Co-ordinates

1. If 2 points (*a*, *b*) and (*c*, *d*) lie in the same quadrant, then *a* and *c* should have the same sign; and *b* and *d* should have the same sign. So here (–a and –b have the same sign and b and a have the same sign, so eventually **GIVEN information says that a and b are of the same sign.**

Asked: are –a, -b, -x of the same sign and are b, a, y of the same sign? Combined we have:

QUESTION: Given a and b are of the same sign, are a, b, x and y all of the same sign?

(1) x and y are of the same sign. NOT SUFFICIENT.
(2) a and x are of the same sign. NOT SUFFICIENT.

Combined a, b, x and y are of the same sign. Ans. (C).

2. From 1, a+b=-1. From 2, x=0, so ab=6. (x+a)*(x+b)=0x^2+(a+b)x+ab=0

So, x=-3, x=2 The answer is C.

3. We need to know whether r^2+s^2=u^2+v^2 or not. From statement 2,

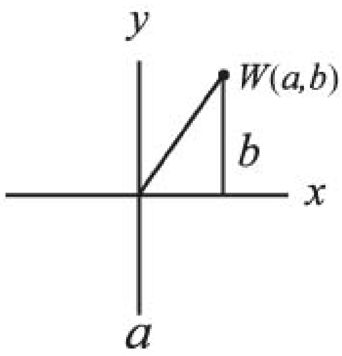
u^2+v^2=(1-r)^2+(1-s)^2=r^2+s^2+2-2(r+s)

Combined statement 1, r+s=1, we can obtain that r^2+s^2=u^2+v^2.

Answer is C.

4 ….

5. To find the distance from the origin to any point in the coordinate plane, we take the square root of the sum of the squares of the point's *x*- and *y*-coordinates. So, for example, the distance from the origin to point *W* is the square root of ($a^2 + b^2$). This is because the distance from the origin to any point can be thought of as the hypotenuse of a right triangle with legs whose lengths have the same values as the *x*- and *y*-coordinates of the point itself:



We can use the Pythagorean Theorem to determine that $a^2 + b^2 = p^2$, where p is the length of the hypotenuse from the origin to point W.

We are also told in the question that $a^2 + b^2 = c^2 + d^2$, therefore point X and point W are equidistant from the origin. And since $e^2 + f^2 = g^2 + h^2$, we know that point Y and point Z are also equidistant from the origin.

If the distance from the origin is the same for points W and X, and for points Z and Y, then the length of WY must be the same as the length of XZ. Therefore, the value of length XZ – length WY must be 0.

The correct answer is C.

6. At the point where a curve intercepts the x-axis (i.e. the x intercept), the y value is equal to 0. If we plug $y = 0$ in the equation of the curve, we get $0 = (x - p)(x - q)$. This product would only be zero when x is equal to p or q. The question is asking us if (2, 0) is an x-intercept, so it is really asking us if either p or q is equal to 2.

(1) INSUFFICIENT: We can't find the value of p or q from this equation.

(2) INSUFFICIENT: We can't find the value of p or q from this equation.

(1) AND (2) SUFFICIENT: Together we have enough information to see if either p or q is equal to 2. To solve the two simultaneous equations, we can plug the p-value from the first equation, $p = -8/q$, into the second equation, to come up with $-2 + 8/q = q$.
This simplifies to $q^2 + 2q - 8 = 0$, which can be factored $(q + 4)(q - 2) = 0$, so $q = 2, -4$.
If $q = 2$, $p = -4$ and if $q = -4$, $p = 2$. Either way either p or q is equal to 2.

The correct answer is C.

7. The distance between any two points (x, y) and (x_2, y_2) in the coordinate plane is defined by the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$= \sqrt{(2A + 4 - A)^2 + \left(\sqrt{2A + 9} - 0\right)^2}$
$= \sqrt{(A + 4)^2 + \left(\sqrt{2A + 9} - 0\right)^2}$
$= \sqrt{A^2 + 8A + 16 + 2A + 9}$
$= \sqrt{A^2 + 10A + 25}$
$= \sqrt{(A + 5)^2}$
$= A + 5$

Thus, the distance between point K and point G is A + 5.

Statement (1) tells us that:

$A - 5A - 6 = 0$
$(A - 6)(A + 1) = 0$

Thus A = 6 or A = –1.

Using this information, the distance between point K and point G is either 11 or 4. This is not sufficient to answer the question.

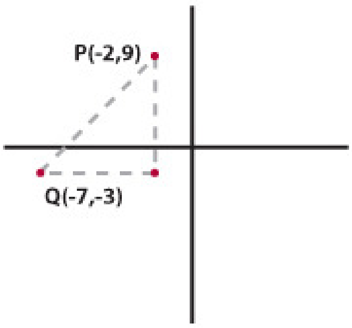
Statement (2) alone tells us that A > 2, which is not sufficient to answer the question.

When we combine both statements, we see that A must be 6, which means the distance between point K and point G is 11. This is a prime number and we are able to answer the question.

The correct answer is C.

8. The formula for the distance between two points (x_1, y_1) and (x_2, y_2) is:
$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

One way to understand this formula is to understand that the distance between any two points on the coordinate plane is equal to the hypotenuse of a right triangle whose legs are the difference of the *x*-values and the difference of the *y*-values (see figure). The difference of the *x*-values of *P* and *Q* is 5 and the difference of the *y*-values is 12. The hypotenuse must be 13 because these leg values are part of the known right triangle triple: 5, 12, 13.



We are told that this length (13) is equal to the height of the equilateral triangle *XYZ*. An equilateral triangle can be cut into two 30-60-90 triangles, where the height of the equilateral triangle is equal to the long leg of each 30-60-90 triangle. We know that the height of *XYZ* is 13 so the long leg of each 30-60-90 triangle is equal to 13. Using the ratio of the sides of a 30-60-90 triangle (1: $\sqrt{3}$: 2), we can determine that the length of the short leg of each 30-60-90 triangle is equal to 13/$\sqrt{3}$. The short leg of each 30-60-90 triangle is equal to half of the base of equilateral triangle *XYZ.* Thus the base of *XYZ* = 2(13/$\sqrt{3}$) = 26/$\sqrt{3}$.

The question asks for the area of *XYZ*, which is equal to 1/2 × base × height:

$$\frac{1}{2} \times \frac{26}{\sqrt{3}} \times 13 = \frac{169}{\sqrt{3}} = \frac{169\left(\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}\right)} = \frac{169\sqrt{3}}{3}$$

The correct answer is A.

9. To find the area of equilateral triangle *ABC*, we need to find the length of one side. The area of an equilateral triangle can be found with just one side since there is a known ratio between the side and the height (using the 30: 60: 90 relationship). Alternatively, we can find the area of an equilateral triangle just knowing the length of its height. **h = a√3 / 2**

(1) INSUFFICIENT: This does not give us the length of a side or the height of the equilateral triangle since we don't have the coordinates of point *A*.

(2) SUFFICIENT: Since *C* has an *x*-coordinate of 6, the height of the equilateral triangle must be 6.

The correct answer is B.

10. Put y = 0, x is positive and put x = 0 and y is positive. So both the x and the y intercepts are positive. By plotting the line, we may say that it does not pass through Quadrant III.

The correct answer is C.

11. To determine in which quadrant the point (*p*, *p* – *q*) lies, we need to know the sign of *p* and the sign of *p* – *q*.
(1) SUFFICIENT: If (*p*, *q*) lies in quadrant IV, *p* is positive and *q* is negative. *p* – *q* must be positive because a positive number minus a negative number is always positive [e.g. 2 – (-3) = 5].

(2) SUFFICIENT: If (*q*, -*p*) lies in quadrant III, *q* is negative and *p* is positive. (This is the same information that was provided in statement 1).

The correct answer is D.

12. By distance formula, the sides are 5, 5, $5\sqrt{2}$. So it is an isosceles triangle. Ans. ½ * 5 * 5 = 12.5.

13. The line passes through (6, 0) and (0, 3). Assume the equation as y = mx + c, substitute these 2 points in the equation of the line and we get m = -1/2 and c = 3. So the equation is y = - ½ x + 3. As we want below this line, the answer is E.

14. If (r, s) lies on the line, then we must have s = 3r + 2.
(1) gives s = 3r + 2 or s = 4r + 9 NS
(2) gives s = 4r – 6 or s 3r + 2 NS
Combined s = 3r + 2 only. Ans. C

15. We must get $2r + 3s \le 6$.
(1) gives $3r + 2s = 6$. Put s = 0 in $3r + 2s = 6$, so r = 2 … put (2, 0) in $2r + 3s \le 6$ and we get $4 \le 6$. YES
Put r = 0 in $3r + 2s = 6$, s = 3. Put (0, 3) in $2r + 3s \le 6$ and we get 9 > 6. NO. So not sufficient.
(2) gives $r \le 3$ and $s \le 2$. Take r = 2, s = 2 so 2r + 3s = 10 > 6. NO
Take r = 0, s = 0 so $2r + 3s = 0 \le 6$. YES. So not sufficient.
Combined: 3r + 2s = 6 and $r \le 3$ and $s \le 2$.
(2, 0), (1, 1.5) both satisfy the equation.
With (2, 0), $2r + 3s = 4 \le 6$. With (1, 1.5), 2r + 3s = 2 + 4.5 = 6.5 > 0. So not sufficient.
Ans. E

16. Just image that, when we let the x-intercept great enough, the line k would not intersect circle c, even the absolute value of its slope is very little. Answer is E

17. y=kx+b 1). k=3b 2). -b/k=-1/3 => k=3b So, answer is E

18. The two intersections: (0,4) and (y, 0) So, 4 * y/ 2 = 12 => y= 6

Slope is positive => y is below the x-axis => y = -6

19. First, let's rewrite both equations in the standard form of the equation of a line:
Equation of line l: $y = 5x + 4$
Equation of line w: $y = -(1/5)x – 2$

Note that the slope of line w, -1/5, is the negative reciprocal of the slope of line l. Therefore, we can conclude that line w is perpendicular to line l.

Next, since line k does not intersect line l, lines k and l must be parallel. Since line w is perpendicular to line l, it must also be perpendicular to line k. Therefore, lines k and w must form a right angle, and its degree measure is equal to 90 degrees.

The correct answer is D.

20. The question asks us to find the slope of the line that goes through the origin and is equidistant from the two points *P*= (1, 11) and *Q*=(7, 7). It's given that the origin is one point on the requested line, so if we can find another point known to be on the line we can calculate its slope. Incredibly the midpoint of the line segment between *P* and *Q* is also on the requested line, so all we have to do is calculate the midpoint between *P* and *Q*! (This proof is given below).

Let's call *R* the midpoint of the line segment between *P* and *Q*. *R*'s coordinates will just be the respective average of *P*'s and *Q*'s coordinates. Therefore *R*'s *x*-coordinate equals 4 , the average of 1 and 7. Its *y*-coordinate equals 9, the average of 11 and 7. So *R*=(4, 9).

Finally, the slope from the (0, 0) to (4, 9) equals 9/4, which equals 2.25 in decimal form.

<u>Proof</u>

To show that the midpoint *R* is on the line through the origin that's equidistant from two points *P* and *Q*, draw a line segment from *P* to *Q* and mark *R* at its midpoint. Since *R* is the midpoint then *PR* = *RQ*.

Now draw a line *L* that goes through the origin and *R*. Finally draw a perpendicular from each of *P* and *Q* to the line *L*. The two triangles so formed are congruent, since they have three equal angles and *PR* equals *RQ*. Since the triangles are congruent their perpendicular distances to the line are equal, so line *L* is equidistant from *P* and *Q*.

The correct answer is B.

21. Lines are said to intersect if they share one or more points. In the graph, line segment *QR* connects points (1, 3) and (2, 2). The slope of a line is the change in *y* divided by the change in *x*, or rise/run. The slope of line segment *QR* is (3 – 2)/(1 – 2) = 1/-1 = -1.

(1) SUFFICIENT: The equation of line *S* is given in $y = mx + b$ format, where *m* is the slope and *b* is the *y*-intercept. The slope of line *S* is therefore -1, the same as the slope of line segment *QR*. Line *S* and line segment *QR* are parallel, so they will not intersect <u>unless</u> line *S* passes through both *Q* and *R*, and thus the entire segment. To determine whether line *S* passes through *QR*, plug the coordinates of *Q* and *R* into the equation of line *S*. If they satisfy the equation, then *QR* lies on line *S*.

Point *Q* is (1, 3):
$y = -x + 4 = -1 + 4 = 3$
Point *Q* is on line *S*.

Point *R* is (2, 2):
$y = -x + 4 = -2 + 4 = 2$

Point *R* is on line *S*.

Line segment *QR* lies on line *S*, so they share many points. Therefore, the answer is "yes," Line *S* intersects line segment *QR*.

(2) INSUFFICIENT: Line *S* has the same slope as line segment *QR*, so they are parallel. They might intersect; for example, if Line *S* passes through points *Q* and *R*. But they might never intersect; for example, if Line *S* passes above or below line segment *QR*.

The correct answer is A.

22. First, we determine the slope of line *L* as follows:

$$L = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{p-2} = \frac{-1}{p-2} = \frac{1}{2-p}$$

If line *m* is perpendicular to line *L*, then its slope is the negative reciprocal of line *L*'s slope. (This is true for all perpendicular lines.) Thus:

If the slope of $L = \frac{a}{b}$, then the slope of $m = -\frac{b}{a}$.

Therefore, the slope of line *m* can be calculated using the slope of line *L* as follows:

$$m = -\left(\frac{1}{\frac{1}{2-p}}\right) = p - 2$$

This slope can be plugged into the slope-intercept equation of a line to form the equation of line *m* as follows:

$y = (p - 2)x + b$

(where $(p - 2)$ is the slope and b is the y-intercept)

This can be rewritten as $y = px - 2x + b$ or $2x + y = px + b$ as in answer choice A.

An alternative method: Plug in a value for *p*. For example, let's say that $p = 4$.

Thus, the slope of line $L = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{4-2} = \frac{-1}{2} = -\frac{1}{2}$

The slope of line *m* is the negative inverse of the slope of line *L*. Thus, the slope of line *m* is 2.

Therefore, the correct equation for line *m* is the answer choice that yields a slope of 2 when the value 4 is plugged in for the variable *p*.

(A) $2x + y = px + 7$ **yields** $y = 2x + 7$

(B) $2x + y = -px$ **yields** $y = -6x$

(C) $x + 2y = px + 7$ **yields** $y = (3/2)x + 7/2$

(D) $y - 7 = x \div (p - 2)$ **yields** $y = (1/2)x + 7$

(E) $2x + y = 7 - px$ **yields** $y = -6x + 7$

Only answer choice A yields a slope of 2. Choice A is therefore the correct answer.

23. First, rewrite the line $y = 4 - 2x$ as $y = -2x + 4$. The equation is now in the form $y = mx + b$ where m represents the slope and b represents the y-intercept. Thus, the slope of this line is -2.

By definition, if line F is the perpendicular bisector of line G, the slope of line F is the negative inverse of the slope of line G. Since we are told that the line $y = -2x + 4$ is the perpendicular bisector of line segment RP, line segment RP must have a slope of $\frac{1}{2}$ (which is the negative inverse of).

Now we know that the slope of the line containing segment RP is $\frac{1}{2}$ but we do not know its y-intercept. We can write the equation of this line as $y = \frac{1}{2}x + b$, where b represents the unknown y-intercept.

To solve for b, we can use the given information that the coordinates of point R are (4, 1). Since point R is on the line $y = \frac{1}{2}x + b$, we can plug 4 in for x and 1 in for y as follows:

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(4) + b$$

$$-1 = b$$

Now we have a complete equation for the line containing segment RP: $y = \frac{1}{2}x - 1$

We also have the equation of the perpendicular bisector of this line: $y = -2x + 4$. To determine the point M at which these two lines intersect, we can set these two equations to equal each other as follows:

$$\frac{1}{2}x - 1 = -2x + 4$$
$$\frac{5}{2}x = 5$$
$$x = 2$$

Thus, the intersection point M has x-coordinate 2. Using this value, we can find the y coordinate of point M:

$$y = -2x + 4$$
$$y = -2(2) + 4$$
$$y = 0$$

Thus the perpendicular bisector intersects line segment RP at point M, which has the coordinates (2, 0). Since point M is on the bisector of RP, point M represents the midpoint on line segment RP; this means that it is equidistant from point R and point P.

We know point R has an x-coordinate of 4. This is two units away from the x-coordinate of midpoint M, 2. Therefore the x-coordinate of point P must also be two units away from 2, which is 0.

We know point R has a y-coordinate of 1. This is one unit away from the y-coordinate of midpoint M, 0. Therefore, the y-coordinate of point P must also be one unit away from 0, which is –1.

The coordinates of point P are $(0, -1)$. The correct answer is D.

24.
We know that the lines L and K intersect in Quadrant I (the top right quadrant).

The products of the slopes can be negative if the slopes are opposite signs. For examples, L = positive slope, K = negative slope, and vice versa.

1) The product of the x-intercepts being positive means that they both cross the x-axis BOTH where $x < 0$, or where $x > 0$. But this does not imply that their slopes have to be the opposite signs of each other.

2) The product of the Y intercepts being negative means that one line crosses the y-axis above the line y=0, and the other line crosses the y-axis below the line where y=0. BUT, this does not imply that they have opposite sign slopes.

If both conditions were true, then the slopes MUST be opposite signs of each other.

Statement (1) means that the x-ints are either both positive or both negative; statement (2) means that the y-ints have opposite signs.

Let's try the 2 possible cases for statement (1):
both negative: this is impossible, because one of the y-ints has to be negative - and, if you have a line with negative x-int and negative y-int, that line doesn't touch the first quadrant.

so, it MUST be true that...
both x-ints are positive: in this case, one line has a positive x-int and a negative y-int, so it must slope upward: positive slope. (if you don't see why this is true, just draw it. please, please don't use algebra to prove this to yourself; that's a tremendous waste of time.) the other line has a positive x-int and a positive y-int, so it must slope downward: negative slope. (again, draw it to see this for yourself.)
so the product is negative.
sufficient.

look at it this way: the actual coordinates (4, 3) don't really matter. the only thing that matters is that the line goes through the 1st quadrant.
the reason we know this is we're only concerned with positives and negatives - and those concepts depend only on quadrants and signs, not the magnitude of the actual coordinates. so, if it helps you to think about (1, 1) - or just the first quadrant in general - instead of (4, 3), then go ahead.

If you have the 2 statements together, you don't even have to have the point (4, 3) anymore.
In the question: If the product of slopes is negative, that means one line has a positive slope and the other has a negative slope.

In statement (2), if the product of y-intercepts is negative, then one is above the x-axis (let's say this is line L) and the other is below the x-axis (let's say this is line K).

It is obvious that line K must have a positive slope. We can take, for example, the y-intercept (0, -3) and (4,3); here our slope is 3/2.

Statement (2) is insufficient because line L can have either a positive or negative slope.

For line L to have a positive slope, then the y-intercept must be below 3. The slope if the y-intercept is 1, is 1/2.

However, if the y-intercept is above 3, then line L will have a negative slope. The slope if the y-intercept is 7 is -1.

Because L can either have a positive or negative slope, then the product of slopes of K and L can be either positive or negative, so statement (2) is insufficient.

ANS. C

25.
Draw the picture. It's much easier to do when you have a visual of what's going on.

In order to determine whether the lines are perpendicular, we need to determine the slopes of the two lines. One way we can rephrase the question is: do we know the slopes of lines k and m?

In order to determine the slopes of each line, we need two points for each line. Another way we can rephrase the question is: do we have two unique points for each line? Note that the problem has one point given for each line; we just need another point for each line.

Statement (1) provides an additional point for line k, but not for line m. We can determine the slope for only line k, but not m. So therefore, this statement is insufficient.

Statement (2) also provides an additional point for line k, but not for line ml. We can determine the slope for only line k, but not m. So therefore, this statement is insufficient.

When combining statements (1) and (2), we now have THREE points for line k, but only ONE point for line m. We still are unable to determine the slope for line m, so combined, these statements are insufficient. **The answer should be E.**

If you look at question Line K passes through point (1,1) and M passes through (1,-1).
If you quickly draw those two points, One is in 1st quadrant and other in 4th. This is what the question stem tells you. And the question is are the two lines prependicular , mathematically : is the product of the slopes of line K & M = -1?

Statement 1:
Tells us that lines interesect at point 1,-1.
If you would have drawn the above two points and now you join (1,1) and (1,-1) , which are the two points of line K, you will know the line K.
Line K is parallel to Y axis.
This is it , nothing else is given.

Whereas, all we know about line M is that it passes through 1,-1. And there can be infinite such lines. We cant conclude whether the two lines are prependicular or not.

INSUFFICIENT.

Statement 2:
This tells us that Line K passes through 1,0. Now if you join 1,1 and 1,0 . Once again you will get a line parallel to Y axis. Again if you extend this line you will relaize that it will cross point 1,-1. But situation is similar to St.1.

INSUFFICIENT

Combining 2 gives us line k. Which we got from both statements as well.
So together INSUFFICIENT as well.

26.

Statement 1 gives us one common point: (5,1)

Statement 2 tells us the y-intercept of n is greater than the y-intercept of p. The y-intercept is where a particular line crosses the y-axis. The corresponding point for that line is (0,y) with y representing the y-intercept.

If n's y-intercept is greater than p's, then the value y is greater for n than for p. If you sketch a coordinate plane, place the point (5,1) on the plane, and then arbitrarily sketch some points along the y-axis:

1) try a pair above the y=1 line, with the higher labeled n and the lower labeled p. In this case, the two slopes are negative, and the slope of n is more negative than the slope of p. That is, n's slope is smaller than p's slope. Answer the question: Yes.
2) try a pair below the y=1 line, with the higher labeled n and the lower labeled p. In this case, the two slopes are positive, and the slope of n is closer to zero than the slope of p. That is, n's slope is smaller than p's slope. Answer the question: Yes.
3) you can also try some pairs where one or the other (n or p) has a y-intercept of 1. In each case, you'll continue to see that n's slope is always smaller than p's slope.

So, together, the statements are sufficient.

Yes, you'll see that the lower (in number) the y-intercept, the larger the slope. Therefore, p's slope is always larger than n's slope.

27.
The 1/6 is irrelevant; all that matters in (1) is that the slope is negative. Lines with negative slopes go up to the left, down to the right. this means that, if you follow ANY negatively sloped line far enough to the left, it will go up into the second quadrant. **Negatively sloped lines MUST hit quadrants 2 and 4. Positively sloped lines MUST hit quadrants 1 and 3.** Zero exceptions. The other quadrants are mutually exclusive options. i.e., if a positively sloped line hits quadrant 2, then it can't hit quadrant 4, and vice versa. Same thing with quadrants 1 and 3 for negatively sloped lines.

With statement (2), a horizontal or positively sloped line through (0, -6) won't hit quadrant 2, but a negatively sloped line will, so that's insufficient. ANS. (A)

28.

"product of slopes is -1" should be IMMEDIATELY translated to "perpendicular".

As with most other coordinate problems, you should try to draw this one first. **DO NOT USE Y = MX + C ON COORDINATE PROBLEMS, UNLESS VISUALIZATION FAILS OR THE PROBLEM IS OBVIOUSLY AN ALGEBRA-BASED PROBLEM.**

This problem boils down to systematically trying different kinds of lines through the given point and seeing whether you can make them perpendicular (it's obvious that you can make them non-perpendicular).

Statement (1)
if both x-intercepts are positive, then the lines could be perpendicular: draw one line ALMOST vertical, but with a positive slope, through (4, 3), so that its x-intercept is slightly less than 4. Then the perpendicular will have a very gentle negative slope, and it will have a huge positive x-intercept.
It's easy to make lines with positive x-intercepts, intersecting at the given point,
that aren't perpendicular.
Insufficient.

Statement (2)
any line with a negative y-intercept that also goes through (4, 3) must have a positive slope.
Since both lines have positive slopes, they can't be perpendicular. (Alternatively, it's also easy to use the non-rephrased question here: if both slopes are positive, then the product of the slopes is positive, so it can't be -1.)
Sufficient. Ans (b)

29.
Correct answer is C. Can you please tell me how to derive a solution (or how to rephrase this and get the answer to the rephrasing)? I lucky guessed C but I don't know why it's right.

Question : Is b > 0?

(1) says (b/a) < 0
This could mean 2 things
a) Either a > 0 & b < 0 OR
b) a < 0 & b > 0

Both are of opposite signs.

This is not sufficient to ans the question. This eliminated A & D

(2) $a < b$
This could mean
a) $a < 0$ & $b > 0$ (eg $-1 < 1$)
b) $a > 0$ & $b > 0$ (eg. $2 < 3$)
c) $a < 0$ & $b < 0$ (eg $-2 < -1$)

This is also not sufficient. WE can eliminate B.

If we combine (1) and (2), we get 1(b) and 2(a) give a definite solution. We can ans the question. So C is the ans.

30.
This is a 30-60-90 triangle… so the sides will be 1, $\sqrt{3}$, 2… area = $\sqrt{3}/2$

31.

In this case, the two sides are known to us… in all such situations when we know 2 sides, the area will be maximum when the included angle between the sides is 90 degrees.
(For those of you who remember Trigonometry, the area of a triangle is ½ ab Sin C… and Sine of an angle is maximum for 90 degrees).

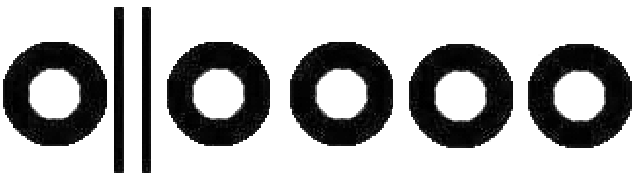
Ans. ½ * 1 * 1 = 1/2

P&C, Probability

1. $({}^{10}P_8 + {}^{10}P_9 + {}^{10}P_{10}) \times 12$ seconds
2. $6! - 5! \times 2!$
3. ${}^2C_1 \times {}^3C_1 \times ({}^7C_4 - {}^5C_2)$
4. $5! / 2!$
5. $4 \times 4 = 16$ (only cheese, only mushroom, both, or neither – focus on the word OPTION)
6. $2 \times {}^5C_4 \times 2 = 20$
7. $3! \times 2!$
8. $1 \times 10 \times 10 \times 5$
9. TTTHHH, HTTTHH, HHTTTH, HHHTTT; total 4
10. The only combination of odd is $5 \times 5 \times 5$. So total required $= 3 \times 3 \times 3 - 1 = 26$.
11. $2 \times 2 \times 2 \times 4 \times 4 - 2 \times 1 \times 1 \times 4 \times 4 = 96$
12. $9! / 5! \times 4!$

13. The key to this problem is to avoid listing all the possibilities. Instead, think of an arrangement of five donuts and two dividers. The placement of the dividers determines which man is allotted which donuts, as pictured below:

In this example, the first man receives one donut, the second man receives three donuts, and the third man receives one donut. Remember that it is possible for either one or two of the men to be allotted no donuts at all. This situation would be modeled with the arrangement below:



Here, the second man receives no donuts. Now all that remains is to calculate the number of ways in which the donuts and dividers can be arranged: There are 7 objects. The number of ways in which 7 objects can be arranged can be computed by taking 7!: $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$ However, the two dividers are identical, and the five donuts are identical. Therefore, we must divide 7! by 2! and by 5!:

$$\frac{5040}{5!2!} = \frac{5040}{(5 \times 4 \times 3 \times 2 \times 1)(2 \times 1)} = \frac{5040}{240} = 21$$

The correct answer choice is A.

14. It is important to first note that our point of reference in this question is all the possible subcommittees that include Michael. We are asked to find what percent of these subcommittees also include Anthony. Let's first find out how many possible subcommittees there are that must include Michael. If Michael must be on each of the three-person committees that we are considering, we are essentially choosing people to fill the two remaining spots of the committee. Therefore, the number of possible committees can be found by considering the number of different two-people groups that can be formed from a pool of 5 candidates (not 6 since Michael was already chosen). Using the anagram method to solve this combinations question, we assign 5 letters to the various board members in the first row. In the second row, two of the board members get assigned a Y to signify that they were chosen and the remaining 3 get an N, to signify that they were not chosen:

A	B	C	D	E
Y	Y	N	N	N

The number of different combinations of two-person committees from a group of 5 board members would be the number of possible anagrams that could be formed from the word YYNNN = 5! / (3!2!) = 10. Therefore there are 10 possible committees that include Michael. Out of these 10 possible committees, of how many will Anthony also be a member? If we assume that Anthony and Michael must be a member of the three-person committee, there is only one remaining place to fill. Since there are four other board members, there are four possible three-person committees with both Anthony and Michael. Of the 10 committees that include Michael, 4/10 or 40% also include Anthony. **The correct answer is C.** As an alternate method, imagine splitting the original six-person board into two equal groups of three. Michael is automatically in one of those groups of three. Now, Anthony could occupy any one of the other 5 positions -- the 2 on Michael's committee and the 3 on the other committee. Since Anthony has an equal chance of winding up in any of those positions, his chance of landing on Michael's committee is 2 out of 5, or 2/5 = 40%. Since that probability must correspond to the ratio of committees asked for in the problem, the answer is achieved. Answer choice C is correct.

15.
Ignoring Frankie's requirement for a moment, observe that the six mobsters can be arranged 6! or 6 x 5 x 4 x 3 x 2 x 1 = 720 different ways in the concession stand line. In each of those 720 arrangements, Frankie must be either ahead of or behind Joey. Logically, since the combinations favor neither Frankie nor Joey, each would be behind the other in precisely half of the arrangements. Therefore, in order to satisfy Frankie's requirement, the six mobsters could be arranged in 720/2 = 360 different ways. The correct answer is D.

16.
This problem cannot be solved through formula. Given that the drawer contains at least three socks of each color, we know that at least one matched pair of each color can be removed. From the first nine socks, we can therefore make three pairs, leaving three ‘orphans.’ To think through the problem, it is useful to conceptualize removing those nine socks from the drawer. We will

need additional information about any socks left in the drawer to solve the problem.

(1) INSUFFICIENT: Once those first nine socks have been removed, only two socks remain, but we do not have sufficient information about the color of the two socks to solve the problem. If the two remaining socks are a matched pair, we can add this final pair to the first three. This scenario results in four pairs and three orphans. However, if the final two socks are mismatched, each will make a new pair with one of the original three orphans, resulting in five pairs and one orphan.

(2) INSUFFICIENT: This statement gives no information about how many socks are in the drawer.

(1) AND (2) INSUFFICIENT: Given that the drawer contains 11 socks and that there are an equal number of black and gray socks, there are two possible scenarios. Three black, three gray, and five blue socks would yield four pairs total. Four black, four gray, and three blue socks would yield five pairs total.

The correct answer is E.

17.

There are $3 \times 2 \times 4 = 24$ possible different shirt-sweater-hat combinations that Kramer can wear. He wears the first one on a Wednesday. The following Wednesday he will wear the 8th combination. The next Wednesday after that he will wear the 15th combination. The next Wednesday after that he will wear the 22nd combination. On Thursday, he will wear the 23rd combination and on Friday he will wear the 24th combination.

Thus, the first day on which it will no longer be possible to wear a new combination is Saturday. The correct answer is E.

18.
There are 2 possible outcomes on each flip: heads or tails. Since the coin is flipped three times, there are $2 \times 2 \times 2 = 8$ total possibilities: HHH, HHT, HTH, HTT, TTT, TTH, THT, THH.
Of these 8 possibilities, how many involve exactly two heads? We can simply count these up: HHT, HTH, THH. We see that there are 3 outcomes that involve exactly two heads. Thus, the correct answer is 3/8.
Alternatively, we can draw an anagram table to calculate the number of outcomes that involve exactly 2 heads.

A	B	C
H	H	T

The top row of the anagram table represents the 3 coin flips: A, B, and C. The bottom row of the anagram table represents one possible way to achieve the desired outcome of exactly two heads. The top row of the anagram yields 3!, which must be divided by 2! since the bottom row of the anagram table contains 2 repetitions of the letter H. There are 3!/2! = 3 different outcomes that contain exactly 2 heads.
The probability of the coin landing on heads exactly twice is (# of two-head results) ÷ (total # of outcomes) = 3/8. The correct answer is B.

19.
Let us say that there are n questions on the exam. Let us also say that p_1 is the probability that Patty will get the first problem right, and p_2 is the probability that Patty will get the second problem right, and so on until p_n , which is the probability of getting the last problem right. Then the probability that Patty will get all the questions right is just $p_1 \times p_2 \times \ldots \times p_n$. We are being asked whether $p_1 \times p_2 \times \ldots \times p_n$ is greater than 50%.
(1) INSUFFICIENT: This tells us that for each question, Patty has a 90% probability of answering correctly. However, without knowing the number of questions, we cannot determine the probability that Patty will get all the questions correct.
(2) INSUFFICIENT: This gives us some information about the number of questions on the exam but no information about the probability that Patty will answer any one question correctly.
(1) AND (2) INSUFFICIENT: Taken together, the statements still do not provide a definitive "yes" or "no" answer to the question. For example, if there are only 2 questions on the exam, Patty's probability of answering all the questions correctly is equal to .90 × .90 = .81 = 81%. On the other hand if there are 7 questions on the exam, Patty's probability of answering all the questions correctly is equal to .90 × .90 × .90 × .90 × .90 × .90 × .90 ≈ 48%. We cannot determine whether Patty's chance of getting a perfect score on the exam is greater than 50%.
The correct answer is E

20.
In order to solve this problem, we have to consider two different scenarios. In the first scenario, a woman is picked from room A and a woman is picked from room B. In the second scenario, a man is picked from room A and a woman is picked from room B.
The probability that a woman is picked from room A is 10/13. If that woman is then added to room B, this means that there are 4 women and 5 men in room B (Originally there were 3 women and 5 men). So, the probability that a woman is picked from room B is 4/9.
Because we are calculating the probability of picking a woman from room A AND then from room B, we need to multiply these two probabilities:
10/13 x 4/9 = 40/117
The probability that a man is picked from room A is 3/13. If that man is then added to room B, this means that there are 3 women and 6 men in room B. So, the probability that a woman is picked from room B is 3/9.
Again, we multiply thse two probabilities:
3/13 x 3/9 = 9/117
To find the total probability that a woman will be picked from room B, we need to take both scenarios into account. In other words, we need to consider the probability of picking a woman and a woman OR a man and a woman. In probabilities, OR means addition. If we add the two probabilities, we get:

40/117 + 9/117 = 49/117
The correct answer is B.

21.
The period from July 4 to July 8, inclusive, contains 8 – 4 + 1 = 5 days, so we can rephrase the question as "What is the probability of having exactly 3 rainy days out of 5?"
Since there are 2 possible outcomes for each day (R = rain or S = shine) and 5 days total, there are 2 x 2 x 2 x 2 x 2 = 32 possible scenarios for the 5 day period (RRRSS, RSRSS, SSRRR, etc…) To find the probability of having exactly three rainy days out of five, we must find the total number of scenarios containing exactly 3 R's and 2 S's, that is the number of possible RRRSS anagrams:
= 5! / 2!3! = (5 x 4)/2 x 1 = 10
The probability then of having exactly 3 rainy days out of five is 10/32 or 5/16.
Note that we were able to calculate the probability this way because the probability that any given scenario would occur was the same. This stemmed from the fact that the probability of rain = shine = 50%. Another way to solve this question would be to find the probability that one of the favorable scenarios would occur and to multiply that by the number of favorable scenarios. In this case, the probability that RRRSS (1st three days rain, last two shine) would occur is (1/2)(1/2)(1/2)(1/2)(1/2) = 1/32. There are 10 such scenarios (different anagrams of RRRSS) so the overall probability of exactly 3 rainy days out of 5 is again 10/32. This latter method works even when the likelihood of rain does not equal the likelihood of shine.
The correct answer is C.

22.
For probability, we always want to find the number of ways the requested event could happen and divide it by the total number of ways that any event could happen.
For this complicated problem, it is easiest to use combinatorics to find our two values. First, we find the total number of outcomes for the triathlon. There are 9 competitors; three will win medals and six will not. We can use the Combinatorics Grid, a counting method that allows us to determine the number of combinations without writing out every possible combination.

A	B	C	D	E	F	G	H	I
Y	Y	Y	N	N	N	N	N	N

Out of our 9 total places, the first three, A, B, and C, win medals, so we label these with a "Y." The final six places (D, E, F, G, H, and I) do not win medals, so we label these with an "N." We translate this into math: 9! / 3!6! = 84. So our total possible number of combinations is 84. (Remember that ! means factorial; for example, 6! = 6 × 5 × 4 × 3 × 2 × 1.)
Note that although the problem seemed to make a point of differentiating the first, second, and third places, our question asks only whether the brothers will medal, not which place they will win. This is why we don't need to worry about labeling first, second, and third place distinctly.
Now, we need to determine the number of instances when at least two brothers win a medal. Practically speaking, this means we want to add the number of instances two brothers win to the number of instances three brothers win.
Let's start with all three brothers winning medals, where B represents a brother.

A	B	C	D	E	F	G	H	I
B	B	B	N	N	N	N	N	N

Since all the brothers win medals, we can ignore the part of the counting grid that includes those who don't win medals. We have 3! / 3! = 1. That is, there is only one instance when all three brothers win medals.

Next, let's calculate the instances when exactly two brothers win medals.

A	B	C	D	E	F	G	H	I
B	B	Y	B'	N	N	N	N	N

Since brothers both win and don't win medals in this scenario, we need to consider both sides of the grid (i.e. the ABC side and the DEFGHI side). First, for the three who win medals, we have 3! / 2! = 3. For the six who don't win medals, we have 6! / 5! = 6. We multiply these two numbers to get our total number: 3 × 6 = 18.

Another way to consider the instances of at least two brothers medaling would be to think of simple combinations with restrictions.

If you are choosing 3 people out of 9 to be winners, how many different ways are there to chose a specific set of 3 from the 9 (i.e. all the brothers)? Just one. Therefore, there is only one scenario of all three brothers medaling.

If you are choosing 3 people out of 9 to be winners, if 2 specific people of the 9 have to be a member of the winning group, how many possible groups are there? It is best to think of this as a problem of choosing 1 out of 7 (2 must be chosen). Choosing 1 out of 7 can be represented as 7! / 1!6! = 7. However, if 1 of the remaining 7 can not be a member of this group (in this case the 3rd brother) there are actually only 6 such scenarios. Since there are 3 different sets of exactly two brothers (B_1B_2, B_1B_3, B_2B_3), we would have to multiply this 6 by 3 to get 18 scenarios of only two brothers medaling.

The brothers win at least two medals in 18 + 1 = 19 circumstances. Our total number of circumstances is 84, so our probability is 19 / 84.

The correct answer is B.

23.

There are four scenarios in which the plane will crash. Determine the probability of each of these scenarios individually:

CASE ONE: Engine 1 fails, Engine 2 fails, Engine 3 works = $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{24}$

CASE TWO: Engine 1 fails, Engine 2 works, Engine 3 fails = $\frac{1}{3} \times \frac{3}{4} \times \frac{1}{2} = \frac{3}{24}$

CASE THREE: Engine 1 works, Engine 2 fails, Engine 3 fails = $\frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{2}{24}$

CASE FOUR: Engine 1 fails, Engine 2 fails, Engine 3 fails = $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{24}$

To determine the probability that any one of these scenarios will occur, sum the four probabilities:

$$\frac{1}{24}+\frac{3}{24}+\frac{2}{24}+\frac{1}{24}=\frac{7}{24}$$

The correct answer is D. There is a 7/24 chance that the plane will crash in any given flight.

24.

The question require us to determine whether Mike's odds of winning are better if he attempts 3 shots instead of 1. For that to be true, his odds of making 2 out of 3 must be better than his odds of making 1 out of 1.

There are two ways for Mike to at least 2 shots: Either he hits 2 and misses 1, or he hits all 3:

Odds of hitting 2 and missing 1 $p \times p \times (1-p)$	**# of ways to hit 2 and miss 1** 3 (HHM, HMH, MHH)	**Total Probability** $3p^2(1-p)$
Odds of hitting all 3 $p \times p \times p$	**# of ways to hit all 3** 1 (HHH)	p^3
Mike's probability of hitting *at least* 2 out of 3 free throws =		$3p^2(1-p)+p^3$

Now, we can rephrase the question as the following inequality:

Is $3p^2(1-p)+p^3 > p$? (Are Mike's odds of hitting at least 2 of 3 greater than his odds of hitting 1 of 1?)

This can be simplified as follows:

$3p^2(1-p)+p^3 > p$

$3p^2-3p^3+p^3 > p$

$3p^2-2p^3 > p$ (we can divide by p since $p>0$)

$3p-2p^2 > 1$

$-2p^2+3p-1 > 0$ (divide by -2, flipping the inequality)

$p^2-1.5p+.5 < 0$

$(p-.5)(p-1) < 0$

In order for this inequality to be true, p must be greater than .5 but less than 1 (since this is the only way to ensure that the left side of the equation is negative). But we already know that p is less than 1 (since Mike occasionally misses some shots). Therefore, we need to know whether p is greater than .5. If it is, then the inequality will be true, which means that Mike will have a better chance of winning if he takes 3 shots.

Statement 1 tells us that $p < .7$. This does not help us to determine whether $p > .5$, so statement 1 is not sufficient.

Statement 2 tells us that p > .6. This means that p must be greater than .5. This is sufficient to answer the question.
The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

25.
In order to determine the probability that the World Series will last *fewer than* 7 games, we can first determine the probability that the World Series WILL last *exactly* 7 games and then subtract this value from 1.

In order for the World Series to last exactly 7 games, the first 6 games of the series must results in 3 wins and 3 losses for each team.

Let's analyze one way this could happen:

Game 1	Game 2	Game 3	Game 4	Game 5	Game 6
T1 Wins	T1 Wins	T1 Wins	T1 Loses	T1 Loses	T1 Loses

There are many other ways this could happen. Using the permutation formula, there are 6!/(3!)(3!) = 20 ways for the two teams to split the first 6 games (3 wins for each).

There are then 2 possible outcomes to break the tie in Game 7. Thus, there are a total of 20 × 2 = 40 ways for the World Series to last the full 7 games.

The probability that any one of these 40 ways occurs can be calculated from the fact that the probability of a team winning a game equals the probability of a team losing a game = 1/2.

Given that 7 distinct events must happen in any 7 game series, and that each of these events has a probability of 1/2, the probability that any one particular 7 game series occurs is $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$.

Since there are 40 possible different 7 game series, the probability that the World Series will last exactly 7 games is:

$$40 \times \frac{1}{128} = \frac{40}{128} = .3125 = 31.25\%$$

Thus the probability that the World Series will last *fewer than* 7 games is 100% - 31.25% = 68.75%.

The correct answer is D.

26.
12 people will be selected from a pool of 15 people: 10 men (2/3 of 15) and 5 women (1/3 of 15). The question asks for the probability that the jury will comprise at least 2/3 men, or at least 8 men (2/3 of 12 jurors = 8 men).

The easiest way to calculate this probability is to use the “1-*x* shortcut.” The only way the jury will have fewer than 8 men is if a jury of 7 men and 5 women (the maximum number of women available) is selected. There cannot be fewer than 7 men on the jury, since the jury must have 12 members and only 5 women are available to serve on the jury.

The total number of juries that could be randomly selected from this jury pool is:

$$\frac{15!}{12!3!} = \frac{(15)(14)(13)}{(3)(2)} = 455$$

The number of ways we could select 7 men from a pool of 10 men is:

$$\frac{10!}{7!3!} = \frac{(10)(9)(8)}{(3)(2)} = 120$$

The number of ways we could select 5 women from a pool of 5 women is:

5!/5! = 1

This makes practical sense, in addition to mathematical sense. All of the women would have to be on the jury, and there is only one way that can happen.

Putting these selections together, the number of ways a jury of 7 men and 5 women could be selected is: 120 × 1 = 120

The probability that the jury will be comprised of fewer than 8 men is thus 120/455 = 24/91.

Therefore, the probability that the jury will be comprised of at least 8 men is 1 – (24/91) = 67/91.

The correct answer is D.

27. 41/50
28. 1/216
29. 2/25
30. C
31. D
32. B
33. A
34. 1/6

PART 2

1.
There are several ways to solve this.
Notation: 1R3W means 1 Right, 3 Wrong

<u>Method 1: The "# of ways method" or "the combinatorics method"</u>
Probability = # of ways to get 1R3W/# of ways total

# of ways total is 4! = 24. Imagine stuffing envelopes randomly. Stacy can put any of 4 letters into the first envelope, any of the remaining 3 into the next, either of the remaining 2 into the next, and has no choice to make on the last, or 4*3*2*1.

# of ways to get 1R3W is more complicated. She could fill the first envelope with the right letter (1 way), then put either of the 2 wrong remaining letters in the next (2 ways), then put a wrong letter in the next (1 way). That's 1*2*1*1 = 2.

But since it doesn't have to be the first envelope that has the Right letter, it could be any of the 4 envelopes (i.e. we could have RWWW, WRWW, WWRW, WWWR), the total ways to get 1R3W is 4*2 = 8.

Probability is 8/24 = 1/3.

<u>Method 2: The successive probability method.</u>
Think first about the RWWW case: The probability that Stacy selects the right letter to put in the first envelope is 1/4. Then, the chance of her picking the wrong letter for the next is 2/3, wrong for the next is 1/2, then definitely the wrong letter goes in the final envelope. Probability of RWWW is 1/4*2/3*1/2 = 1/12.

As noted above, the RWWW chance is the same as the chance of WRWW or WWRW or WWWR, so the total chance is 4*1/12 = 1/3.
Hey people - common-sense guideline: if you're going to post anything, then post the EXPLANATIONS for your answers. If you just post some numbers and say 'these are the possibilities,' then no one is really being helped.

This is one of those problems on which, if you're stuck, you can just make a list and count things: the total number of possibilities is only 4! = 24. You can list that many possibilities in well under a minute, especially if you arrange them in an organized manner.

Here we go: Use ALPHABETICAL ORDER.

ABCD ABDC ACBD ACDB* ADBC* ADCB
BACD BADC BCAD* BCDA BDAC BDCA*
CABD* CADB CBAD CBDA* CDAB CDBA
DABC DACB* DBAC* DBCA DCAB DCBA

I've marked with asterisks those arrangements in which exactly one letter goes in the matching envelope (this is a lot easier to see if you arrange the letters vertically, with the heading ABCD at the top, as you might do on the yellow dry-erase pad).

What you should get from this discussion: If you're stuck on a problem, don't be afraid to just list things and do the problem the 'brute-force way'. Really, don't. Every second you spend just staring at a problem is a second wasted.