Set 9

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1. The population of City X is 50 percent of the population of City Y. The population of City X is what percent of the total population of City X and City Y?

A. 25%

B. $(33\frac{1}{3})\%$

C. 40%

D. 50%

E. $(66\frac{2}{3})$

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2. What is the greatest prime factor of $2^{100}-2^{96}$?

A. 2

B. 3

C. 5

D. 7

E. 11

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3. How many liters of apple juice were added to the cranberry juice in a certain container?

(1) The amount of apple juice that was added was $\frac{3}{2}$ the amount of cranberry juice in the container.

(2) There were 5 liters of cranberry juice in the container.

4. In a demographic study, the population and total income of a certain region were estimated from other data, and both estimates had lower and upper limits. At the time of the estimates, was the per capita income for the region greater than $16,500?

(1) The lower limit for the estimate of the population was 330,000 people.

(2) The lower limit for the estimate of the total income was $5,500,000,000.

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5. Which of the following fractions has a decimal equivalent that is a terminating decimal?

A. $\frac{10}{84}$

B. $\frac{15}{196}$

C. $\frac{16}{147}$

D. $\frac{75}{432}$

E. $\frac{39}{250}$

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6. A box contains 10 light bulbs, fewer than half of which are defective. Two bulbs are to be drawn simultaneously from the box. If n of the bulbs in box are defective, what is the value of n?

(1) The probability that the two bulbs to be drawn will be defective is $\frac{1}{15}$.

(2) The probability that one of the bulbs to be drawn will be defective and the Other will not be defective is $\frac{7}{15}$.

7. There are 11 women and 9 men in a certain club. If the club is to select a committee of 2 women and 2 men, how many different such committees are possible?

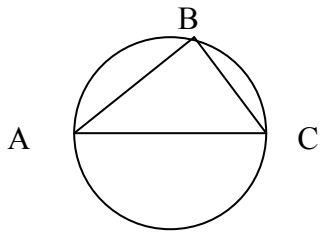
A. 120

B. 720

C. 1,060

D. 1,520

E. 1,980



8. In the figure shown, the triangle is inscribed in the circle. AC is the diameter of the circle. If the length of line segment AB is 8 and the length of line segment BC is 6, what is the length of arc ABC?

A. 15π

B. 12π

C. 10π

D. 7π

E. 5π

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9. If $5x = 2y$ and $xy \neq 0$, then x is what percent of y?

A. 2%

B. 4%

C. 10%

D. 40%

E. 50%

10. Six cards numbered from 1 to 6 are placed in an empty bowl. First one card is drawn and then put back into the bowl; then a second card is drawn. If the cards are drawn at random and if the sum of the numbers on the cards is 8, what is the probability that one of the two cards drawn is numbered 5?

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. $\frac{2}{5}$

E. $\frac{2}{3}$

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11. The product of the units digit, the tens digit, and the hundreds digit of the positive integer m is 96. What is the units' digit of m?

(1) m is odd.

(2) The hundreds digit of m is 8.

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12. Division R of Company Q has 1,000 employees. What is the average (arithmetic mean) annual salary of the employees at Company Q?

(1) The average annual salary of the employees in Division R is $30,000.

(2) The average annual salary of the employees at Company Q who are <u>not</u> in Division R is $35,000.

13. The sequence $a_1, a_2 \ldots, a_n$, is such that $a_n = 2a_{n-1}$ - x for all positive integers and for a certain x. If $a_5 = 99$ and $a_3 = 27$, what is the value of x?

A. 3

B. 9

C. 18

D. 36

E. 45

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14. Box W and Box V each contain several blue sticks and red sticks, and all of the red sticks have the same length. The length of each red stick is 18 inches less than the average length of the sticks in Box W and 6 inches greater than the average length of the sticks in Box V. What is the average (arithmetic mean) length, in inches, of the sticks in Box W minus the average length, in inches, of the sticks in Box V?

A. 3

B. 6

C. 12

D. 18

E. 24

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15. If $x \neq 0$, then $(x+\frac{1}{x})^2 - (x-\frac{1}{x})^2 =$

A. $\frac{1}{x}$

B. x

C. 1

D. 2

E. 4

16. If m, p, and t are positive integers and m<p<t, is the product mpt an even integer?

(1) t – p = p –m

(2) t – m = 16

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17. If a certain coin is flipped, the probability that the coin will land heads is $\frac{1}{2}$. If the coin is flipped 5 times, what is the probability that it will land heads up on the first 3 flips and not on the last 2 flips?

A. $\frac{3}{5}$

B. $\frac{1}{2}$

C. $\frac{1}{5}$

D. $\frac{1}{8}$

E. $\frac{1}{32}$

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18. The function f is defined by f(x) = $\sqrt{x-10}$ for all positive numbers x. If u = f($\sqrt{t}$) for positive numbers t and u, what is t in terms of u?

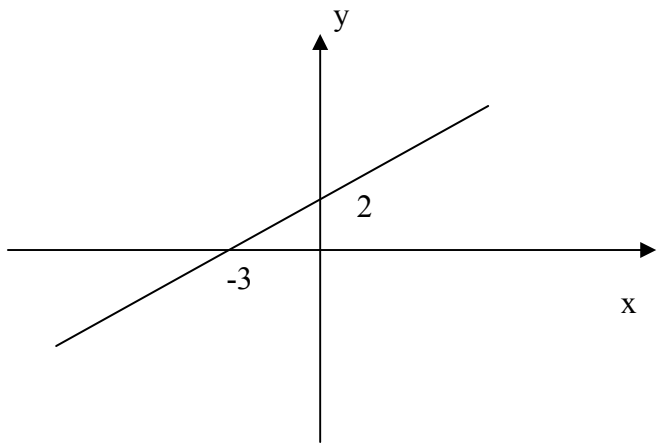
A. $\sqrt{(u+10)}$

B. $\sqrt{(u+10)^2}$

C. $\sqrt{(u^2+10)}$

D. $(u + 10)^2$

E. $(u^2+10)^2$



19. The graph of which of the following equations is a straight line that is parallel to line l in the figure above?

A. $3y - 2x = 0$

B. $3y + 2x = 0$

C. $3y + 2x = 6$

D. $2y - 3x = 6$

E. $2y + 3x = -6$

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20. What is the total number of executives at Company P?

(1) The number of male executives is $\frac{3}{5}$ the number of female executives.

(2) There are 4 more female executives than male executives.

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21. The ratio of the number of boys to the number of girls in a certain school club is 3:4. If there are 5 more girls than boys in the club, how many girls are in the club?

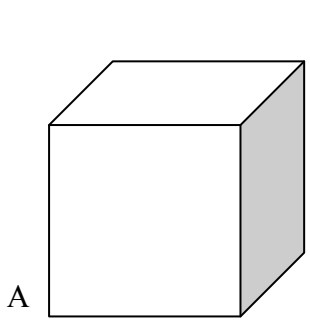
A. 7

B. 8

C. 12

D. 15

E. 20



22. What is the volume of the cube above?

(1) The surface area of the cube is 600 square inches.

(2) The length of diagonal AB is $10\sqrt{3}$ inches.

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23. If x and y are positive integers, and $x^3 y^4 = 2{,}000$, which of the following is the value of xy?

A. 2

B. 4

C. 8

D. 10

E. 20

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24. If ab = 1, what is the value of (axb)(ayb)?

(1) ax = by = 2

(2) 2xy = 8

25. Company P had 15 percent more employees in December than it had in January. If Company P had 460 employees in December, how many employees did it have in January?

A. 391
B. 400
C. 410
D. 423
E. 445

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26. If positive integer x is a multiple of 6 and positive integer y is a multiple of 14, is xy a multiple of 105?

(1) x is a multiple of 9.
(2) y is a multiple of 25.

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27. R, S, T and U are points on a line, and U is the midpoint of line segment ST. If the lengths of line segments RS, RT, and ST are 20, 4, and 24, respectively, what is the length of line segment RU?

A. 6
B. 8
C. 12
D. 14
E. 16

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28. Each of the 30 boxes in a certain shipment with either 10 pounds or 20 pounds, and the average (arithmetic mean) weight of the boxes in the shipment is 18 pounds. If the average weight of the boxes in the shipment is to be reduced to 14 pounds by removing some of the 20-pounds boxes, how many 20-pounds boxes must be removed?

A. 4 B. 6 C. 10 D. 20 E. 24

29. The sum of all the integers k such that $-26<k<24$ is

A. 0

B. –2

C. –25

D. –49

E. –51

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30. A rectangular photograph is surrounded by a border that is 1 inch wide on each side. The total area of the photograph and the border is M square inches. If the border had been 2 inches wide on each side, the total area would have been (M + 52) square inches. What is the perimeter of the photograph, in inches?

A. 34

B. 36

C. 38

D. 40

E. 42

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31. A small, rectangular park has a perimeter of 560 feet and a diagonal measurement of 200 feet. What is its area, in square feet?

A. 19,200

B. 19,600

C. 20,000

D. 20,400

E. 20,800

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32. Seven different numbers are selected from the integers 1 to 100, and each number is divided by 7. What is the sum of the remainders?

(1) The range of the seven remainders is 6.

(2) The seven numbers selected are consecutive integers.

33. 150, 200, 250, n

Which of the following could be the median of the 4 integers listed above?

I. 175

II. 215

III. 235

A. I only
B. II only
C. I and II only
D. II and III only
E. I, II and III

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34. The operation Θ is defined by x Θ y = $\frac{1}{x}+\frac{1}{y}$ for all nonzero numbers x and y. If z is a number greater than 1, which of the following must be true?

I. z Θ (-z) = 0

II. z Θ $\frac{z}{z-1}$ = 1

III. $\frac{2}{z}$ Θ $\frac{2}{z}$ = z

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II and III

35. If $xy \neq 0$ and $x^2 y^2 - xy = 6$, which of the following could be y in terms of x?

I. $\frac{1}{2x}$

II. $-\frac{2}{x}$

III. $\frac{3}{x}$

A. I only

B. II only

C. I and II

D. II+III

E. None

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36. If a, b, c, and d are positive integers, is $(\frac{a}{b})(\frac{c}{d}) > \frac{c}{b}$?

(1) c > b

(2) a > d

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37. In a meeting of 3 representatives from each of 6 different companies, each person shook hands with every person not from his or her own company. If the representatives did not shake hands with people from their own company, how many handshakes took place?

A. 45

B. 135

C. 144

D. 270

E. 288

Answers for set 9

1. The best answer is B.

The population of City Y is unknown. Take 100 as an example ➔ the population of City X is 50. 50 are 33.33% from 150 (the total population).

2. The best answer is C.

$2^{100} - 2^{96} = 2^{96}(2^4 - 1) = 2^{96}(3 \times 5)$ = the greatest prime factor is 5.

3. The best answer is C.

(1) Insufficient, we do not know the amount of cranberry juice that was in the container.

(2) Insufficient, we do not know the amount of apple juice that was added to the container.

(1+2) Sufficient. $\left(5 * \frac{3}{2} = 7.5\right)$

4. The best answer is E.

(1) Insufficient, we do not know the estimate of the total income.

(2) Insufficient, we do not know the estimate of the population.

(1+2) Insufficient, e.g. if the estimate of the population was 330,000 people and the estimate of the total income was $5,500,000,000 then the per capita income would be 16,666.666. If the estimate of the population was 400,000 people and the estimate of the total income was $5,500,000,000 then the per capita income would be 13,750.

5. The best answer is E.

A fraction has a decimal equivalent that is a terminating decimal if the different prime factors of the denominator (after cancellation) are 2, 5 or both of them. You should check each of the denominators (after cancellation) ➔ $250 = 2 \times 5^3$. Notice that $\frac{39 * 3}{250 * 3}$ also has a decimal equivalent that is a terminating decimal.

6. The best answer is D.

(1) Sufficient, $\frac{n}{10}\times\frac{n-1}{9}=\frac{1}{15}$

(2) Sufficient, $\frac{n}{10}\times\frac{10-n}{9}+\frac{10-n}{10}\times\frac{n}{9}=\frac{7}{15}$

7. The best answer is E.

$\frac{11!}{9!\times 2!}\times\frac{9!}{7!\times 2!}=1980.$

8. The best answer is E.

AC is the diameter ➔ inscribed angle ABC = 90° ➔ AC =10 (Pythagorean Theorem) ➔ Radius=5 ➔ the circumference equals 10π ➔ the length of arc ABC is 5π.

9. The best answer is D.

5x = 2y ➔ x = 0.4y.

10. The best answer is D.

There are 5 options that the sum of the number on the cards will be 8 [(2,6), (3,5) (4,4), (5,3) and (6,2)], two of them include card number 5.

11. The best answer is A.

(1) Sufficient, since $96 = 2^5\times 3$ ➔ therefore, if m is odd its units digit has to be 3.

(2) Insufficient, e.g. 834 or 843.

12. The best answer is E.

(1) Insufficient

(2) Insufficient

(1+2) Insufficient, since we do not know how many employees are in Division R, nor the ratio between the number of workers in division R, to the total number of workers.

13. The best answer is A.

$a_4 = 2a_3 - x = 54 - x$ ➔ $a_5 = 2a_4 - x = 2(54 - x) - x$ ➔ $a_5 = 99 = 108 - 3x$ (1) ➔ x=3.

14. The best answer is E.

The length of each red stick is 18 inches less than the average length of the sticks in box W and 6 inches greater than the average length of the sticks in box V. Therefore the difference between the average lengths of the sticks in the boxes must be 24.

15. The best answer is E.

$$(x+\frac{1}{x})^2 - (x-\frac{1}{x})^2 = (x^2+2+\frac{1}{x^2}) - (x^2-2+\frac{1}{x^2}) = 4$$

16. The best answer is E.

(1) Insufficient, using this statement we know that t and m are both even or both odd

(2) Insufficient, using this statement we know that t and m are both even or both odd

(1+2) Insufficient, e.g. t, p and m are odd integers and the product mpt is odd, or t, p and m are even integers and the product mpt is even.

17. The best answer is E.

The probability that it will land heads up on flips 1, 2 and 3, and tails up on flips 4 and 5 is $(\frac{1}{2})^5 = \frac{1}{32}$. Notice that if we were asked about the probability that in 5 flips we will get 3 heads and two tails with no importance for order, it would have been a different answer $\left[\left(\frac{1}{2}\right)^5 * \frac{5!}{2!3!}\right]$

18. The best answer is E.

$u = \sqrt{\sqrt{t} - 10}$ ➔ $u^2 = \sqrt{t} - 10$ ➔ $t = (u^2 + 10)^2$

19. The answer is A.

Line l is passing though points (0,2) and (-3,0) ➔ The slope of line l is: $\frac{2-0}{0-(-3)}=\frac{2}{3}$.

It is recommended to find the slopes of the given equations by transforming them to the following template: y = Ax + B, were A would be the slope of the line.

20. The best answer is C.

(1) Insufficient

(2) Insufficient

(1+2) Sufficient, take x as the number of female executives ➔ $\frac{3}{5}x = x - 4$.

21. The best answer is E.

Take 3x as the number of boys and 4x as the number of girls. Therefore 4x – 3x = 5 ➔ x = 5 ➔ 4x = 20.

22. The best answer is D.

(1) Sufficient, $6a^2 = 600$ ➔ $a^3 = 1000$.

(2) Sufficient, $AB = 10\sqrt{3}$ ➔ a = 10 ➔ $a^3 = 1000$.

23. The best answer is D.

$2{,}000 = 2^4 5^3$ ➔ $xy = 5 \times 2 = 10$.

24. The best answer is D.

Since ab = 1, (axb)(ayb) = xy. We need to find the value of xy.

(1) Sufficient, x = $\frac{2}{a}$ and y = $\frac{2}{b}$. Therefore xy = $\frac{4}{ab}=\frac{4}{1}=4$

(2) Sufficient, xy = 4.

25. The best answer is B.

Take y as the number of employees in January ➔ $460 = \frac{115}{100} y$ ➔ y = 400.

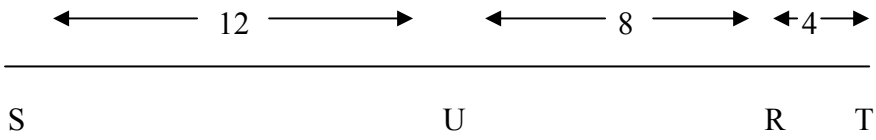
26. The best answer is B.

$105 = 3 \times 5 \times 7$ ➔ therefore xy has to be a multiple of those 3 prime numbers. Using the data in the question we know that xy has at least 2, 3 and 7, as factors.

(1) Insufficient, Using this statement we know that the prime factors of xy are at least 2, 3, 3 and 7➔ we don't know whether 5 is also one of the factors.

(2) Sufficient, Using this statement we know that the prime factors of xy are at least 2, 3, 5, 5 and 7➔ xy is a multiple of 105

27. The best answer is B.



28. The best answer is D.

Take x as the number of the 10-pounds boxes ➔ $10x + 20(30 - x) = 18 \times 30$ ➔ x = 6.

Take y as the number of 20-pounds boxes that have been removed ➔ $60 + 20(24 - y) = 14(30 - y)$ ➔ y = 20.

29. The best answer is D.

0+ [1+ (-1)]+ [2+ (-2)]+…+ [23+ (-23)]+ (-24)+ (-25) =0+ 0+ 0…+ (-24)+ (-25) =-49.

30. The best answer is D.

Take x as the width of the photograph and y as the length of the photograph. We need to find the value of 2x + 2y.

The total area if the border has been 1 inch wide: 2x + 2(y + 2) + xy = M (1).

The total area if the border has been 2 inches wide: 4x + 4(y + 4) + xy = M + 52 (2).

From (1) and (2) 4x + 4y +16 - 52 = 2x + 2y + 4 ➔ 2x + 2y = 40.

31. The best answer is A.

We need to find the value of xy. (x + y) = 280 and $x^2+y^2 = 40{,}000$ ➔ $(x + y)^2 =$ x^2+y^2+2xy ➔ 280^2=40,000+2xy ➔ $xy = \frac{78{,}400 - 40{,}000}{2} = 19{,}200.$

32. The best answer is B.

(1) Insufficient, e.g. the remainders can be 0, 1, 2, 3, 4, 5, 6 or 3, 3, 6, 0, 3, 3, 3.

(2) Sufficient, since the remainders have to be 0, 1, 2, 3, 4, 5, 6.

33. The best answer is C.

The range of the median is between 175 (when $n \leq 150$) and 225 (when $n \geq 250$).

If n = 230, the median will be 215. If $n \leq 150$, the median will be 175.

34. The best answer is E.

I. $z \Theta (-z) = \frac{1}{z} + \frac{1}{(-z)} = \frac{z - z}{-z^2} = 0$

II. $z \Theta \frac{z}{z-1} = \frac{1}{z} + \frac{z-1}{z} = \frac{z}{z} = 1$

III. $\frac{2}{z} \Theta \frac{2}{z} = \frac{z}{2} + \frac{z}{2} = z$

35. The best answer is D

Plug in the answers:

I) If y = $\frac{1}{2x}$ ➔ $\frac{x^2}{4x^2} - \frac{1}{2} = -\frac{1}{4} \neq 6.$

II) If y = $-\frac{2}{x}$ ➔ $\frac{4x^2}{x^2} + \frac{2x}{x} = 6.$

III) If y = $\frac{3}{x}$ ➔ $\frac{9x^2}{x^2} - \frac{3x}{x} = 6.$

36. The best answer is B.

We need to check if $\frac{a}{b}\frac{c}{d} = \frac{c}{b}\frac{a}{d}$ ➔ $\frac{c}{b}\frac{a}{d} > \frac{c}{b}$ ➔ it is given that $\frac{c}{b} > 0$ ➔ we can divide the inequality by $\frac{c}{b}$ ➔ we need to check if $\frac{a}{d} > 1$.

(1) Insufficient

(2) Sufficient

37. The best answer is B.

Each of the representatives shook hands with other 15 people. Every handshake was between two people at once, and it would not have to happen again ➔ $\frac{15 \times 18}{2} = 135.$