

Concept of Percentage

What is a Percentage?

A fraction with denominator 100 is called percentage. To convert a fraction into percentage, multiply by 100 and put % sign.

Expressing one quantity as percentage of other:

Ex: What percent is number 5 of number 25?

Out of 25 → 5,

Out of 1 → $\frac{5}{25}$,

Out of 100 → $(\frac{5}{25}) * 100 = 20\%$.

Percentage Change:

General Formula: $Percent\ change = (\frac{Change\ in\ Quantity}{Original\ Quantity} * 100) \%$

$Percentage\ increase = \frac{Increased\ value - original\ value}{Original\ value} * 100$

$Percentage\ decreased = \frac{Original\ value - Decreased\ value}{Original\ value} * 100$

Percentage Change Advance:

1. If a value R is increased by x%, then decrease the increased resultant value $(R + \frac{Rx}{100})$ by $(\frac{x}{x+100} * 100)\%$ to get back the original value R.
2. If a value R is decreased by x%, then increase the decreased resultant value $(R - \frac{Rx}{100})$ by $(\frac{x}{100-x} * 100)\%$ to get back the original value R.

Ex: 1000 is increased by 20% then resultant value is 1200.

$$\frac{20}{120} * 100 = 16.67\% \text{ (Rounded to nearest digit)}$$

$$\text{So, } 1200 \text{ is reduced by } 16.66\% = 1200 * \frac{16.67}{100} = 200.04$$

1200 - 200.04 = 1000 – the approximation due to the round off- If we use 16.6666666666667 we will get the exactly 200.

Percentage Increased/ Reduced BY and Percentage Increased/ Reduced TO:

If a quantity is reduced by $x\%$ then result will $(100-x)\%$ of original, and if a quantity is reduced to $x\%$, then the new value is $x\%$ of original value.

By represent difference and to represents the final value.

Ex: if a rate of product is reduced by 30% , then the new rate will be 70% of original, and if the rate of the product is reduced to 30% , then new rate will be 30% of original.

Percentage in POPULATIONS:

If the original population of region is A , and annual growth is $x\%$. Then population after R years is $A \left(1 + \frac{x}{100}\right)^R$.

i. Increase in Population – $A \left[\left(1 + \frac{x}{100}\right)^R - 1 \right]$

If the original population of region is A , and decrease in population is $x\%$. Then population after R years is $A \left(1 - \frac{x}{100}\right)^R$.

i. Decrease in Population- $A \left[1 - \left(1 - \frac{x}{100}\right)^R \right]$.

If a value of a number is first increased by $R\%$ and then decreased by $R\%$, the net change is always a decrease or loss in original value.

Hence, % Loss or decrease = $\left(\frac{R}{10}\right)^2 \%$.

Other Important Notes:

- If a value R is increased by $x\%$ to S , and S is again decreased to R by $y\%$, Then x is always greater than y (Positive values only).
- If a value R is decreased by $x\%$ to S , and S is again increased to R by $y\%$, then x is always less than y .

- c. If a value P is increased by x% then by y% and then by z%, the final value will be same even if we reverse the order of x,y,z. I.e., P is increased by z% first, then by y% and then by x%.
(Same for the decrease)- Successive increase or decrease in value.
- d. Similarly if value P is increased by X% then by Y% and then decreased by Z%. the final value will be same when P is decreased by Z% first, then increased by Y%, and then by X%.
- e. If there is an increase of $\frac{x}{y}$ in any value A, then the increased value will be $A \left(1 + \frac{x}{y}\right)$.
- f. If there is a decrease of $\frac{x}{y}$ in any value A, then the increased value will be $A \left(1 - \frac{x}{y}\right)$.