770GMAT Ivy GMAT

Quant Topics – Co-ordinate Geometry, P&C, Probability

Co-ordinates:

- If 2 points (a, b) and (c, d) lie in the same quadrant, then a and c should have the same sign; and b and d should have the same sign.
- Distance between 2 points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Distance of the point (a, b) from the origin $(0, 0)$ is given by $\sqrt{a^2 + b^2}$
- Any point on the X axis can be taken as $(a, 0)$
- Any point on the Y axis can be taken as $(0, b)$
- In order to find the X–intercept of a line, put Y = 0 in the equation of the line and find X
- In order to find the Y intercept of a line, put X = 0 and find Y.
- To plot a line, first put y = 0, find the point on x axis; then put x = 0, fins the point on y axis. Join the two points to get the desired graph.

Straight line (slope):

- An equation of the form AX + BY + C = 0 is called the general equation of a straight line, where X and Y are variables and A, B, C are constants.
- Any point lying on this line will satisfy the equation of the line. i.e., the coordinates of the point when substituted by X & Y resp. in the above equation will make the LHS vanish.
- If AB is a straight line on the XY plane, then the ratio of y intercept to x intercept (with signs) is called its slope and is denoted by 'm'. The lengths OP and OQ are respectively called the intercepts on X and Y axes, made by the line.
- So slope = RISE / RUN So, Slope = $(y_2 - y_1) / (x_2 - x_1)$
- If 'm' is the slope of the line and 'c' the intercept made by the line on Y axis, the equation is Y = mX + c
- If the slope of the line is m and it passes through (X1, Y1), the equation is $(Y - Y_1) = m (X - X_1)$
- General form: AX + BY + C = 0 In this slope = m = –A/B, X intercept = –C/A Y intercept = –C/B
- If two lines are parallel then their slopes are equal ($m_1 = m_2$). If two lines do not intersect, they are parallel.
- If the slopes of two lines are equal ($m_1 = m_2$), then either the lines are parallel (no intersection) or they are co-incident (infinite points of intersection).
- If two lines are perpendicular to each other, the product of their slopes is –1. ($m_1 m_2 = -1$). **(There is one exception to this rule – when two lines are parallel to x and y axis respectively, then $m_1 m_2 \neq -1$, even if the lines are perpendicular).**
- $a_1X + b_1Y + c_1 = 0$ and $a_2X + b_2Y + c_2 = 0$ will represent the same straight lines if $a_1/a_2 = b_1/b_2 = c_1/c_2$. In this case, the lines are coincident and theoretically intersect at infinite points.
- The point of intersection of two lines (X, Y) is obtained by simultaneously solving both the equations.
- The length of perpendicular (p) from (X1, Y1) on the line AX + BY + C = 0 is $P = |ax_1 + by_1 + c / \sqrt{(a^2 + b^2)}|$
  Equation of a line parallel to X axis is Y = b (b is a constant)
- Equation of a line parallel to Y axis is X = a (a is a constant)
- Equation of X and Y axes are Y = 0 and X = 0 respectively
- The image of the point (a, b) in x axis is (a, -b)
- The image of the point (a, b) in y axis is (-a, b)
- The image of the point (a, b) in the line y = x is (b, a)
- To plot a line, first put y = 0, find the point on x axis; then put x = 0, fins the point on y axis. Join the two points to get the desired graph.

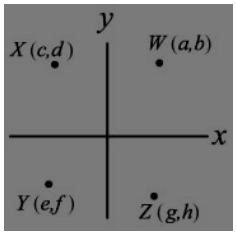
1. If $ab \neq 0$ and points $(-a, b)$ and $(-b, a)$ are in the same quadrant of the xy–plane, is point $(-x, y)$ in this same quadrant? (1) $xy > 0$ (2) $ax > 0$

2. In the xy–plane, at what two points does the graph of y = (x + a) (x + b) intersect the x–axis?
(1) a + b = –1 (2) The graph intersects the y–axis at (0, –6).

3. In the rectangular coordinate system, are the points (r, s) and (u, v) equidistant from the origin?
(1) r + s = 1 (2) u = 1 – r and v = 1 – s

4. What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?

5. In the rectangular coordinate plane points *X* and *Z* lie on the same line through the origin and points *W* and *Y* lie on the same line through the origin. If $a^2 + b^2 = c^2 + d^2$ and $e^2 + f^2 = g^2 + h^2$, what is the value of length *XZ* – length *WY*?



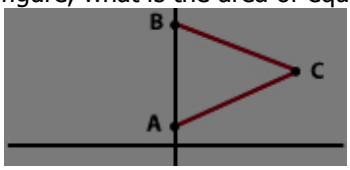
–2 –1 0 1 2

6. Does the equation $y = (x - p)(x - q)$ intercept the *x*–axis at the point (2,0)?
(1) $pq = -8$ (2) $-2 - p = q$

7. Point K = (A, 0), Point G = $\left(2A+4, \sqrt{2A+9}\right)$. Is the distance between point K and G prime?
(1) $A^2 - 5A - 6 = 0$ (2) A > 2

8. The (x, y) coordinates of points *P* and *Q* are (–2, 9) and (–7, –3), respectively. The height of equilateral triangle *XYZ* is the same as the length of line segment *PQ*. What is the area of triangle *XYZ*?
169/√3 84.5 75√3 169√3 /4 225√3 /4

9. If points *A* and *B* are on the *y*–axis in the figure, what is the area of equilateral triangle *ABC*?

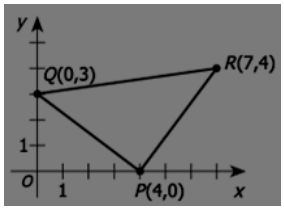


(1) Coordinates of point *B* are (0, 5√3). (2) Coordinates of point *C* are (6, 3√3).

10. The line $3x + 4y = 8$ passes through all of the quadrants in the coordinate plane except:
I II III IV II and IV.

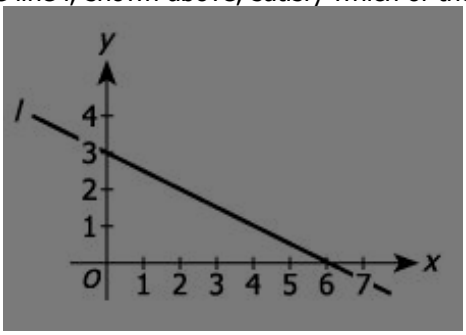
11. If *p* and *q* are nonzero numbers, and *p* is not equal to *q*, in which quadrant of the coordinate system does point $(p, p - q)$ lie?
(1) (p, q) lies in quadrant IV. (2) $(q, -p)$ lies in quadrant III.

12. The area of the triangle PQR as shown above will be:



12.5 14 21 28 35

13. All points (x, y) that lie below the line *l*, shown above, satisfy which of the following inequalities?



y < 2x + 3 y < –2x + 3 y < –x + 3 y < ½ x + 3 y < -½ x + 3

14. In the XY plane, does the line with equation y = 3x + 2 contain the point (r, s)?
(1) (3r + 2 - s) (4r + 9 - s) = 0 (2) (4r - 6 - s) (3r + 2 - s) = 0

15. In the xy-plane, region *R* consists of all the points (*x*, *y*) such that $2x + 3y \leq 6$. Is the point (*r*, *s*) in region *R*?
(1) $3r + 2s = 6$ (2) $r \leq 3$ and $s \leq 2$

16. Circle C and line k lie in the xy–plane. If circle C is centered at the origin and has radius 1, does line k intersect circle C?
(1) The x–intercept of line k is greater than 1 (2) the slope of line k is –1/10

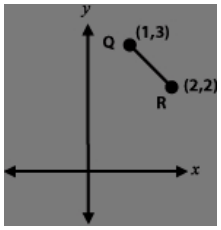
17. In the x–y plane, what is the y–intercept of the line l?
(1) Slope of the line l is 3 times its y intercept. (2) The x–intercept of line l is –1/3

18. In the xy–plane, line k has positive slope and x–intercept 4. If the area of the triangle formed by line k and the two axes is 12, what of the y– intercept of line?

19. Line *l* is defined by the equation $y - 5x = 4$ and line *w* is defined by the equation $10y + 2x + 20 = 0$. If line *k* does not intersect line *l*, what is the degree measure of the angle formed by line *k* and line *w*?
0 30 60 90 It cannot be determined from the information given.

20. In the *xy*–coordinate system, what is the slope of the line that goes through the origin and is equidistant from the two points *P* = (1, 11) and *Q* = (7, 7)?
2 2.25 2.50 2.75 3

21. Does line *S* intersect line segment *QR*?



(1) The equation of line *S* is $y = -x + 4$. (2) The slope of line *S* is –1.

22. Line L contains the points (2, 3) and (p, q). If $q = 2$, which of the following could be the equation of line m, which is perpendicular to line L?
(A) $2x + y = px + 7$ (B) $2x + y = -px$ (C) $x + 2y = px + 7$
(D) $y - 7 = x \div (p - 2)$ (E) $2x + y = 7 - px$

23. The line represented by the equation y = 4 − 2x is the perpendicular bisector of line segment RP. If R has the coordinates (4, 1), what are the coordinates of point P?
(A) (–4, 1) (B) (–2, 2) (C) (0, 1) (D) (0, –1) (E) (2, 0)

24. In the XY-coordinate plane, line L and line K intersect at the point (4, 3). Is the product of their slopes negative?
(1) The product of the x-intercepts of line L and K is positive.
(2) The product of the y-intercepts of line L and k is negative.

25. In the XY-plane, line K passes through the point (1, 1) and line M passes through point (1, -1). Are lines K and m perpendicular to each other?
(1) Lines K and m intersect at the point (1, -1) (2) Line K intersect the x-axis at the point (1, 0)

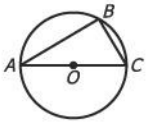
26. Lines N and P lie in the x-y plane. Is the slope of the line N less than the slope of line P?
(1) Lines N and N intersect at (5, 1) (2) The y-intercept of line N is greater than y-intercept of line P.

27. In the rectangular coordinate system, does the line k (not shown) intersect quadrant II?
(1) Slope of k is -1/6 (2) The y-intercept of k is -6

28. Two line l and k intersect at a point (4, 3). Is the product of their slopes -1?
(1) x intercepts of line l and k are positive (2) y intercept of line l and k are negative

29. In the xy-plane, the line k passes through the origin and through point (a, b), where $ab \neq 0$. Is b positive?
(1) The slope of k is negative (2) a < b

30. In the figure, the radius of the circle with center O is 1 and BC = 1. What is the area of triangular region ABC?



P&C Theory

Permutations and Combinations concepts are used for Complex Counting

Arrangements - keywords – seating, sitting, sequence, order, alphabets, schedule, ranking, itinerary, codes

Order important – gives unique arrangements

For e.g. A and B sitting on chair can be AB or BA so these are two distinct arrangements

It is basically selection followed by arrangement. So ${}^{n}P_{r} = {}^{n}C_{r} \times r!$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Selection - keywords – team, committee, balls, handshakes, matches, picking

Order not important – For example choosing A and B from a group of 3 or four alphabets. The order does not matter. India playing a match against Australia is the same as Australia playing against India.

$${}^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

Different formulae

1. $${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

   When to use? When n distinct items present and r have to be selected and then arranged.

   E.g – how many ways can you arrange 4 people in 5 chairs = ${}^{5}P_{4}$

2. n^{r}

   All n distinct selection of r but repetition is allowed.

   In how many ways can you wear three different rings on four fingers?
   4^{3}

3. $$\frac{n!}{p!\,q!\,r!}$$

   Arranging n things in which p are of one type, q of a second type and r of third type:

Ex: In how many ways can you arrange the letters of word Banana?
Ans. $\frac{6!}{3!\,2!}$

4. Special Cases

   5 people A, B, C, D, E to be arranges in which A and B are together.
   $4! \times 2!$

5 people A, B, C, D, E to be arranged in which A and B are not together.

$5! - 4! \times 2!$

5. **Block diagrams** - Some problems can not be done with any formula but with a block diagram

Combinations

1. Select 5 people out of 10 Ans. ${}^{10}C_5$

Particular Cases – Select 5 out of 10 people such that A and B are always selected. This means only 3 of the remaining 8 are to be selected ${}^{8}C_3$

Select 5 out of 10 such that A and B are never selected. This means that out of remaining 8, 5 have to be selected so it is ${}^{8}C_5$

2. Select 5 out of 10 so that A and B are never together.
   = Total – Together = ${}^{10}C_5 - {}^{8}C_3$

AND denotes Multiplication

OR denotes Addition

Circular Permutations: (n – 1) !.

1. A password contains at least 8 distinct digits. It takes 12 seconds to try one combination, what is the minimum amount of time required to guarantee access to the database?

2. Greg, Marcia, Peter, Jan, Bobby and Cindy go to a movie and sit next to each other in 6 adjacent seats in the front row of the theater. If Marcia and Jan will not sit next to each other, in how many ways different arrangements can the 6 people sit?

3. An engagement team consists of a project manager, team leader, and four consultants. There are 2 candidates for the position of project manager, 3 candidates for the position of team leader, and 7 candidates for the 4 consultant slots. If 2 out of 7 consultants refuse to be on the same team, how many different teams are possible?

4. How many different five-letter combinations can be created from the word TWIST?

5. A university cafeteria offers 4 flavors of pizza – pepperoni, chicken, Hawaiian and vegetarian. If a customer has an option to add, extra cheese, mushrooms, or both to any kind of pizza, how many different pizza varieties are available?

6. Mario's Pizza has two choices of crust: deep dish and thin-and-crispy. The restaurant also has a choice of 5 toppings: tomatoes, sausage, peppers, onions, and pepperoni. Finally, Mario's offers every pizza in extra-cheese as well as 'regular'. If Linda's volleyball team decides to order a pizza with four toppings, how many different choices do the teammates have at Mario's Pizza?

7. A group of 5 students bought movie tickets in one row next to each other. If Bob and Lisa are in this group, what is the number of ways of seating if both of them will sit next to only one other student from the group?

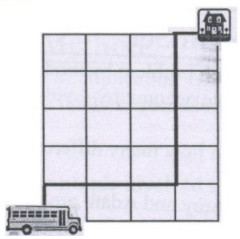
8. Mark's clothing store uses a bar-code system to identify every item. Each item is marked by a combination of 2 letters followed by 3 digits. Additionally, the three-digit number must be even for male products and odd for female products. If all apparel products start with the letter combination AP, how many male apparel items can be identified with the bar code?

9. If 6 fair coins are tossed, how many different coin sequences will have exactly 3 tails, if all tails have to occur in a row?

10. A telephone company needs to create a set of 3-digit area codes. The company is entitled to use only digits 2, 4 and 5, which can be repeated. If the product of the digits in the area code must be even, how many different codes can be created?

11. In how many ways can a teacher write an answer key for a mini-quiz that contains 3 true-false questions followed by 2 multiples-choice questions with 4 answer choices each, if the correct answers to all true-false questions cannot be the same?

12. Every morning, Casey walks from her house to the bus stop. She always travels exactly nine blocks from her house to the bus, but she varies the route she takes every day. (One sample route is shown.) How many days can Casey walk from her house to the bus stop without repeating the same route?



13. Larry, Michael, and Doug have five donuts to share. If any one of the men can be given any whole number of donuts from 0 to 5, in how many different ways can the donuts be distributed?
(A) 21 (B) 42 (C) 120 (D) 504 (E) 5040

14. Anthony and Michael sit on the six-member board of directors for company X. If the board is to be split up into 2 three-person subcommittees, what percent of all the *possible* subcommittees that include Michael also include Anthony?
    20% 30% 40% 50% 60%

15. Six mobsters have arrived at the theater for the premiere of the film "Goodbuddies." One of the mobsters, Frankie, is an informer, and he's afraid that another member of his crew, Joey, is on to him. Frankie, wanting to keep Joey in his sights, insists upon standing behind Joey in line at the concession stand. How many ways can the six arrange themselves in line such that Frankie's requirement is satisfied?
    6 24 120 360 720

16. From a drawer containing black, blue and gray solid-color socks, including at least three socks of each color, how many matched pairs can be removed?
    (1) The drawer contains 11 socks.
    (2) The drawer contains an equal number of black and gray socks.

17. On Tuesday, Kramer purchases exactly 3 new shirts, 2 new sweaters, and 4 new hats, On the following day and each subsequent day thereafter, Kramer wears one of his new shirts together with one of his new sweaters and one of his new hats. Kramer avoids wearing the exact same combination of shirt, sweater, and hat for as long as possible. On which day is this no longer possible?
    Tuesday Wednesday Thursday Friday Saturday

18. A fair coin is flipped three times. What is the probability that the coin lands on heads exactly twice?
    (A) 1/8 (B) 3/8 (C) 1/2 (D) 5/8 (E) 7/8

19. Is the probability that Patty will answer all of the questions on her chemistry exam correctly greater than 50%?
    (1) For each question on the chemistry exam, Patty has a 90% chance of answering the question correctly.
    (2) There are fewer than 10 questions on Patty's chemistry exam.

20. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B, where there are already 3 women and 5 men. If a single person is then to be picked from room B, what is the probability that a woman will be picked?
    (A) 13/21 (B) 49/117 (C) 15/52 (D) 5/18 (E) 40/117

21. If the probability of rain on any given day in Chicago during the summer is 50%, independent of what happens on any other day, what is the probability of having exactly 3 rainy days from July 4 through July 8, inclusive?
    (A) 1/32 (B) 2/25 (C) 5/16 (D) 8/25 (E) ¾

22. Triplets Adam, Bruce, and Charlie enter a triathlon. If there are 9 competitors in the triathlon and medals are awarded for first, second, and third place, what is the probability that at least two of the triplets will win a medal?
    (A) 3/14 (B) 19/84 (C) 11/42 (D) 15/28 (E) ¾

23. A small, experimental plane has three engines, one of which is redundant. That is, as long as two of the engines are working, the plane will stay in the air. Over the course of a typical flight, there is a 1/3 chance that engine one will fail. There is a 75% probability that engine two will work. The third engine works only half the time. What is the probability that the plane will crash in any given flight?
    (A) 7/12 (B) ¼ (C) ½ (D) 7/24 (E) 17/24

24. Mike recently won a contest in which he will have the opportunity to shoot free throws in order to win $10,000. In order to win the money Mike can either shoot 1 free throw and make it, or shoot 3 free throws and make at least 2 of them. Mike occasionally makes shots and occasionally misses shots. He knows that his probability of making a single free throw is p, and that this probability doesn't change. Would Mike have a better chance of winning if he chose to attempt 3 free throws?
    (1) $p < 0.7$ (2) $p > 0.6$

25. Baseball's World Series matches 2 teams against each other in a best-of-seven series. The first team to win four games wins the series and no subsequent games are played. If you have no special information about either of the teams, what is the probability that the World Series will consist of fewer than 7 games?
(A) 12.5% (B) 25% (C) 31.25% (D) 68.75% (E) 75%

26. If a jury of 12 people is to be selected randomly from a pool of 15 potential jurors, and the jury pool consists of 2/3 men and 1/3 women, what is the probability that the jury will comprise at least 2/3 men?
24/91 45/91 2/3 67/91 84/91

27. At a certain car dealership, the 40 vehicles equipped with air conditioning represent 80% of all cars available for sale. Among all the cars, there are 15 convertibles, 14 of which are equipped with an air-conditioning system. If a customer is willing to purchase either a convertible or a car equipped with air conditioning, what is the probability that a randomly selected vehicle will fit customer specifications?

28. In a certain game of dice, the player's score is determined as a sum of three throws of a single die. The player with the highest score wins the round. If more than one player has the highest score, the winnings of the round are divided equally among these players. If Jim plays this game against 21 other players, what is the probability of the minimum score that will guarantee Jim some monetary payoff?

29. A telephone number contains 10 digits, including a 3-digit area code. Bob remembers the area code and the next 5 digits of the number. He also remembers that the remaining digits are not 0, 1, 2, 5, or 7. If Bob tries to find the number by guessing the remaining digits at random, the find probability that he will be able to find the correct number in at most 2 attempts.

30. There were initially no black marbles in a jar. Subsequently, new marbles were added to the jar. If marbles are drawn at random and selected marbles are not returned to the jar, what is the probability of selecting 2 black marbles in a row?
    (1) After the new marbles are added, 50% of all marbles are black.
    (2) Among the 10 added marbles, 8 are black.

31. What is the probability that it will rain on each of the next 3 days if the probability of raining on any single day is the same in that period?
    (1) The probability of no rain throughout the first two days is 36%.
    (2) The probability of rain on the third day is 40%.

32. If a number is drawn at random from the first 1000 positive integers, what is the probability of selecting a refined number?
    (1) Any refined number must be divisible by 22.
    (2) A refined number is any even multiple of 11.

33. A certain jar contains only B black marbles, W white marbles, and R red marbles, if one marble is to be chosen at random from the jar, is the probability that the marble chosen will be red greater than the probability that marble chosen will be white?
    (1) $R / (B + W) > W / (B + R)$ (2) $B - W > R$

34. What is the probability that a number selected from (-10, -6, -5, -4, -2.5, -1, 0, 2.5, 4, 6, 7, 10) can fulfil $(x-5)(x+10)(2x-5)=0$?
    1/12 1/6 ¼ 1/3 1/2

35. Tanya prepared 4 different letters to be sent to 4 addresses. For each letter she prepared an envelope with its correct address. If the 4 letters to be put in to 4 envelopes at random, what is the probability that only one letter will be put in to the envelope with the correct address?