Total Gadha's Complete Book of

NUMBER SYSTEM

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TYPES OF NUMBERS

Natural Numbers

The group of numbers starting from 1 and including 1, 2, 3, 4, 5, and so on, are known as natural numbers. Zero, negative numbers, and decimals are not included this group.

If n is an odd natural number, what is the highest number that always divides $n \times (n^2 - 1)$?

Answer: $n \times (n^2 - 1) = (n - 1) \times n \times (n + 1)$, which is a product of three consecutive numbers. Since n is odd, the numbers (n – 1) and (n + 1) are both even. As they are two consecutive even numbers one of these numbers will be a multiple of 2 and the other will be a multiple of 4. Hence, their product is a multiple of 8. Since one out of every three consecutive numbers is a multiple of 3, one of the three numbers will be a multiple of three. Hence, the product of three numbers will be a multiple of $8 \times 3 = 24$.

Hence, the highest number that always divides $n \times (n^2 - 1)$ is 24.

The product of n consecutive natural numbers is always divisible by n!, where $n! = 1 \times 2 \times 3 \times 4 \times 5.... \times n$

For every natural number n, the highest number that $n \times (n^2 - 1) \times (5n + 2)$ is always divisible by is
(a) 6 (b) 24 (c) 36 (d) 48

Answer:

Case 1: If n is odd, $n \times (n^2 - 1)$ is divisible by 24 as proved in the earlier question.

Case 2: If n is even, both (n – 1) and (n + 1) are odd. Since product of three consecutive natural numbers is always a multiple of 3 and n is even, the product $n \times (n^2 - 1)$ is divisible by 6. Since n is even 5n is even. If n is a multiple of 2, 5n is a multiple of 2 and hence 5n + 2 is a multiple of 4. If n is a multiple of 4, 5n + 2 is a multiple of 2. Hence, the product $n \times (5n + 2)$ is a multiple of 8.

Hence, the product $n \times (n^2 - 1) \times (5n + 2)$ is a multiple of 24.

Prove that $(2n)!$ is divisible by $(n!)^2$.

Answer: $(2n)! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n - 1) \times n \times (n + 1) \times \ldots \times 2n = (n)! \times (n + 1) \times (n + 2) \times \ldots \times 2n$. Since $(n + 1) \times (n + 2) \times \ldots \times 2n$ is a product of n consecutive numbers, it is divisible by n!. Hence, the product $(n)! \times (n + 1) \times (n + 2) \times \ldots \times 2n$ is divisible by $n! \times n! = (n!)^2$.

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

Sum of cubes of first n natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$

Number of single-digit natural numbers: 1- 9 = 9
Number of two-digit natural numbers: 10- 99 = 90
Number of three-digit natural numbers: 100- 999 = 900
Square of every natural number can be written in the form **3n** or **3n + 1**.
Square of every natural number can be written in the form **4n** or **4n + 1**.

If you write 1st 252 natural numbers in a straight line, how many times do you write the digit 4?

Answer: In the 1st 99 natural numbers, digit 4 comes 20 times. Similarly, from 100 to 199, digit 4 comes 20 times. Now from 200 to 299, digit 4 comes again 20 times out of which we need to subtract 5 numbers (254, 264, 274, 284 and 294). Therefore, total number of times that we write the digit 4 = 20 + 20 + 20 – 5 = 55.

If a book has 252 ages, how many digits have been used to number the pages?

Answer: From page number 1 to age number 9, we will use 1 digit per page $\Rightarrow$ digits used = 9.
From page number 10 to age number 99, we will use 2 digits per page $\Rightarrow$ digits used = $2 \times 90 = 180$.
From page number 100 to age number 252, we will use 3 digits per page $\Rightarrow$ digits used = $3 \times 153 = 459$.
Therefore, total number of digits used = 9 + 180 + 459 = 648

There are three consecutive natural numbers such that the square of the second minus twelve times the first is three less than twice the third. What is the largest of the three numbers?

Answer: Let the consecutive natural number be n, n + 1, n + 2.
$\Rightarrow (n + 1)^2 - 12n = 2(n + 2) - 3$. Solving, we get n = 12 and n + 2 = 14.

What is the smallest natural number which is cube of a natural number and fourth power of a **different** natural number?

Answer: Let $N = x^3$ and $N = y^4$. Therefore, N will contain 12th power (LCM of 3 and 4) of a natural number. Therefore, $N = a^{12} = (a^4)^3 = (a^3)^4$. The smallest such number is $2^{12} = 4096$.

1 and 8 are the first two natural numbers for which 1 + 2 + 3 + ... + n is a perfect square. Which number is the 4th such number?

Answer: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} = M^2$ (say) $\Rightarrow n(n + 1) = 2M^2$

Now n and n + 1 will have no factor in common. Since RHS is twice the square of a natural number, one of n and n + 1 will be twice of a perfect square and the other will be a perfect square. As twice of a perfect square will be even, the other square will be odd. We start investigating the odd squares and their neighbours. The fourth such numbers we get is 288×289.

Let N = 999 999 999 999 999 999. How many 9's are there in N^2?

Answer: $N^2 = (10^{18} - 1)^2 = 10^{36} + 1 - 2 \times 10^{18} = \underbrace{100000\ldots00001}_{35\text{ zeroes}} - \underbrace{2000..0000}_{18\text{ zeroes}} = \underbrace{99999..9998000000\ldots0001}_{17\ 9\text{'s}}$

Whole Numbers

All Natural Numbers plus the number 0 are called as Whole Numbers.

Integers

All Whole Numbers and their negatives are included in this group.

For how many integers n is $n^4 + 6n < 6n^3 + n^2$?

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Answer: $n^4 + 6n - 6n^3 - n^2 < 0 \Rightarrow n(n^2 - 1)(n - 6) < 0$. n cannot be equal to 1 or 0 because LHS becomes 0. Now n2 – 1 will always be positive, therefore, n (n – 6) should be negative $\Rightarrow$ n = 2, 3, 4 and 5.

Find the sum of all two-digit positive integers which exceed the product of their digits by 12.

Answer: Let the two-digit integer be ab. Therefore, $10a + b = ab + 12 \Rightarrow (a - 1)(10 - b) = 2 \Rightarrow$ numbers are 28 or 39.

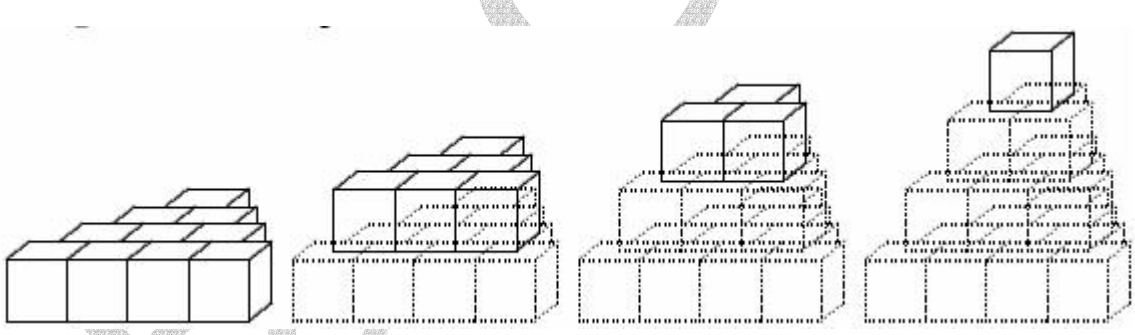
For which integer n is $2^8 + 2^{11} + 2^n$ is a perfect square?

Answer: In order to write the above expression in the form $(a + b)^2 = a^2 + 2ab + b^2$, we note that $2^8 = (2^4)^2$ and $2^{11} = 2 \times 2^4 \times 2^6$. Therefore, we need the square of $2^6 \Rightarrow 2^n = (2^6)^2 = 2^{12} \Rightarrow n = 12$.

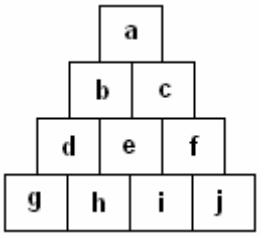
Find the smallest positive integer n for which $(2^2 - 1)(3^2 - 1)(4^2 - 1)\ldots (n^2 - 1)$ is a perfect square.

Answer: Nth term $= (n^2 - 1) = (n + 1)(n - 1) \Rightarrow$ series $= 1 \times 3 \times 2 \times 4 \times 3 \times 5 \ldots \times (n - 2) \times (n) \times (n - 1) \times (n + 1) = 2\,n\,(n + 1) \times k^2$ because all the other terms are squared. The fist value of n which 2 n (n + 1) as perfect square is n = 8.

Twenty cubical blocks are arranged as shown. First, 10 are arranged in a triangular pattern; then a layer of 6, arranged in a triangular pattern, is centered on the 10; then a layer of 3, arranged in a triangular pattern, is centered on the 6; and finally one block is centered on top of the third layer. The blocks in the bottom layer are numbered 1 through 10 in some order. Each block in layers 2, 3 and 4 is assigned the number which is the sum of the numbers assigned to the three blocks on which it rests. Find the smallest possible number which could be assigned to the top block.



Answer: Let the numbers on the bottom 10 cubes be a, b, c, d, e, f, g, h, I, and j, as shown in the figure below:



The sum on the top cube will come out to be a + g + j + 3(b + c + d + f + h + i) + 6e. Giving the lowest value to highest occurring number (e = 1) and highest values to lowest occurring numbers (a = 8, g = 9, j = 10), we get the minimum value = 114.

Rational Numbers

Any number that can be expressed as a ratio of two integers is called a rational number.

This group contains decimal that either do not exist (as in 6 which is 6/1), or terminate (as in 3.4 which is 34/10), or repeat with a pattern (as in 2.333... which is 7/3).

Express 0.212121..in rational form.

Answer: Let A = 0.21212121 $\Rightarrow$ 100A = 21.212121.. = 21 + A $\Rightarrow$ A = $\frac{21}{99}$.

Rule: To express a recurring fraction in rational form, write the recurring digits once in the numerator and write as many 9s in the denominator as are the number of recurring digits. For example,

0.abcabcabc.. = $\frac{abc}{999}$ and 0.abcdabcdabcd.. = $\frac{abcd}{9999}$

Express 2.1434343…in rational form.

Answer: Let R = 2.1434343.. $\Rightarrow$ 10R = 21.43434343…= 21 + $\frac{43}{99} \Rightarrow$ R = $\frac{2122}{990}$.

Rule: To write a fraction, which has both recurring and non-recurring parts, in a rational form, do the following steps:

Numerator: (Number formed by writing all the digits once) – (Number formed by writing all the non-recurring part once) = 2143 – 21 = 2122.

Denominator: Number of 9s equal to number of recurring digits followed by number of zeroes equal to non-recurring digits after the decimal.

What is the value of $\frac{2^{2004}+2^{2001}}{2^{2003}-2^{2000}}$?

Answer: $\frac{2^{2004}+2^{2001}}{2^{2003}-2^{2000}} = \frac{2^{2001}(2^3+1)}{2^{2000}(2^3-1)} = \frac{18}{7}$

Irrational Numbers

Any number that can not be expressed as the ratio of two integers is called an irrational number (imaginary or complex numbers are not included in irrational numbers).
These numbers have decimals that never terminate and never repeat with a pattern.
Examples include pi, e, and √2. 2 + √3, 5 - √2 etc. are also irrational quantities called **Surds.**

Express the value of $\frac{1}{\sqrt{5}+\sqrt{6}-\sqrt{11}}$ as a fraction whose denominator is rational.

Answer: $\frac{1}{\sqrt{5}+\sqrt{6}-\sqrt{11}} = \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{(\sqrt{5}+\sqrt{6}-\sqrt{11})\times(\sqrt{5}+\sqrt{6}+\sqrt{11})}$

$= \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{[(\sqrt{5}+\sqrt{6})^2-(\sqrt{11})^2]} = \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{2\sqrt{30}} = \frac{\sqrt{30}(\sqrt{5}+\sqrt{6}+\sqrt{11})}{60}$

If p = $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$ and q = $\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$, then the value of $p^2 + pq + q^2$ is

Answer: p = $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} = \frac{(\sqrt{8}+\sqrt{7})^2}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})} = (\sqrt{8}+\sqrt{7})^2 = 15+2\sqrt{56}$

Similarly, q = $(\sqrt{8}-\sqrt{7})^2 = 15-2\sqrt{56}$

$p^2 + pq + q^2 = (15+2\sqrt{56})^2 + (15)^2 - (2\sqrt{56})^2 + (15-2\sqrt{56})^2 = 675 + 224 = 899$

Real Numbers

This group is made up of all the Rational and Irrational Numbers. The ordinary number line encountered when studying algebra holds real numbers.

Imaginary Numbers

These numbers are formed by the imaginary number i (i = $\sqrt{-1}$). Any real number times i is an imaginary number.
Examples include i, 3i, −9.3i, and (pi)i. Now $i^2 = -1$, $i^3 = i^2 \times i = -i$, $i^4 = 1$.

What is the value of $\frac{i^4+i^6+i^8+i^{10}+i^{12}}{i^{14}+i^{16}+i^{18}+i^{20}+i^{22}}$?

Answer: $i^4 = 1$, $i^6 = i^4 \times i^2 = -1$, $i^8 = 1$, $i^{10} = -1$, and so on.

Hence, $\frac{i^4+i^6+i^8+i^{10}+i^{12}}{i^{14}+i^{16}+i^{18}+i^{20}+i^{22}} = \frac{1-1+1-1+1}{-1+1-1+1-1} = -1$

Complex Numbers

A Complex Numbers is a combination of a real number and an imaginary number in the form a + bi. a is called the real part and b is called the imaginary part.

Examples include 3 + 6i, 8 + (−5)i, (often written as 8 - 5i).

Note: a number in the form $\frac{1}{a+ib}$ is written in the form of a complex number by multiplying both numerator and denominator by the conjugate of a + ib, i.e. a − ib.
Hence, $\frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{ib}{a^2+b^2}$, which is in the form p + iq.

The value of $(\frac{1+i}{1-i})^7$ is

Answer: $\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2} = 2i/2 = i$

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Hence, $(\frac{1+i}{1-i})^7 = (i)^7 = -i$

Prime Numbers

All the numbers that have only two divisors, 1 and the number itself, are called prime numbers. Hence, a prime number can only be written as the product of 1 and itself. The numbers 2, 3, 5, 7, 11…37, etc. are prime numbers.

Note: 1 is not a prime number.

EXAMPLE

If $x^2 - y^2 = 101$, find the value of $x^2 + y^2$, given that x and y are natural numbers.

Answer: $x^2 - y^2 = (x + y)(x - y) = 101$. But 101 is a prime number and cannot be written as product of two numbers unless one of the numbers is 1 and the other is 101 itself. Hence, $x + y = 101$ and $x - y = 1$. → $x = 51$, $y = 50$.
→ $x^2 + y^2 = 51^2 + 50^2 = 5101$.

What numbers have exactly three divisors?

Answer: The squares of prime numbers have exactly three divisors, i.e. 1, the prime number, and the square itself.

For how many prime numbers p, is $p^2 + 3p - 1$ a prime number?

Answer: When p = 3, the expression gives a prime number (17). When p is not equal to 3, p^2 will be of the form 3k + 1 as every square number is of the form 3n or 3n + 1. Therefore, $p^2 + 3p - 1 = 3k + 1 + 3p - 1 \Rightarrow$ a multiple of 3. Therefore, for only p = 3, do we get a prime number from the expression.

The number of positive integers n in the range $12 \le n \le 40$ such that the product (n – 1) (n – 2) (n – 3)..3·2·1 is not divisible by n is **(CAT 2003)**

Answer: The product (n – 1) (n – 2) (n – 3)..3·2·1 will not be divisible by n only when this product does not contain factors of n, i.e. n is a prime number. The prime numbers in the range given are 13, 17, 19, 23, 29, 31, and 37. 7 numbers in all.

To find whether a number N is prime or not

Find the root R (approximate) of the number N, i.e. $R = \sqrt{N}$. Divide N by every prime number less than or equal to R. If N is divisible by at least one of those prime numbers it is not a prime number. If N is not divisible by any of those prime numbers, it is a prime number.

Odd and Even Numbers

All the numbers divisible by 2 are called even numbers whereas all the numbers not divisible by 2 are called odd numbers. 2, 4, 6, 8… etc. are even numbers and 1, 3, 5, 7.. etc. are odd numbers.

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Remember!	$(Even)^{Odd}$ = Even
Odd + Odd = Even	Even × Odd = Even
Even + Even = Even	Even × Even = Even
Odd + Even = Odd	Odd × Odd = Odd
$(Odd)^{Even}$ = Odd	$(Odd)^{Even} \times (Even)^{Odd}$ = Even
	$(Odd)^{Even} + (Even)^{Odd}$ = Odd

REMAINDERS

Suppose the numbers N_1, N_2, N_3… give quotients Q_1, Q_2, Q_3… and remainders R_1, R_2, R_3..., respectively, when divided by a common divisor D.

Therefore $N_1 = D \times Q_1 + R_1$,
$N_2 = D \times Q_2 + R_2$,
$N_3 = D \times Q_3 + R_3$.. and so on.

Let P be the product of N_1, N_2, N_3…

Therefore, $P = N_1N_2N_3.. = (D \times Q_1 + R_1)(D \times Q_2 + R_2)(D \times Q_3 + R_3)..$
$= D \times K + R_1R_2R_3...$ where K is some number ---- (1)

➔ In the above equation, since only the product $R_1R_2R_3$… is free of D, therefore the remainder when P is divided by D is the remainder when the product $R_1R_2R_3$… is divided by D.

Let S be the sum of N_1, N_2, N_3…
Therefore, $S = (N_1) + (N_2) + (N_3) +... = (D \times Q_1 + R_1) + (D \times Q_2 + R_2) + (D \times Q_3 + R_3)..$
$= D \times K + R_1 + R_2 + R_3$… where K is some number--- (2)

➔ Hence the remainder when S is divided by D is the remainder when $R_1 + R_2 + R_3$ is divided by D.

What is the remainder when the product 1998 × 1999 × 2000 is divided by 7?

Answer: the remainders when 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively. Hence the final remainder is the remainder when the product 3 × 4 × 5 = 60 is divided by 7. Therefore, remainder = 4

What is the remainder when 2^{2004} is divided by 7?

Answer: 2^{2004} is again a product (2 × 2 × 2... (2004 times)). Since 2 is a number less than 7 we try to convert the product into product of numbers higher than 7. Notice that 8 = 2 × 2 × 2. Therefore we convert the product in the following manner- $2^{2004} = 8^{668} = 8 \times 8 \times 8...$ (668 times).
The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{668} is divided by 7 is the remainder obtained when the product 1 × 1 × 1... is divided by 7. Therefore, remainder = 1

What is the remainder when 2^{2006} is divided by 7?

Answer: This problem is like the previous one, except that 2006 is not an exact multiple of 3 so we cannot convert it completely into the form 8^x. We will write it in following manner- $2^{2006} = 8^{668} \times 4$.
Now, 8^{668} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2006} is divided by 7 is the remainder when the product 1 × 4 is divided by 7. Therefore, remainder = 4

What is the remainder when 25^{25} is divided by 9?

Answer: Again $25^{25} = (18 + 7)^{25} = (18 + 7)(18 + 7)...25 \text{ times} = 18K + 7^{25}$
Hence remainder when 25^{25} is divided by 9 is the remainder when 7^{25} is divided by 9.
Now $7^{25} = 7^3 \times 7^3 \times 7^3$.. (8 times) $\times$ 7 = 343 $\times$ 343 $\times$ 343... (8 times) $\times$ 7.

The remainder when 343 is divided by 9 is 1 and the remainder when 7 is divided by 9 is 7.
Hence the remainder when 7^{25} is divided by 9 is the remainder we obtain when the product $1 \times 1 \times 1$... (8 times) $\times$ 7 is divided by 9. The remainder is 7 in this case. Hence the remainder when 25^{25} is divided by 9 is 7.

What is the remainder when $32^{32^{32}}$ is divided by 7?

Let me put up the steps for finding remainder when X^{Y^Z} is divided by D.

1. Divide X by D. Let the remainder be R. Therefore, you have to find the remainder when R^{Y^Z} is divided by D. 32 gives a remainder 4 when divided by 7. Therefore, you are trying to find the remainder when $4^{32^{32}}$ is divided by 7.
2. Find a power of R that gives a remainder of + 1 or – 1 with D. If you find a power that gives a remainder – 1, twice of that power will give a remainder of + 1. Now I know that $4^3 = 64$ gives a remainder 1 when divided by 7.
3. Find the remainder when Y^Z is divided by that power. Here, find the remainder when 32^{32} is divided by 3. The remainder is 1. Therefore, 32^{32} can be written as 3k + 1 and $4^{32^{32}}$ can be written as 4^{3k+1} or $(4^3)^k \times 4$.
4. Now 4^3 gives a remainder 1 when divided by 7. Therefore, we need to find the remainder when 4 is divided by 7. Therefore, the remainder is 4.

SOME SPECIAL CASES:

When both the dividend and the divisor have a factor in common.

Let N be a number and Q and R be the quotient and the remainder when N is divided by the divisor D.
Hence, $N = Q \times D + R$.
Let $N = k \times A$ and $D = k \times B$ where k is the HCF of N and D and $k > 1$. Hence $kA = Q \times kB + R$.
Let Q_1 and R_1 be the quotient and the remainder when A is divided by B. Hence $A = B \times Q_1 + R_1$.
Putting the value of A in the previous equation and comparing we get-

$k(B \times Q_1 + R_1) = Q \times kB + R \rightarrow R = kR_1$.

Hence to find the remainder when both the dividend and the divisor have a factor in common,

- Take out the common factor (i.e. divide the numbers by the common factor)
- Divide the resulting dividend (A) by resulting divisor (B) and find the remainder (R_1).
- The real remainder R is this remainder R1 multiplied by the common factor (k).

What the remainder when 2^{96} is divided by 96?

The common factor between 2^{96} and 96 is $32 = 2^5$.
Removing 32 from the dividend and the divisor we get the numbers 2^{91} and 3 respectively.
The remainder when 2^{91} is divided by 3 is 2.
Hence the real remainder will be 2 multiplied by common factor 32.
Remainder = 64

The concept of negative remainder

$15 = 16 \times 0 + 15$ or $15 = 16 \times 1 - 1$.

The remainder when 15 is divided by 16 is 15 the first case and −1 in the second case. Hence, the remainder when 15 is divided by 16 is 15 or −1.

➔ When a number N < D gives a remainder R (= N) when divided by D, it gives a negative remainder of R − D.

For example, when a number gives a remainder of −2 with 23, it means that the number gives a remainder of 23 − 2 = 21 with 23.

Find the remainder when 7^{52} is divided by 2402.

Answer: $7^{52} = (7^4)^{13} = (2401)^{13} = (2402 - 1)^{13} = 2402K + (-1)^{13} = 2402K - 1$.

Hence, the remainder when 7^{52} is divided by 2402 is equal to −1 or 2402 − 1 = 2401.
Remainder = 2401.

When dividend is of the form $a^n + b^n$ or $a^n - b^n$

Theorem1: $a^n + b^n$ is divisible by a + b when n is **ODD**.
Theorem 2: $a^n - b^n$ is divisible by a + b when n is **EVEN**.
Theorem 3: $a^n - b^n$ is **ALWAYS** divisible by a − b.

What is the remainder when $3^{444} + 4^{333}$ is divided by 5?

Answer: The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.
$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$. Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$. Since the number is divisible by 145 it will certainly be divisible by 5. Hence, the remainder is 0.

What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?

Answer: The remainders when 5555 and 2222 are divided by 7 are 4 and 3 respectively. Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$. Now $(16)^{1111} + (243)^{1111}$ is divisible by 16 + 243 or it is divisible by 259, which is a multiple of 7. Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

$20^{2004} + 16^{2004} - 3^{2004} - 1$ is divisible by:
(a) 317 (b) 323 (c) 253 (d) 91

Answer: $20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 3^{2004}) + (16^{2004} - 1^{2004})$. Now $20^{2004} - 3^{2004}$ is divisible by 17 (Theorem 3) and $16^{2004} - 1^{2004}$ is divisible by 17 (Theorem 2). Hence the complete expression is divisible by 17.

$20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 1^{2004}) + (16^{2004} - 3^{2004})$. Now $20^{2004} - 1^{2004}$ is divisible by 19 (Theorem 3) and $16^{2004} - 3^{2004}$ is divisible by 19 (Theorem 2). Hence the complete expression is also divisible by 19.

Hence the complete expression is divisible by $17 \times 19 = 323$.

When $f(x) = a + bx + cx^2 + dx^3 + \ldots$ is divided by $x - a$

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The remainder when $f(x) = a + bx + cx^2 + dx^3 + ..$ is divided by $x - a$ is $f(a)$. Therefore, If $f(a) = 0$, $(x - a)$ is a factor of $f(x)$.

What is the remainder when $x^3 + 2x^2 + 5x + 3$ is divided by $x + 1$?

Answer: The remainder when the expression is divided by $(x - (-1))$ will be $f(-1)$. Remainder $= (-1)^3 + 2(-1)^2 + 5(-1) + 3 = -1$

If $2x^3 - 3x^2 + 4x + c$ is divisible by $x - 1$, find the value of c.

Answer: Since the expression is divisible by $x - 1$, the remainder $f(1)$ should be equal to zero $\Rightarrow 2 - 3 + 4 + c = 0$, or $c = -3$.

Euler's Theorem

If M and N are two numbers coprime to each other, i.e. $HCF(M,N) = 1$ and $N = a^p b^q c^r ...$, $\text{Remainder}[\frac{M^{\phi(N)}}{N}] = 1$, where $\phi(N) = N(1 - \frac{1}{a})(1 - \frac{1}{b})(1 - \frac{1}{c})...$ and is known as Euler's Totient function.. $\phi(N)$ is also the number of numbers less than and prime to N.

Find the remainder when 5^{37} is divided by 63.

Answer: 5 and 63 are coprime to each other, therefore we can apply Euler's theorem here.

$63 = 32 \times 7 \Rightarrow \phi(63) = 63(1 - \frac{1}{3})(1 - \frac{1}{7}) = 36$

Therefore, $\text{Remainder}[\frac{5^{37}}{63}] = \text{Remainder}[\frac{5^{36} \times 5}{63}] = 5$

Find the last three digits of 57^{802}.

Answer: Many a times (not always), the quicker way to calculate the last three digits is to calculate the remainder by 1 000. We can see that 57 and 1 000 are coprime to each other. Therefore, we can use Euler's theorem here if it's useful.

$1\,000 = 2^3 \times 5^3 \Rightarrow \phi(1000) = 1000(1 - \frac{1}{2})(1 - \frac{1}{4}) = 400$

Therefore,

$$\text{Remainder}[\frac{57^{400}}{1000}] = 1 \Rightarrow \text{Remainder}[\frac{57^{400} \times 57^{400}}{1000}] = \text{Remainder}[\frac{57^{800}}{1000}] = 1$$

$$\Rightarrow \text{Remainder}[\frac{57^{802}}{1000}] = \text{Remainder}[\frac{57^{800} \times 57^2}{1000}] = 249$$

Hence, the last two digits of 57^{802} are 249.

Fermat's Theorem

If p is a prime number and N is prime to p, then $N^p - N$ is divisible by p.

What is the remainder when $n^7 - n$ is divided by 42?

Answer: Since 7 is prime, $n^7 - n$ is divisible by 7. $n^7 - n = n(n^6 - 1) = n(n + 1)(n - 1)(n^4 + n^2 + 1)$. Now $(n - 1)(n)(n + 1)$ is divisible by 3! = 6. Hence $n^7 - n$ is divisible by 6 x 7 = 42. Hence the remainder is 0.

Fermat's Little Theorem

If N in the above Euler's theorem is a prime number, then $\phi(N) = N(1 - \frac{1}{N}) = N - 1$. Therefore, if M and N are coprime to each other and N is a prime number, $\text{Remainder}[\frac{M^{N-1}}{N}] = 1$

Find the remainder when 52^{60} is divided by 31.

Answer: 31 is a prime number therefore $\phi(N) = 30$. 52 and 31 are prime to each other. Therefore, by Fermat's theorem:

$$\text{Remainder}[\frac{52^{30}}{31}] = 1 \Rightarrow \text{Remainder}[\frac{52^{60}}{31}] = 1$$

Wilson's Theorem

If P is a prime number then $\text{Remainder}[\frac{(P-1)!+1}{P}] = 0$. In other words, (P – 1)! + 1 is divisible by P if P is a prime number. It also means that the remainder when (P – 1)! Is divided by P is P – 1 when P is prime.

Find the remainder when 40! is divided by 41.

Answer: By Wilson's theorem, we can see that 40! + 1 is divisible by 41 $\Rightarrow \text{Remainder}[\frac{40!}{41}] = 41 - 1 = 40$

Find the remainder when 39! is divided by 41.

Answer: In the above example, we saw that the remainder when 40! is divided by 41 is 40.
$\Rightarrow 40! = 41k + 40 \Rightarrow 40 \times 39! = 41k + 40$. The R.H.S. gives remainder 40 with 41 therefore L.H.S. should also give remainder 40 with 41. L.H.S. = $40 \times 39!$ where 40 gives remainder 40 with 41. Therefore, 39! should give remainder 1 with 41.

Chinese Remainder Theorem

This is a very useful result. It might take a little time to understand and master Chinese remainder theorem completely but once understood, it is an asset.

If a number N = a × b, where a and b are prime to each other, i.e., hcf(a, b) = 1, and M is a number such that $\text{Remainder}[\frac{M}{a}] = r_1$ and $\text{Remainder}[\frac{M}{b}] = r_2$ then $\text{Remainder}[\frac{M}{N}] = ar_2x + br_1y$, where $ax + by = 1$

Confused?

Following example will make it clear.

Find the remainder when 3^{101} is divided by 77.

Answer: $77 = 11 \times 7$.

By Fermat's little theorem, $\text{Remainder}[\frac{3^6}{7}] = 1$ AND $\text{Remainder}[\frac{3^{10}}{11}] = 1$

$$\text{Remainder}[\frac{3^{101}}{7}] = \text{Remainder}[\frac{3^{96} \times 3^5}{7}] = \text{Remainder}[\frac{(3^6)^{16} \times 3^5}{7}] = \text{Remainder}[\frac{1 \times 3^5}{7}] = 5 = r_1$$

$$\text{Remainder}[\frac{3^{101}}{11}] = \text{Remainder}[\frac{3^{100} \times 3}{11}] = \text{Remainder}[\frac{(3^{10})^{10} \times 3}{11}] = \text{Remainder}[\frac{1 \times 3}{11}] = 3 = r_2$$

Now we will find x and y such that $7x + 11y = 1$. By observation we can find out, $x = -3$ and $y = 2$.

Now we can say that $\text{Remainder}[\frac{3^{101}}{77}] = 7 \times 3 \times -3 + 11 \times 5 \times 2 = 47$

We can also solve this problem by Euler's theorem and this is the method I follow most of the time. No confusion remains thereby.

Find the remainder when 3^{101} is divided by 77.

Answer: $\phi(77) = 77(1 - \frac{1}{7})(1 - \frac{1}{11}) = 60$

$$\text{Remainder}[\frac{3^{60}}{77}] = 1 \Rightarrow \text{Remainder}[\frac{3^{101}}{77}] = \text{Remainder}[\frac{3^{60} \times 3^{41}}{77}] = \text{Remainder}[\frac{1 \times 3^{41}}{77}] = \text{Remainder}[\frac{3^{41}}{77}]$$

$$\text{Remainder}[\frac{3^4}{77}] = \text{Remainder}[\frac{81}{77}] = 4$$

$$\Rightarrow \text{Remainder}[\frac{3^{41}}{77}] = \text{Remainder}[\frac{(3^4)^{10} \times 3}{77}]\text{Remainder}[\frac{4^{10} \times 3}{77}] = \text{Remainder}[\frac{4^4 \times 4^4 \times 4^2 \times 3}{77}] = \text{Remainder}[\frac{256 \times 256 \times 48}{77}]$$

$$= \text{Remainder}[\frac{25 \times 25 \times 48}{77}] = \text{Remainder}[\frac{9 \times 48}{77}] = 47$$

Find the smallest number that when divided by 7, 8 and 9 leave a remainder of 5, 4 and 3.

Answer: Let N be the number. Therefore, $N = 7a + 5$, $N = 8b + 4$, $N = 9c + 3$.

Find the remainder when $32^{32^{32}}$ is divided by 9.

Answer: Notice that 32 and 9 are coprime. $\phi(9) = 9(1 - \frac{1}{3}) = 6$

Hence by Euler's theorem, $\text{Remainder}[\frac{32^6}{9}] = 1$. Since the power is 32^{32}, we will have to simplify this power in terms of $6k + r$. Therefore, we need to find the remainder when 32^{32} is divided by 6.

$$\text{Remainder}[\frac{32^{32}}{6}] = \text{Remainder}[\frac{2^{32}}{6}] = \text{Remainder}[\frac{(2^8)^4}{6}] = \text{Remainder}[\frac{256 \times 256 \times 256 \times 256}{6}] = \text{Remainder}[\frac{256}{6}] = 4$$

Therefore, $32^{32^{32}} = 32^{6k+4} = (32^6)^k \times 32^4$

$\Rightarrow \text{Remainder}[\frac{(32^6)^k \times 32^4}{9}] = \text{Remainder}[\frac{32^4}{9}] = \text{Remainder}[\frac{5\times5\times5\times5}{9}] = \text{Remainder}[\frac{625}{9}] = 4$

What will be the remainder when $N = 10^{10} + 10^{100} + 10^{1000} + \ldots\ldots\ldots + 10^{10000000000}$ is divided by 7?

Answer: By Fermat's Little Theorem 10^6 will give remainder as 1 with 7.

$\text{Remainder}[\frac{10^{10}}{7}] = \text{Remainder}[\frac{10^6 \times 10^4}{7}] = \text{Remainder}[\frac{10^4}{7}] = \text{Remainder}[\frac{3^4}{7}] = 4$

Similarly, all the other terms give remainder of 4 with 7. Therefore, total remainder = 4 + 4 + 4… (10 times) = 40.

Remainder of 40 with 7 = 5

What is the remainder when $N = 2222^{5555} + 5555^{2222}$ is divided by 7?

2222^6 will give remainder 1 when divided by 7.

$5555 = 6K+5 \Rightarrow 2222^{5555} = 2222^{6k+5} \Rightarrow \text{Remainder}[\frac{2222^{5555}}{7}] = \text{Remainder}[\frac{2222^5}{7}] = \text{Remainder}[\frac{3^5}{7}] = 5$

Also 55556 will give remainder 1 when divided by 7.

$5555^{2222} = 5555^{6k+2} \Rightarrow \text{Remainder}[\frac{5555^{2222}}{7}] = \text{Remainder}[\frac{5555^2}{7}] = \text{Remainder}[\frac{4^2}{7}] = 2$

So final remainder is (5 + 2) divided by 7 = 0

Find the remainder when 8^{643} is divided by 132.

Answer: Note that here 8 and 132 are not coprime as HCF (8, 132) = 4 and not 1. Therefore, we cannot apply Euler's theorem directly.

$\text{Remainder}[\frac{8^{643}}{132}] = \text{Remainder}[\frac{2^{1929}}{132}] = 4 \times \text{Remainder}[\frac{2^{1927}}{33}]$. Now we can apply Euler's theorem.

$\phi(33) = 33(1-\frac{1}{3})(1-\frac{1}{11}) = 20 \Rightarrow \text{Remainder}[\frac{2^{20}}{33}] = 1 \Rightarrow \text{Remainder}[\frac{2^{1927}}{33}] = \text{Remainder}[\frac{2^7}{33}] = 29$

$\Rightarrow \text{Real remainder} = 4 \times 29 = 116$

To find the number of numbers that are less than or equal to a certain natural number n, and that are divisible by a certain integer

To find the number of numbers, less than or equal to n, and that are divisible by a certain integer p, we divide n by p. The quotient of the division gives us the number of numbers divisible by p and less than or equal to n.

How many numbers less than 400 are divisible by 12?

Answer: Dividing 400 by 12, we get the quotient as 33. Hence the number of numbers that are below 400 and divisible by 12 is 33.

How many numbers between 1 and 400, both included, are not divisible either by 3 or 5?

Answer: We first find the numbers that are divisible by 3 or 5. Dividing 400 by 3 and 5, we get the quotients as 133 and 80 respectively. Among these numbers divisible by 3 and 5, there are also numbers which are divisible both by 3 and 5 i.e. divisible by 3 x 5 = 15. We have counted these numbers twice. Dividing 400 by 15, we get the quotient as 26.

Hence the number divisible by 3 or 5 = 133 + 80 – 26 = 187
Hence, the numbers not divisible by 3 or 5 are = 400 – 187 = 213.

How many numbers between 1 and 1200, both included, are not divisible by any of the numbers 2, 3 and 5?

Answer: as in the previous example, we first find the number of numbers divisible by 2, 3, or 5. from set theory we have

$n(AUBUC) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$n(2U3U5) = n(2) + n(3) + n(5) - n(6) - n(15) - n(10) + n(30)$
$\rightarrow n(2U3U5) = 600 + 400 + 240 - 200 - 80 - 120 + 40 = 880$
Hence number of numbers not divisible by any of the numbers 2, 3, and 5 = 1200 – 880 = 320.

Some Special Problems:

Find the remainder when 123456789101112……..40 is divided by 36.

Answer: $36 = 9 \times 4$. Therefore, we first find the remainders when this number is divided by 9 and 4.
The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits = $4 \times (1 + 2 + 3 + 4 + \ldots + 9) + 10 \times (1 + 2 + 3) + 4 = 180 + 60 + 4 = 244 \Rightarrow$ remainder by 9 = 1.
The remainder by 4 would be the remainder when the last two digits are divided by 4 $\Rightarrow$ remainder by 4 = 0.

Therefore, to find the remainder we need to find the smallest multiple of 4 that gives remainder 1 with 9. The smallest such number = 28. Therefore, remainder = 28.

Find the remainder when 112123123412345…12345678 is divided by 36.

Answer: $36 = 9 \times 4$. Therefore, we first find the remainders when this number is divided by 9 and 4.
The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits = $1 \times 8 + 2 \times 7 + 3 \times 6 + \ldots + 8 \times 1 = 120 \Rightarrow$ remainder by 9 = 3.
The remainder by 4 would be the remainder when the last two digits are divided by 4 $\Rightarrow$ remainder by 4 = 2.

The overall remainder would be the smallest number that gives remainder 3 with 9 and remainder 2 with 4. Therefore, the number would satisfy the equation $9a + 3 = 4b + 2 \Rightarrow 4b - 9a = 1 \Rightarrow (a, b) = (3, 7)$ and the number = 30. Therefore, remainder = 30.

Let $n!$ = $1 \times 2 \times 3 \times \ldots \times n$ for integer $n \geq 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p+2$ when divided by 11! leaves a remainder of **(CAT 2005)**

1. 10 2. 0 3. 7 4. 1

Answer: Nth term of the series = $n \times n! = (n + 1 - 1) \times n! = (n + 1)! - n!$
Therefore, $p = 2! - 1! + 3! - 2! + 4! - 3! + \ldots + 11! - 10! = 11! - 1! \Rightarrow p + 2 = 11! + 1 \Rightarrow$ remainder by 11! = 1

Find the remainder when $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + 98 \times 99 + 99 \times 100$ is divided by 101.

Answer: Nth term of the series = $n \times (n + 1) = n^2 + n$.

Therefore, sum of the series $\sum(n^2 + n) = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6} = \frac{99 \times 100 \times 101}{6} \Rightarrow$ remainder by 101 = 0.

A number when divided by 8 leaves remainder 3 and quotient Q. The number when divided by 5 leaves remainder 2 and quotient Q + 8. What is the number?

Answer: Let the number be $N \Rightarrow N = 8Q + 3$ and $N = 5(Q + 8) + 2 = 5Q + 42$
$8Q + 3 = 5Q + 42 \Rightarrow Q = 13 \Rightarrow N = 107$

Find the largest natural number that divides 364, 414, and 539 and leaves the same remainder in each case.

Answer: Let the divisor be D and the remainder be R. Therefore, $364 = Da + R$, $414 = Db + R$, $539 = Dc + R$
Subtracting first equation from the second and the second equation from the third we get

$50 = D(b - a)$ and $125 = D(c - b)$. As D is the common factor in RHS of both the equation, it should be the common factor on the LHS of both the equation. The HCF of 50 and 125 is 25. Therefore, the highest number can be 25.

What is the remainder when $\underbrace{11111\ldots11111}_{243 \text{ times}}$ is divided by 243?

Answer: It can be proved that a number formed by writing any single digit 3^n times will be divisible by 3^n. This is left to students to check it out.

How many numbers between 1 and 1000 are there such that $n^2 + 3n + 5$ is divisible by 121?

Answer: 0 values. $n^2 + 3n + 5 = (n - 4)(n + 7) + 33$. Now, 33 is divisible by 11 but not 121. n + 7 and n - 4 are two numbers with a difference of 11, therefore either both are divisible by 11 or both are not divisible by 11. If both are divisible by 11, their product is divisible by 121 but 33 is divisible only by 11 therefore the expression is not divisible by 121. If both are not divisible by 11, the expression is again not divisible by 121.

Find the remainder when $1^{39} + 2^{39} + 3^{39} + 4^{39} + \ldots + 12^{39}$ is divided by 39.

Answer: $1^p + 2^p + 3^p + \ldots n^p$ is divisible by $1 + 2 + 3 + \ldots + n$ if p is odd. Therefore, remainder = 0 as $1 + 2 + 3 + \ldots + 12 = 78$ which is a factor of 13.

DIVISORS OF A NUMBER

Divisors:

Total Gadha

For a natural number N, all the numbers, including 1 and N itself, which divide N completely are called divisors of N.

Example: The number 24 is divisible by 1, 2, 3, 4, 6, 8, 12, and 24. Hence all these numbers are divisors of 24.

How to find the number of divisors of a number:

Let us the find the number of divisors of 60.

$60 = 2^2 \times 3 \times 5$.
Any divisors of 60 will have powers of 2 equal to either 2^0 or 2^1 or 2^2.
Similarly, any divisor of 60 will have powers of 3 equal to either 3^0 or 3^1, and powers of 5 equal to either 5^0 or 5^1.

To make a divisor of 60, we will have to choose a power of 2, a power of 3 and a power of 5. A power of 2 can be chosen in 3 ways out of 2^0 or 2^1, or 2^2. Similarly, a power of 3 can be chosen in 2 ways and a power of 5 can be chosen in 2 ways.

Therefore, the number of divisors = $3 \times 2 \times 2 = 12$.

Notice that we have added 1 each to powers of 2, 3 and 5 and multiplied.

Now for the formula:

Let N be a composite number such that $N = (x)^a(y)^b(z)^c..$ where x, y, z.. are prime factors. Then, the number of divisors of $N = (a + 1)(b + 1)(c + 1)..$

Find the number of divisors of 21600.
Answer: $21600 = 2^5 \times 3^3 \times 5^2 \Rightarrow$ Number of divisors = $(5 + 1) \times (3 + 1) \times (2 + 1) = 6 \times 4 \times 3 = 72$.

How many divisors of 21600 are odd numbers?
Answer: An odd number does not have a factor of 2 in it. Therefore, we will consider all the divisors having powers of 3 and 5 but not 2. Therefore, ignoring the powers of 2, the number of odd divisors = $(3 + 1) \times (2 + 1) = 4 \times 3 = 12$.

How many divisors of 21600 are even numbers?
Answer: Total number of divisors of 21600 = 72.
Number of odd divisors of 21600 = 12.
$\Rightarrow$ Number of even divisors of 21600 = 72 – 12 = 60.

How many divisors of 360 are not divisors of 540 and how many divisors of 540 are not divisors of 360?
Answer: The best option here is to find the number of common divisors of 360 and 540. For that we find the highest common powers of all the common prime factors in 360 and 540.
Now, $360 = 2^3 \times 3^2 \times 5$ and $540 = 2^2 \times 3^3 \times 5$.

The number of common factors would be made by $2^2 \times 3^2 \times 5$. The number of factors made by this = $3 \times 3 \times 2 = 18$. Therefore, the two numbers will have 18 factors in common.

Number of factors of $360 = 4 \times 3 \times 2 = 24 \Rightarrow$ Number of factors of 360 which are not factors of 540 = 24 – 18 = 6.
Number of factors of $540 = 3 \times 4 \times 2 = 24 \Rightarrow$ Number of factors of 540 which are not factors of 360 = 24 – 18 = 6.

How many divisors of the number $2^7 \times 3^5 \times 5^4$ have unit digit equal to 5?
Answer: For unit digit equal to 5, the number has to be a multiple of 5 and it should not be a multiple of 2 otherwise the unit digit will be 0. To be a multiple of 5, the powers of 5 that it can have is 5^1, 5^2, 5^3 or 5^4. The powers of 3 can be 3^0, 3^1, 3^2, 3^3, 3^4 or 3^5.

Therefore, the number of divisors which have a unit digit of 5 = $4 \times 6 = 24$.

How many divisors of 36^{36} are perfect cubes?

Answer: $36^{36} = 2^{72}3^{72}$. To find the divisors which are perfect cubes, we need to take those powers of prime factors which are multiples of 3. Therefore, powers of 2 will be 2^0, 2^3, 2^6, 2^9, … 2^{72} and similarly, powers of 3 will be 3^0, 3^3, 3^6, 3^9, … 3^{72}. Both are 25 in number. Therefore, number of divisors = $25 \times 25 = 625$.

Reverse Operations on Divisors:

Find all the numbers less than 100 which have exactly 8 divisors.

Answer: To find the number of divisors of a number, we used to add 1 to powers of all the prime factors and then multiply them together. Now, given the number of divisors, we will express this number as a product and then subtract 1 from every multiplicand to obtain the powers.

$8 = 2 \times 2 \times 2 = (1 + 1) \times (1 + 1) \times (1 + 1)$. Therefore, the number is of the form $a^1b^1c^1$, where a, b and c are prime. The numbers can be $2 \times 3 \times 5 =$ **30**, $2 \times 3 \times 7 =$ **42**, $2 \times 3 \times 11 =$ **66**, $2 \times 3 \times 13 =$ **78**, $2 \times 5 \times 7 =$ **70**.

$8 = 4 \times 2 = (3 + 1) \times (1 + 1)$. Therefore, the number is of the form a^3b, where a and b are prime. The numbers can be $2^3 \times 3 =$ **24**, $2^3 \times 5 =$ **40**, $2^3 \times 7 =$ **56**, $2^3 \times 11 =$ **88**, $3^3 \times 2 =$ **54**.

The number can also be of the form a^7, but there is no such number less than 100.

Find the smallest number with 15 divisors.

Answer: $15 = 3 \times 5 = (2 + 1)(4 + 1) \Rightarrow$ The number is of the form a^2b^4, where a and b are prime. To find the smallest such number, we give the highest power to smallest rime factor, i.e. 2, and the next highest power to next smallest prime number, i.e. 3, and so on. Therefore, the smallest number = $2^4 \times 3^2 = 144$.

Let N be a composite number such that $N = (2)^a(y)^b(z)^c..$ where y, z.. are prime factors. Then, the number of even divisors of N = (a)(b + 1)(c + 1) and number of even divisors of N = (b + 1)(c + 1)

How many divisors of 21600 are perfect squares?
Answer: In a perfect square, all the prime factors have even powers. For example, $2^5 \times 6^8$ will not be a perfect square as the power of 2 is odd whereas $2^4 \times 6^8$ will be a perfect square because all the prime factors have even powers. $21600 = 2^5 \times 3^3 \times 5^2$ therefore, all the divisors made by even powers of 2, 3 and 5 will be perfect squares.
The even powers of 2 are 2^0, 2^2, 2^4, even powers of 3 are 3^0 and 3^2, and even powers of 5 are 5^0 and 5^2. We can select an even power of 2 in 3 ways, even power of 3 in 2 ways, and even power of 5 in 2 ways. Therefore, the number of combinations = $3 \times 2 \times 2 = 12$.

Let N be a composite number such that $N = (x)^a(y)^b(z)^c..$ where x, y, z.. are prime factors. Then, the sum of divisors of N = $\frac{x^{a+1}-1}{x-1} \times \frac{y^{b+1}-1}{y-1} \times \frac{z^{c+1}-1}{z-1} \ldots$

Total Gadha

What is the sum of divisors of 60?

Answer: $60 = 2^2 \times 3 \times 5 \Rightarrow$ Sum of the divisors $= \frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} = 168$

Find the sum of even divisors of $2^5 \times 3^5 \times 5^4$

Answer: All the even divisors of the number will have powers of 2 equal to one of 2, 2^2, 2^3, 2^4, or 2^5. Therefore, sum of even divisors = $(2 + 2^2 + 2^3 + 2^4 + 2^5) \times (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5) \times (1 + 5 + 5^2 + 5^3 + 5^4)$

$= \frac{2(2^5-1)}{2-1} \times \frac{3^6-1}{3-1} \times \frac{5^5-1}{5-1} = 17625608$

Let N be a composite number such that $N = (x)^a(y)^b(z)^c..$ where x, y, z.. are prime factors. Then, the product of divisors of $N = (N)^{\frac{(a+1)(b+1)(c+1)}{2}} = \left(x^a y^b z^c\right)^{\frac{(a+1)(b+1)(c+1)}{2}} = \sqrt{x^{a(a+1)} y^{b(b+1)} z^{c(c+1)}}$

What is the product of divisors of 60?

Answer: $60 = 2^2 \times 3 \times 5 \Rightarrow$ product of divisors of $60 = (60)^{\frac{3\times2\times2}{2}} = 60^6 = 2^{12} \times 3^6 \times 5^6$

Let A = set of all divisors of 8100 and B = set of all divisors of 21600. What is the product of the elements of AUB?

Answer: $8100 = 2^2 \times 3^4 \times 5^2$ and $21600 = 2^5 \times 3^3 \times 5^2$. AUB will have all the divisors of 8100 and 21600 with the common divisors written only once. Therefore, these common divisors will be multiplied only once. The common divisors will come from $2^2 \times 3^3 \times 5^2$ and are 36 in number. Their product will be $(2^2 \times 3^3 \times 5^2)^{18} = 2^{36} \times 3^{54} \times 5^{36}$

Required product=

$$\frac{\text{product of divisors of 8100} \times \text{product of divisors of 21600}}{\text{product of common divisors}} = \frac{(2^2 \times 3^4 \times 5^2)^{\frac{45}{2}} \times (2^5 \times 3^3 \times 5^2)^{36}}{2^{36} \times 3^{54} \times 5^{36}} = 2^{189} \times 3^{144} \times 5^{81}$$

Let N be a composite number such that $N = (x)^a(y)^b(z)^c..$ where x, y, z.. are prime factors.

If N is not a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)}{2} = \frac{\text{Number of divisors}}{2}$

If N is a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)+1}{2} = \frac{\text{Number of divisors}+1}{2}$

For example, the divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. Now, $60 = 1 \times 60 = 2 \times 30 = 3 \times 20 = 4 \times 15 = 5 \times 12 = 6 \times 10$. Therefore, **divisors occur in pairs for numbers which are not perfect squares**.

The divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.
$36 = 1 \times 36 = 2 \times 18 = 3 \times 12 = 4 \times 9 = 6 \times 6$. Therefore, **divisors occur in pairs except for the square root for numbers which are perfect squares**.

How many ordered pairs of integers, (x, y) satisfy the equation xy = 110?
Answer: $110 = 2 \times 5 \times 11$. Hence, the number of divisors of 110 is $= 2 \times 2 \times 2 = 8$. Hence, the number of positive ordered pairs of x and y = 8 (as (2, 55) is not same as (55, 2)). Also, since we are asked for integers, the pair consisting of two negative integers will also suffice. Hence the total number of ordered pairs $= 2 \times 8 = 16$.

The number of ways in which a composite number can be resolved into two factors which are prime to each other $= 2^{n-1}$, where n is the number of different prime factors of the number.

For example, let the number $N = 2^{10} \times 3^7 \times 5^6 \times 7^4$. We have to assign these prime factors and their powers to one of the two factors. As the two factors will be prime to each other, we will have to assign a prime factor with its power (for example 210) completely to one of the factors. For every prime factor, we have two ways of assigning it. Therefore, the total number of ways $= 2 \times 2 \times 2 \times 2 = 16$. As we are not looking for ordered pairs, the required number of ways $= \frac{16}{2} = 8$.

Number of numbers less than or prime to a given number:

If N is a natural number such that $N = a^p \times b^q \times c^r$, where a, b, c are different prime factors and p, q, r are positive integers, then the number of positive integers less than and prime to N $= N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$. Therefore, $N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ numbers have **no** factor in common with N.

The above formula is extremely versatile as it lets us find not only the numbers which do not contain any of the prime factors of N but also the numbers which do not contain some selected prime factors of N. The following examples will make it clear:

How many of the first 1200 natural numbers are not divisible by any of 2, 3 and 5?
Answer: 1200 is a multiple of 2, 3 and 5. Therefore, we need to find the number of numbers which are less than and prime to 1200. Number of numbers $= 1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 320$

How many of the first 1200 natural numbers are not divisible by any of 2 and 5?
Answer: Unlike the previous problem, this problem only asks for number not divisible by only 2 factors of 1200, i.e. 2 and 5. Therefore, in the formula we remove the part containing the factor of 3 and calculate the numbers of numbers prime to 1200 with respect to prime factors 2 and 5. The required number $= 1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 480$

How many of the first 1200 natural numbers are either prime to 6 or to 15?
Answer: Number of numbers prime to 6 are $1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 400$ and numbers prime to 15 are $1200\left(1-\frac{1}{5}\right)\left(1-\frac{1}{3}\right) = 640$ Out of these numbers, we will have to subtract numbers which are prime to both 6 and 15 (the question asks for either..or). These are 320 in numbers in all (we have already calculated it).
Therefore, the required number $= 400 + 640 - 320 = 720$

The number of divisors of every natural number from 1 to 1000 is calculated. Which natural number has the highest number of divisors?

Answer: The number less than 1000 which can incorporate highest number of prime factor is = $2 \times 3 \times 5 \times 7 = 210$. Now we are looking for highest multiple of 210 that is less than 1000. The multiple is $210 \times 4 = 840$ which has 32 divisors.

UNIT'S DIGIT

To find the units digit of x^y we only consider the units digits of the number x.
To calculate units digit of 237^{234} we only consider the units digit of 237. Hence, we find the units digit of 7^{234}.
To find the units digit of $a \times b$, we only consider the units digits of the numbers a and b.
To calculate units digit of 233×254, we only consider the units digit of 233 and 254 i.e. 3 and 4, respectively. Hence, we find the units digit of 3 x 4, respectively.

To calculate units digit of x^y where x is a single digit number

To calculate units digit of numbers in the form x^y such 7^{253}, 8^{93}, 3^{74} etc.

Case 1: When y is NOT a multiple of 4
We find the remainder when y is divided by 4. Let $y = 4q + r$ where r is the remainder when y is divided by 4, and
$0 < r < 4$. **The units digit of x^y is the units digit of x^r.**

Case 2: When y is a multiple of 4
We observe the following conditions:
Even numbers 2, 4, 6, 8 when raised to powers which are multiple of 4 give the units digit as 6.
Odd numbers 3, 7, and 9 when raised to powers which are multiple of 4 give the units digit as 1.

Find the units digit of 7^{33}.

Answer: The remainder when 33 is divided by 4 is 1. Hence the units digit of 7^{33} is the unit digit of $7^1 = 7$

Find the units digit of 43^{47}.

Answer: The units digit of 43^{47} can be found by finding the units digit of 3^{47}. 47 gives a remainder of 3 when divided by 4. Hence units digit = units digit of $3^3 = 7$

Find the units digit of $28^{28} - 24^{24}$.

Answer: We have to find the units digit of $8^{28} - 4^{24}$. Since 28 and 24 are both multiples of 4, the units digits of both 8^{28} and 4^{24} will be 6. Hence the units digit of the difference will be 0.

Find the units digit of $43^{43} - 22^{22}$.

Answer: Units digit of 43^{43} is 7 and units digit of 22^{22} is 4. Hence the units digit of the expression will be $7 - 4 = 3$.

Find the units digit of 3^{3^3}

Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when 3^3 is divided by 4. Now, $3^3 = 27$, remainder by 4 = 3.

Therefore, units digit of 3^{3^3} = units digit of $3^3 = 7$.

Find the units digit of $7^{11^{13^{17}}}$.

Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when $11^{13^{17}}$ is divided by 4. Now $11 = 12 - 1 \Rightarrow$ Remainder $[11^{Odd}]$ = Remainder$[(-1)^{Odd}] = -1 = 3$.

Therefore, units digit of $7^{11^{13^{17}}}$ = units digit of $7^3 = 3$.

Find the units digit of $1^3 + 2^3 + 3^3 + \ldots + 98^3 + 99^3$

Answer: Unit digit of 1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, 9^3 are 1, 8, 7, 4, 5, 6, 3, 2, and 9, respectively. The sum of these units digits gives a unit digit of 5. Now these units digit will repeat 10 times each. Therefore, units digit of the sum = $5 \times 10 = 0$.

LAST TWO DIGITS

Before we start, let me mention binomial theorem in brief as we will need it for our calculations.

$$(x + a)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \ldots \text{ where } {}^nC_r = \frac{n!}{r!(n-r)!}$$

Last two digits of numbers ending in 1

Let's start with an example.

What are the last two digits of 31^{786}?

Solution: $31^{786} = (30 + 1)^{786} = {}^{786}C_0 \times 1^{786} + {}^{786}C_1 \times 1^{785} \times (30) + {}^{786}C_2 \times 1^{784} \times 30^2 + \ldots$, Note that all the terms after the second term will end in two or more zeroes. The first two terms are ${}^{786}C_0 \times 1^{786}$ and ${}^{786}C_1 \times 1^{785} \times (30)$. Now, the second term will end with one zero and the tens digit of the second term will be the product of 786 and 3 i.e. 8. Therefore, the last two digits of the second term will be 80. The last digit of the first term is 1. So the last two digits of 31^{786} are 81.

Now, here is the shortcut:

Multiply the tens digit of the number (3 here) with the last digit of the exponent (6 here) to get the tens digit. The units digit is equal to one.

Here are some more examples:

Find the last two digits of 41^{2789}

In no time at all you can calculate the answer to be 61 ($4 \times 9 = 36$. Therefore, 6 will be the tens digit and one will be the units digit)

Find the last two digits of 71^{56747}

Last two digits will be 91 (7×7 gives 9 and 1 as units digit)

Now try to get the answer to this question within 10 s:

Find the last two digits of $51^{456} \times 61^{567}$

The last two digits of 51^{456} will be 01 and the last two digits of 61^{567} will be 21. Therefore, the last two digits of $51^{456} \times 61^{567}$ will be the last two digits of $01 \times 21 = 21$

Total Gadha

Last two digits of numbers ending in 3, 7 or 9

Find the last two digits of 19^{266}.

$19^{266} = (19^2)^{133}$. Now, 19^2 ends in 61 ($19^2 = 361$) therefore, we need to find the last two digits of $(61)^{133}$.
Once the number is ending in 1 we can straight away get the last two digits with the help of the previous method. The last two digits are 81 ($6 \times 3 = 18$, so the tens digit will be 8 and last digit will be 1)

Find the last two digits of 33^{288}.

$33^{288} = (33^4)^{72}$. Now 33^4 ends in 21 ($33^4 = 33^2 \times 33^2 = 1089 \times 1089 = xxxxx21$) therefore, we need to find the last two digits of 21^{72}. By the previous method, the last two digits of $21^{72} = 41$ (tens digit = $2 \times 2 = 4$, unit digit = 1)

So here's the rule for finding the last two digits of numbers ending in 3, 7 and 9:

Convert the number till the number gives 1 as the last digit and then find the last two digits according to the previous method.

Now try the method with a number ending in 7:

Find the last two digits of 87^{474}.

$87^{474} = 87^{472} \times 87^2 = (87^4)^{118} \times 87^2 = (69 \times 69)^{118} \times 69$ (The last two digits of 87^2 are 69) $= 61^{118} \times 69 = 81 \times 69 = 89$

If you understood the method then try your hands on these questions:

Find the last two digits of:

1. 27^{456}
2. 79^{83}
3. 583^{512}

Last two digits of numbers ending in 2, 4, 6 or 8

There is only one even two-digit number which always ends in itself (last two digits) - 76 i.e. 76 raised to any power gives the last two digits as 76. Therefore, our purpose is to get 76 as last two digits for even numbers. We know that 24^2 ends in 76 and 2^{10} ends in 24. Also, 24 raised to an even power always ends with 76 and 24 raised to an odd power always ends with 24. Therefore, 24^{34} will end in 76 and 24^{53} will end in 24.

Let's apply this funda:

Find the last two digits of 2^{543}.

$2^{543} = (2^{10})^{54} \times 2^3 = (24)^{54}$ (24 raised to an even power) $\times 2^3 = 76 \times 8 = 08$

Total Gadha

(NOTE: Here if you need to multiply 76 with 2^n, then you can straightaway write the last two digits of 2^n because when 76 is multiplied with 2^n the last two digits remain the same as the last two digits of 2^n. Therefore, the last two digits of 76×2^7 will be the last two digits of $2^7 = 28$. Note that this funda does works for power of $2 \geq 2$)

Same method we can use for any number which is of the form 2^n. Here is an example:

Find the last two digits of 64^{236}.
$64^{236} = (2^6)^{236} = 2^{1416} = (2^{10})^{141} \times 2^6 = 24^{141}$ (24 raised to odd power) $\times$ 64 = 24 $\times$ 64 = 36

Now those numbers which are not in the form of 2n can be broken down into the form 2n $\times$ odd number. We can find the last two digits of both the parts separately.

Here are some examples:

Find the last two digits of 62^{586}.
$62^{586} = (2 \times 31)^{586} = 2^{586} \times 3^{586} = (2^{10})^{58} \times 2^6 \times 31^{586} = 76 \times 64 \times 81 = 84$

Find the last two digits of 54^{380}.
$54^{380} = (2 \times 3^3)^{380} = 2^{380} \times 3^{1140} = (2^{10})^{38} \times (3^4)^{285} = 76 \times 81^{285} = 76 \times 01 = 76.$

Find the last two digits of 56^{283}.
$56^{283} = (2^3 \times 7)^{283} = 2^{849} \times 7^{283} = (2^{10})^{84} \times 2^9 \times (7^4)^{70} \times 7^3 = 76 \times 12 \times (01)^{70} \times 43 = 16$

Find the last two digits of 78^{379}.
$78^{379} = (2 \times 39)^{379} = 2^{379} \times 39^{379} = (2^{10})^{37} \times 2^9 \times (39^2)^{189} \times 39 = 24 \times 12 \times 81 \times 39 = 92$

POWERS OF A NUMBER CONTAINED IN A FACTORIAL

Highest power of prime number p in n! = $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \ldots$ where [x] denotes the greatest integer less than or equal to x.

Find the highest power of 2 in 50!

The highest power of 2 in 50! = $\left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$ = 25 + 12 + 6 + 3 + 1 = 47

Find the highest power of 30 in 50!

30 = 2 $\times$ 3 $\times$ 5. Now 5 is the largest prime factor of 30, therefore, the powers of 5 in 50! will be less than those of 2 and 3. Therefore, there cannot be more 30s than there are 5 in 50! So we find the highest power of 5 in 50! The highest power of 5 in 50! = $\left[\frac{50}{5}\right] + \left[\frac{50}{25}\right]$ = 10 + 2 = 12. Hence the highest power of 30 in 50! = 12

Find the number of zeroes present at the end of 100!

We get a zero at the end of a number when we multiply that number by 10. So, to calculate the number of zeroes at the end of 100!, we have to find the highest power of 10 present in the number. Since 10 =

2×5, we have to find the highest power of 5 in 100! The highest power of 5 in 100! = $\left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] = 20 + 4 = 24$

Therefore, the number of zeroes at the end of 100! = 24

Find the number of divisors of 15!

Answer: To find the number of divisors of 15!, we will have to first find out powers of every prime factor in 15!. The prime factors in 15! Are , 3, 5, 7, 11 and 13.

Powers of 2 in 15! = $\left[\frac{15}{2}\right] + \left[\frac{15}{4}\right] + \left[\frac{15}{8}\right] = 7 + 3 + 1 = 11$

Powers of 3 in 15! = $\left[\frac{15}{3}\right] + \left[\frac{15}{9}\right] = 5 + 1 = 6$

Powers of 5 in 15! = $\left[\frac{15}{5}\right] = 3$

Powers of 7 in 15! = 2

Powers of 11 in 15! = 1

Powers of 13 in 15! = 1

Therefore, $15! = 2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13 \Rightarrow$ Number of divisors = $12 \times 7 \times 4 \times 3 \times 2 \times 2 = 4032$.

What is the rightmost non-zero digit in 15!?

Answer: We saw that $15! = 2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$. Now $2^3 \times 5^3$ will give 10^3 or 3 zeroes at the end. Removing $2^3 \times 5^3$, we will be left with $2^8 \times 3^6 \times 7^2 \times 11 \times 13$. Calculating units digit of each prime factor separately, the units digit of the product $2^8 \times 3^6 \times 7^2 \times 11 \times 13$ = units digit of $6 \times 9 \times 9 \times 1 \times 3 = 8$.
Therefore, rightmost non-zero digit = 8

To find the powers of p^a in n! where p is a prime number and a is a natural number.

Highest power of prime number p^a in n! = $\left[\frac{\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \ldots}{a}\right]$ where p is a prime number, a is a natural number and [x] denotes the greatest integer less than or equal to x.

Find the highest power of 72 in 100!

$72 = 8 \times 9$. Therefore, we need to find the highest power of 8 and 9 in 72!.

$8 = 2^3 \Rightarrow$ highest power of 8 in 100! = $\left[\frac{\left[\frac{100}{2}\right] + \left[\frac{100}{4}\right] + \left[\frac{100}{8}\right] + \left[\frac{100}{16}\right] + \left[\frac{100}{32}\right] + \left[\frac{100}{64}\right]}{3}\right] = 32$

$9 = 3^2 \Rightarrow$ highest power of 9 in 100!= $\left[\frac{\left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right]}{2}\right] = 24$

As powers of 9 are less, therefore, powers of 72 in 100! = 24

Find the highest power of 24 in 150!

24 = 8 × 3. Therefore, we need to find the highest power of 8 and 3 in 150!

$8 = 2^3 \Rightarrow$ highest power of 8 in 150! $= \left[\frac{\left[\frac{150}{2}\right]+\left[\frac{150}{4}\right]+\left[\frac{150}{8}\right]+\left[\frac{150}{16}\right]+\left[\frac{150}{32}\right]+\left[\frac{150}{64}\right]+\left[\frac{150}{128}\right]}{3}\right] = 48$

Highest power of 3 in 150! $= \left[\frac{150}{3}\right]+\left[\frac{150}{9}\right]+\left[\frac{150}{27}\right]+\left[\frac{150}{81}\right] = 72$

As the powers of 8 are less, powers of 24 in 150! = 48.

n! has x number of zeroes at the end and (n + 1)! has x + 3 zeroes at the end. Find the number of possible values of n if n is a three digit number.

Answer: We can see that increasing the natural number by 1, we are gathering 3 more powers of 5. Therefore, n + 1 is a multiple of 125 but not a multiple of 625 as it would result in 4 powers of 5. Therefore, n + 1 will be equal to all the multiples of 125 minus 625.

DIVISIBILITY

Divisibility by 2, 4, 8, 16, 32..
A number is divisible by 2, 4, 8, 16, 32,.. 2^n when the number formed by the last one, two, three, four, five...n digits is divisible by 2, 4, 8, 16, 32,..2^n respectively.
Example: 1246384 is divisible by 8 because the number formed by the last three digits i.e. 384 is divisible by 8. The number 89764 is divisible by 4 because the number formed by the last two digits, 64 is divisible by 4.

A 101 digit number is formed by writing first 55 natural numbers next to each other. Find the remainder when the number is divided by 16.

Answer: to find remainder by 16 we only divide the number formed by the last 4 digits by 16. The last 4 digits would be 5455 $\Rightarrow$ remainder by 16 = 15.

Divisibility by 3 and 9
A number is divisible by 3 or 9 when the sum of the digits of the number is divisible by 3 or 9 respectively.
Example: 313644 is divisible by 3 because the sum of the digits- 3 + 1 + 3 + 6 + 4 + 4 = 21 is divisible by 3.
The number 212364 is divisible by 9 because the sum of the digit- 2 + 1 + 2 + 3 + 6 + 4 = 18 is divisible by 9.

The six-digit number 73A998 is divisible by 6. How many values of A are possible?

Answer: Since the number is ending in an even digit, the number is divisible by 2. To find divisibility by 3, we need to consider sum of the digits of the number. The sum of the digits = 7 + 3 + A + 9 + 9 + 8 = 36 + A.
For the number to be divisible by 3, the sum of the digits should be divisible by 3. Hence A can take values equal to 0, 3, 6, and 9. Therefore, number of values possible = 4

Divisibily by 6, 12, 14, 15, 18..
Whenever we have to check the divisibility of a number N by a composite number C, the number N should be divisible by all the prime factors (the highest power of every prime factor) present in C .
divisibility by 6: the number should be divisible by both 2 and 3.

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divisibility by 12: the number should be divisible by both 3 and 4.
divisibility by 14: the number should be divisible by both 2 and 7.
divisibility by 15: the number should be divisible by both 3 and 5.
divisibility by 18: the number should be divisible by both 2 and 9.

Divisibility by 7, 11, and 13

Let there be a 6- digit number abcdef. We can write

abcdef = abc000 + def = abc × 1000 + def = abc × 1000 + abc – abc + def = abc × 1001 + def – abc.

1001 = 7 × 11 × 13 ⇒ abc × 1001 is divisible by 7, 11 and 13. Therefore,
⇒ If the number def – abc is divisible by 7, the number abcdef is divisible by 7.
⇒ If the number def – abc is divisible by 11, the number abcdef is divisible by 11.
⇒ If the number def – abc is divisible by 13, the number abcdef is divisible by 13.

Now let's see the application

Let a number be ....kjlhgfedcba where a, b, c, d, are respectively units digits, tens digits, hundreds digits, thousands digits and so on.

Starting from right to left, make groups of three digit numbers successively and continue till the end. It is not necessary that the leftmost group has three digits. Grouping of the above number in groups of three, from right to left, is done in the following manner→ kj,ihg,fed,cba

Add the alternate groups (1^{st}, 3^{rd}, 5^{th} etc.. and 2^{nd}, 4^{th}, 6^{th}, etc..) to obtain two sets of numbers, N_1 and N_2. In the above example, N_1 = cba + ihg and N_2 = fed + kj

Let D be difference of two numbers, N_1 and N_2 i.e. $D = N_1 - N_2$.
→ If D is divisible by 7, then the original number is divisible by 7.
→ If D is divisible by 11, then the original number is divisible by 11
→ If D is divisible by 13 then the original number is divisible by 13.

Corollary:
Any six-digit, or twelve-digit, or eighteen-digit, or any such number with number of digits equal to multiple of 6, is divisible by ***EACH*** of 7, 11 and 13 if all of its digits are **same**.

For example 666666, 888888888888 etc. are all divisible by 7, 11, and 13.

Find if the number 29088276 is divisible by 7.

Answer: We make the groups of three as said above- 29,088,276. N_1 = 29 + 276 = 305 and N_2 = 88. $D = N_1 - N_2$ = 305 – 88 = 217. We can see that D is divisible by 7. Hence, the original number is divisible by 7.

Find the digit A if the number 888…888A999…999 is divisible by 7, where both the digits 8 and 9 are 50 in number.

Answer: We know that 888888 and 999999 will be divisible by 7. Hence 8 written 48 times in a row and 9 written 48 times in a row will be divisible by 7. Hence we need to find the value of A for which the number 88A99 is divisible by 7. By trial we can find A is = 5.

Find a four-digit number **abcd** with distinct digits which is divisible by 4, such that **bacd** is divisible by 7, **acbd** is divisible by 5, and **abdc** is divisible by 9.

Answer: acbd is divisible by 5, therefore, d is either 5 or 0. As abcd is divisible by 4, d cannot be 5 ⇒ d = 0.
As abcd is divisible by 4 and d = 0 ⇒ c = 2, 4, 6 or 8. Now, sum of the digits should be equal to 9, 18 or 27 as the number abdc is divisible by 9.
Case 1: c = 2, d = 0 ⇒ (a, b) = (1, 6), (3, 4), (7, 9). ⇒ No result.
Case 2: c = 4, d = 0 ⇒ (a, b) = (2, 3), (6, 8), (5, 9) ⇒ No result.
Case 3: c = 6, d = 0 ⇒ (a, b) = (1, 2), (5, 7), (4, 8), (3, 9). Checking for divisibility by 7, **abcd = 2160 and 5760**.
Case 4: c = 8, d = 0 ⇒ (a, b) = (4, 6), (3, 7), (1, 9). ⇒ No result.

A number consisting entirely of the digit one is called a repunit; for example, 11111. Find the smallest repunit that is divisible by 63.

Answer: 333333. A number formed by repeating a single digit 6 times is divisible by 7. Also, the sum of digits is divisible by 9.

The product of a two-digit number by a number consisting of the same digits written in the reverse order is equal to 2430. Find the lower number?

The product of the number and its reciprocal is ending in 0, therefore the number is to be of the form x5 where x is even, such that x5 × 5x = 2430. As the R.H.S. is a multiple of 9, the L.H.S. should also be a multiple of 9. Only x = 4 yields the result. Therefore, the number is 45.

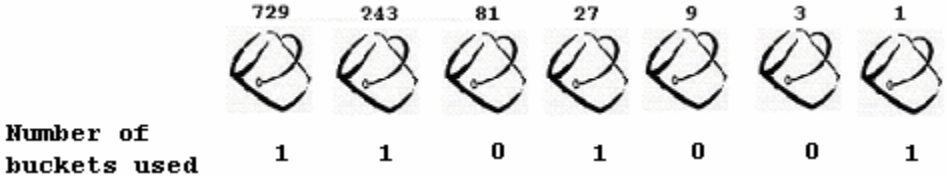
The number 523abc is divisible by 7, 8 and 9. Then a × b × c is equal to

Answer: The LCM of 7, 8, and 9 is 504. Therefore, 523abc should be divisible by 504. Now 523abc = 504 + 19abc. Therefore, 19abc should be divisible by 504.
19abc = 19000 + abc = 18648 + 352 + abc. 18648 is divisible by 504 ⇒ 352 + abc should be divisible by 504. Therefore, abc = 504 – 352 or 2 × 504 – 352 = 152 or 656. Therefore, a × b × c = 1 × 5 × 2 = 10 or 6 × 5 × 6 = 180.

BASE SYSTEM

Suppose you have a 1 000 L tank to be filled with water. The buckets that are available to you all have sizes that are powers of 3, i.e. 1, 3, 9, 27, 81, 243, and 729 L. Which buckets do you use to fill the tank in the minimum possible time?

You will certainly tell me that the first bucket you will use is of 729 L. That will leave 271 L of the tank still empty. The next few buckets you will use will 243 L, 27 L and 1 L. The use of buckets can be shown as below



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We can say that 1 000 = 729 + 243 + 27 + 1
$= 1 \times 3^6 + 1 \times 3^5 + 0 \times 3^4 + 1 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + 0 \times 3^0$.

The number 1 000 has been written in increasing powers of 3. Therefore, 3 is known as the 'base' in which we are expressing 1 000.

For example, The number 7368 can be written as $8 + 6 \times 10 + 3 \times (10)^2 + 7 \times (10)^3$.
The number 10 is called the 'base' in which this number was written.

Let a number abcde be written in base p, where a, b, c, d and e are single digits less than p. The value of the number abcde = $e + d \times p + c \times p^2 + b \times p^3 + a \times p^4$

For example, if the number 7368 is written in base 9,
The value of $(7368)_9 = 8 + 6 \times 9 + 3 \times 9^2 + 7 \times 9^3 = 5408$ (this value is in base 10)

There are two kinds of operations associated with conversion of bases:

Conversion from any base to base ten

The number $(pqrstu)_b$ is converted to base 10 by finding the value of the number. i.e. $(pqrstu)_b = u + tb + sb^2 + rb^3 + qb^4 + pb^5$.

Convert $(21344)_5$ to base 10.

Answer: $(21344)_5 = 4 + 4 \times 5 + 3 \times 25 + 1 \times 125 + 2 \times 625 = 1474$

Conversion from base 10 to any base

A number written in base 10 can be converted to any base 'b' by first dividing the number by 'b', and then successively dividing the quotients by 'b'. The remainders, written in reverse order, give the equivalent number in base 'b'.

Write the number 25 in base 4.

4	25	
	6	1
	1	2
	0	1

Writing the remainders in reverse order the number 25 in base 10 is the number 121 in base 4.

Addition, subtraction and multiplication in bases:

Add the numbers $(4235)_7$ and $(2354)_7$

Answers: The numbers are written as

4 2 3 5
2 3 5 4

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The addition of 5 and 4 (at the units place) is 9, which being more than 7 would be written as $9 = 7 \times 1 + 2$. The Quotient is 1 and written is 2. The Remainder is placed at the units place of the answer and the Quotient gets carried over to the ten's place. We obtain

$$\begin{array}{r} {\scriptstyle +1\ +1\ \ \ \ } \\ 4\ 2\ 3\ 5 \\ 2\ 3\ 5\ 4 \\ \hline 6\ 6\ 2\ 2 \end{array}$$

At the tens place: 3 + 5 + 1 (carry) = 9

Similar procedure is to be followed when multiply numbers in the same base

Multiply $(43)_8 \times (67)_8$

Answer:

$7 \times 3 = 21 = 8 \times 2 + 5 \Rightarrow$ we write 5 and carry 2 $\times$ base (8)
$7 \times 4 + 2$ (carry) $= 30 = 8 \times 3 + 6$ we write 6 and carry 3 $\times$ base (8)
$6 \times 3 = 18 = 8 \times 2 + 2 \Rightarrow$ we write 2 and carry 2 $\times$ base (8)
$6 \times 4 + 2$ (carry) $= 26 = 8 \times 3 + 2 \Rightarrow$ we write 2 and carry 3 $\times$ base (8)

$$\begin{array}{r} (4\ \ 3)_8 \\ (6\ \ 7)_8 \\ \hline 365 \\ 322 \\ \hline 3605 \end{array}$$

For subtraction the procedure is same for any ordinary subtraction in base 10 except for the fact that whenever we need to carry to the right we carry the value equal to the base.

Subtract 45026 from 51231 in base 7.

Answer:

$$\begin{array}{r} 5\ 1\ 2\ 3\ 1 \\ -\ 4\ 5\ 0\ 2\ 6 \\ \hline 3\ 2\ 0\ 2 \end{array}$$

In the units column since 1 is smaller than 6, we carry the value equal to the base from the number on the left. Since the base is 7 we carry 7. Now, 1 + 7 = 8 and 8 − 6 = 2. Hence we write 2 in the units column. We proceed the same way in the rest of the columns.

Important rules about bases

A number in base N is divisible by N − 1 when the sum of the digits of the number in base N is divisible by N − 1.

When the digits of a k-digit number N_1, written in base N are rearranged in any order to form a new k-digit number N_2, the difference $N_1 - N_2$ is divisible by N − 1.

If a number has even number of digits in base N, the number is divisible by base N + 1 if the digits

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equidistant from each end are the same, i.e. the number is a palindrome.

The number 35A246772 is in base 9. This number is divisible by 8. Find the value of digit A.

Answer: The number will be divisible by 8 when the sum of the digits is divisible by 8.
Sum of digits = 3 + 5 + A + 2 + 4 + 6 + 7 + 7 + 2 = 36 + A. The sum will be divisible by 8 when A = 4.

A four-digit number N_1 is written in base 13. A new four-digit number N_2 is formed by rearranging the digits of N_1 in any order. Then the difference $N_1 - N_2$ is divisible by

(a) 9 (b) 10 (c) 12
(d) 13

Answer: The difference is divisible by 13 – 1 = 12.

In what base is the equation 53 × 15 = 732 valid?

Answer: Let the base be b. $(53)_b = 3 + 5b$, $(15)_b = b + 5$, $(732)_b = 7b^2 + 3b + 2$
$\Rightarrow (3 + 5b) \times (b + 5) = 7b^2 + 3b + 2 \Rightarrow b = 13$.

A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals **(CAT 2003)**

Answer: Whenever we change a number from base 10 to any other base, the units digit is the first remainder when the number is divided by that base. Therefore, M when divided by 2, 3 and 5 gives remainder 1 in each case. LCM of 2, 3 and 5 is 30. Therefore, M = 30k + 1 = 31, 61 and 91. Out of these 3 numbers, only the number 91 satisfies the second criterion of leading digit (last remainder).

A palindromic number reads the same forward and backward. A 10-digit palindromic number in base 16 will always be divisible by

Answer: If a number has even number of digits in base N, the number is divisible by base N + 1 if the digits equidistant from each end are the same. Therefore, the number will be divisible by 16 + 1 = 17.

If n is a natural number, find the possible terminating digits of $n^2 + n$ in base 5.
Answer: For a natural number n, $n^2 + n$ will always end in either 0, or 2 or 6. Therefore, when written in base 5, the first remainders will be 0, 1 or 2. Therefore, the units digit of $n^2 + n$ written in base 5 will be 0, 1 or 2.

Solve in base 7, the pair of equations 2x - 4y = 33 and 3x + y = 31, where x, y and the coefficients are in base 7.

Answer: Working in base 7:
2x - 4y = 33 --- (1)
3x + y = 31 --- (2)
Multiply (2) by 4, noting that 3 × 4 = 15 in base 7 and 31 × 4 = 154 in base 7. We obtain
15x + 4y = 154 --- (3)
Adding (1) and (3) we get $20x = 220 \Rightarrow x = 11 \Rightarrow y = -2$.

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HCF AND LCM

What is highest common factor (HCF) and least common multiple (LCM)? How do you calculate HCF and LCM of two or more numbers? Are you looking for problems on HCF and LCM? This chapter will answer all these questions.

Highest Common Factor (HCF)

The largest number that divides two or more given numbers is called the highest common factor (HCF) of those numbers. There are two methods to find HCF of the given numbers:

Prime Factorization Method- When a number is written as the product of prime numbers, the factorization is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

To find the HCF of given numbers by this method, we perform the prime factorization of all the numbers and then check for the **common** prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given number. The HCF is product of all such prime factors with their respective least indices.

Find the HCF of 72, 288, and 1080

Answer: $72 = 2^3 \times 3^2$, $288 = 2^5 \times 3^2$, $1080 = 2^3 \times 3^3 \times 5$
The prime factors common to all the numbers are 2 and 3. The lowest indices of 2 and 3 in the given numbers are 3 and 2 respectively.
Hence, HCF = $2^3 \times 3^2 = 72$.

Find the HCF of $36x^3y^2$ and $24x^4y$.

Answer: $36x^3y^2 = 2^2 \cdot 3^2 \cdot x^3 \cdot y^2$, $24x^4y = 2^3 \cdot 3 \cdot x^4 \cdot y$. The least index of 2, 3, x and y in the numbers are 2, 1, 3 and 1 respectively. Hence the HCF = $2^2 \cdot 3 \cdot x^2 \cdot y = 12x^2y$.

Division method- To find HCF of two numbers by division method, we divide the higher number by the lower number. Then we divide the lower number by the first remainder, the first remainder by the second remainder... and so on, till the remainder is 0. The last divisor is the required HCF.

Find the HCF of 288 and 1080 by the division method.

Answer:

```
288|1080|3
  864
 216|288|1
    216
     72|216|3
       216
         0
```

Hence, the last divisor 72 is the HCF of 288 and 1080.

Three company of soldiers containing 120, 192, and 144 soldiers are to be broken down into smaller groups such that each group contains soldiers from one company only and all the groups have equal number of soldiers. What is the least number of total groups formed?

Answer: The least number of groups will be formed when each group has number of soldiers equal to the HCF. The HCF of 120, 192 and 144 is 24. Therefore, the numbers of groups formed for the three

companies will be 5, 8, and 6, respectively. Therefore, the least number of total groups formed = 5 + 8 + 6 = 19.

The numbers 2604, 1020 and 4812 when divided by a number N give the same remainder of 12. Find the highest such number N.

Answer: Since all the numbers give a remainder of 12 when divided by N, hence (2604 – 12), (1020 – 12) and (4812 – 12) are all divisible by N. Hence, N is the HCF of 2592, 1008 and 4800. Now $2592 = 2^5 \times 3^4$, $1008 = 2^4 \times 3^2 \times 7$ and $4800 = 2^6 \times 3 \times 5^2$. Hence, the number N = HCF = $2^4 \times 3 = 48$.

The numbers 400, 536 and 645, when divided by a number N, give the remainders of 22, 23 and 24 respectively. Find the greatest such number N.

Answer: N will be the HCF of (400 – 22), (536 – 23) and (645 – 24). Hence, N will be the HCF of 378, 513 and 621. → N = 27.

The HCF of two numbers is 12 and their sum is 288. How many pairs of such numbers are possible?

Answer: If the HCF if 12, the numbers can be written as 12x and 12y, where x and y are co-prime to each other. Therefore, 12x + 12y = 288 → x + y = 24.

The pair of numbers that are co-prime to each other and sum up to 24 are (1, 23), (5, 19), (7, 17) and (11, 13). Hence, only four pairs of such numbers are possible. The numbers are (12, 276), (60, 228), (84, 204) and (132, 156).

The HCF of two numbers is 12 and their product is 31104. How many such numbers are possible?

Answer: Let the numbers be 12x and 12y, where x and y are co-prime to each other. Therefore, $12x \times 12y = 31104$ → xy = 216. Now we need to find co-prime pairs whose product is 216.

$216 = 2^3 \times 3^3$. Therefore, the co-prime pairs will be (1, 216) and (8, 27). Therefore, only two such numbers are possible.

Find the HCF of $2^{100} - 1$ and $2^{120} - 1$

Answer: $2^{100} - 1 = (2^{20})^5 - 1 \Rightarrow$ divisible by $2^{20} - 1$ ($a^n - b^n$ is always divisible by a – b)
Similarly, $2^{120} - 1 = (2^{20})^6 - 1 \Rightarrow$ divisible by $2^{20} - 1$ ($a^n - b^n$ is always divisible by a – b)
$\Rightarrow$ HCF = $2^{20} - 1$

Least Common Multiple (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

To calculate the LCM of two or more numbers, we use the following two methods:

Prime Factorization Method: After performing the prime factorization of the numbers, i.e. breaking the numbers into product of prime numbers, we find the highest index, among the given numbers, of all the prime numbers. The LCM is the product of all these prime numbers with their respective highest indices.

Find the LCM of 72, 288 and 1080.

Answer: $72 = 2^3 \times 3^2$, $288 = 2^5 \times 3^2$, $1080 = 2^3 \times 3^3 \times 5$. The prime numbers present are 2, 3 and 5. The highest indices (powers) of 2, 3 and 5 are 5, 3 and 1, respectively.
Hence the LCM = $2^5 \times 3^3 \times 5 = 4320$.

Find the LCM of $36x^3y^2$ and $24x^4y$.

Answer: $36x^3y^2 = 2^2 \cdot 3^2 \cdot x^3 \cdot y^2$ $24x^4y = 2^3 \cdot 3 \cdot x^4 \cdot y$. The highest indices of 2, 3, x and y are 3, 2, 4 and 2 respectively.
Hence, the LCM = $2^3 \cdot 3^2 \cdot x^4 \cdot y^2 = 72x^4y^2$.

Division Method: To find the LCM of 72, 196 and 240, we use the division method in the following way:

2 | 72, 240, 196
2 | 36, 120, 98
2 | 18, 60 , 49
3 | 9 , 30 , 49
  | 3 , 10 , 49

L.C.M. of the given numbers = product of divisors and the remaining numbers = $2 \times 2 \times 2 \times 3 \times 3 \times 10 \times 49 = 72 \times 10 \times 49 = 35280$.

Remember!
For **TWO** numbers, HCF x LCM = product of the two numbers

For example, the HCF of 288 and 1020 is 72 and the LCM of these two numbers is 4320. We can see that
72 x 4320 = 288 x 1080 = 311040.

Note- This formula is applicable only for two numbers.

PROPERTIES OF HCF AND LCM

- The HCF of two or more numbers is smaller than or equal to the smallest of those numbers.
- The LCM of two or more numbers is greater than or equal to the largest of those numbers
- If numbers N_1, N_2, N_3, N_4 etc. give remainders R_1, R_2, R_3, R_4, respectively, when divided by the same number P, then P is the HCF of $(N_1 - R_1)$, $(N_2 - R_2)$, $(N_3 - R_3)$, $(N_4 - R_4)$ etc.
- If the HCF of numbers N_1, N_2, N_3 … is H, then N_1, N_2, N_3... can be written as multiples of H (Hx, Hy, Hz..). Since the HCF divides all the numbers, every number will be a multiple of the HCF.
- If the HCF of two numbers N_1 and N_2 is H, then, the numbers $(N_1 + N_2)$ and $(N_1 - N_2)$ are also divisible by H. Let $N_1 = Hx$ and $N_2 = Hy$, since the numbers will be multiples of H. Then, $N_1 + N_2 = Hx + Hy = H(x + y)$, and
  $N_1 - N_2 = Hx - Hy = H(x - y)$. Hence both the sum and differences of the two numbers are divisible by the HCF.
- If numbers N_1, N_2, N_3, N_4 etc. give an equal remainder when divided by the same number P, then P is a factor of $(N_1 - N_2)$, $(N_2 - N_3)$, $(N_3 - N_4)$…
- If L is the LCM of N_1, N_2, N_3, N_4.. all the multiples of L are divisible by these numbers.

- If a number P always leaves a remainder R when divided by the numbers N_1, N_2, N_3, N_4 etc., then P = LCM (or a multiple of LCM) of N_1, N_2, N_3, N_4.. + R.

Find the highest four-digit number that is divisible by each of the numbers 24, 36, 45 and 60.

Answer: $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2$, $45 = 3^2 \times 5$ and $60 = 2^3 \times 3^2 \times 5$. Hence, the LCM of 24, 36, 45 and $60 = 2^3 \times 3^2 \times 5 = 360$. The highest four-digit number is 9999. 9999 when divided by 360 gives the Remainder 279. Hence, the number (9999 – 279 = 9720) will be divisible by 360. Hence the highest four-digit number divisible by 24, 36, 45 and 60 = 9720.

Find the highest number less than 1800 that is divisible by each of the numbers 2, 3, 4, 5, 6 and 7.

Answer: The LCM of 2, 3, 4, 5, 6 and 7 is 420. Hence 420, and every multiple of 420, is divisible by each of these numbers. Hence, the number 420, 840, 1260, and 1680 are all divisible by each of these numbers. We can see that 1680 is the highest number less than 1800 which is multiple of 420. Hence, the highest number divisible by each one of 2, 3, 4, 5, 6 and 7, and less than 1800 is 1680.

Find the lowest number which gives a remainder of 5 when divided by any of the numbers 6, 7, and 8.

Answer: The LCM of 6, 7 and 8 is 168. Hence, 168 is divisible by 6, 7 and 8. Therefore, 168 + 5 = 173 will give a remainder of 5 when divided by these numbers.

What is the smallest number which when divided by 9, 18, 24 leaves a remainder of 5, 14 and 20 respectively?

Answer: The common difference between the divisor and the remainder is 4 (9 – 5 = 4, 18 – 14 = 4, 24 – 20 = 4). Now the LCM of 9, 18, and 24 is 72. Now 72 – 4 = 72 – 9 + 5 = 72 – 18 + 14 = 72 – 24 + 20. Therefore, if we subtract 4 from 72, the resulting number will give remainders of 5, 14, and 20 with 9, 18, and 24.

Hence, the number = 72 – 4 = 68.

A number when divided by 3, 4, 5, and 6 always leaves a remainder of 2, but leaves no remainder when divided by 7. What is the lowest such number possible?

Answer: the LCM of 3, 4, 5 and 6 is 60. Therefore, the number is of the form 60k + 2, i.e. 62, 122, 182, 242 etc. We can see that 182 is divisible by 7. Therefore, the lowest such number possible = 182.

For how many pairs (***a***, ***b***) of natural numbers is the LCM of ***a*** and ***b*** is $2^3 5^7 11^{13}$?

Let's solve for the powers of 2. One of the number will have 2^3 in it, as the LCM has 2^3. Now the other number can have the powers of 2 as 2^0, 2^1, 2^2, and 2^3. Therefore, number of pairs will be 4: (2^3, 2^0), (2^3, 2^1), (2^3, 2^2), and (2^3, 2^3) and the number of ordered pairs will be $2 \times 4 - 1 = 7$ (we cannot count the pair (2^3, 2^3) twice.

Similarly ordered pairs for powers of $5 = 2 \times 8 - 1 = 15$.
Number of ordered pairs for powers of $7 = 2 \times 14 - 1 = 27$.
Total ordered pairs (a, b) $= 7 \times 15 \times 27 = 2835$

DIGIT-SUM RULE

Given a number N_1, all the digits of N_1 are added to obtain a number N_2. All the digits of N_2 are added to obtain a number N_3, and so on, till we obtain a single digit number N. This single digit number N is called the digit sum of the original number N_1.

What is the digit sum of 123456789?

Answer: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 → 4 + 5 = 9. Hence, the digit sum of the number is 9.

Note: In finding the digit-Sum of a number we can ignore the digit 9 or the digits that add up to 9. For example, in finding the digit-sum of the number 246819, we can ignore the digits 2, 6, 1, and 9. Hence, the digit-sum of 246819 is = 4 + 8 = 12 = 1 + 2 = 3.

Digit-Sum Rule of Multiplication: The digit-sum of the product of two numbers is equal to the digit sum of the product of the digit sums of the two numbers!

Example: The product of 129 and 35 is 4515.
Digit sum of 129 = 3 and digit sum 35 = 8. Product of the digit sums = 3 × 8 = 24 → Digit-sum = 6. Digit-sum of 4515 is = 4 + 5 + 1 + 5 = 15 = 1 + 5 = 6. Digit-sum of the product of the digit sums = digit sum of 24 = 6 → Digit sum of the product (4515) = Digit-sum of the product of the digit sums (24) = 6

Applications of Digit-Sum

Rapid checking of calculations while multiplying numbers

Suppose a student is trying to find the product 316 × 234 × 356, and he obtains the number 26525064. A quick check will show that the digit-sum of the product is 3. The digit-sums of the individual numbers (316, 234 and 356) are 1, 9, and 5. The digit-sum of the product of the digit sum is 1 × 9 × 5 = 45 = 4 + 5 = 9. ⇒ the digit-sum of the product of the digit-sums (9) ≠ digit-sum of the 26525064 (3). Hence, the answer obtained by multiplication is not correct.

Note: Although the answer of multiplication will not be correct if the digit-sum of the product of the digit-sums is not equal to digit-sum of the product, but the reverse is not true i.e. the answer of multiplication **may or may not be** correct if the digit-sum of the product of the digit-sums is equal to digit-sum of the product

Finding the sum of the digits of a number raised to a power

The digits of the number $(4)^{24}$ are summed up continually till a single digit number is obtained. What is that number?

Answer: $4^3 = 64$. Digit sum of 64 is = 1. $4^{24} = 4^3 \times 4^3 \times 4^3 \ldots \times 4^3$ (8 times). Digit sums on both sides will be the same. ⇒ digit sum of 4^{24} = digit sum of 1 × 1 × 1 × 1… (8 times) = 1

Remember!
If one of the multiplicand is 9, the digit sum is always 9. 235 × 9 = 2115 → digit sum = 9
6224 × 9 = 56016 → digit sum = 9

Find the sum of the sum of the sum of the digits of 25!

Answer: 25! = 1 × 2 × 3 × … × 24 × 25. As one of the multiplicands is 9, the digit sum will be 9.

Determining if a number is a perfect square or not

S. No.	Number	Square	Digit-Sum of the		S. No.	Number	Square	Digit-Sum of the square

			square					
1	1	1	**1**		10	10	100	**1**
2	2	4	**4**		11	11	121	**4**
3	3	9	**9**		12	12	144	**9**
4	4	16	**7**		13	13	169	**7**
5	5	25	**7**		14	14	196	**7**
6	6	36	**9**		15	15	225	**9**
7	7	49	**4**		16	16	256	**4**
8	8	64	**1**		17	17	289	**1**
9	9	81	**9**		18	18	324	**9**

It can be seen from the table that the digit-sum of the numbers which are perfect squares will always be 1, 4, 9, or 7.

Note: A number will **NOT** be a perfect square if its digit-sum is **NOT** 1, 4, 7, or 9, but it **may or may not** be a perfect square if its digit-sum is 1, 4, 7, or 9.

Is the number 323321 a perfect square?

Answer: the digit-sum of the number 323321 is 5. Hence, the number cannot be a perfect square.

A 10-digit number *N* has among its digits one 1, two 2's, three 3's, and four 4's. Is *N* be a perfect square?

Answer: We can see that the digit sum of a perfect square is always 1, 4, 7, or 9. As the digit sum of the number is 3, it cannot be a perfect square.

If a five-digit number N is such that the sum of the digits is 29, can N be the square of an integer?

GREATEST INTEGER FUNCTION

The greatest integer function, denoted by [x], gives the greatest integer less than or equal to the given number x.

To put it simply, if the given number is an integer, then the greatest integer gives the number itself, otherwise it gives the first integer towards the left of the number of x on the number line.

For example, [1.4] = 1, [4]= 4, [3.4] = 3, [- 2.3] = -3, [- 5.6] = - 6, and so on.

NOTE: We can see that [1.4] = 1 + 0.4 or x = [x] + {x}, where {x} is the fractional part of x. For x = - 2.3, [x] = - 3 and {x} = 0.7

In the following figure, we can see that the greatest integer function gives the number itself (when the given number is an integer) or the first integer to the left of the number on the number line.

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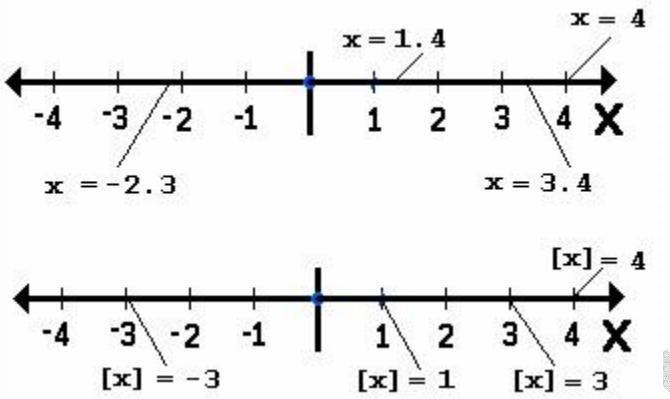
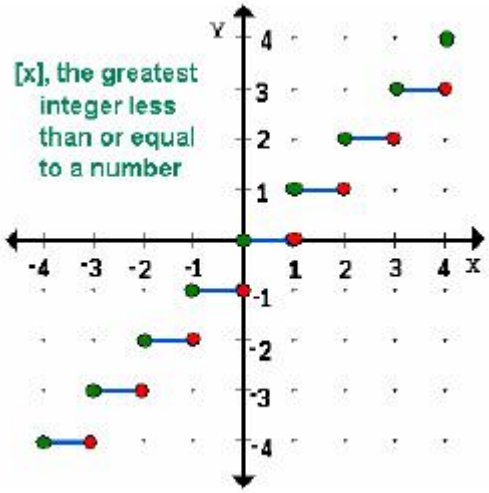


The graph of greatest integer function is given below. Note that the **red** dot indicates that integer value on the number line is not included while the **green** dot indicates that the integer value is included.



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Examples:

- What is the value of $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\cdots+[\sqrt{49}]+[\sqrt{50}]$ where $[x]$ denotes the greatest integer function?
  Answer: It can be seen that
  $[\sqrt{1}]=1,[\sqrt{2}]=[1.41]=1,[\sqrt{3}]=[1.73]=1,[\sqrt{4}]=2$ and so on..

  Therefore, from $[\sqrt{1}]$ to $[\sqrt{3}]$, the value will be 1, from $[\sqrt{4}]$ to $[\sqrt{8}]$ the value will be 2, from $[\sqrt{9}]$ to $[\sqrt{15}]$ the value will be 3 and so on..
  Therefore, the total value = $3 \times 1 + 5 \times 2 + 7 \times 3 + \ldots + 13 \times 6 + 2 \times 7$
  = 217.

- If $[\sqrt{x}]=5$ and $[\sqrt{y}]=6$, where x and y are natural numbers, what can be the greatest possible value of $x + y$?
  Answer: It is clear that $[\sqrt{25}]=5,[\sqrt{26}]=5,[\sqrt{27}]=5$ and so on. The highest value of x that we can take is 35, since $[\sqrt{35}]=5$ but $[\sqrt{36}]=6$.
  Similarly, the highest value of y we can take is 48, since
  $[\sqrt{48}]=6$ but $[\sqrt{49}]=7$
  Therefore, the greatest value of $x + y = 35 + 48 = 83$.

- What is the value of x for which $x[x] = 28$?
  Answer: If the value of x is 5, $x[x] = 25$, and if the value of x is 6 $x[x]=36$.
  Therefore, the value of x lies between 5 and 6.
  If x lies between 5 and 6, $[x] = 5$.
  $\Rightarrow x = \frac{28}{[x]} = \frac{28}{5} = 5.6$

- Let x be number and $[x]$ and $\{x\}$ denote the greatest integer less than x and fractional part of x. If $[x]^3 + \{x\}^2 = -63.64$, then the value of x is
  Answer: Since the value of $[x]^3 + \{x\}^2$ is negative, the value of $[x]$ is negative. The cube lying near –63.64 is –64. Therefore, $[x]^3 = -64$ or $[x] = -4$. Therefore, $\{x\}^2 = 0.36$ or $\{x\} = 0.6$
  Hence, $x = [x] + \{x\} = -4 + 0.6 = -3.4$

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1. The product of three consecutive odd numbers is 531117. What is the sum of the three numbers?

A. 183
B. 213
C. 243
D. 273

2. N is the smallest number such that N/2 is a perfect square and N/3 is a perfect cube. Then, the number of divisors of N is

A. 12
B. 16
C. 20
D. 24

3. In the value of the number 30!, all the zeroes at the end are erased. Then, the unit digit of the number that is left is

A. 2
B. 4
C. 6
D. 8

4. N is a number such that $200 < N < 300$ and it has exactly 6 positive divisors. How many different values of N are possible?

A. 12
B. 13
C. 14
D. 15

5. In 1936, my age was equal to the last two digits of my birth year. My grandfather said that it was true for him also. Then, the sum of my age and my grandfather's age in 1936 was

A. 84
B. 86
C. 90
D. 94

6. If n is a natural number such that $10^{12} < n < 10^{13}$ and the sum of the digits of n is 2, then the number of values n can take is

A. 13
B. 12
C. 11
D. 10

7. The highest number that certainly divides $2^{4n} - 15n - 1$ is

A. 15
B. 60
C. 120
D. 225

8. N is the sum of the squares of three consecutive odd numbers such that all the digits of N are the same. If N is a four-digit number, then the value of N is

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A. 3333
B. 5555
C. 7777
D. 9999

Read the information given below and answer the question that follows.

While driving on a straight road, Jason passed a milestone with a two-digit number. After exactly an hour, he passed a second milestone with the same two digits written in reverse order. Exactly one more hour after that, he passed a third milestone with the same two digits reversed and separated by a zero.

9. What is the sum of the two digits?
A. 5
B. 6
C. 7
D. 8

10. The squares of the natural numbers are written in a straight line 149162536… to form a 200-digits number. What is the 100^{th} digit from the left?
A. 2
B. 5
C. 6
D. 9

11. $18^{2000} + 12^{2000} - 5^{2000} - 1$ is divisible by

A. 323

B. 221

C. 299

D. 237

12. Let n be the smallest positive number such that the number $S = (8^n)(5^{600})$ has 604 digits. Then the sum of the digits of S is

A. 19

B. 8

C. 10

D. 11

13. Swadesh threw five standard dice simultaneously. He found that the product of the numbers on the top faces was 216. Which of the following could not be the sum of the numbers on the top five faces?

A. 17

B. 18

C. 19

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D. 20

14. If $S=\frac{(10^4+324)(22^4+324)(34^4+324)(46^4+324)(58^4+324)}{(4^4+324)(16^4+324)(28^4+324)(40^4+324)(52^4+324)}$, then the value of *S* is

A. 373

B. 256

C. 504

D. 588

15. Let S = $(3 + 3^2 + 3^3 + \ldots + 3^{400}) - (7 + 7^2 + 7^3 + \ldots + 7^{201})$. The last two digits of S are

A. 00

B. 07

C. 43

D. 93

16. For how many integers S is $\frac{S}{20-S}$ square of an integer?

A. 5

B. 4

C. 3

D. 2

17. How many natural numbers between 1 and 900 are NOT multiples of any of the numbers 2, 3, or 5?

A. 240

B. 250

C. 270

D. 300

18. If N = $(63)^{1! + 2! + 3! + ..+ 63!} + (18)^{1! + 2! + 3!..+ 18!} + (37)^{1! + 2! + 3! + ...+ 37!}$, then the unit digit of N is

A. 2

B. 4

C. 6

D. 8

19. The last two digits of 4^{1997} are

A. 96

Total Gadha

B. 36

C. 84

D. 24

20. The numbers 123 456 789 and 999 999 999 are multiplied. How many times does digit '9' come in the product?

A. 0

B. 1

C. 2

D. 3

21. All the divisors of 72 are multiplied. The product can be written in the form 2^a*3^b. Then the value of a + b is

A. 28

B. 30

C. 34

D. 40

22. The number A4531B, where A and B are single-digit numbers, is divisible by 72. Then A + B is equal to

A. 5

B. 7

C. 8

D. 4

23. N is the smallest natural number which when multiplied by 7 gives a product P. Every digit of P is one. The product when N is multiplied by 8 is

A. 127 784

B. 125 384

C. 126 984

D. 181 384

24. The remainder when 7^{77} is divided by 9 is

A. 2

Total Gadha

B. 4

C. 6

D. 8

25. In the nineteenth century a person was X years old in the year X^2. How old was he in 1884?

A. 43

B. 58

C. 68

D. 78

26. The average of the nine numbers 9, 99, 999,…, 999999999 is A. Then A does not contain the digit

A. 0

B. 1

C. 5

D. 9

27. How many natural numbers less than 65 have odd number of divisors (including 1 and those numbers themselves)?

A. 8

B. 12

C. 15

D. 21

Read the information below and answer the question that follows.

In a mathematical game, one hundred people are standing in a line and they are required to count off in fives as 'one, two, three, four, five, one, two, three, four, five,' and so on from the first person in the line. The person who says 'five' is taken out of the line. Those remaining repeat this procedure until only four people remain in the line.

28. What was the original position in the line of the last person to leave?

A. 93

B. 96

C. 97

Total Gadha

D. 98

29. Let A = 333…333 (51 digits) and B = 666…666 (51 digits). Then the 52^{nd} digit (counting from the right) in the product A $\times$ B is

A. 9

B. 7

C. 2

D. 1

30. What is the sum of the sum of the sum of the digits of 55!?

A. 5

B. 9

C. 27

D. 36

31. An Indian king was born in a year that was a square number, lived a square number of years and died in a year that was also a square number. Then the year he could have been born in was

A. 1936

B. 1764

C. 1600

D. 1444

32. To number the pages of a book, exactly 300 digits were used. How many pages did the book have?

A. 136

B. 137

C. 138

D. 139

33. If $[3(150 + S)]^2 = 22752S$ then the value of single-digit number S is

A. 0

B. 3

Total Gadha

C. 5

D. 9

34. If $S = 5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ where n is any whole number, then S is always divisible by

A. 7

B. 17

C. 19

D. 33

35. The highest power of 12 that can divide $5^{36} - 1$ is

A. 1

B. 2

C. 3

D. 4

36. What is the largest prime whose cube divides 1!2!…1001!?

A. 997

B. 991

C. 977

D. 973

37. Let $S = p^2 + q^2 + r^2$, where p and q are consecutive positive integers and $r = p \times q$. Then $\sqrt{S}$ is

A. an even integer

B. an odd integer

C. always irrational

D. sometimes irrational

38. The number (2n)! is divisible by

I. $(n!)^2$
II. $((n-1)!)^2$
III. $n! \times (n+1)!$

Total Gadha

A. I and II only

B. II and III only

C. I and III only

D. I, II and III

Use the following information to answer the next question.

Baghira, the oldest inmate of Tihar Jail is learning mathematics. He notices the following facts about his prisoner number:
I. It is a three digit number not bigger than 500.
II. If you sum the cube of the digits of the number, you get the number itself.
III. The number is the sum of consecutive factorials.

39. Then, the sum of the digits of Baghira's prisoner number is

A. 12

B. 10

C. 9

D. 6

40. The number 6162 in base 10 is written as $(222)_b$. Then, the base b is equal to

A. 60

B. 55

C. 45

D. 42

Read the information given below and answer the question that follows.

Kallu Kallan Kalia's nightclub number is a three-digit perfect square. This number when written in the reverse order also gives a perfect square and is telephone extension of Kallu's office. His Mercedes registration number is a four-digit perfect square formed by repeating the rightmost digit of his nightclub number.

41. The sum of the digits of Kallu's nightclub number is

A. a perfect square

B. an even number

C. a prime number

D. a perfect number

Total Gadha

42. $53^{53} - 27^{27}$ is certainly divisible by

A. 7

B. 9

C. 10

D. 11

43. $43^{444} + 34^{333}$ is divisible by

A. 2

B. 5

C. 9

D. 11

Read the information given below and answer the question that follows.

The teacher of Confusius, the confused soul, told him: "my age is not prime but odd and if you reverse the digits of my age and add that number to my age, you obtain a number that is a perfect square. If you reverse the digits of my age and subtract the number from my age you again have a perfect square."

44. The age of Confucius' teacher was a number divisible by

A. 7

B. 9

C. 13

D. 15

45. If in some base x, $(563)_x + (544)_x + (433)_x = (2203)_x$ then x is equal to

A. 6

B. 7

C. 9

D. 11

46. The remainder when 2^{2005} is divided by 13 is

A. 2

B. 3

Total Gadha

C. 4

D. 8

47. What is the sum of the coefficients of all the terms of $(1 + 2x + 3x^2 - 4x^3)^{10}$ when it is expanded?

A. 16

B. 256

C. 512

D. 1024

48. How many positive integers less than or equal to 120 are relatively prime to 120?

A. 24

B. 32

C. 36

D. 40

49. What is the remainder when S = 1! + 2! + 3! +… + 19! + 20! is divided by 20?

A. 0

B. 1

C. 33

D. 13

50. If $3^{27^x} = 27^{3^x}$, then x is equal to

A. −1

B. 1/2

C. 1

D. 2

51. If S = $\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots + \frac{1}{\sqrt{121}-\sqrt{120}}$, then the value of S is

A. 12

Total Gadha

B. 11

C. 10

D. 9

52. The difference between the cubes of two consecutive positive integers is 1027. Then the product of these integers is

A. 552

B. 342

C. 306

D. 132

53. For how many integer values of x is $\frac{2x^2-10x-4}{x^2-4x+3}$ an integer?

A. 4

B. 5

C. 6

D. 7

54. How many integers between 1 and 1000, both inclusive, can be expressed as the difference of the squares of two non negative integers?

A. 750

B. 748

C. 300

D. 250

55. The product P of three positive integers is 9 times their sum, and one of the integers is the sum of the other two. The sum of all possible values of P is

A. 621

B. 702

C. 540

D. 336

Total Gadha

56. Let M be the greatest number divisible by 8, such that no digit from 0 to 9 is repeated in M. What is the remainder when M is divided by 1000?

A. 320

B. 120

C. 104

D. 96

57. Let q and r be the quotient and remainder when M, a five digit number, is divided by 100. For how many values of M is $q + r$ divisible by 99?

A. 900

B. 908

C. 909

D. 989

58. Let $[x]$ = greatest integer less than or equal to x. Let A = $[2x]$, B = $2[x]$ and C = $[x + 1/2] + [x - 1/2]$. Then

I. A, B and C can be equal for some value of x.

II. A, B and C can all take different values for some value of x.

A. I is true but II is false

B. II is true but I is false

C. Both I and II are true

D. Both I and II are false

59. How many perfect squares are the divisors of the product $1! \cdot 2! \cdot 3! \cdots 8!$?

A. 120

B. 240

C. 360

D. 720

60. The number 2006! is written in base 22. How many zeroes are there at the end?

A. 500

B. 450

Total Gadha

C. 200

D. 199

61. $25^{25^{25}}$ when divided by 9 gives the remainder

A. 2

B. 4

C. 5

D. 7

62. The unit digit of 7^{7^7} is

A. 1

B. 3

C. 7

D. 9

63. For how many positive integers x between 1 and 1000, both inclusive, is $4x^6 + x^3 + 5$ is divisible by 7?

A. 0

B. 4

C. 11

D. 36

64. In how many ways can the number 105 be written as a sum of two or more consecutive positive integers?

A. 4

B. 5

C. 6

D. 7

Use the information given below and answer the question that follows.

It is given that m and n are two smallest natural numbers satisfying following conditions

I. $5 < n < m$

II. both $mn - 3$ and $mn + 3$ are prime numbers

Total Gadha

65. Then $m + n$ is equal to

A. 15

B. 17

C. 19

D. 21

66. Five consecutive integers are chosen. Let S denote their sum and let P denote their product. If P/S is divisible by 100, what can be the smallest value of S?

A. 115

B. 125

C. 615

D. 625

67. If the product of four positive integers is 10! What is the smallest possible value their sum can have?

A. 175

B. 176

C. 180

D. 181

68. S is a six digit number beginning with 1. If the digit 1 is moved from the leftmost place to the rightmost place the number obtained is three times of S. Then the sum of the digits of S is

A. 21

B. 24

C. 26

D. 27

69. Given that x and y are integers and $5x^2 + 2y^2 = 5922$, what can be the unit digit of y?

A. 3

B. 5

C. 7

Total Gadha

D. 9

70. Which of the following numbers can be written as the sum of the squares of three odd natural numbers?

A. 5021

B. 4445

C. 3339

D. 1233

71. If S = $\frac{10^{33}+2}{3}$, then, the sum of digits of S is

A. 97

B. 100

C. 103

D. 106

Read the information given below and answer the question that follows.

In a bag, some slips of paper are kept with the numbers thirteen or fourteen written on them. The slips with number thirteen written on them are five more than the slips with number fourteen written on them.

72. Which of the following can be the sum of the numbers in the bag?

A. 254

B. 300

C. 327

D. 353

Read the information given below and answer the question that follows.

Two natural numbers *a* and *b* are given in base 10. The number *a* can be written as 212 in base *b* and 128 in base *b* + 2.

73. The value of *a* + *b* in base 10 is

A. 219

B. 125

C. 114

D. 107

Total Gadha

74. How many two-digit positive integers are there which are one and a half times larger than the product of their digits?

A. 0

B. 1

C. 2

D. 3

75. A three-digit number *abc* is divisible by 7 if

A. 3a + b + c is divisible by 7

B. a + 2b + c is divisible by 7

C. 2a + 3b + c is divisible by 7

D. 2a + 2b + c is divisible by 7

76. For how many values of k is 12^{12} the least common multiple of 6^6, 8^8, and k?

A. 1

B. 12

C. 24

D. 25

Read the information given below and answer the question that follows.

For single digit numbers *a*, *b* and *c*, $(abc)_7 = (cba)_9$

77. The value of $a + b + c$

A. = 8

B. = 11

C. = 16

D. cannot be determined

78. What is the largest natural number which divides $n^5 - 5n^3 + 4n$ for every natural number *n*?

A. 120

B. 60

C. 30

Total Gadha

D. 24

79. The value of $x + y$ such that $x^2 - y^2 = 343$, is

A. 343

B. 49

C. 7

D. A or B

80. What is the remainder when $6^{83} + 8^{83}$ is divided by 49?

A. 0

B. 14

C. 35

D. 42

81. What is the remainder when $(81)^{21} + (27)^{21} + (9)^{21} + (3)^{21} + 1$ is divided by $3^{20} + 1$?

A. 0

B. 1

C. 61

D. 121

82. All the divisors of 360, including 1 and the number itself, are summed up. The sum is 1170. What is the sum of the reciprocals of all the divisors of 360?

A. 3.25

B. 2.75

C. 2.5

D. 1.75

83. Let $S = \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$ and $T = \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{d}{|d|} + \frac{abcd}{|abcd|}$, where a, b, c, and d are not equal to zero. Then the set of intersection of all values of S and T is

A. {0}

B. {ϕ}

Total Gadha

C. {1}

D. {4}

84. What is the sum of the real values of x satisfying the equation $4 \times 3^{2x+2} - 9^{2x} = 243$?

A. 1

B. 3/2

C. 5/2

D. 3

85. What is the remainder when the number $\underbrace{123123123.....123123}_{300 digits}$ is divided by 99?

A. 18

B. 27

C. 33

D. 36

86. A 10-digit number N has among its digits one 1, two 2's, three 3's, and four 4's. Is N be a perfect square?

A. Yes

B. No

C. Maybe

D. Can't say

87. Vinay has 128 boxes with him. He has to put least 120 oranges in one box and 144 oranges at the most. Then the least number of boxes containing the same number of oranges is

A. 5

B. 6

C. 24

D. 103

88. There were 90 questions in an exam. If 1 mark was awarded for every correct answer and 1/3rd mark was deducted for every wrong answer, how many different net scores were possible in the exam?

A. 120

B. 358

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C. 359

D. 360

Use the following information to answer the next question.

In a class, the teacher wrote a set of consecutive integers beginning with 1 on the blackboard. Little Johnny came and erased one number. The average of the remaining numbers was $27\frac{4}{13}$.

89. What was the number that little Johnny erased?

A. 7

B. 11

C. 13

D. 15

90. What is the remainder when $1+(11)^{11}+(111)^{111}+(1111)^{1111}+\ldots\underbrace{(111\ldots111)}_{10 digits}^{\underbrace{111\ldots111}_{10 digits}}$ is divided by 100?

A. 40

B. 30

C. 10

D. 0

91. For how many ordered pairs (x, y), where x and y are non-negative integers, is the equation $\sqrt{x}+\sqrt{y}=\sqrt{1332}$ satisfied?

A. 3
B. 5
C. 7
D. 9

92. For how many prime number p, is $p^2 + 15p - 1$ also a prime number?

A. 0
B. 1
C. 2
D. 3

93. The remainder when $x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1$ is divided by $(x - 1)(x - 2)$ is

A. 7
B. $2x - 3$
C. $6x - 5$
D. 0

94. If 11 sweets are distributed among four boys, then, which of the following is true?

A. Two boys each received more than 1 sweet
B. One of the boys received more than 3 sweets

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C. One of the boys received fewer than 3 sweets
D. One of the boys received exactly 2 sweets

95. The least common multiple of $2^6 - 1$ and $2^9 - 1$ is
A. $2^{12} + 27 \times 2^9 - 217$
B. $2^{12} + 63 \times 2^3 - 1$
C. $2^{12} + 5 \times 2^9 - 1$
D. $2^{12} + 9 \times 2^8 - 1$

96. All possible pairs are formed from the divisors of 21600. How many such pairs have HCF of 45?
A. 8
B. 276
C. 34
D. 49

97. Let H(x, y) = Highest common factor of x and y, and L(x, y) = Least common multiple of x and y. For three numbers, a, b and c it is given that

H(a, b) = 6	L(a, b) = 180
H(b, c) = 30	L(b, c) = 90
H(a, c) = 6	L(c, a) = 60

The value of a × b × c is
A. 32 400
B. 97 200
C. 16 200
D. 97 200

98. If the remainder when x^{100} is divided by $x^2 - 3x + 2$ is $ax + b$, then the values of a and b are
A. 2^{100} and 1
B. 2^{100} and $1 - 2^{100}$
C. 2^{100} and $2 - 2^{100}$
D. $2^{100} - 1$ and $2 - 2^{100}$ *

99. How many natural numbers between 1 and 100 have exactly four factors?
A. 30
B. 32
C. 300
D. 33

100. S is a set of *n* natural numbers such that at least two numbers in S give the same remainder when divided by 3, and there exist at least two numbers in S whose sum is divisible by 3. What is the minimum value of *n* which ensures that both the conditions for S are fulfilled?
A. 2
B. 3
C. 4*
D. 5

101. If four hens lay five eggs in six days, how many days will it take twenty-four hens to lay sixty eggs?
A. 3
B. 12
C. 24
D. 72

Total Gadha

102. 'Privileged Number' is a natural number which has two prime numbers as its neighbors on the number line. For example, 4 and 12 are privileged numbers. What is the mean of all privileged numbers less than 100?

A. 30.5
B. 24
C. 28.5
D. 26

103. What is the remainder when $1+(11)^{11}+(111)^{111}+(1111)^{1111}+\ldots(\underbrace{111\ldots111}_{10digits})^{\frac{111\ldots111}{10digits}}$ is divided by 100?

A. 40
B. 10
C. 30
D. 0

104. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of **(CAT 2005)**

A. 69
B. 35
C. 0
D. 1

105. If $R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then **(CAT 2005)**

A. R > 1.0
B. 0 < R ≤ 0.1
C. 0.1 < R ≤ 0.5
D. 0.5 < R ≤ 1.0

106. The digits of a three-digit number ***A*** are written in the reverse order to form another three-digit number ***B***. If ***B*** > ***A*** and ***B***−***A*** is perfectly divisible by 7, then which of the following is necessarily true? **(CAT 2005)**

A. 112 < A < 311
B. 100 < A < 299
C. 106 < A < 305
D. 118 < A < 317

107. The rightmost non-zero digit of the number 30^{2720} is

A. 1
B. 3
C. 7
D. 9

108. The total number of integer pairs (x, y) satisfying the equation x + y = xy is

A. 0
B. 1
C. 2
D. none of the above

109. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

A. 4
B. 15

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C. 0
D. 18

110. What is the remainder when 4^{96} is divided by 6? **(CAT 2003)**
A. 0
B. 3
C. 2
D. 4

111. Let *x* and *y* be positive integers such that *x* is prime and *y* is composite. Then,
A. y − x cannot be an even integer.
B. (x + y)/x cannot be an even integer.
C. xy cannot be an even integer.
D. None of the other statements is true.

112. If $|b| > 1$ and $x = -|a|\ b$, then which one of the following is necessarily true? **(CAT 2003)**
A. $a - xb \leq 0$
B. $a - xb > 0$
C. $a - xb \geq 0$
D. $a - xb < 0$

113. If *a*, *a* + 2, and *a* + 4 are prime numbers, then the number of possible solutions for *a* is **(CAT 2003)**
A. 1
B. 2
C. 3
D. more than 3

114. 'Jumpy,' our favorite grasshopper is sitting at the origin. He starts jumping from one lattice point to another. At each jump, Jumpy moves one unit to the right, or one unit to the left, or one unit up, or one unit down. If Jumpy takes 12 jumps in all, how many lattice points could he finally end up at?

115. A faulty odometer of a car always jumps from digit 4 to digit 6, always skipping the digit 5, regardless of the position. For example, after traveling for one kilometer the odometer reading changed from 000149 to 000160. If the odometer showed 000000 when the car was bought and now it shows 001000, how many kilometers has the car traveled?

116. In how many zeroes does $\frac{10000!}{(100!)^2}$ end?

117. A number N when divided by a divisor D gives a remainder of 52. The number 5N when divided by D gives a remainder of 4. How many values of D are possible?

118. Which one of the following numbers is a perfect square?
A. $35! \times 36!$
B. $37! \times 38!$
C. $34! \times 37!$
D. $36! \times 37!$

119. What is the remainder when $(17)^{36} + (19)^{36}$ is divided by 111?

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120. If $3^{36} - 1 = 1\mathbf{A}009463\mathbf{A}296999120$, where A is a single digit whole number, then the value of A is

121. In how many ways can 7^{13} be written as product of 3 natural numbers?

122. N has f factors, 2N has 2f factors, 6N has 4f factors, 15N has 3f factors. How many factors 30N has?

123. How many zeroes are present at the end of 25! + 26! + 27! + 28! + 30!?

124. How many zeroes does 5^5! End in?
- **A.** 781
- **B.** 100
- **C.** 50
- **D.** 3906

125. S is a set of 10 consecutive two-digit integers such that the product of these 10 integers has the highest power of 2 contained in it. How many such sets S are possible?
- **A.** 10
- **B.** 8
- **C.** 4
- **D.** 24

126. N is a number such that the ratio of sum of its digit to product of its digits is 3: 40. If N is divisible by 37 and N is the smallest such number, how many factors does N have?
- **A.** 8
- **B.** 16
- **C.** 30
- **D.** 20

127. In a national hockey single elimination tournament, 303 teams are participating. How many games will be played before a team becomes the national champion?
- **A.** 303
- **B.** 302
- **C.** 152
- **D.** 77

128. A two-digit number is divided by the sum of its digits. The answer is 6. What is the product of the digits?
- **A.** 24
- **B.** 54
- **C.** 20
- **D.** 18

129. The sum of 20 distinct numbers is 801. What is their minimum LCM possible?
- **A.** 480
- **B.** 360
- **C.** 840
- **D.** 42

130. What is the smallest positive composite number generated by the expression $p^2 - p - 1$ where p is a prime number?
- **A.** 13
- **B.** 155
- **C.** 40

Total Gadha

D. 270

131. N the least positive integer that is eleven times the sum of its digits. Then N is divisible by
A. 4
B. 7
C. 15
D. 9

132. The value of A + B that satisfies $(6^{30} + 6^{-30})(6^{30} - 6^{-30}) = 3^A8^B - 3^{-A}8^{-B}$ is
A. 20
B. 60
C. 80
D. 40

133. $10000! = (100!)^K \times P$, where P and K are integers. What can be the maximum value of K?
A. 105
B. 102
C. 103
D. 104

134. The value of $\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ is
A. 2
B. 1
C. $\sqrt{7}$
D. $\sqrt{2}$

135. Consider the set S = {1, 2, 3, ..., 1000}. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements?
A. 4
B. 3
C. 7
D. 6
E. 8

136. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers?
A. 25
B. 67
C. 41
D. 73
E. 21

137. The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduate and above, to that of employees who are not, can possibly be
A. 97: 84
B. 87: 100
C. 85: 98
D. 101: 88
E. 110: 111

138. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?

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A. 6
B. 8
C. 10
D. 5
E. 7

139. The four numbers x, y, x + y and x – y are all prime numbers. Then the sum of these four prime numbers is
A. divisible by 7
B. divisible by 5
C. even
D. prime
E. divisible by 3

140. A three-digit number in base 10 is written in base 9 and base 11 to give two numbers N_1 and N_2,respectively. What is the probability that $N_1$and N_2 are also three-digit numbers?
A. 0.33
B. 0.88
C. 0.67
D. 0.55
E. 0.42

141. The single digits *a* and *b a*re neither both nine nor both zero. The repeating decimal 0.abababab... is expressed as a fraction in lowest terms. How many different denominators are possible?
A. 4
B. 6
C. 5
D. 3

142. The last three digits of a number N are *x*25. For how many values of *x* can N be the square of an integer?
A. 2
B. 5
C. 4
D. 3

143. What is the remainder when 7^{7^7} is divided by 13?
A. 10
B. 6
C. 7
D. 1

144. What is the value of n such that n! = 3! × 5! × 7!
A. 10
B. 11
C. 8
D. 9

145. What is the value of N such that N × [N] = 27, where [N] represents the greatest integer less than or equal to N?
A. 5.4
B. 5.8
C. 6.1

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D. 5.6

146. How many integers between 100 and 900 have the sum of their digits equal to 12?

A. 80
B. 62
C. 66
D. 82

147. The digits 1, 2, 3, 4, and 5 are each used once to compose a five-digit number ***abcde*** such that the three-digit number ***abc*** is divisible by 4, ***bcd*** is divisble by 5, and ***cde*** is divisble by 3. Find the digit **a**.

A. 1
B. 2
C. 3
D. 4

148. A gadha never lives up to 100 years because its stupidity gets it killed. Dhondu and Bhondu are the cutest gadhas in Donkeyland. When you write Dhondu's age followed by Bhondu's age, you get a four-digit perfect square. After 31 years, if you write their ages in the same order you again obtain a four-digit perfect square. How old is Dhondu?

A. 20
B. 12
C. 14
D. 10

149. Let $N = 2^{15} \times 3^{12}$. How many factors of N^2 are less than N but do not divide N completely?

A. 387
B. 180
C. 208
D. 310

150. In how many ways can 2004 be written as a sum of two or more consecutive positive integers?

A. 2
B. 5
C. 4
D. 3

151. The smallest positive integer N such that $\sqrt{N} - \sqrt{N-1}$ is less than 0.01 is

A. 2502
B. 2500
C. 2501
D. 2499
E. 2498

152. The product of the ages of some teenagers is 10584000. The sum of their ages is equal to

A. 86
B. 88
C. 85
D. 89
E. 87

153. In a village of 2029 inhabitants, at least x villagers have the same English initials for their first name and their surname. The least possible value of x is

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A. 4
B. 3
C. 6
D. 5
E. 2

154. What are the last two digits of $\frac{1}{5^{903}}$?

A. 08
B. 48
C. 36
D. 18

155. A Number N is divisible by 10, 90, 98 and 882 but it is not divisible by 50 or 270 or 686 or 1764. It is also known that N is a factor of 9261000. What is N?

A. 4410
B. 22050
C. 13230
D. 8820

156. Every digit of a number n is equal to 1, i.e. n = 1111…Given that every digit of $X = an^2 + bn + c$ is also 1, for any value of n (a, b, c are constant integers and $a > 0$), then b

A. is equal to 3
B. is equal to 2
C. is equal to 1
D. can have infinite values

157. My grandfather said he was 84 years old but he was not counting the Sundays. How old my grandfather really was?

A. 64
B. 97
C. 96
D. 98

158. What are the last **two** digits of $2^{2^{2003}}$?

A. 56
B. 36
C. 76
D. 16

159. Let S = 1630/4542. If you swap two digits of the numerator of S with two digits of the denominator, the fraction is equal to 1/3. What is the sum of the digits of the denominator of the new fraction?

A. 12
B. 8
C. 9
D. 15

Answers:

1. C.

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The unit digits of three consecutive odd numbers would be (1, 3, 5), (3, 5, 7), (5, 7, 9), (7, 9, 1) and (9, 1, 3). As the unit digit of the product is 7, only the last triplet of units digits, i.e. (9, 1, 3) will qualify. Therefore, we need to find three consecutive odd numbers ending in 9, 1 and 3 such that there product is 531117. We can see that $80^3 = 512000$. Therefore, the numbers would be lying around 80. The numbers are 79, 81 and 83 and the sum is 243.

2. C.
$\frac{N}{2}$ is a perfect square $\Rightarrow \frac{N}{2} = x^2$. $\frac{N}{3}$ is a perfect cube $\Rightarrow \frac{N}{3} = y^3$. $\frac{N}{2} \times \frac{N}{3} = x^2y^3 \Rightarrow N = \sqrt{6x^2y^3} = xy\sqrt{6y}$. For N to be a natural number, 6y has to be a perfect square $\Rightarrow y = 6 \Rightarrow N = 2^3 \times 3^4 \Rightarrow$ number of divisors = 20.

3. D
Finding the powers of all the prime factors in 30! We obtain, $30! = 2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$. Removing 27×57 for the number of zeroes, we obtain $2^{19} \times 3^{14} \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$. We need to find the units digit of this product to find the rightmost non-zero digit. The units digit of the product = units digit of $8 \times 9 \times 1 \times 1 \times 9 \times 7 \times 9 \times 3 \times 9 = 8$.

4. B
The number which has 6 divisors will be of the form ab^2 or c^5, where a, b and c are prime numbers. The numbers are 2×11^2, 5×7^2, 11×5^2, 23×3^2, 29×3^2, 31×3^2, 53×2^2, 59×2^2, 61×2^2, 67×2^2, 71×2^2, 73×2^2 and 3^5.

5. B
The ages will be lying on either side of 1900 for us to have two solutions to same situation. Let the birth year of grandfather be 18ab. Therefore, his age in 1936 = 1936 – 18ab = 36 + 1900 – 18ab = 36 + 100 – ab. This should be equal to ab $\Rightarrow$ ab = 36 + 100 – ab $\Rightarrow$ ab = 68. Similarly, the birth year for grandson is 1918. The sum of ages = 68 + 18 = 86

6. A
10^{12} = 1 000 000 000 000. Since the number is greater than 10^{12} and the sum of the digits is 2, one of the zeroes in
1 000 000 000 000 will be replaced by 1. Therefore, there will be 12 numbers generated this way. Apart from this, the number 2 000 000 000 000 also satisfies the criterion. Hence, 13 numbers are possible.

7. D
$(1 + x)^n - 1 - nx$ is always divisible by n2. $2^{4n} - 15n - 1 = (1 + 15)^n - 15n - 1 \Rightarrow$ the expression is divisible by $15^2 = 225$.

8. B
Every odd square can be written in the form 4k + 1. Let the three squares be 4a + 1, 4b + 1, 4c + 1. Therefore, sum = 4(a + b + c) + 3 = 4k + 3. Therefore, subtracting 3 from these options we should get a multiple of 4. Only B and D fulfill the criterion. Also, every square will be of the form 3n or 3n + 1. Out of three consecutive odd numbers, one will be a multiple of 3. Therefore, the squares will be of the form 3p + 1, 3q + 1 and 3r. Their sum is of the form 3k + 2. Therefore, subtracting two from the options, we should get a multiple of 3. Out of B and D, only B satisfies the criterion.

9. C
The numbers are ab, ba and a0b and the difference is constant between two consecutive numbers. 10b + a – (10a + b) = 100a + b – (10b + a) $\Rightarrow$ b – a = 11a – b $\Rightarrow$ b = 6a. Therefore, the numbers are 16, 61 and 106. The sum of digits = 7

10.D

Single digit square = 3 (1, 3, 9) Digits written = 3
Two-digit squares = 6 (16, 25, … 81) Digits written = $2 \times 6 = 12$.
Three-digit squares = 22 (100, 121, … 961) Digits written = $3 \times 22 = 66$
Total digits written so far = 3 + 12 + 66 = 81. Digits left = 100 – 81 = 19.
After 961, we will start writing four-digit squares – 1024, 1089… with every square we cover four digits. Since we need to cover 19 digits, we will have to write 4 four-digit squares and then we will see the 3^{rd} digit of the fifth four-digit square. The fifth four-digit square = $36^2 = 1296 \Rightarrow 3^{rd}$ digit = 9. Therefore, 100^{th} digit = 9.

11.B

Rule: $a^n - b^n$ is divisible by both a + b and a – b when n is even.

$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 5^{2000} + 12^{2000} - 1^{2000} \Rightarrow 18^{2000} - 5^{2000}$ is divisible by 13 and 23. Similarly, $12^{2000} - 1^{2000}$ is divisible by 11 and 13. As 13 is the common factor, the whole expression is divisible by 13.
$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 1 + 12^{2000} - 5^{2000} \Rightarrow 18^{2000} - 1$ is divisible by 17 and 19. Similarly, $12^{2000} - 5^{2000}$ is divisible by 7 and 17. As 17 is the common factor, the whole expression is divisible by 17.
As the expression is divisible by both 13 and 17, it is divisible by 221.

12.A

$2^{600} \times 5^{600}$ will have 601 digits (1 followed by 600 zeroes). To get 604 digits we will have to increase the power of 2 such that we get a four digit number followed by 600 zeroes. The power should also be a multiple of 3 as $8 = 2^3$. The smallest such power is $2^{12} = 8^4$. Therefore, $S = 8^{204} \times 5^{600} = 4096000000\ldots$ (604 digits). The sum of the digits of S = 19.

13.C

$216 = 6 \times 6 \times 6 \times 1 \times 1 \Rightarrow$ Sum = 6 + 6 + 6 + 1 + 1 = 20
$216 = 3 \times 6 \times 6 \times 2 \times 1 \Rightarrow$ Sum = 3 + 6 + 6 + 2 + 1 = 18
$216 = 3 \times 6 \times 3 \times 2 \times 2 \Rightarrow$ Sum = 3 + 6 + 3 + 2 + 2 = 16
$216 = 3 \times 6 \times 3 \times 4 \times 1 \Rightarrow$ Sum = 3 + 6 + 3 + 4 + 1 = 17

We can check that the sum of 19 cannot be obtained.

14.A

$10^4 + 324 = (10^2 + 18)^2 - 2 \times 10^2 \times 18 = (10^2 + 18)^2 - 36 \times 10^2 = (10^2 + 18)^2 - (6 \times 10)^2 = (10^2 + 18 + 60) \times (10^2 + 18 - 60) = 178 \times 58$.
Simplify the rest similarly. Many terms will cancel out.

15.D

Let's see the last two digits of summation in the groups of 4.
$3 + 3^2 + 3^3 + 3^4 = 03 + 09 + 27 + 81 = 20$
$3^5 + 3^6 + 3^7 + 3^8 = 43 + 29 + 87 + 61 = 20$
$3^9 + 3^{10} + 3^{11} + 3^{12} = 83 + 49 + 47 + 41 = 20$

Similarly, for 7,
$7 + 7^2 + 7^3 + 7^4 = 07 + 49 + 43 + 01 = 00$
$7^5 + 7^6 + 7^7 + 7^8 = 07 + 49 + 43 + 01 = 00$

$(3 + 3^2 + 3^3 + 3^4) + (3^5 + (3^6 + 3^7 + 3^8) + \ldots + 3^{400} = (20) + (20) + \ldots + 20 = 00$
$(7 + 7^2 + 7^3 + 7^4) + (7^5 + 7^6 + 7^7 + 7^8) + \ldots + 7^{200}) + 7201 = 00 + 00 + 00 + \ldots + 00 + 07 = 07$

Therefore, difference = 00 – 07 = 93.

16.B

4 integers- 0, 10, 16, and 18.

17.A

The number of numbers prime to and less than 900 = $900(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 240$

18.D

The units digit is found by finding the remainder of the exponent with 4. 4! and onwards the remainder with 4 will be 0. Therefore, we need to find the remainder of 1! + 2! + 3! with 4 $\Rightarrow$ remainder = 1. Therefore, units digit of the expression = units digit of $63^1 + 18^1 + 37^1 = 8$

19.C

Let's see the cycle of the last two digits for powers of 4

$4^2 = 16$
$4^3 = 64$
$4^4 = 56$
$4^5 = 24$
$4^6 = 96$
$4^7 = 84$
$4^8 = 36$
$4^9 = 44$
$4^{10} = 76$
$4^{11} = 04$.

Therefore, 4^1 and 4^{11} have the same last two digits, i.e. the last two digits of 4 repeat after every increase of 10 in the exponent. Therefore, 4^{1991} will have the same last two digits. Therefore, 4^{1997} will have the same last two digits as $4^7 = 84$.

20.A

$123456789 \times 999999999 = 123456789 \times (1000000000 - 1) = 123456789000000000 - 123456789 = 123456788876543211$

21.B

When the divisors of $N = X^aY^bZ^c$ are multiplied the product is equal to $(N)^{\frac{(a+1)(b+1)(c+1)}{2}}$. When the divisors of $72 = 2^3 \times 3^2$ are multiplied, the product is equal to $(2^3 \times 3^2)^{\frac{(2+1)(3+1)}{2}} = 2^{18} \times 3^{12}$.
Therefore, a + b =30

22.A

If a number is divisible by 72, it is divisible by 8 and 9. To be divisible by 8, the number formed by the last three digits should be divisible by. Therefore, 31B should be divisible by 8 $\Rightarrow$ B = 2. To be divisible by 9, the sum of the digits of the number should be divisible by 9 $\Rightarrow$ A = 3. Therefore, A + B = 5

23.C

A digit when repeated 6 times is divisible by 7. Therefore, 111 111 would be a multiple of 7 $\Rightarrow$ = 111 111. Therefore, N = P/7 = 15873 $\Rightarrow$ 8P = 126984.

24.B

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$7^3 = 343$ gives remainder 1 with 9. $7^{77} = 7^{75} \times 7^2$ = remainder with 9 $(1 \times 49) = 4$.

25.D
There is only one perfect square in the nineteenth century 432 = 1849. Therefore, the man was 43 years old in 1849. Therefore, he was 78 years old in 1884.

26.A
The average = $\frac{9+99+999+...+999999999}{9} = 1+11+111+...+111111111 = 123456789$. Therefore, the average does not contain the digit 0.

27.A
Only perfect divisors have odd number of divisors. Number of perfect divisors less than 65 = 8 (1, 4, 9, 16, 25, 36, 49, 64)

28.D
After the first cycle, 80 people will remain and 99^{th} person will be the last. After the second cycle 64 people will remain and 98^{th} person will be the last. Now, even after the subsequent cycles, the 98^{th} person will not leave the queue as the number of remaining people will never be a multiple of 5. Note that the last person leaves the queue only if the number of remaining people is a multiple of 5.

29.D
Do it by induction, multiplying 33 and 66, 333 and 666 and so on..

30.B
Ensure that the sum of the sum of the sum of the digits will result in a single digit number. As 55! is a multiple of 9, the digit will be 9 only.

31.C
The king was born in 1600 (40^2) and died in 1681 (41^2). He lived for 81 years.

32.A
For page number 1 to 9, single digits will be used per page. Total digits used = 9
From page number 10 to 99, two digits will be used per page. Total digits used = $2 \times 90 = 180$.
Total digits used till page number 99 = 9 + 180 = 189.
Digits left = 300 – 189 = 111.
From page 100 onwards, three digits will be used per page. Therefore, number of pages required to cover 111 digits = 111/3 = 37.
Therefore, the book has 136 pages.

33.D
As the L.H.S. is a multiple of 32 = 9, R.H.S. will also be a multiple of 9. Therefore, 2 + 2 + 7 + 5 + 2 + S = 18 + S will be divisible by 9. Therefore, S = 0 or 9. But if S is zero 3(150 + S) will end in a zero and its square will end in two zeroes. But the tens digit of R.H.S is not zero. Therefore, S = 9.

34.D
$a^n + b^n$ is divisible by a + b when n is **odd** $\Rightarrow 5^{2n+1} + 17^{2n+1}$ is divisible y 17 + 5 = 22 $\Rightarrow 5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ is divisible by 11. The expression is also divisible by 3. Therefore, it is divisible by 33. **NOTE:** Keeping n = 0 will give the answer straightaway.

35.B

$5^{36} - 1 = (4 + 1)^{36} - 1 = 4^{36} + 36 \times 4^{35} + \ldots + 36 \times 4 + 1 - 1 = 4^2(4^{34} + 9 \times 4^{34} + \ldots + 9)$. Therefore, the highest power of 4 is 2.
$5^{36} - 1 = (6 - 1)^{36} - 1 = 6^{36} - 36 \times 6^{35} + \ldots - 36 \times 6 + 1 - 1 = 6^3(633 - 6^{34} + \ldots - 1)$. Therefore, the highest power of 3 is 3. Therefore, the highest power of 12 is 2.

36.A

997 occurs in 997!, 998!, 999!, 1000!, and 1001!

37.B

Let $p = n$, $q = n + 1$ and $r = n(n + 1) \Rightarrow p^2 + q^2 + r^2 = n^2 + n^2 + 1 + 2n + n^2(n^2 + 2n + 1) = n^4 + 2n^3 + 3n^2 + 2n + 1 = (n^2 + n + 1)^2 \Rightarrow S = n^2 + n + 1 = n(n + 1) + 1$ = even number + odd number = odd number.

38.D

39.C

The number is the sum of consecutive factorial and it is less than 500. Therefore, the number cannot have 6! Or above in the sum of consecutive factorial as 6! = 720 > 500. As the number is a three-digit number it will certainly have 5! in the sum of consecutive factorial. Now the following possibilities are there for the number-
5! + 4! = 144
5! + 4! + 3! = 150
5! + 4! + 3! + 2! = 152
5! + 4! + 3! + 2! + 1! = 153.
Now the number is the sum of the cube of its digits. Only 153 satisfies the criterion. Sum of the digits of 153 = 9.

40.B

$(222)_b = 6162 \Rightarrow 2b^2 + 2b + 2 = 6162 \Rightarrow b = 55$

41. A

The number is 144 which is a perfect square. 441 is also a perfect square. 1444 is also a perfect square.

42. C

The unit digit of $53^{53} = 3$. Unit digit of $27^{27} = 3$. Therefore, unit digit of $53^{53} - 27^{27} = 0$. Therefore, the expression is divisible by 10.

43.B

Units digit of $43^{444} = 1$. Units digit of $34^{333} = 4$. Therefore, units digit of expression = 5 $\Rightarrow$ the expression is divisible by 5.

44.C

Let the two digit number be ab. The value of this number = 10a + b. By reversing the digits, the number becomes ba, with value 10b + a.
$10a + b + 10b + a = 11(a + b)$ = a perfect square $\Rightarrow a + b = 11 \Rightarrow$ The pairs are (2, 9), (3, 8), (4, 7), (5, 6).
$10a + b - (10b + a) = 9(a - b)$ = a perfect square $\Rightarrow a - b = 1$, 4 or 9. Only the pair (5, 6) satisfies one of the three differences. Therefore, 65 is the age of the teacher and it is divisible by 13.

45.B

Adding the unit digit of the numbers on L.H.S., we get 3 + 4 + 3 = 10. But the unit digit is 3. Therefore, base x is 3 less than 10 (the sum of units digit) or x = 7.

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46.A

The number of numbers less than and prime to 13 = 12. Therefore, 2^{12} gives remainder 1 with 13. $2^{2005} = 2^{2004} \times 2 = (2^{12})^{167} \times 2 \Rightarrow$ Remainder $(1 \times 2) = 2$.

47.D

Let $(1 + 2x + 3x^2 - 4x^3)^{10} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{30}x^{30}$. Keeping x = 1, we get $2^{10} = a_0 + a_1 + a_2 + \ldots + a_{30}$.

48.B

The number of numbers less than and prime to 120 = $120\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 32$

49.D

Numbers from 5! and onwards are divisible by 20. Therefore we need to find the remainder when 1! + 2! + 3! + 4! is divided by 20. 1! + 2! + 3! + 4! = 33 $\Rightarrow$ remainder = 13.

50. B

$3^{27^x} = 3^{(3^3)^x} = 3^{3^{3x}}$

$27^{3^x} = (3^3)^{3^x} = 3^{3\times 3^x} = 3^{3^{x+1}}$

$\Rightarrow 3x = x + 1 \Rightarrow x = 1/2$.

51. A

$$\frac{1}{\sqrt{2}-\sqrt{1}} = \frac{1}{\sqrt{2}-\sqrt{1}} \times \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}} = \sqrt{2}+\sqrt{1}$$

$$\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots + \frac{1}{\sqrt{121}-\sqrt{120}} \sqrt{1}+\sqrt{2}-\sqrt{2}-\sqrt{3}+\sqrt{3}+\sqrt{4}-\sqrt{4}-\sqrt{5}+\ldots+\sqrt{121}+\sqrt{120} = 11$$

+ 1 = 12

52.B

$(n + 1)^3 - n^3 = 1027 \Rightarrow 3n^2 + 3n - 1026 = 0 \Rightarrow n^2 + n - 342 = 0 \Rightarrow n = 18$.

53. B

54. A

All multiples of 4 and all odd numbers can be written as difference of squares of non-negative integers. Odd numbers in first 1000 natural numbers = 500. Multiples of 4 = 250. Therefore, total numbers = 750.

55.B

P = abc = 9(a + b + c). Let a = b + c $\Rightarrow$ abc = 18a $\Rightarrow$ bc = 18 $\Rightarrow$ b, c = (1, 18), (2, 9), (3, 6) $\Rightarrow$ a = 19, 11, 9 $\Rightarrow$ abc = 342, 198, 162 $\Rightarrow$ Sum = 702

56.B

As every digit is used only once, the highest number would have 9876543 as the first 7 digits. Now we need to use 0, 1, 2 at the end such that the number is divisible by 8. The only number we can form is 120. Therefore, M = 9876543120 $\Rightarrow$ remainder by 1000 = 120

57.C

Total Gadha

Let M = 100q + r = 99q + q + r $\Rightarrow$ q + r = M – 99q. As 99q is divisible by 99, we need to find values of M that are divisible by 99. The smallest five digit number divisible by 99 = 10098. The highest five-digit number divisible by 99 = 99990. Therefore, the number of value of M = $\frac{99990-10098}{99}+1=909$

58.D

Keep the values. Neither of the statements are true.

59.C

We first do the prime factorization of the product by finding the powers of the prime factors in all the factorials.

Powers of 2: 2! = 1, 3! = 1, 4! = 3, 5! = 3, 6! = 4, 7! = 4, 8! = 7, Total = 23

Powers of 3: 3! = 1, 4! = 1, 5! = 1, 6! = 2, 7! = 2, 8! = 2. Total = 9

Powers of 5: 5! = 1, 6! = 1, 7! = 1, 8! = 1. Total = 4

Powers of 7: 7! = 1, 8! = 1. Total = 2.

Therefore, the product = $2^{23} \times 3^9 \times 5^4 \times 7^2$.

Number of divisors that are perfect square = (number of even powers of 2) $\times$ (number of even powers of 3) $\times$ (number of even powers of 5) $\times$ (number of even powers of 7) = $12 \times 5 \times 3 \times 2$ = 360.

60.D

When we write 2006! In base 22, we successively divide 2006! by 22 and keep writing down the remainders. The first remainder will become the units digit, the second remainder will become the tens digit, the third remainder will become the hundreds digit and so on. Therefore, the number of zeroes that 2006! written in base 22 will have will be equal to the number of times 22 divides 2006! completely. The number of times 22 divides 2006! completely is equal to the highest power of 22 in 2006! or equal to highest power of 11 in 2006!.

The highest power of 11 in 2006! = $\left[\frac{2006}{11}\right]+\left[\frac{2006}{121}\right]+\left[\frac{2006}{1331}\right]=182+16+1=199$

61.D

Number of numbers less than and prime to 9 = $9\left(1-\frac{1}{3}\right)=6$. Therefore, 256 will give remainder 1 with 6. $25^{25^{25}}=25^{(24+1)^{25}}=25^{6k+1}=(25^6)^k\times 25 \Rightarrow$ Remainder = $1 \times 7 = 7$

62.B

7^4 gives a unit digit of 1. $7^{7^7}=7^{(8-1)^7}=7^{4k+3}=(7^4)^k\times 7^3=$ units digit $1\times$ units digit $3=3$

63.A

For any natural number x, x^6 will give remainder 1 with 7 whereas x^3 will give remainder 1 or 6 with 7. The given expression will never give remainder 0 with 7 and hence will never be divisible by 7.

64.D

Number of ways 105 can be written as a sum of two or more consecutive positive integers = number of odd divisors of 105 – 1 = 8 – 1 = 7.

65.A

m and n are 7 and 8. Therefore, m + n = 15

66.C

Let the 5 number be a – 2, a – 1, a, a + 1, a + 2.their sum = 5a which is a multiple of 5. As P/S is divisible by 100 (a multiple of 25) and S is a multiple of 5, P should have a factor of 5^3 in it.

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Therefore, 125 is a multiple of P. Therefore, the five consecutive numbers will be 121, 122, 123, 124, and 125. Their sum = 615.

67.A

$10! = 2^8 \times 3^4 \times 5^2 \times 7$. Let the numbers be a, b, c and d. therefore, $a \times b \times c \times d = 10! = 2^8 \times 3^4 \times 5^2 \times 7$. For minimum sum, we take all the four numbers to be equal or nearly equal. Therefore, $a4 = 2^8 \times 3^4 \times 5^2 \times 7 \Rightarrow a = 2^2 \times 3 \times (5^2 \times 7)^{1/4} \approx 43.64$. We have to take 4 numbers near 43. $42 = 2 \times 3 \times 7$, $40 = 2^3 \times 5$, $45 = 3^2 \times 5$ and $48 = 2^4 \times 3$. Therefore, $40 \times 42 \times 45 \times 48 = 2^8 \times 3^4 \times 5^2 \times 7$. Their sum = 175.

68.D

$$\begin{array}{r} 1ABCDE \\ \times\ 3 \\ \hline ABCDE1 \end{array}$$

Going from right to left, E = 7, D = 5, C = 8, B = 2, A = 4. The sum of the digits = 27

69.D

$5x^2 + 2y^2 = 5922$, the R.H.S. is even and therefore, L.H.S. should also be even $2y^2$ is even therefore, $5x^2$ should also be even, therefore x will be even $\Rightarrow 5x^2$ will end in 0. As the unit digit of R.H.S. is 2, the unit digit of $2y^2$ should also be 2 $\Rightarrow$ unit digit of y^2 should be 1 $\Rightarrow$ unit digit of y can be 9.

70.C

The sum of the squares of three odd numbers will be in the form 4k + 3. Only C satisfies the criterion.

71.B

S = 333…3334 (33 digits). Sum of digits = $3 \times 32 + 4 = 100$

72.A

Let the number of slips with 14 written on them be x. Therefore, sum = 14x + 13(x + 5) = 27x + 65. Therefore, the number should be of the form 27k + 65. Only A satisfies the option.

73.C

$(212)_b = (128)_{b+2} \Rightarrow 2b^2 + b + 2 = (b + 2)^2 + 2(b + 2) + 8 \Rightarrow b^2 - 5b - 14 = 0 \Rightarrow b = 7 \Rightarrow 114$

74.B

75.C

$abc = 100a + 10b + c = 98a + 2a + 7b + 3b + c = 7k + 2a + 3b + c$.

76.D

$12^{12} = 2^{24} \times 3^{12}$, $6^6 = 2^6 \times 3^6$, $8^8 = 2^{24}$. The LCM is $12^{12} = 2^{24} \times 3^{12}$. 2^{24} will come from 8^8 but 3^{12} will have to come from k. Therefore, k will contain 3^{12}. Also, the powers of 2 in k can be from 0 to 24. Therefore, k can be equal to 3^{12}, 2×3^{12}, $2^2 \times 3^{12}$, …$2^{24} \times 3^{12}$. In total there 25 values.

77.D

$(abc)_7 = (cba)_9 \Rightarrow 49a + 7b + c = 81c + 9b + a \Rightarrow b = 24a - 40c = 8(3a - 5c)$. The solution to these equations are a , c = (2, 1) and (5, 3). Therefore b = 8 or 0. Therefore, a + b + c = 11 or 8

78.A

Total Gadha

$n^5 - 5n^3 + 4n = n(n^4 - 5n^2 + 4) = (n-2)(n-1)n(n+1)(n+2)$. These are five consecutive numbers therefore they will be divisible by 5! = 120.

79. D

$(x + y)(x - y) = 343$

80. C

$6^{83} + 8^{83} = (7+1)^{83} + (7-1)^{83} = (1 + 83 \times 7 + \ldots + 7^{83}) + (-1 + 83 \times 7 - \ldots + 7^{83}) = 2 \times 83 \times 7$ + (a term divisible by 49). Remainder = 35.

81. C

Let 320 = x. Therefore the given expression is $f(x) = 81x^4 + 27x^3 + 9x^2 + 3x + 1$. To find the remainder by x + 1, keep x = –1.

82. A

The divisors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180 and 360. We can see that $360 = 1 \times 360 = 2 \times 180 = 3 \times 120 = \ldots = 18 \times 20$.

Therefore, $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{180}+\frac{1}{360}=\frac{360}{360}+\frac{180}{360}+\frac{120}{360}+\ldots+\frac{2}{360}+\frac{1}{360}=\frac{1+2+3+4+5\ldots+180+360}{360}=\frac{1170}{360}=3.25$

83. B

$\frac{x}{|x|}$ is equal to either 1 or –1, depending on whether x is positive or negative. Taking zero negative, one negative, two negative, three negative or four negative out of a, b, c, and d, we obtain the values of $S=\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}$ to be {4, 0, –4} and the values of $T=\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{d}{|d|}+\frac{abcd}{|abcd|}$ equal to {5, 1, –1, –3}. Hence the intersection of S and T is a null set.

84. C

85. C

$\underbrace{123123123.....123123}_{300 \text{digits}} = 123\underbrace{1001001....1001}_{298 \text{ digits}} = 123(10^{297} + 10^{294} + 10^{291} + \ldots + 10^3 + 1)$. 10^{odd} gives a remainder of 10 with 99 whereas 10^{even} gives a remainder of 1 with 99. Therefore, in the expression $10^{297} + 10^{294} + 10^{291} + \ldots + 10^3 + 1$, we will get 50 and 50 ones when we divide by 99. Also, 123 gives remainder 24 with 99.

Therefore, remainder = $24(50 \times 10 + 50)$ = remainder when 24×550 is divided by 99 = 33.

86. B

The digit sum of the number = $1 + 2 \times 2 + 3 \times 3 + 4 \times 4 = 30 = 3 + 0 = 3$. Therefore, the number cannot be a perfect square as the digit sum of perfect squares are always 1, 4, 7 or 9.

87. A

Total Gadha

Box	Oranges	Box	Oranges	Box	Oranges	Box	Oranges	Box	Oranges	Box	Oranges
1	120	26	120	51	120	76	120	101	120	126	120
2	121	27	121	52	121	77	121	102	121	127	121
3	122	28	122	53	122	78	122	103	122	128	122
4	123	29	123	54	123	79	123	104	123		
5	124	30	124	55	124	80	124	105	124		
6	125	31	125	56	125	81	125	106	125		
7	126	32	126	57	126	82	126	107	126		
8	127	33	127	58	127	83	127	108	127		
9	128	34	128	59	128	84	128	109	128		
10	129	35	129	60	129	85	129	110	129		
11	130	36	130	61	130	86	130	111	130		
12	131	37	131	62	131	87	131	112	131		
13	132	38	132	63	132	88	132	113	132		
14	133	39	133	64	133	89	133	114	133		
15	134	40	134	65	134	90	134	115	134		
16	135	41	135	66	135	91	135	116	135		
17	136	42	136	67	136	92	136	117	136		
18	137	43	137	68	137	93	137	118	137		
19	138	44	138	69	138	94	138	119	138		
20	139	45	139	70	139	95	139	120	139		
21	140	46	140	71	140	96	140	121	140		
22	141	47	141	72	141	97	141	122	141		
23	142	48	142	73	142	98	142	123	142		
24	143	49	143	74	143	99	143	124	143		
25	144	50	144	75	144	100	144	125	144		

Therefore, minimum 6 boxes have same number of oranges

88.B

The maximum marks that can be achieved is 90 and the minimum marks that can be achieved is –30. Technically, from –30 to 90, we should have all the scores at a difference of $\frac{1}{3}$, i.e. $-30, -29\frac{2}{3}, -29\frac{1}{3}, -29, \ldots 0, \ldots 89, 89\frac{1}{3}, 89\frac{2}{3}, 90$. Therefore, there should be 90 + 1 + 270 = 361 scores possible. But we cannot achieve scores of $89\frac{2}{3}, 89\frac{1}{3}$ and $88\frac{1}{3}$ (think why). You can obtain rest of the scores.

Therefore, total number of scores possible = 361 – 3 = 358.

89.B

Total Gadha

The sum of n consecutive numbers starting with 1, i.e. $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

Now the average of these n numbers = $\frac{n(n+1)}{2n} = \frac{n+1}{2}$ which is definitely less than n. Now when we remove o

of the numbers the average is $\frac{355}{13} = 27.30$. Therefore, the number of numbers is definitely more than 27. No

look at the denominator, i.e. 13. The fraction $\frac{355}{13}$ is certainly a reduced fraction. Therefore, the initial fraction

got after using the formula Average = $\frac{\text{Sum of all numbers}}{\text{Number of numbers}}$ would be some whole number multiplied to both

numerator and denominator of $\frac{355}{13}$. Therefore, the original fraction can be $\frac{1065}{39}$, $\frac{1420}{52}$, $\frac{1775}{65}$, and so on...a

the original number of numbers that we took was one more than the denominator as the denominator represe
the number of numbers after removing one number.

If the original fraction was $\frac{1065}{39}$, you had originally taken 40 consecutive numbers. (sum = $\frac{40 \times 41}{2} = 820$. N

possible.)

If the original fraction was $\frac{1420}{52}$, you had originally taken 53 consecutive numbers. (sum = $\frac{53 \times 54}{2} = 1431$. T

missing number = 1431 – 1420 = 13)

If the original fraction was $\frac{1775}{65}$, you had originally taken 66 consecutive numbers. (sum = $\frac{66 \times 67}{2} = 2211$. T

missing number = 2211 – 1775 = 436. Not possible.)

Therefore, the missing number is 13.

90.D
We can solve this by finding the last two digits of every number. The last two digits of every term (except the first one) will be 11. Therefore, last two digits of the expression = $1 + 9 \times 11 = 1 + 99 = 00$. Therefore, the expression is divisible by 100.

91.C
$\sqrt{1332} = 6\sqrt{37}$. Therefore, $\sqrt{x}$ and $\sqrt{y}$ should be of the form $a\sqrt{37}$ and $b\sqrt{37}$ where a + b = 6.
Therefore, pairs of (a, b) = (0, 6), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0).

92.B
Every square can be written in the form 3k or 3k + 1. $p^2 + 15p - 1 = (3k + 1) + 15p - 1 = a$ multiple of 3. Therefore, it cannot be a prime number. If p = 3, $p^2 + 15p - 1 = 53$. Therefore, the expression is prime for only one value of p.

93. Let $f(x) = x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1$.
Remainder by $x - 1 = 1 \Rightarrow f(x) = G(x)(x - 1) + 1$ --- (1)
Similarly, $f(x) = H(x)(x - 2) + 7$. --- (2)
Multiply 1 by (x – 2) and 2 by (x – 1), and subtract.

94.C

Total

95. $2^6 - 1 = (2^3)^2 - 1$ which is a multiple of $2^3 - 1$ as $a^n - b^n$ is always divisible by $a - b$. similarly $2^9 - 1$ is also a multiple of $2^3 - 1$. Therefore, HCF = $2^3 - 1$. Now HCF $\times$ LCM = product of two numbers.
$\Rightarrow (2^3 - 1) \times LCM = (2^6 - 1) \times (2^9 - 1) \Rightarrow LCM = 2^{12} + 63 \times 2^3 - 1$.

96. The pair will be (45a, 45b) where a and b will be co-prime to each other. Now $21600 = 2^5 \square 3^3 \square 5^2$. To find a and b, we first take the factor of 45 from 21600, which leaves $2^5 \square 3 \square 5$. Now we need to find the number of co-prime pairs (a, b) that we can make out of $2^5 \square 3 \square 5$. Let's write down the powers of the prime factors in order to find the co-prime factors: $(2, 2^2, 2^3, 2^4, 2^5)$, 3, 5

Therefore, the number of co-prime pairs is found by various combinations of these prime factors:

(Prime factor, Prime factor) – $(2, 3), (2^2, 3), (2^3, 3) \ldots (2^4, 5), (2^5, 5), (3, 5)$ ----- 11 in number
(Two prime factors, prime factor) – $(2 \times 3, 5), (2^2 \times 3, 5), \ldots (2^4 \times 5, 3), (2^5 \times 5, 3), (3 \times 5, 2), \ldots (3 \times 5, 2^5)$ --- 15 in number
(1, prime factor) – $(1, 2), (1, 2^2) \ldots (1, 2^5), (1, 3), (1, 5)$ --- 7 in number
(1, two prime factors) – $(1, 2 \times 3), (1, 2^2 \times 3), \ldots (1, 2^4 \times 5), (1, 2^5 \times 5), (1, 3 \times 5)$ --- 11 in number
(1, three prime factors)- $(1, 2 \times 3 \times 5), (1, 2^2 \times 3 \times 5), \ldots (1, 2^5 \times 3 \times 5)$ --- 5 in number.

Therefore, total number of co-prime pairs (a, b) = 49.

97. A

98. $x^2 - 3x + 2 = (x - 1)(x - 2)$
Remainder when x^{100} is divided by $x - 1 = 1$. Remainder when x^{100} is divided by $x - 2 = 2^{100}$. Therefore, $x^{100} = Q(x)(x - 1) + 1$ and $x^{100} = R(x)(x - 2) + 2^{100}$. Multiply first equation by $x - 2$ and the second equation by $x - 1$ and subtract. The result is
$X^{100} = H(x)(x - 1)(x - 2) + x(2^{100} - 1) + 2 - 2^{100}$. Therefore, we get the values of a and b.

99. B
The number will be of the form $a \times b$ or c^3, where a, b and c are prime factors. Count all the numbers below 100 of the above form.

100. C
The problem text is "What is the minimum value of *n* which **ensures** that both the conditions for S are fulfilled?"
Suppose you have 6 red balls and 6 black balls in a bag. If I blindfold you and ask you to pick out balls from the bag without seeing their colours, how many balls you will have to take out of the bag in order to ensure that you have at least one black ball?

You can give the answer to this question as 1, i.e. you pick out only one ball and it's black. But to '**ensure**' you will have to assume the **worst-case-scenario**. Assume that you keep on picking out red balls. Therefore, you will choose 6 red balls first. The seventh ball will certainly be black. Therefore, you need to choose 7 balls in order to ensure the condition. You **MIGHT** pick out a black ball before picking up the 7th ball **but by picking up 7 balls you are certainly ensuring that you have at least one black ball**.

Similarly, you might satisfy the condition through two or three numbers only, but by picking up four numbers you are certainly ensuring that you will satisfy the conditions in all possible scenarios.

TotalGadha

101. B

102. A

103. D

The last two digit of $11^{11} = 11$, last two digits of $111^{111} = 11$..and so on. The last two digits of every term except the first term are 11. Therefore, the sum of the last two digits = 01 + 11 + 11 + 11… + 11 (9 times) = 01 + 99 = 100. As the last two digits of the expression are 00, the expression is divisible by 100. Therefore, remainder = 0.

104. C

$16^3 + 19^3$ is divisible by 16 + 19 = 35, ($a^n + b^n$ is divisible by a + b is n is **ODD**). Similarly, $17^3 + 18^3$ is divisible by 35. Also, the expression contains two even and two odd terms. Therefore, the sum is even. Therefore, the whole expression is also divisible by 2 $\Rightarrow$ it is divisible by 70.

105. A

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})$

$\Rightarrow 30^{65} - 29^{65} = (30 - 29)(30^{64} + 30^{63}29 + 30^{62}29^2 + \ldots + 29^{64}) > 30^{64} + 29^{64}$.

106. C

Let A = abc and B = cba. Therefore, B – A = 100c + 10b + a – (100a + 10b + c) = 99(c – a). B – A is a multiple of 7 $\Rightarrow$ c – a = 7 $\Rightarrow$ (a, c) = (1, 8) or (2, 9). Therefore, the number can be from 108 to 19 or from 209 to 299.

107. A

108. C

109. C

110. D

111. D

112. $a - xb = a + |a|\, b^2$. $|a| \geq a$ and $b^2 > 1$. Therefore, $a - xb > 0$.

113. A

114. When jumpy takes two jumps, he can end up at (2, 0), (1, 1), (0, 2), (–1, 1), (–2, 0), (–1, –1), (0, –2), (1, –1) or (0, 0). In short he can end up at the coordinates satisfying the equations $|x| + |y| = 2$ and $|x| + |y| = 0$. The number of integer points satisfying the equation $|x| + |y| = n$, where $n \neq 0$ is equal to 4n.

When Jumpy takes n jumps the integer points he can end up at are the solutions of the equations $|x| + |y| = n, n - 2, n - 4, \ldots$. If n is even, then the number of solutions are $4n + 4(n - 2), \ldots + 8$ and 1 (for 0, 0) = $n(n + 2) + 1 = (n + 1)^2$.

If n is odd, then the number of lattice points jumpy can end up at = $4 \times 1 + 4 \times 3 + 4 \times 5 + \ldots 4n = (n + 1)^2$.

Therefore, number of lattice points Jumpy can end up at in 12 jumps = 169.

115. The number of kilometers that the car has traveled = 1 000 – number of numbers which contain the digit 5 = 1000 – (1 + 18 + 252) = 729.

116. Number of zeroes at the end of 10 000! = $\left[\frac{10000}{5}\right]+\left[\frac{10000}{25}\right]+\left[\frac{10000}{125}\right]+\left[\frac{10000}{625}\right]+\left[\frac{10000}{3125}\right] = 2499$. Number of zeroes at the end of $(100!)^2$ = 2 × number of zeroes at the end of 100! = 2 × 24 = 48. Therefore, the number of zeroes at the end of given expression = 2499 − 48 = 2451.

117. N = D × Q + 52 ⇒ 5N = 5D × Q + 260. 5N gives remainder 4 with D. As 5DQ is divisible by D, 260 should give remainder 4 with D. Therefore, 260 − 4 = 256 should be divisible by D. Now D can have values equal to divisors of 256 which are greater than 52 (as D gives a remainder 52 it has to be greater than 52). The only values are 64, 128 and 256.

118. A

119. 111 = 37 × 3. Therefore we find the remainder when the expression is divided by both 3 and 37. Now we know that if x is prime to p, where p is a prime number, $(x)^{p-1} - 1$ is divisible by p. In other words, $(x)^{p-1}$ gives remainder 1 when divided by p. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 37. Therefore, total remainder with 37 = 1 + 1 = 2. Also, $17^{36} = (17^{18})^2 \Rightarrow$ will give remainder 1 with 3. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 3. Therefore, total remainder with 37 = 1 + 1 = 2. Therefore, remainder with 111 = 2.

120. $3^{36} - 1 = (3^3)^{12} - 1^{12} \Rightarrow$ divisible by $3^3 + 1 = 28 \Rightarrow$ divisible by 7. To find divisibility of a number by 7, we make groups of three from right to left, add odd-numbered and even-numbered groups separately, and then find the difference between the two sums. This difference should be divisible by 7.
1**A**009463**A**296999120 = (1**A**0)(094)(63**A**)(296)(999)(120). Sum of odd numbered groups = 120 + 296 + 094 = 510. Sum of even numbered groups = 999 + 63A + 1A0. Difference = 489 + 63A + 1A0. Remainder of 489 with 7 = 6 ⇒ 63A + 1A0 should give remainder 1 with 7 ⇒ A = 5.

121. Let $7^{13} = 7^a \times 7^b \times 7^c$. The powers should add upto 13 ⇒ (a, b, c)
= (0, 0, 13), (0, 1, 12), (0, 2, 11), (0, 3, 10)… (0, 6, 7),
(1, 1, 11), (1, 2, 10),… (1, 6, 6),
(2, 2, 9), (2, 3, 8)…. (2, 5, 6),
(3, 3, 7), (3, 4, 6), (3, 5, 5)
(4, 4, 5)
Total number of ways= 7 + 6 + 4 + 3 + 1 = 21

122. It can be seen that N is not a multiple of 2 and 3 as introducing a factor of 2 or 6 doubles and quadruples the number of factors. But N is a multiple of 5 as introducing 15 does not quadruples the number of factors. We see that taking N = 5 satisfies the given options. Now $30N = 2 \times 3 \times 5^2$ ⇒ factors = 12 = 6f.

123. 25! + 26! + 27! + 28! + 30! = 25! (1 + 26 + 27 × 26 + 28 × 27 × 26) + 30! = 25! × a number ending in 5 + 30! ⇒ 6 zeroes.

124. A
The number of zeroes = the highest power of 5 =
$\left[\frac{5^5}{5}\right]+\left[\frac{5^5}{25}\right]+\left[\frac{5^5}{125}\right]+\left[\frac{5^5}{625}\right]+\left[\frac{5^5}{3125}\right] = 625+125+25+5+1 = 781$

125. C
In two digit numbers, the highest power of 2 is contained in 64. Now we need to find ten consecutive two-digit numbers such that they include 64 and the power of 2 is highest in their product.

Total Gadha

By observation, we can see that 56 also contains a high power of 2, i.e. 23. Similarly, 72 also contains 23. Therefore, we can form groups of 10 consecutive numbers including 56 or 72. The groups are:
55, 56, 57, … 64
56, 57, 58, … 64, 65
63, 64, 65, … 71, 72
64, 65, 66, … 72, 73

4 groups in all.

126. C

$\frac{3}{40} = \frac{6}{80} = \frac{9}{120} = \frac{12}{160} = \frac{15}{200} = \frac{18}{240} = \ldots$

The numerator is going to be the sum of the digits and the denominator is going to be the product. To know the digits of the number, we can factorize the denominator. We can quickly check and see that we can find single digit numbers satisfying the conditions only for the ratio $\frac{18}{240}$ (2, 3, 8, 5). Therefore, we need to find a four-digit number made by 2, 3, 8, and 5, and which is divisible by 37.

The rule for divisibility of 37 is that we make groups of 3 from right to left and keep adding them. The number thus obtained should be divisible by 37. Mentally I got the number as 5328 because (328 + 5 = 333 --> divisible by 37).
$5328 = 2^4 \times 3^2 \times 37 \Rightarrow$ number of factors $= 5 \times 3 \times 2 = 30$

127. B

128. B

$10a + b = 6(a + b) \Rightarrow 4a = 5b \Rightarrow (a, b) = (5, 4)$

129. B

Find a number with more than 20 divisors, maybe 21, 22, 24 divisors. Then, remove some of the divisors such the sum of the rest of the divisors was 801. And Voila! I got my answer!

360 is the number with 24 divisors. The sum of these divisors is 1170. If you remove 360, 2, 3 and 4 from these divisors, you have 20 numbers whose sum is 801. You cannot find a number smaller than this.

Therefore, the smallest LCM is 360 and the 20 numbers are all the divisors of 360 minus 360, 2, 3, and 4.

130. B

131. Let N be a two-digit number ab $\Rightarrow 10a + b = 11(a + b) \Rightarrow$ no solution for positive a and b.
Let N be a three-digit number abc $\Rightarrow 100a + 10b + c = 11(a + b + c) \Rightarrow 89a = b + 10c \Rightarrow$ (a, b, c) = (1, 9, 8). Therefore, the number is 198 and it is divisible by 9.

132. C

133. In this case, we check the powers of both the highest and the lowest prime number contained in 100!. The highest and the lowest prime numbers in 100! are 97 and , respectively, and their highest power in 100! are 1 and 97 respectively. The highest powers of 2 and 97 in 10000! are

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9995 and 104, respectively. Therefore, the values of K according to 2 and 9 are $\left[\frac{9995}{97}\right] = 103$ and $\left[\frac{104}{1}\right] = 104$, respectively. As the value of 2 cannot exceed 103, K = 103.

134. B

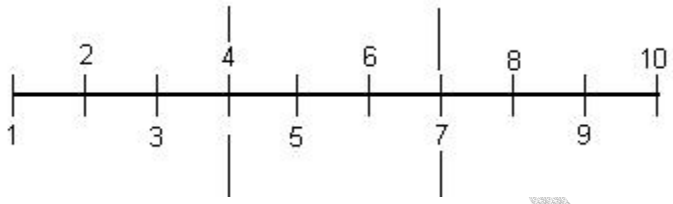
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2^2-\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)^2} = \sqrt{2-\sqrt{2+\sqrt{3}}}$$

$$\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2-\sqrt{2+\sqrt{3}}} = \sqrt{2-\sqrt{3}}$$

$$\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2+\sqrt{3}}\sqrt{2-\sqrt{3}} = 1$$

135. C

Let's start small in this case. In place of 1 000 numbers, let us take the first 10 natural numbers. They are shown below:



Notice that two consecutive numbers have a common difference of 1 between them. If you take this common difference as an interval, you have 9 such intervals. Now we choose 3 or more numbers in AP with first and last terms being 1 and 10, respectively, by putting partitions, as shown. The partitions will have to be put such that there are equal number of intervals between any two consecutive partitions (since the numbers are to be in AP, equal number of intervals mean same common difference).

In essence we are trying to find the number of ways of dividing 9 intervals equally. This will be nothing but the number of divisors of 9 i.e. 1, 3, and 9. Since we cannot take 1, we can only take 3 and 9.

Similarly, we are trying to find the number ways of dividing 999 intervals equally, or the number of divisors of 999. Now $999 = 3^3 \times 37$ therefore number of divisors = $4 \times 2 = 8$. And not counting 1, the number of ways = 7.

136. C

Let the 4 odd numbers be 2n + 1, 2n + 3, 2n + 5, and 2n + 7. The sum of these four numbers is equal to 8n + 16 or 8(n + 2). This is a multiple of 8. Since the sum is divisible by 10, it is also a multiple of 10. Therefore, the sum is a multiple of 40 (LCM of 8 and 10).
Let the sum = 40k. Now 40k when divided by 10, leaves 4k which is a perfect square. 4 is a square therefore k also has to be a perfect square.
Therefore k = 1, 4, 9, 16, 25 $\Rightarrow$ 40k = 40, 160, 360, 1000..etc.
8n + 16 = 40, 160, 360, 1000 $\Rightarrow$ n = 3, 18, 43, 123.
$\Rightarrow$ 2n + 1 = 7, 37, 87…
numbers are (7, 9, 11, 13) or (37, 39, 41, 43), (87, 89, 91, 93) etc..
Only 41 is given in the options.

TotalGadha

137. A

138. A

10b + a – (10a + b) = 18 ⇒ 9(b – a) = 18 ⇒ b – a = 2. Apart from 13 there are 6 such numbers which satisfy this criterion. 24, 35, 46, 57, 68 and 79.

139. D

x = 5, y = 2. the four numbers are 2, 3, 5 and 7. There sum is 17 which is prime.

140. C

Number of three digit numbers in base 10 = 900.
In base 9, the highest three-digit numbers is $(888)_9$ = 728.
In base 11, the lowest three-digit number is (100)11 = 121. From 121 to 728, all the numbers converted to base 9 and 11 will be three digit numbers. These are 608 numbers in all.
Therefore probability = 608/900 = 0.67

141. C

Denominator can be 99, 33, 11, 9 and 3.

142. D

The integer, the last three digits of whose square are x25, will have units digit as 5. Let the last three digit of the number be ab5. Now the last three digits of the square will come from the square of the last three digits of the numbers only. Therefore, the last three digits of the square = last three digits of ab5 × ab5. Multiplying, we can see that digit x = units digit of $b^2 + b$ = 0, 2 or 6.

143. The number of numbers less than and prime to 13 = 12. Therefore, 712 will give remainder 1 when divided by 13.

$$7^{7^7} = 7^{7\times7\times7\times7\times7\times7\times7} = 7^{49\times49\times49\times7} = 7^{(48+1)\times(48+1)\times(48+1)\times7} = 7^{12k+7}$$

$$\Rightarrow \text{Remainder by } 13 = \text{Remainder when } 7^7 \text{ is divided by } 13.$$

$$7^7 = 49\times49\times49\times7 = \text{Remainder}[-4\times-4\times-4\times7] = 6$$

144. Since R.H.S. has 7! Multiplied by a positive quantity, L.H.S will be greater than 7!. As R.H.S. does not have the prime factor of 11, L.H.S. will be less than 11!. As R.H.S. has only one power of 5 (in 5!), L.H.S. will be less than 10!. Therefore, only possibilities are 8! And 9!. Considering powers of 3 on both sides, n = 9.

145. We can see that $5 \le N \le 6$ as $25 \le N^2 \le 36$. Therefore, N is between 5 and 6 ⇒ [N] = 5. Now N × [N] = 27

$$\Rightarrow N = \frac{27}{5} = 5.4$$

146. Let the integer be abc. Therefore, $1 \le a \le 8$ as the number lies between 100 and 900.
Method 1: Let a = 1, therefore, (b, c) = (2, 9), (3, 8), …(9, 2) ⇒ 8 numbers
Let a = 2 therefore, (b, c) = (1, 9), (2, 8), …(9, 1) ⇒ 9 numbers
Let a = 3 therefore, (b, c) = (0, 9), (1, 8), …(9, 0) ⇒ 10 numbers
Let a = 4 therefore, (b, c) = (0, 8), (1, 7), …(8, 0) ⇒ 9 numbers
Let a = 5 therefore, (b, c) = (0, 7), (1, 6), …(7, 0) ⇒ 8 numbers
Let a = 6 therefore, (b, c) = (0, 6), (1, 5), …(6, 0) ⇒ 7 numbers

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Let a = 7 therefore, (b, c) = (0, 5), (1, 4), …(5, 0) ⇒ 6 numbers
Let a = 8 therefore, (b, c) = (0, 4), (1, 3), …(4, 0) ⇒ 5 numbers
Therefore, total number of numbers = 62

Method 2: Total number of numbers = coefficient of x^{12} in $(x + x^2 + x^3 + \ldots + x^8)(1 + x + x^2 + \ldots + x9)(1 + x + x^2 + \ldots + x^9) = 62$

147. **bcd** is divisible by 5 ⇒ d = 5. **abc** is divisible y 4 ⇒ bc = 12, 32, 52 or 24. As b cannot be equal to 5, only options are 12, 32, or 24 ⇒ c can be 2 or 4. **cde** is divisible by 3 ⇒ **c5e** is divisible by 3 ⇒ (c, e) = (1, 3),
(3, 1), (3, 4) or (4, 3). Only the last value satisfies the condition that c can be 2 or 4. Therefore, c = 4 and
e = 3. Also, this gives b = 2. Therefore, a = 1

148. Let the ages of Dhondu and Bhondu be **ab** and **cd**, respectively. Therefore, **abcd** = m^2. Also **pqrs** = n^2, **pq** = **ab** + 31 and **rs** = **cd** + 31. **pqrs** – **abcd** = 3131 = $n^2 - m^2$ ⇒ 31 × 101 = (n + m)(n – m) ⇒ n = 66 and m = 35. Therefore, abcd = 35^2 = 1225 ⇒ Dhondu's age = 12.

149. Number of factors of N = 16 × 13 = 208. Number of factors of N^2 = 31 × 25 = 775. Now, for N^2, leaving aside N, half of the factors of N^2 will be lying below N and the other half will be lying above N. Therefore, number of factors of N^2 lying below N = 774/2 = 387. Therefore, numbers which are multiples of N but not multiples of N^2 = 387 – 207 (not considering N) = 180.

150. Number of ways 2004 can be written as a sum of two or more consecutive positive integers = number of odd divisors of 2004 – 1 = 4 – 1 = 3.

151. **C**

$$\sqrt{N} - \sqrt{N-1} \le 0.01 \Rightarrow \frac{1}{0.01} = 100 \le \frac{1}{\sqrt{N} - \sqrt{N-1}} \Rightarrow 100 \le \frac{1}{\sqrt{N} - \sqrt{N-1}} \times \frac{\sqrt{N} + \sqrt{N-1}}{\sqrt{N} + \sqrt{N-1}} \Rightarrow \sqrt{N} + \sqrt{N-1} \ge 100 \Rightarrow \sqrt{N} > 50 \Rightarrow N \ge 2501$$

152. **C**

153. The number of combinations of Initial names and surnames = 26 × 26 = 676. Therefore, a group of 676 people can be given sets of different initial names and surnames. Then another group of 676 people can be given same sets of initial names and surnames. In all, three groups of 676 people each can be given combinations of different names and surnames such that three persons, one from each group, will have the same combination. This makes 2028 people. Then the remaining one person can be given any one of the 676 combinations. Therefore, at least 4 people will have the same combination.

154. The last two digits of $\frac{1}{5^{903}}$ = The last two digits of 2^{903} = 08.

155. **D**

156. B
Start Small. Take n = 1, 2 etc. You will see that a = 9, b = 2 and c = 0

157. **D**
For every 7 days the grandfather is counting only 6 days. Therefore, the actual number of years = $\frac{84 \times 7}{6} = 98$

Total Gadha

158. Since we look for 210 to determine the last two digits in case of 2, we first need to find the last two digits of the power, i.e. 2^{2003}. Now, $2^{2003} = (2^{10})^{200} \times 2^3 = 76 \times 08 = 08$. Therefore $2^{2003} = N08$ where N is some large number.

$2^{2^{2003}} = 2^{N08} = 2^{N00+8} = (2^{10})^{N0} \times 2^8 \Rightarrow$ Last two digits $= 76 \times 56 = 56$

Total Gadha