Overlapping sets :-

f.1 $\text{Total} = A+B+C - (\text{Sum of 2 grp overlaps}) + (\text{All three}) + \text{Neither}$

f.2 $\text{Total} = A+B+C - (\text{Sum of Exactly 2 grp overlaps}) - 2(\text{All three}) + \text{Neither}$

Trick — A+1, E+2

Successive change in %age :- $\text{Total} = A\left(1 \pm \frac{x_1}{100}\right)\left(1 \pm \frac{x_2}{100}\right) \ldots\ldots$

Population of town or Value of a machine :

Inc. by x% every year | Dec. by y% every year

(i) Pop. after n years $= P\left(1+\frac{x}{100}\right)^n$ $= P\left(1-\frac{y}{100}\right)^n$

(ii) Pop. n years ago $= \frac{P}{\left(1+\frac{x}{100}\right)^n}$ $= \frac{P}{\left(1-\frac{y}{100}\right)^n}$

Compound Interest :

$$\text{Amount} = P\left(1+\frac{R/n}{100}\right)^{nt}$$

R - Rate, t - no. of yrs.
n - no. of times interest is Compounded per annum.

→ Annually $\text{Amt} = P\left(1+\frac{R}{100}\right)^t$

→ Half yearly $\text{Amt} = P\left(1+\frac{R/2}{100}\right)^{2t}$

→ Quarterly $\text{Amt} = P\left(1+\frac{R/4}{100}\right)^{4t}$

→ Annually for $3\frac{2}{5}$ yrs $\text{Amt} = P\left(1+\frac{R}{100}\right)^3\left(1+\frac{2}{5}\frac{R}{100}\right)$

when rates are diff.

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) + \ldots\ldots$$

WORK/RATE :

A can do a work in X days
B Y days
Together they can do a work in $= \frac{XY}{X+Y}$ days

A → X
B → Y
C → Z
Together $= \frac{XYZ}{XY+YZ+XZ}$

Time to fill / empty a tank : $\frac{XY}{X \sim Y}$

S = D/T
T = D/S

if three men's speed is a : b : c
time taken would be 1/a : 1/b : 1/c

A goes from X to Y @ u km/hr & Y to X @ v km/hr

⇒ Avg speed $= \frac{2uv}{u+v}$ km/hr

Speed of Boat (Man) in still water = X
Speed of the stream (Current) = Y

1. speed of the Boat D/S = X + Y
2. speed of the Boat U/S = X − Y
3. speed of the Boat in still water = (speed of the Boat D/S + speed of the Boat U/S)/2
4. speed of the stream = (speed of the Boat D/S − speed of the Boat U/S)/2

STATISTICS :

Mean → Average $\bar{x} = \frac{\Sigma x}{N}$

* for Consecutive integers or an A.P

Mean = (First term + Last term)/2 = Median

Median → Middle Value {Could be more, less or equal to Mean}

Mode → The value the occurs most no. of times.
(Can be so, that more than one modes are there)

Range → $X_{max} - X_{min}$ (greatest − smallest) ∴ +ve

Standard Deviation → $\sigma = \sqrt{\frac{\Sigma d^2}{N}}$ $d = (x - \bar{x})$
d is the SPREAD

* if Avg spread is more when comparing two sets, the Standard Deviation will be more.

ex- S.D for (0, 7, 8, 10, 10)

Mean $(\bar{x}) = \frac{0+7+8+10+10}{5} = 7$

x	$\bar{x}$	d	d^2
0	7	−7	49
7	7	0	0
8	7	1	1
10	7	3	9
10	7	3	9

$\Rightarrow \sqrt{(49+0+1+9+9)/5}$

$\sigma = \sqrt{68/5}$

ex- S.D is more for ? (2, 4, 6, 8) or (5, 5, 5, 5)

(2, 4, 6, 8)	(5, 5, 5, 5)
$\bar{x} = 5$	$\bar{x} = 5$
Spread (3, 1, 1, 3)	Spread (0, 0, 0, 0)
Avg. spread $\frac{3+1+1+3}{4} = 2$	Avg. spread $\frac{0}{4} = 0$

∴ S.D is more ✓.

Variance = σ^2

Inequalities :

* Preferred values to check in a no. line.

$-2 \quad -1 \quad -1/2 \quad 0 \quad 1/2 \quad 1 \quad 2$

* $|x| > a \Rightarrow x > a$ or $x < -a$

$|x| < a \Rightarrow -a < x < a$ $-a$ ——— a

* $|x-a| > b \Rightarrow (x-a) > b$ or $(x-a) < -b$

$|x-a| < b \Rightarrow -b < (x-a) < b$

* $|a+b| \leq |a| + |b|$

$|a-b| \geq ||a| - |b||$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$; $b \neq 0$

$|a^2| = a^2$

* $(x-a)(x-b) < 0$

$\Rightarrow a < x < b$

* $(x-a)(x-b) > 0$

$\Rightarrow x < a$ or $x > b$

Here $b > a$

Not necessary
if $x > y$ then
$x^2 > y^2$ & $\sqrt{x} > \sqrt{y}$

Necessary.
if $x > y$ then.
$x^3 > y^3$ & $\sqrt[3]{x} > \sqrt[3]{y}$

$\frac{a+x}{b+x} > \frac{a}{b}$ if $\frac{a}{b} < 1$

$\frac{a+x}{b+x} < \frac{a}{b}$ if $\frac{a}{b} > 1$

NUMBERS : Real – ✓✓, Imaginary – xx

Real → Rational, Irrational

Rational: 0·5, 0·76, 0·7366666… (terminating or repeating decimals)

Irrational: π, 8·145767……

* b/w n & 2n there will be a prime no. $n < p < 2n$
* Natural No. that is not prime is Composite.
* Natural No., Sum of whose proper divisors is equal to the no. itself is a Perfect no. $6 = 1+2+3$
* Even no :- $(2n)\ (2n-2)\ (2n+2)$
* odd no :- $(2n-3)\ (2n-1)\ (2n+1)\ (2n+3)$
* If a no. has no factor equal to or less than its sq. root, then the no. is prime.
* All prime no. above 3 are $6n-1$ or $6n+1$

No. of factors & Sum of factors. n & S

$N = a^p \times b^q \times c^r$; a, b, c are prime factors.

$n = (p+1)(q+1)(r+1)$

$$S = \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$$

ex- $450 \to 2^1 \times 3^2 \times 5^2$

$n = (1+1)(2+1)(2+1) = 18$

∴ 450 has 18 factors.

$$S = \frac{(2^{1+1}-1)(3^{2+1}-1)(5^{2+1}-1)}{(2-1)(3-1)(5-1)} = 1209$$

∴ Sum of the factors is 1209

* if n = odd
  N will be a perfect Square

Divisibility (also see 4 pg. Right Corner).

3 – Sum of the digit div by 3

4 – Last two digit ——

6 – both 3 & 2

7 – take the last digit, double it & substract it from the Rest of the no. if the ans. is div by 7, no. is div by 7.

$364 \to 4 \times 2 = 8 \Rightarrow 36 - 8 = 28$
∴ div by 7

8 – last three digit div by 8

9 – Sum ——

11 – Add all alternate no. in gps. & substract, if 0 or div by 11 then no. is.

ex- 9488699 – $9+8+6+9 = 32$
$4+8+9 = 21$
Diff = 11 ∴ no. is div.

12 – if div by 3 & 4.

* Any no. with even no. of digits when added to its reverse, the sum is always div. by 11.

Remainders : $d \overline{)\,D\,}(\,Q$ … R $\Rightarrow$

$\boxed{dQ + R = D}$ → Dividend

$\boxed{Q + \frac{R}{d} = \frac{D}{d}}$ → divisor

ex- if $\frac{A}{B} = 4·35$
which could be remainder amongst 14, 15, 16, 17, 18

$Q = 4$

$\frac{R}{d} = 0·35 = \frac{35}{100} = \frac{7}{20}$

$\Rightarrow \frac{20R}{7} = d$

for d to be integer
$R \to 14$

ex. when divided by 18 remainder is 7, what is the remainder when divided by 6

No. is $18Q + 7$

∴ when $18Q + 7$ is divided by 6

$\frac{18Q}{6} + \frac{7}{6}$ 1

Remainder is 1

** Decimal Portion is nothing but $\frac{\text{Remainder}}{\text{Divisor}}$

* **FACTORIALS : ($n!$)**

\# No. of trailing zeroes $= \frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3} + \cdots$

ex. $32!$ $\Rightarrow \frac{32}{5} + \frac{32}{5^2} + \frac{32}{5^3} + \cdots$

$= 6 + 1 + 0 + \cdots$

$= 7$ zeroes.

\# Power of Prime factor P $= \frac{n}{P} + \frac{n}{P^2} + \frac{n}{P^3} + \cdots$

ex. 2 in $32!$ $\Rightarrow \frac{32}{2} + \frac{32}{2^2} + \cdots$

$= 16 + 8 + 4 + 2 + 1 + 0 + \cdots$

$= 31$

* Power of a Non Prime no. (factor) say 12, break in to prime nos $3^1 \times 2^2$ then proceed & constitute the powers req. to for 12

12 in $32!$ $\Rightarrow \frac{32}{3} + \frac{32}{3^2} + \frac{32}{3^3} + \cdots$

$= 10 + 3 + 1 + \cdots$

$= 14$

$\frac{32}{2} + \frac{32}{2^2} + \cdots$

$= 16 + 8 + 4 + 2 + 1 + \cdots$

$= 31$

for 12 one 3 & two 2 are required.

So $\Rightarrow$ lesser value of 14 & 31/2

$\therefore$ 14

Decimal Representation :

Reduced fraction a/b can be expressed as terminating decimal if b's prime factors have only 2 & 5

i.e $b = 2^m \times 5^n$ ex. $\frac{7}{250} \Rightarrow 250 = 2^1 \times 5^3 \therefore$ it is terminating decimal.

Decimal to fraction Conversion :

* $0.393939 = \frac{39}{99} = \frac{13}{33}$

* $0.4576161 = \frac{45761 - 457}{99000}$

* $0.3471515 = \frac{34715 - 347}{99000}$

* $0.38467467 = \frac{38467 - 38}{99900}$

\# Last two digit of a product :- $345 \times 9512 \times 408 \times 613$

$\Rightarrow 45 \times 12 \times 08 \times 13$

$\Rightarrow 540 \times 104$

$\Rightarrow 40 \times 04 = 160 \Rightarrow 60$

* Pick last ones for last digit.

* Last digit of $(xyz)^n$ is same as $(z)^n$

Divisibility Continued :-

ex- if a is div by 12 & 10 is a div by 24

$12 \rightarrow 2 \times 3 \times 2$

$10 \rightarrow 2 \times 5$

2 3 2 5 $\boxed{2,3,2,5}$

24 cant be made out of these nos in the prime box, So no.

ex- if a is div by 7 & 18, is a div by 42.

$7 \rightarrow 7$

$18 \rightarrow 3 \times 3 \times 2$

7 3 3 2 $\boxed{7,3,3,2}$

42 can be made

ex- if a is div by 9 & 21, is a div by 63

$9 \rightarrow 3 \times 3$

$21 \rightarrow 7 \times 3$

3 3 7 $\boxed{3,3,7}$ 63 can be made. So OK.

L.C.M :- Pick highest powers of all the factors.

H.C.F :- Pick lowest powers of Common factors.

H.C.F of fraction = H.C.F of N / L.C.M of D & vice versa.

Progressions : H.C.F $\ngtr |m-n|$ H.C.F × L.C.M = m × n

AP $a_n = a+(n-1)d$ mean = median $= \frac{a_1+a_n}{2}$

Sum $= (1^{st}\text{ term} + \text{last term})\frac{n}{2} = (a_1+a_n)\frac{n}{2}$

Sum $= (2a+(n-1)d)\frac{n}{2}$

GP $\frac{a_n}{a_{n-1}} = r$; $a_n = ar^{n-1}$

Sum of an infinite G.P will be finite if $r<1$

Sum $= \frac{a}{1-r}$

Sum (general) $= \frac{a(1-r^n)}{(1-r)}$

**

Three nos in AP $a-d,\ a,\ a+d$

four nos in AP $a-3d,\ a-d,\ a+d,\ a+3d$

Three nos in GP $a/r,\ a,\ ar$

four nos in GP $a/r^3,\ a/r,\ ar,\ ar^3$

HP Every term if reversed form A.P

A.M $= \frac{a+b}{2} = \frac{a_1+a_2+\dots a_n}{n}$

G.M $= \sqrt{ab} = (a_1 \times a_2 \times a_3 \times \dots a_n)^{1/n}$

H.M $= \frac{2ab}{a+b} = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\dots\frac{1}{a_n}}$

* A.M $\geq$ G.M $\geq$ H.M

* for two nos $\sqrt{A.M \times H.M} = G.M$

Algebra : Quadratic eq:- $ax^2+bx+c=0$

Solⁿ $= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac>0$ two Solⁿ

<0 no Solⁿ

$=0$ one Solⁿ

Sum of roots $= -b/a$

Product of roots $= c/a$

* roots x_1 & x_2

eq:- $(x-x_1)(x-x_2)=0$

Mixed items :

Consequtive nos →

$\{9,10,11\}$ Sum = 30

div by 3

$\{9,10,11,12\}$ Sum = 42

not div by 4

$\sqrt{2}$	1.414
$\sqrt{3}$	1.732
$\sqrt{5}$	2.24
$\sqrt{6}$	2.45
$\sqrt{7}$	2.65
$\sqrt{8}$	2.83
$\sqrt{9}$	3.0
$\sqrt{10}$	3.16

Roots.

if n is odd, Sum of Consequtive integers is alway div by n

if n is even, ———— never div by n

Also. Product of n Consequtive int. is always div by n!

* if P is a Prime no.

$1+(P-1)!$ is div by P

eg $1+16!$ is div by 17.

$\Rightarrow 1+(17-1)!$

Ex $1+22!$ is div by 23

$1+(23-1)!$

* if P is a prime no.

(a^P-a) is div by P

$\Rightarrow (n^3-n)$ is div by 3

(n^5-n) is div by 5

(n^7-n) is div by 7

also. $(a^P-a) = a(a^{P-1}-1)$

is same

Pure Surd :- $\sqrt{6}, \sqrt{7}, \sqrt{8}$ made of only irrational no.

Mixed Surd :- partly rational & partly irrational. $2\sqrt{6}$, $3\sqrt{8}$

Convert $\sqrt{27}$ into mixed surd $\Rightarrow 3\sqrt{3}$

$2\sqrt{8}$ into Pure surd $\Rightarrow \sqrt{32}$

Surds

Remainder & Factor Theorem :

if $f(x)$ is divided by $(x-a)$, then $f(a)$ is the remainder & if $f(a)=0$ then $(x-a)$ is a factor of $f(x)$.

Let $f(x) = x^3 + 3x^2 - 5x + 4$, find $(x-1)$ is a factor or not, if not find the Remainder

$\Rightarrow$ put $x=1 \Rightarrow f(1) = 3$, so, remainder is 3.

Sum :

Sum of first N Natural Nos. $= \frac{n(n+1)}{2}$

Sum of squares of first N " " $= \frac{n(n+1)(2n+1)}{6}$

Sum of Cubes " ——— $= \left[\frac{n(n+1)}{2}\right]^2$

Sum of first n odd natural nos. $= n^2$

Sum of — " even — " — $= n(n+1)$

Cyclicity of the Powers

2	2	2	2
	2^2		4
	2^3		8
	2^4		6
	2^5	2	

7	7	7	7
	7^2	49	9
	7^3	3	3
	7^4	1	1
	7^5	7	7

3	3	3	3
	3^2	9	9
	3^3	27	7
	3^4	81	1
	3^5	243	3

4	4	4	4
	4^2	16	6
	4^3	64	4
	4^4	256	6

formulas :-

* $(a+b)^3 = a^3+b^3+3ab(a+b)$ * $a^3+b^3 = (a+b)(a^2+b^2-ab)$

* $(a-b)^3 = a^3-b^3-3ab(a-b)$ * $a^3-b^3 = (a-b)(a^2+b^2+ab)$

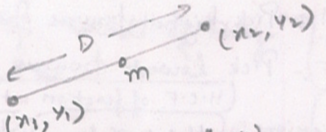
* $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

* $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

if $(a+b+c)=0$ then $a^3+b^3+c^3 = 3abc$

GIVE IT ALL

Co-ordinate Geometry.

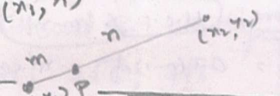


① $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

② $m = x_m, y_m$

$x_m = \frac{x_1 + x_2}{2}$

$y_m = \frac{y_1 + y_2}{2}$

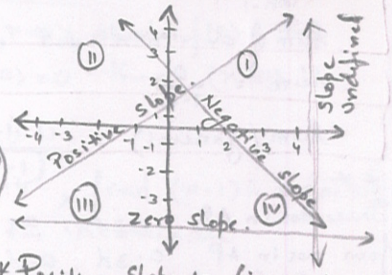


③ $P_x = \frac{m x_2 + n x_1}{m+n}$

$P_y = \frac{m y_2 + n y_1}{m+n}$

LINES

* $ax + by + c = 0$
* $y = mx + c$
* $\frac{x}{a} + \frac{y}{b} = 1$
* $(y - y_1) = m(x - x_1)$
* $(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$
* Dis. b/w two // lines
  $y = mx + b$
  $y = mx + c$ $\quad D = \frac{|b-c|}{\sqrt{1+m^2}}$
* $\perp^{ar}$ lines $y = mx + b$
  $y = m'x + c$
  $\Rightarrow m = \frac{-1}{m'}$

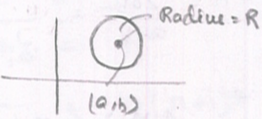


* Positive Slope :- Line intersects Quadrants ① & Ⅲ
  X-Y intercepts will have opp sign
* Negative Slope :- Line intersects Quadrants Ⅱ & Ⅳ
  X-Y intercepts will have Same Sign

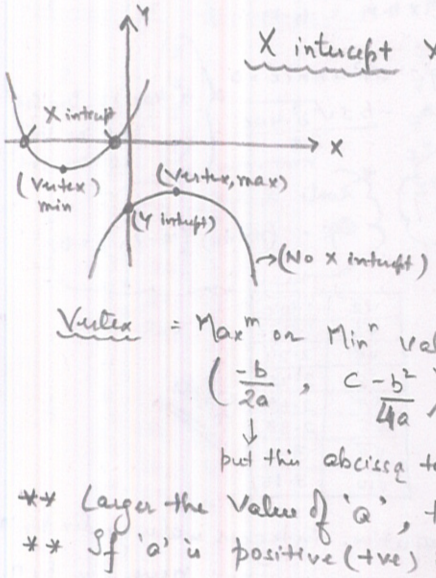
* Dis of a point to a line (x_0, y_0) line $ax + by + c = 0$

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

Circle :- $(x-a)^2 + (y-b)^2 = R^2$



Parabola :- $y = ax^2 + bx + c$



X intercept $\quad x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

If D +ve two X-intercept
D -ve No X-intercept
$D = 0$ One X-intercept

Y intercept $= c$

Vertex = Max^m or Min^n value of the function.

$$\left(\frac{-b}{2a}, \; c - \frac{b^2}{4a}\right)$$

↓

put this abcissa to get the co-ordinate

** Larger the value of 'a', thinner is the parabola
** If 'a' is positive (+ve) parabola opens up & Vice Versa

TRIANGLES :-

$Area = \frac{1}{2} b \times h$

$Area = \frac{1}{2} P \times r$

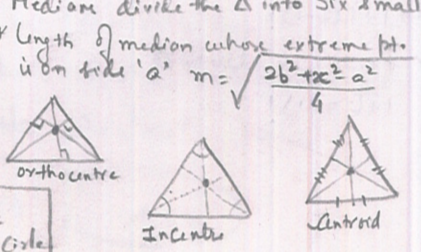
$Area = \frac{1}{4} \frac{abc}{R}$

$Area = \sqrt{s(s-a)(s-b)(s-c)}$

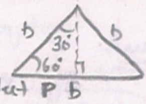
here, $s = \frac{a+b+c}{2}$

b = base
h = altitude
P = Perimeter
r = Radius of inscribed Circle
R = Radius of Circumscribed Circle

* Intersection of altitudes – Orthocentre
* Intersection of Bisector's of Angles – Incenter (Centre of the Incircle of Δ)
* Intersection of the Medians of the sides is CENTROID
* Medians divide the Δ into Six smaller Δ
* Length of median whose extreme pt. is on side 'a' $m = \sqrt{\frac{2b^2 + 2c^2 - a^2}{4}}$



Equilatreal Triangle : $\boxed{A = \frac{\sqrt{3}}{4} b^2}$



* for given area, equilatreal Δ has smallest P b
* for given P [Perimeter], equilateral Δ has largest Area (A)
* of all the Δ's inside a O, equi Δ has largest Area
* Area of the Outer Circle is 4 times the area of the Inner Circle.

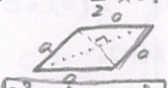
if $c^2 < a^2 + b^2$ acute angle Δ
$c^2 = a^2 + b^2$ Right angle Δ
$c^2 > a^2 + b^2$ obtuse angle Δ

Rectangle

* Diagonals bisect each other
* Joining mid point will form a Rhombus

Rhombus

* Diagonals bisect each other at 90°
* Area $= \frac{1}{2} \times D_1 \times D_2$

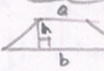


$\boxed{D_1^2 + D_2^2 = 4a^2}$

Square

* Diagonals bisect each other @ 90°

Trapezium



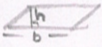
$A = \frac{1}{2}(a+b)h$

Quadilaterals

Sum of all angles = 360°

Parallelogram

* Diagonals bisect
* $A = b \times h$



POLYGONS :
Sum of interior angles $= (2n-4)90°$
Sum of Exterior angles $= 360°$
Each interior angle $= \frac{(2n-4)90°}{n}$
Each Exterior angle $= \frac{360°}{n}$

Permutation.

Each of the different orders of arrangements, obtained by taking some, or all, of a number of things, is called a Permutation.

Combination.

Each of the different gps., or collections, that can be formed by taking some, or all, of a number of things irrespective of the order in which they appear in the gp, is called a Combination.

Suppose, A, B, C, D →
order of Arrangement :

ABC, BCA, BAC, BCA, CAB, CBA — 1
BCD - - - - - - - - - - - - 2
ACD - - - - - - - - - - - - 3
ABD - - - - - - - - - - - - 4

Thus each of 24 arrangements is Permutation & 4 types of gps they formed is Combination

$nP_r = \frac{n!}{(n-r)!}$

$nC_r = \frac{n!}{(n-r)!\, r!}$