

# Manipulating Algebraic Expressions

| Problem                                                                         | Example                                                                                                                              |
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| If you have to add/subtract numbers with the same power, but different basis... |                                                                                                                                      |
| Factor into expressions with "digestable" exponents                             | $3^8 - 2^8 = (3^4 - 2^4)(3^4 + 2^4)$ $= (3^2 - 2^2)(3^2 + 2^2)(3^4 + 2^4)$ $= (9 - 4)(9 + 4)(81 + 16)$                               |
| Special case: Add two numbers with 7 as a basis                                 | $7^3 + 7^2 = 7(7^2 + 1) = 7(50)$                                                                                                     |
| Prime factorize                                                                 | $\sqrt{80} - \sqrt{45} = \sqrt{(2)(2)(2)(2)(5)} - \sqrt{(3)(3)(5)}$ $= 4\sqrt{5} - 3\sqrt{5}$                                        |
| If you are confronted with a sum in the denominator...                          |                                                                                                                                      |
| Factor out the same term in the numerator                                       | $\frac{5x - 5y}{x - y} = \frac{5(x - y)}{x - y} = 5$                                                                                 |
| Cross-multiply with the other term in the numerator                             | $\frac{1}{x} - \frac{1}{(x + 1)} = \frac{(x + 1)}{x(x + 1)} - \frac{x}{x(x + 1)}$                                                    |
| If you are confronted with different exponents in the denominator...            |                                                                                                                                      |
| Extent with "1"                                                                 | $\frac{1}{3^8} + \frac{1}{3^9} + \frac{1}{3^9} = \frac{1(3)}{3^8(3)} + \frac{1}{3^9} + \frac{1}{3^9} = \frac{5}{3^9}$                |
| Look for a factor of 10 (+ extent with 1)                                       | $\frac{1}{2^3 5^7} = \frac{2^4}{2^3 5^7 2^4} = \frac{2^4}{2^7 5^7} = \frac{2^4}{10^7}$                                               |
| If you are confronted with a root in the denominator...                         |                                                                                                                                      |
| Extent the fraction with "1"                                                    | $\frac{20}{\sqrt{2}} = \frac{20\sqrt{2}}{\sqrt{2}\sqrt{2}}$                                                                          |
| Use the conjugate                                                               | $\frac{4}{3 - \sqrt{2}} = \frac{4}{3 - \sqrt{2}} \left( \frac{3 + \sqrt{2}}{3 + \sqrt{2}} \right)$ $= \frac{4(3 + \sqrt{2})}{9 - 2}$ |
| If you have to take the reciprocal of a sum fraction...                         |                                                                                                                                      |
| Always combine the terms first                                                  | $1 - \frac{1}{x} = \frac{x}{x} - \frac{1}{x} = \frac{x - 1}{x} \rightarrow \frac{x}{x - 1}$                                          |

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| If you have to solve for x in a 3rd power equation...      |                                                                                                                                                                                                                                                                                                                         |
| Factor out one variable to turn into quadratic equation    | $x^3 + 11x^2 + 30x = x(x^2 + 11x + 30)$ $= x(6 + x)(5 + x) = 0$                                                                                                                                                                                                                                                         |
| Substitute with a lower degree variable                    | $x^6 - 3x^3 + 2 = 0$ $= y^2 - 3y + 2$                                                                                                                                                                                                                                                                                   |
| If you have to simplify expressions that are close to 1... |                                                                                                                                                                                                                                                                                                                         |
| Use powers of 10                                           | $\frac{0.9999999}{1 - 10^{-8}} - \frac{0.99999991}{1 - 9 * 10^{-8}}$ $= \frac{1.0001}{1 - 10^{-8}} - \frac{1.003}{1 - 9 * 10^{-8}}$ $= \frac{1 + 10^{-4}}{(1 + 10^{-4})(1 - 10^{-4})} - \frac{1 + 3 * 10^{-4}}{(1 + 3 * 10^{-4})(1 - 3 * 10^{-4})}$ $= \frac{1}{1 - 10^{-4}} - \frac{1}{1 - 3 * 10^{-4}} = 2 * 10^{-4}$ |