Powers of 10: Shifting the Decimal

To multiply number by positive power of 10, move decimal right specified number of places:

$$3.9752 \times 10^3 = 3{,}972.2 \qquad 89.507 \times 10 = 895.07$$

To divide number by positive power of 10, move decimal left specified number of places:

$$4{,}169.2 \div 10^2 = 41.692 \qquad 89.507 \div 10 = 8.9507$$

Negative powers of 10 reverse regular process:

$$6{,}782.01 \times 10^{-3} = 6.78201 \qquad 53.0447 \div 10^{-2} = 5{,}304.47$$

This is essentially **trading decimal places for powers of 10.**

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Raising a Decimal to a Higher Power

Rewrite decimal as product of an integer and power of 10, and then apply exponent:

$$(0.5)^4 = ?$$

$$0.5 = 5 \times 10^{-1} \quad \text{rewrite decimal}$$

$$(5 \times 10^{-1})^4 = 5^4 \times 10^{-4}$$

$$5^4 = 25^2 = 625$$

$$625 \times 10^{-4} = 0.0625$$

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Solving for Roots of Decimals

A root is a number raised to a fractional power. A square root is a number raised to the 1/2 power, a cube root is a number raised to the 1/3 power, etc:

$$\sqrt[3]{0.000027} = ?$$

$$0.000027 = 27 \times 10^{-6}$$

$$(0.000027)^{1/3} = (27 \times 10^{-6})^{1/3}$$

$$(27)^{1/3} \times (10^{-6})^{1/3} = (27)^{1/3} \times 10^{-2}$$

$$3 \times 10^{-2} = 0.03$$

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Powers & Roots Shortcut

The number of decimal places in the result of a cubed decimal is 3 times the number of decimals places in original decimal:

$$(0.04)^3 = 0.000064$$

The number of decimal places in a cube root is 1/3 the number of decimal places in original decimal:

$$\sqrt[3]{0.000000008} = 0.002$$

Make sure to work with powers of 10 using exponent rules.

Decimal Operations: Multiplication & Division

To simplify multiplication, move decimals in **opposite direction** same number of places:

$$0.0003 \times 40{,}000 = 3 \times 4 = 12$$

This is multiplying and dividing by same power of 10, i.e., **trading decimal places** in one number for those in another.

Simplify division problems by shifting decimals **in the same direction:**

$$\frac{0.0045}{0.09} = \frac{45}{900}$$

This is essentially simplifying a fraction.

Numerator & Denominator Rules for Positive Fractions

As the **numerator** goes up, the fraction **increases** in value. As the **denominator** goes up, the fraction **decreases** in value.

$$\frac{1}{8} < \frac{2}{8} < \frac{3}{8} < \frac{4}{8} < \frac{5}{8} \ldots \qquad \frac{3}{2} > \frac{3}{3} > \frac{3}{4} > \frac{3}{5} > \frac{3}{6} \ldots$$

Adding same number to numerator and denominator brings a fraction **closer** to 1, regardless of its value.

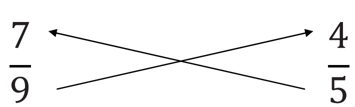
If a **proper fraction** (less than 1), value **increases** as it approaches 1. If an **improper fraction** (more than 1), value **decreases** as it approaches 1:

$$\frac{1}{2} < \frac{1+1}{2+1} = \frac{2}{3} < \frac{2+9}{3+9} = \frac{11}{12} \qquad \frac{3}{2} > \frac{3+1}{2+1} = \frac{4}{3} > \frac{4+9}{3+9} = \frac{13}{12}$$

Compare Fractions: Cross-Multiplication

Cross-multiply fractions and put each answer by the corresponding **numerator:**

$$(7 \times 5) = 35 \qquad (9 \times 4) = 36$$



Reciprocals

The product of a number and its reciprocal **always equals 1:**

$$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$$

$$5 \times \frac{1}{5} = \frac{5}{1} \times \frac{1}{5} = \frac{5}{5} = 1$$

$$\sqrt{7} \times \frac{\sqrt{7}}{7} = \frac{\sqrt{7}}{1} \times \frac{\sqrt{7}}{7} = 1$$

Percents as Decimals: Multiplication Shortcut

Convert percents into decimals by moving decimal point two spaces to the left:

$525\% = 5.25 \qquad 52.5\% = 0.525 \qquad 5.25\% = 0.0525$

Convert decimals into percents by moving decimal point two spaces to the right:

$0.6 = 60\% \qquad 0.28 = 28\% \qquad 0.459 = 45.9\%$

The percentage is always bigger than the decimal.

Translating Percent Questions

Percent	=	divide by 100	$(/100)$
Of	=	multiply	$(\times)$
Is	=	equals	$(=)$
What	=	unknown value	(x)

What is 70 percent of 120? $\rightarrow x = \frac{70}{100} \times 120$

30 is what percent of 50? $\rightarrow 30 = \frac{x}{100} \times 50$

Solve percent problems by isolating x.

Percent Increase & Decrease

Use this equation to find a change in percent:

$$\text{Original} + \text{Change} = \text{New}$$

To find **change** in percent/value:

$$\text{Percent Change} = \frac{\text{Change in Value}}{\text{Original Value}}$$

To find **new** percent/value:

$$\text{New Percent} = \frac{\text{New Value}}{\text{Original Value}}$$

The Original percent is *always* 100%.

Successive Percents

Successive percents (increases and/or decreases) **cannot** be added together. Always rephrase successive percents as percents of the original.

A price is increase by 25%. Later, the price is reduced by 20%. What is the overall percentage change?

A 25% increase followed by a 20% decrease is the same as 125% of 80% of the original number:

$$\left(\frac{125}{100}\right)\left(\frac{80}{100}\right)x = ? \qquad \left(\frac{5}{4}\right)\left(\frac{4}{5}\right)x = x$$

Calculating Compound Interest

$$\text{Compound Interest} = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = principal, r = rate (in decimal form), n = number of times per year, t = number of years. It's simpler to think of compound interest as a successive percents problem:

$200 earns 5% annual interest, compounded annually. Value after two years? This is a 5% increase each year, i.e., 105% of 105% of $200:

$$1.05 \times 1.05 \times 200 = 220.50$$

$100 earns 8% annual interest, compounded quarterly. Value after 6 months? Compounds every 3 months, each time 1/4 of total interest:

$$1.02 \times 1.02 \times 100 = 104.04$$

Ratios

Ratios can be expressed with "to" as in 3 to 4; with a colon, as in 3:4; and as a fraction, 3/4 (only if 2 quantities).

Ratios express a part-part or part-whole relationship:

- The ratio of men to women is 3:4
- There are 3 men for every 7 employees

Ratios express division. If the ratio of men to women is 3:4, then the number of men **divided by** the number of women equals 3/4, or 0.75.

If two quantities have a **constant ratio**, they are in direct proportion to each other.

Proportions

Solve simple ratio problems with a proportion:

Ratio of girls to boys is 4 to 7. If 35 boys, how many girls?

Set up labeled proportion and cross-multiply to solve.

Save time by canceling factors either vertically within a fraction or horizontally across an equals sign. **Never cancel factors diagonally across an equals sign.**

$$\frac{4 \text{ girls}}{7 \text{ boys}} = \frac{x \text{ girls}}{35 \text{ boys}} \rightarrow \frac{4 \text{ girls}}{1 \text{ boy}} = \frac{x \text{ girls}}{5 \text{ boys}} \rightarrow x = 20$$

The Unknown Multiplier

Use a variable for an unknown multiplier in a ratio, when no quantity is already equal to a number or variable expression.

Ratio of men to women is 3:4. If 56 total, how many men?

$$\text{Men} + \text{Women} = \text{Total} = 56$$
$$3x + 4x = 56$$
$$7x = 56$$
$$X = 8$$
$$\text{Men} = 3x = 3(8) = 24$$

Multiple Ratios: Make a Common Term

Fractions can be changed to have a common denominator. Similarly, all sides of a ratio can be multiplied by the same number to have common terms corresponding to same quantity.

3 of C for every 2 of A, and 5 of C for every 4 of L

C : A : L		C : A : L
3 : 2	→ multiply by 5 →	15 : 10 :
5 : : 4	→ multiply by 3 →	15 : : 12
	combined ratio:	15 : 10 : 12

Fractions, Decimals, & Percents

FDPs are three ways of expressing a part-whole relationship;

A **fraction** expresses a part-whole relationship in terms of a numerator (the part) and a denominator (the whole).

A **decimal** expresses a part-whole relationship in terms of place value (a tenth, hundredth, thousandth, etc).

A **percent** expresses the special part-whole relationship between a number (the part) and one hundred (the whole).

Common FDP Equivalents

Fraction	Decimal	Percent
1/100	0.01	1%
1/50	0.02	2%
1/25	0.04	4%
1/20	0.05	5%
1/10	0.10	10%
1/9	0.11	≈ 11.1%
1/8	0.125	12.5%
1/6	$0.1\overline{6} \approx 0.167$	≈ 16.7%
1/5	0.2	20%
1/4	0.25	25%
3/10	0.3	30%
1/3	$0.\overline{3} \approx 0.333$	≈ 33.3%
3/8	0.375	37.5%
2/5	0.4	40%
1/2	0.5	50%

Fraction	Decimal	Percent
3/5	0.6	60%
5/8	0.625	62.5%
2/3	$0.\overline{6} \approx 0.667$	≈ 66.7%
7/10	0.7	70%
3/4	0.75	75%
4/5	0.8	80%
5/6	$0.8\overline{3} \approx 0.833$	≈ 83.3%
7/8	0.875	87.5%
9/10	0.9	90%
1/1	1	100%
5/4	1.25	125%
4/3	$1.\overline{3} \approx 1.33$	133%
3/2	1.5	150%
7/4	1.75	175%

Converting Among FDPs

TO → FROM ∈	Fraction 3/8	Decimal 0.375	Percent 37.5%
Fraction 3/8		Divide numerator by denominator $3 \div 8 = 0.375$	Divide numerator by denominator and move decimal two places right $3 \div 8 = 0.375 \rightarrow 37.5\%$
Decimal 0.375	Use place value of last digit as denominator, put decimal's digits in numerator. Simplify. $\frac{375}{1000} = \frac{3}{8}$		Move decimal two places right $0.375 \rightarrow 37.5\%$
Percent 37.5%	Use digits of percent for numerator and 100 for denominator. Simplify. $\frac{37.5}{100} = \frac{3}{8}$	Find percent's decimal and move two places left $37.5\% \rightarrow 0.375$	

Concrete vs. Relative Values

1. **Concrete values** are actual amounts.
2. **Relative values** relate two quantities using fractions, decimals, percents, or ratios.

If a DS question asks for the **relative value** of two pieces of a ratio, **any** statement that gives the relative value of **any** two pieces of the ratio will be sufficient.

If a DS question asks for the **concrete value** of one element of a ratio, you will need **both** the concrete value of another element of the ratio **and** the relative value of two elements of the ratio.

Smart Numbers

Pick **Smart Numbers** equal to common multiples of denominators to stand in for unspecified numerical amounts. **Do not** pick smart numbers when *any* amount or total is given!

To solve percent problems with unspecified amounts, pick 100:

Initial cost d, *on sale 20% off.*
Sale price s, d *is what percent of* s*?*

Initial cost \$100, then $d = 100$. Sale 20% off, then new price \$80. Therefore, d is 125% of s.

Estimating Fractions with Benchmark Values

Which is greater: $\frac{127}{255}$ or $\frac{162}{320}$?

Recognizing that 127 is less than half of 255, and 162 more than half of 320 saves times.

Which is $\frac{10}{22}$ of $\frac{5}{18}$ of 2000?

These fractions are close to Benchmark Values $\frac{1}{2}$ and $\frac{1}{4}$, therefore $\frac{1}{2}$ of $\frac{1}{2}$ of ≈ 250. Round some numbers up and other down to compensate for rounding errors.

Heavy Division Shortcut

What is $1{,}530{,}794 \div (31.49 \times 10)$ to nearest whole number?

Set up problem in fraction form: $\frac{1{,}530{,}794}{31.49 \times 10^4}$

Eliminate powers of 10: $\frac{1{,}530{,}794}{314{,}900}$

Get to a single digit to the left of decimal in denominator. In this case, move decimal left 5 spaces (dividing top and bottom by same power of 10: 100,000). If too imprecise, keep one more decimal place.

Thus: $\frac{1{,}530{,}794}{314{,}900} = \frac{15.30794}{3.14900} \approx \frac{15}{3} = 5$