. t < –4 Insufficient since this only provides us that z > 0, but does not given any indication

whether z > 4
for example t = –6, z = 2 , zt = –12
t = –5, z = 5 zt = –25

Now taking them together: z < 9 and t < –4
If one has to be negative, for the 1st statement to hold true than z is +ve between 1 and 9 and t –ive –4 to –infinity.
z = 8, t = –5 ; zt = –40
z = 3, t = –5 ; zt = –15
Thus one cannot conclusively state whether z< 4, therefore the correct answer is E.

If you simplify the stem as below:
zt < –3
z < –3/t
and since t < –4 from Statement 2:
if t = –5 —> z < –3/–5 = 0.6 —> z < 0.6 is less than 4
if t = –10 —> z < –3/–10 = 0.3 —> z < 0.3 which is less than 4

WRONG…
you can't divide by t unless you've ascertained whether t is positive or negative. and moreover, if t is negative, then you have to switch around the inequality sign ("<" becomes ">").
so if t < –4, then you actually have z **>** –3/t, not "<".

67.

remember that if c is positive, then the statement is guaranteed to be true (because b is already greater than a, so adding something positive will keep it that way).

statement (1)
in this case, you know that each of b and c is greater than a, but that's all you know.
if b and c are positive, then this is good enough, because then b + c will be greater than either b or c (both of which are already greater than a). so that's a Yes.
with negative values, though, you can get a No. if b = –2, c = –3, and a = –4, then b > a and c > a, but b + c < a. that's a No.
insufficient.

(2) means one of the following: (a) all three are positive, (b) two are negative and one is positive.
if all 3 are positive, then *a fortiori* c is positive, so b + c must be greater than a.
but
if b is positive, and a and c are negative, then it's possible that b + c is not greater than a. if you

don't like plugging actual numbers, then consider the idea that c could be a REALLY BIG NEGATIVE that cancels out the positive–ness of b. for instance, if a = –1, b = 2, and c = –100, then b > a, but b + c <<<< a.
insufficient.

(together)
in this case, a is the smallest of the 3 numbers.
3 cases:
* all positive: this is a yes, as established before
* a < b < 0 < c: this is also a yes, because c is positive
* a < c < 0 < b:
rearrange the inequality to "is b > a – c ?"
in this case, notice that b is positive and a – c is negative, so this is still a yes.
always yes
sufficient

ans = c

68.

OA is B

the best way to solve this problem is to notice that its subject matter is POSITIVES AND NEGATIVES. how do you know this? because it deals *only* with absolute values and inequality signs – no other numbers or non–absolute values in sight to mess things up.

there is no way to 'quickly solve' this one algebraically, unfortunately. in fact, even at the highest levels of mathematics, the best (and really the only) way to solve problems like these is case–wise, considering the different possibilities for + and – one case at a time.

A) y < x
a case for >
x = 2, y = –3
LHS = 5 > RHS = –1
Case for = or <
x = anything, y = 0
LHS = RHS
Insufficient

B) xy < 0. x and y are on opposite sides of 0 on the number line
| x – y | – distance of x from y
|x| – distance of x from 0
|y| – distance of y from 0
If you imagine a number line

like this
x————0—————y
or
y————0—————x

you can conclude that the distance between x and y is greater than the difference between x,0 and y,0.

OR

Lets take (1)
y<x

y 2 –4
x 4 2

Substitute in the equation

|x–y|>|x|–|y|

Using the above values,
|2|>|4|–|2| First Column Values

Not true
|2–(–4)|>|2|–|–4| Second Column Values
|6|>2–4
6>–2 —True

Hence A is Insuff

(2)
xy<0 which means either x or y should be negative

x 7 –7
y –3 3

Substitute the values in the eq
|x–y|>|x|–|y|

|10|>|7|–|–3|

$10>4$

$|-7-3|>|-7|-|3|$
$|-10|>-7-3$
$10>-10$

Hence Suff

Hence B.

69.

REPHRASE
is $y <1$
statement 1) doesnt tell us anything about Y
statement 2) if $y<0$ then y must be <1
B is the answer

70.

Ans: E

1) $7x-2y>0$

$7(8)-2(3)$ = positive
$7(8)-2(-3)$ = positive

y can be neg or pos. NOT sufficient.

2) $-y<x$

$x = 8$, y can equal 3
$x = 8$, $-3<8$, so y can be positive

$x = 8$, y can equal -3
$x=8$, $-(-3)<8$, $6<8$, y can be negative too. Not sufficient.

c) combined: Look at the breakdown from statement (1) with numbers that also satisfy statement (2)'s criteria. y can be neg OR positive.

Answer is Choice E.

71.

1. If that is that case.

   i) is insufficient. there is not enough information on the values of x,y,z.
   ii) is sufficient. there are 3 possibilities: $x<y=z$, $x=y=z$, $y=z<x$. in any case z is the median.

   Hence the answer is B

If you need convincing about answer a, then remember that your goal on these types of problems is to try to prove 'maybe' (i.e., insufficient). so try to find 2 groups of numbers, one of which gives a 'yes' answer and one of which gives a 'no' answer:
(x, y, z) = (1, 3, 5): z is not the median
(x, y, z) = (1, 5, 3): z is the median
insufficient.

important note:
make sure that you understand that this is the direction of the logic in *all* data sufficiency questions. you are *always* trying to prove/disprove the prompt question, based on the evidence given in the 2 statements. you have written the question backwards – your 'if' and 'then' construct a logic that runs in precisely the opposite direction – which will make it essentially impossible for you to answer questions correctly.

—

treatment of the question:

rephrase of initial question:
this is the same as asking: **is *z* equal to the middle number of the three numbers?**

statement (1)
this statement tells nothing about the order of the three numbers. it could be true regardless of the order of the 3 numbers, and, more to the point, regardless of the position of z in the ordered list.
examples:
x = 1, y = 2, z = 3: z is not the median
x = 1, y = 3, z = 2: z is the median
insufficient

statement (2)
if y and z are equal, there are three possibilities:
—— they are the two largest #s in the list. in this case, both of them equal the median of the list.
—— they are the two smallest #s in the list. in this case, both of them equal the median of the list.
—— all three numbers in the list are the same. in this case, all of them equal the median.
in any of these cases, z is the median.
sufficient

answer = b

72.

OA = E
try to draw out the cases. for example

(1) $xyz < 0$ and $|xy| > |xz|$. this implies the following are possible (with the Y–axis indicated as the vertical bar)

yx | z
z | xy
xz | y
yz | x

(2) $xy < 0$ means that x & y are on opposite sides of the vertical axis. and $|xy| > |xz|$

y | zx
y | xz
x | zy

the cases illustrate the answer

1. $xyz < 0$ — so either all three or one of the three is negative
2. $xy < 0$ — either x or y is negative, BUT not both.

Scenario A) where $XYZ < 0$.

2 positives 1 negative
3 negatives.

For three negatives on the number the order can be z, x, y, 0...or x, z, y, 0.

So A is insufficient.

Scenario B) hwere $XY < 0$.

One negative one positive. So the order can go. x 0 z y...or z x 0 y. So B is insufficient.

Scenario A+B.

If X and Y are both of oppositie signs ($XY<0$)..then Z has to be postive in order for $XYZ < 0$.

So One Negative and two positives.

Order can be.....x 0 z y...or y 0 x z.

Thus E.

73.

x + y = 1

If y>=0.15, then try some:
If y=0.15, x = 0.85 in which case, no, x is not less than 0.8.
If y=0.3, then x=0.7, in which case, yes, x is less than 0.8.
Contradictory answers = indufficient.

y = 1–x
If C >=7.30, then try some:
If C=7.30, then 7.3 = 6.5x + 8.5(1–x)
7.3 = 6.5x + 8.5 – 8.5x
–1.2 = –2x
1.2/2 = x
x = 0.6, so yes, $x < 0.8$.

If x+y=1, and I need to create a larger C, I have to make x smaller and y larger (because I multiply y by 8.5 in the formula while only multiplying x by 6.5). So, x will decreas as C increases. As a result, I can always say that $x < 0.8$. Sufficient.

74.

The correct answer is D

(1) if $m<p$, $mv<pv<0$ will only hold if V is +ve and M and P are –ve. For other two cases that you mentioned $mv>pv$ so they are not valid.

Hence, for $mv<pv<0$ V to hold, V has to +ve. So (1) is sufficient.
Luci, you were on the right track with your process, but you didn't actually work out the numbers, so you didn't notice that some of them were inconsistent with the given condition ($mv<pv<0$). I love trying numbers as a technique, but make sure you follow through just a bit more on the calculations.

If you had, you would have seen:
ex. m=3, p=5, and let's make v =–2

$mv = -6$
$pv = -10$
$-6 < -10 < 0$
Which is not true, so that combination is invalid – I can't use it to test the statement. The only way to make it work would be to make p less than m, but statement one gives the condition $m < p$, so I can't do it.

75.

E