

Christoph's Math Notes for GMAT:

1. Cyclicity of numbers (What is the last digit of 2^3 or 5^4 or 6^9 ?):

0 1 5 6 $\Rightarrow$ cycle repeats every time $\Rightarrow$ i.e. $5^2 = 25$ (last digit is 5); $5^3 = 125$ (last digit is 5), etc..

2 3 7 8 $\Rightarrow$ cycle repeats every fourth time $\Rightarrow$ i.e. $2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $2^5 = 32$ (cycle repeats after the fourth time)

4 9 $\Rightarrow$ cycle repeats every second time

2. Division of numbers:

odd/odd $\Rightarrow$ quotient = even or odd; remainder = even or odd

odd/even $\Rightarrow$ quotient = even or odd; remainder = odd !

even/odd $\Rightarrow$ quotient = even or odd; remainder = even or odd

even/even $\Rightarrow$ quotient = even or odd; remainder = even !

3. LCM:

Product of ALL prime factors of the given numbers, but only in their highest degrees !

4. HCF:

Product of the COMMON prime numbers in their lowest degrees !

5. Powers:

Same base $\Rightarrow$ add the exponent: $a^n \cdot a^m = a^{(n+m)}$

Same exponent $\Rightarrow$ multiply the base: $a^n \cdot b^n = (a \cdot b)^n$

6. Sum of 5 consecutive numbers is always div. by 5

7. Product of 3 consecutive numbers is div. by 6

8. Product of 3 consecutive numbers, the 1st of which is even, is div. by 24

9. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

10. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

11. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

12. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

13. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

14. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$

15. Number of ways of expressing a given number (N) as a product of 2 factors:

$\frac{1}{2}[(p+1)(q+1)(r+1)] \Rightarrow$ excl. $\sqrt{N} \times \sqrt{N}$

incl. $\sqrt{N} \times \sqrt{N} \Rightarrow +1 \Rightarrow$ only when q, p and r all are even

16. Sum of all factors of a number:

$$a^p b^q = \frac{a^{(p+1)} - 1}{(a-1)} * \frac{b^{(q+1)} - 1}{(b-1)}$$

17. Arithmetic progression:

a, a+d, a+2d, a+3d

nth term of an a.p.: $a + (n-1)d$

sum of the 1st n terms: $\frac{n}{2}(2a + (n-1)d)$

18. Geometric progression:

a, a*r, a*r², a*r³

nth term of a g.p.: $a * r^{(n-1)}$

sum of the 1st n terms: $\frac{a(1-r^n)}{(1-r)} \Rightarrow$ only if $r \neq 1$

19. Same numbers: When 2nd divisor is a factor of the 1st divisor and is more than the 1st remainder, then the remainders will be the same for both the numbers !

20. Same numbers: When 2nd divisor is a factor of the 1st divisor, the 2nd remainder is obtained by div. the 1st remainder by the 2nd div..

21. $ax^2 + bx + c > 0$ lies NOT b/w the roots

22. $ax^2 + bx + c < 0$ lies b/w the roots

23. $\frac{(ax+b)}{(px+q)} > 0$ lies NOT b/w the roots

24. $\frac{(ax+b)}{(px+q)} < 0$ lies b/w the roots

25. Height of an equilateral triangle: $\frac{\sqrt{3}}{2} * \text{side}$

26. Area of an equilateral triangle: $\frac{\sqrt{3}}{4} * \text{side}^2$

27. Similar triangles (corresponding sides are proportional):

3 angles equal or

2 angles equal or

2 sides + included angle are equal

28. Median: bisects the area of an triangle

29. Centroid: divides each median in ratio 2:1

30. Segment is parallel to base and goes through midpoints of triangle $\Rightarrow \frac{1}{2}$ of base

31. Segment is parallel to base of triangle $\Rightarrow$ resulting triangles are similar

32. Rhombus: diagonals bisect each other at right angles, but are not equal

Area: $\frac{1}{2} * d_1 * d_2$

$$(\frac{1}{2}d_1)^2 + (\frac{1}{2}d_2)^2 = (\text{side})^2$$

33. Distance b/w 2 points: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

34. Distance from 1 point to origin: $\sqrt{x_1^2 + y_1^2}$

35. Slope of a line: $m = (y_1 - y_2) / (x_1 - x_2)$

36. Slope + 1 point is known, then the equation of line is $y - y_1 = m(x - x_1)$

37. x-intercept and y-intercept is known, then: $x/a + y/b = 1$

38. Longest diagonal of a rectangular box: $\sqrt{l^2 + b^2 + h^2}$

39. Longest diagonal of a cube: $\sqrt{3} * \text{side}$

40. How many integers are there from 73 through 419, inclusive ?

$$419 - 73 + 1 = 347$$

41. Midpoint formula: $(x, y) = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$

42. $|x| = x$ if $x \geq 0$ but $|x| < x$ if $x < 0$

43. Sum of the interior angles of any polygon is: $180(n - 2)$, where n is the number of sides

44. Inscribed angle is half of central angle of the arc it intercepts

45. Triangle that is inscribed in a circle is right when one side goes through the center and acts as the diameter

46. Number of factors of a given number: $(p+1) * (q+1) * (r+1)$

47. Vertical line: $x = a$

48. Horizontal line: $y = a$

49. x-intercept: $(x, 0)$

50. y-intercept: $(0, y)$

51. A central angle describes an arc that is proportional to a fractional part of 360°

52. The sum of any 2 sides of a triangle must be greater than the 3rd side

53. Triplets: 3-4-5 and 5-12-13

54. 2 points are equidistant to the origin: (x, y) $(a, b) \Rightarrow x^2 + y^2 = a^2 + b^2$

55. Simple interest: $p * r * t$

56. Compound interest: $p(1+r/c)^{(c*t)}$; p = principal and t=time and c=compounded annually and r=interest

57. 2 Numbers are div. by the same divisor and the remainders are the same in both cases, then the diff. b/w the 2 numbers is div. by the divisor

58. 2 Numbers are div. by the same divisor and the remainders are r_1 and r_2 , then when the sum of the 2 numbers is div. by the same divisor, the remainder is $r_1 + r_2$

if r_1+r_2 is greater than d, the remainder will be $r_1+r_2-d=r_3$

59. 2 numbers are div. by the same divisor and the remainders are r_1 and r_2 , then when the product of the 2 numbers is div. by the same divisor the remainder will be r_1*r_2

if r_1*r_2 is greater than d, the remainder will be $(r_1*r_2)/d=r_3$

60. Volume cylinder: $\pi*r^2*h$

61. Surface cylinder: $2(\pi*r^2)+2*\pi*r*h$

62. a^n-b^n always div. by $a-b$

63. a^n+b^n never div. by $a-b$

64. a^n-b^n div. by $a+b$ when n is even

65. a^n+b^n div. by $a+b$ when n is odd

66. a^n-b^n not div. by $a+b$ when n is odd

67. a^n+b^n not div. by $a+b$ when n is even

68. Do not multiply or divide inequalities by unknowns, unless you know the sign of the unknowns

69. Picking numbers:

never pick 1 or 0

all numbers are different

pick small numbers

70. Largest power of prime number contained in number: divide number and add the quotient

71. Finding the number of zeros in a number:

find out how many 2's and 5's appear in the number

72. $1+2+3...n=(n*(n+1))/2 \Rightarrow$ summing the first n natural number

73. $(n*(n+1)(2n+1))/6 \Rightarrow$ summing the first n natural square numbers

74. Direct variation: 2 quantities vary directly $\Rightarrow x/y=z/w$

75. Inverse variation: 2 quantities vary inversely $\Rightarrow x*y=z*w$

76. Percentile: 45 out of 50 students had test scores less than 80. $45/50=90\%$. If you had a score of 80, you are in the 90th percentile.

77. In evenly spaced sequence: median = average

78. Change in average: $(\text{the new term} - \text{existing average})/(\text{the new \# of terms})$

79. Special product: product of n consecutive numbers is div. by n

80. Special sum: a series of odd consecutive numbers is div. by the number of terms

81. Remainders of a power follow a certain pattern

82. Sum of exterior angles: 360°

83. Diagonal of square: $\text{side} \cdot \sqrt{2}$

84. Diagonal of rectangle: $\sqrt{w^2 + l^2}$

85. Area of trapeze: $(\text{sum of parallel sides}/2) \cdot \text{height}$

86. Equal distances and diff. speed: $\text{av. speed} = (2 \cdot s_1 \cdot s_2)/(s_1 + s_2)$

87. Equal time and diff. distances: $\text{av. speed} = (s_1 + s_2)/2$

88. Mixtures-replacement:

$300 \cdot 20\% - x \cdot 20\% (\text{drained off}) + x \cdot 80\% (\text{replaced}) = 300 \cdot 25\%$

89. Successive discount: sum of discounts – product of discounts = overall discount

90. Rule of alligation:

$(\text{larger number} - \text{mean})/(\text{mean} - \text{smaller number}) = \text{smaller/larger}$

91. Replacement n-times:

$A_r/A_o = (1-r/t)$; A_r =Amount after the replacement and A_o =Amount originally and R =Amount taken out and T =Total volume

92. In series of consecutive numbers the sum of n-odd numbers is div. by 3 or by the number of terms

93. The average of n-consecutive-odd numbers is never an integer

94. $\text{angle } 1 + \text{angle } 2 = \text{angle } 3$ means the triangle is right

95. Work: $(x \cdot y)/(x+y) \Rightarrow$ combined work for rate 1 and rate 2

96. Area of regular polygon: $\frac{1}{2} \cdot \text{perimeter} \cdot \text{height}$

97. Volume sphere: $\frac{4}{3} \cdot \pi \cdot r^3$

98. X contains only A and A is taken out and replace by y units of B. What amount of A is left ?

Amount of A left: $T((1-y/T)^n)$

99. Sum of 1st n-odd number: $1+3+5\dots n=n^2$
100. Sum of 1st n-even numbers $2+4+6\dots n=n(n+1)$
101. Convert x km/h in x m/s: $x*5/18$
102. Convert x m/s in x km/h: $x*18/5$
103. Increase by x% and decrease by x% of the same number always results in a decrease: $x^2/100$
104. Volume cone: $1/3*\pi*r^2*h$
105. Volume pyramid: $1/3*b*h$
106. LCM of fraction: LCM/HCF
107. HCF of fraction: HCF/LCM
108. Permutation is arrangement of objects
109. Combination is selection of objects
110. Sets
- $A+B+C-AB-AC-CB+ABC$
- $A+B+C-(\text{exactly 2 items})-2(ABC)$
- $A+B+C-(\text{at least 2 items})-ABC$
111. Surface of sphere: $4*\pi*r^2$
112. Number of ppl in exactly 1 set:
- $A+B+C-2AB-2AC-2BC+3ABC$
113. Number of ppl in exactly 2 sets:
- $AB+AC+BC-3ABC$
114. Number of ppl in exactly 3 sets:
- ABC
115. Number of ppl in 2 or more sets:
- $AB+AC+BC+2ABC$
116. Normal distribution: 68 95 99
117. Area hexagon: $\sqrt{3}/4*side^2*6$
118. Area octagon: $2(1+\sqrt{2})*side^2$
119. In geometric progression the ratio of successive terms is constant

120. In arithmetic progression the diff. is constant

GOOD LUCK !!!