DS

1.3066Ashok and Brian are both walking east along the same path; Ashok walks at a faster constant speed than does Brian. If Brian starts 30 miles east of Ashok and both begin walking at the same time, how many miles will Brian walk before Ashok catches up with him?

(1) Brian's walking speed is twice the difference between Ashok's walking speed and his own.

(2) If Ashok's walking speed were five times as great, it would be three times the sum of his and Brian's actual walking speeds.

You can solve this algebraically. Alternatively, the question stem doesn't provide any real numbers for rate, time, or distance. The only real number given relates to position: Brian and Ashok are 30 miles apart to start. The statements provide relationships between the rates but, again, no real numbers. Therefore, you can pick your own smart numbers to test the statements. Remember that Ashok's walking speed has to be faster than Brian's.

Both solutions are shown below.

Algebraic solution

(1) SUFFICIENT: Write an equation: $B = 2(A - B)$. Note that rate = distance / time. Time is the same for both men; call that t. If Brian's distance is D, then Ashok's distance is $D + 30$ (since Ashok has to walk the same distance plus an additional 30 miles in order to catch up to Brian).

Substitute into the equation. Brian's rate is D / t. Ashok's rate is $(D + 30) / t$.

$\frac{D}{t} = 2\left(\frac{D+30}{t} - \frac{D}{t}\right)$ multiply both sides by t

$D = 2(D + 30 - D)$

$D = 2(30)$

$D = 60$

Brian walked 60 miles. (Note: stop whenever you realize that you will get a single numerical value as your answer—that's sufficient!)

(2) SUFFICIENT: Write an equation: $5A = 3(A + B)$. As with statement 1, Brian's rate can be written D / t and Ashok's rate can be written $(D + 30) / t$. Plug these into the equation.

$5\left(\frac{D+30}{t}\right) = 3\left(\frac{D+30}{t} + \frac{D}{t}\right)$ multiply both sides by t

$5(D + 30) = 3(D + 30 + D)$

$5D + 150 = 6D + 90$

$60 = D$

Brian walked 60 miles. (Note: stop whenever you realize that you will get a single numerical value as your answer—that's sufficient!)

Smart Numbers solution:

(1) SUFFICIENT: Write an equation: $B = 2(A - B)$. Because B is in the equation twice, pick for that number. If Brian's walking speed is 2 miles per hour, then find Ashok's: $2 = 2(A - 2)$, so $A = 3$.

If Ashok walks 3 miles per hour and Brian walks 2 miles per hour, then for every hour they walk, Ashok gets 1 mile closer to Brian. Therefore, it would take Ashok 30 hours to catch up to Brian. Brian walks for the same length of time at a rate of 2 miles per hour, so in 30 hours, Brian walks 60 miles.

If you're not sure whether different numbers would give the same result (they will), pick another rate for Brian to see. If Brian's walking speed is 3 miles per hour, then Ashok's is 3 = 2(A – 3), or A = 4.5. For every hour that they walk, Ashok catches up to Brian by 1.5 miles. In order to catch up the full 30 miles, Ashok will have to walk for 30 / 1.5 = 20 hours. If Brian also walks for 20 hours, he will walk a total of (20)(3) = 60 miles.

(2) SUFFICIENT: Write an equation: $5A = 3(A + B)$. Because A is in the equation twice, pick for that number. If Ashok's walking speed is 2 miles per hour, then find Brian's: $5(2) = 3(2 + B)$, so… ugh, that would give 10/3 = 2 + B. Change A to 3 miles per hour instead. $5(3) = 3(3 + B)$, so $B = 2$.

Do the numbers look familiar? You used $B = 2$ and $A = 3$ for statement 1! You already know that this combination means that they walked for 30 hours, so Brian walked 60 miles. This statement is also sufficient.

The correct answer is (D).

2.3065The next number in a certain sequence is defined by multiplying the previous term by some positive constant k, where $k \neq 1$. How many of the first nine terms in this sequence are greater than 1?

(1) The ninth term in this sequence is 81.

(2) The fifth term in this sequence is 1.

Note that the daily increase is consistent: he always earns $10 more each day. This is essentially a consecutive integer problem (although, in this case, there are increments of 10, not 1). Use f to represent Dan's earnings on the first day. Express his earnings on subsequent days as $f + 10$, $f + 20$, $f + 30$, … $f + 140$.

(1) SUFFICIENT: This statement provides the sum of the most recent three days:

$(f + 120) + (f + 130) + (f + 140) = \$690$
$3f + 390 = \$690$
$3f = \$300$
$f = \$100$

Dan earned $100 on his first day.

(2) INSUFFICIENT: This statement provides the difference between the totals for the first three days and the last three days:

First three days = $(f) + (f + 10) + (f + 20) = 3f + 30$
Last three days = $(f + 120) + (f + 130) + (f + 140) = 3f + 390$
Last three days − First three days = $(3f + 390) - (3f + 30) = \$360$

This statement is always true, regardless of how much Dan made on the first day. (This information can be determined from the question stem alone!)

The correct answer is (A).

3.3064The next number in a certain sequence is defined by multiplying the previous term by some positive constant k, where $k \neq 1$. How many of the first nine terms in this sequence are greater than 1?

(1) The ninth term in this sequence is 81.

(2) The fifth term in this sequence is 1.

Begin by rephrasing the question. Each term in the sequence can be found by multiplying the previous term by a positive constant *k*. This is an example of a geometric sequence; in sequence notation, this can be written as:

$a_{n+1} = ka_n$

where a_n represents a term in the sequence and a_{n+1} represents the following term. In normal language, the next number in the sequence equals the previous number multiplied by *k*.

The problems asks you to determine how many of the first nine terms in the sequence are greater than 1. Note that if the positive constant *k* is greater than 1, the sequence will be increasing, and if the positive constant *k* is less than 1, the sequence will be decreasing. Note that *k* cannot equal 1, 0, or any negative number.

(1) INSUFFICIENT: The ninth term is 81. Consider the possible cases for k: $k > 1$ and $k < 1$. For example, if $k = 0.5$, then the eighth term would be 162, the seventh term would be 324, and so on; that is, the first 9 terms would all be greater than 81 and thus greater than 1 as well. Alternatively, if $k = 100$, then the eighth term would be 0.81, the seventh would be 0.0081, and so on. In this case, only the ninth term would be greater than 1. Because there are multiple possibilities for the number of terms that are greater than 1, this statement is insufficient.

(2) SUFFICIENT: The fifth term is 1. Consider the two possible cases for k: $k > 1$ and $k < 1$. If $k < 1$, then the sequence will be decreasing, meaning the terms one through four will be greater than 1 and the terms six through nine will be less than 1. In this case, a total of 4 terms will be greater than 1. Alternatively, if $k > 1$, then the sequence will be increasing, meaning the terms one through four must be less than 1 and the terms six through nine must be greater than 1. In this case, again, a total of 4 terms will be greater than 1. Because 4 out of the 9 terms are greater than 1 regardless of the value of *k*, this statement is sufficient.

The correct answer is (B).

4.3063Point Q lies in the interior of a particular circle. Is point Q the center of the circle?

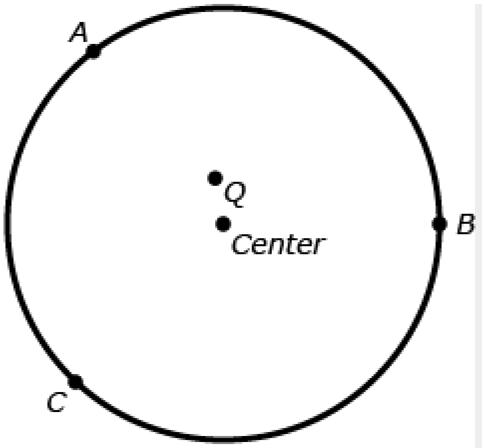
(1) There exist points A, B, and C, all distinct points on the circle's circumference, such that the distances QA, QB, and QC are identical.

(2) At least two different diameters of the circle contain point Q.

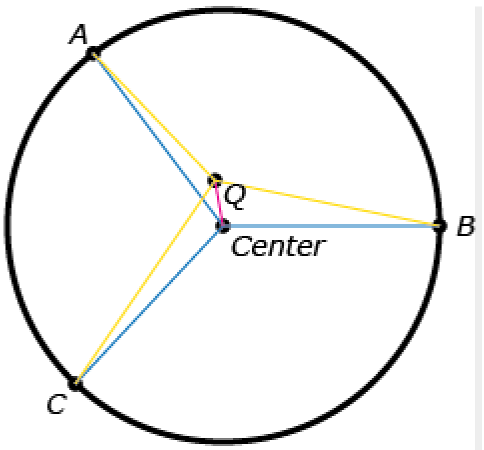
(1) SUFFICIENT: According to this statement, *Q* is equidistant from three different points on the boundary of the circle.

This can be true if *Q* *is* the center of the circle; all three lengths would be radii of the circle. Could it also be true if *Q* is not the center of the circle? Try to create a situation satisfying this criterion for which *Q* is *not* the center of the circle.

Draw a diagram that shows *Q* not at the center of the circle:



According to statement (1), the distances *QA, QB,* and *QC* are all the same; these are the yellow lines in the diagram below.
Also, as always, the radii of the circle are all equal, so the Center is equidistant from *A*, *B*, and *C* as well; these are the blue lines in the diagram.
Finally, draw a line between the Center and *Q*.



These lines create three triangles (call the center *O*): *AQO*, *BQO*, and *CQO*. According to statement 1, the yellow sides are the same, so all three of these triangles should contain three identical side lengths. If the triangles are congruent (have the same 3 side lengths), then the matching angles must also be the same. This is not the case, though! For example, angles *QOC* and *QOA* are *not* the same in the diagram, since *QOA* is only part of*QOC*. In order for the two to have the same angle, points A and C would have to be in the same place, but the statement says that all three points are distinct (or different).

No matter where the three points are placed, at least one such impossible situation will occur where, according to the statement, the sides are all the same, but the angles are not actually the same. It is impossible, then, to create a situation where statement 1 is true but *Q* is not the center of the circle.

Q is the center of the circle. Statement (1) is sufficient.

(2) SUFFICIENT: All diameters of a circle intersect at the circle's center and nowhere else. Therefore, if *Q* is the point of intersection of two or more of the circle's diameters, then *Q* must be the center of the circle. The statement is sufficient.

The correct answer is D.

5.3062Of the birds in a particular aviary, 30 are both spotted and crested and 10 are neither spotted nor crested. What is the total number of birds in the aviary?

(1) The number of birds that are spotted but not crested is 10 greater than the number of birds that are crested but not spotted.

(2) More than 3/5 of the birds in the aviary are spotted.

The information presented can be set up in a Double-Set Matrix:

	C	No C	Total
S	30		
No S		10	
Total			?

(1) INSUFFICIENT: If x represents the number of birds that are crested but not spotted, this statement yields the following:

	C	No C	Total
S	30	$x + 10$	$x + 40$
No S	x	10	$x + 10$
Total	$x + 30$	$x + 20$	$2x + 50 = ?$

Without a value for x, it is not possible to find the total.

(2) INSUFFICIENT: By itself, this statement does not provide values for any unknowns in the problem.

	C	No C	Total
S	30		> 3/5 Total
No S		10	< 2/5 Total
Total			Total = ?

(1) and (2) INSUFFICIENT: The two statements together provide a filled-in matrix and a relationship between the total number of spotted birds and the total number of birds in the aviary. However, this only provides a range for the total:

	C	No C	Total
S	30	$x + 10$	$x + 40$

No S	x	10	$x + 10$
Total	$x + 30$	$x + 20$	$2x + 50 = ?$

Total S > 3/5 of Total Birds
$x + 40 > 3/5\ (2x + 50)$
$x + 40 > 6x/5 + 30$
$10 > x/5$
$50 > x$

If $x < 50$, the total number of birds, $2x + 50$, must be less than 150, but this does not provide a definitive value for the total.

The correct answer is (E).

6.3056Guests at a recent party ate a total of fifteen hamburgers. Each guest who was neither a student nor a vegetarian ate exactly one hamburger; no other guests ate hamburgers. If half of the guests were vegetarians, how many guests attended the party?

(1) The vegetarians attended the party at a rate of 2 students to every 3 non-students, half the rate for non-vegetarians.

(2) 30% of the guests were vegetarian non-students.

For this overlapping set problem, set up a double-set matrix to test the possibilities. The first set is vegetarians vs. non-vegetarians; the second set is students vs. non-students.

	V	NON-V	TOTAL
S			
NON-S		15	
TOTAL	x	x	$2x = ?$

Each non-vegetarian non-student ate exactly one of the 15 hamburgers, and nobody else ate any of the 15 hamburgers. Therefore, there are exactly 15 people in the non-vegetarian non-student category. The question stem also indicates that half of the group is vegetarian, so the other half must be non-vegetarian. The two groups are equal in size, then! Put an x in both boxes in the table.

The second statement is easier than the first statement; start there.

(2) INSUFFICIENT: This statement provides information about one group: vegetarian non-students. The figure given, though, is a percentage of another unknown figure; a real number can't be calculated from this. The statement is insufficient.

(1) SUFFICIENT: This statement provides two pieces of information. First, for every 2 vegetarian students in attendance, there were 3 vegetarian non-students. Next, this 2:3 rate is half the rate for non-vegetarians. To double a rate, double the first number (4:3). The rate is 4 non-vegetarian students to 3 non-vegetarian non-students. The problem provides a real number for one of these categories! There are 15 non-vegetarian non-students. If that's the case, then there must be 20 non-vegetarian students. Add this info to the table.

	V	NON-V	TOTAL
S		20	

NON-S		15	
TOTAL	x or 35	x or 35	$2x$ = ?

Therefore, there were 20 + 15 = 35 total non-vegetarians (see the chart below). Since the same number of vegetarians and non-vegetarians attended the party, there were also 35 vegetarians, for a total of 70 guests.

The correct answer is A.

7.3040If n is an integer between 100 and 900, what is the tens digit of n?

(1) If n is rounded to the nearest ten and the result is then rounded to the nearest hundred, the final value differs from the result of rounding n to the nearest hundred.

(2) The tens digit of n is half the units digit of n and is twice the hundreds digit of n.

(1) SUFFICIENT: Try a real number to understand how the statement works. If n = 113, then first rounding to the nearest ten would give 110, and then to the nearest hundred would give 100. This is NOT a different result than rounding 113 to the nearest hundred (so n can't equal 113). What would need to be true of the value of n in order to make this statement true?

The number would need to be somewhere near the midpoint (150, 250, 350, etc) so that it could round up one way and down the other. If n = 146, then rounding to the nearest tens digit gives 150 and then rounding to the nearest hundreds digit gives 200. If 146 is rounded directly by the hundreds digit, though, then the number rounds down to 100.

What if n = 152? Rounding down by the tens digit gives 150; rounding by the hundreds digit then gives 200. Rounding 152 directly by the hundreds digit also gives 200. This case doesn't work.

Only something in the 145 to 149 range will work. (Or 245 to 249, 345 to 349, and so on.) The tens digit, then, must equal 4. The statement is sufficient.

Alternatively, think through the theory. In the following discussion, the underdash _ stands for the unknown hundreds digit of *n*.

Consider the results of rounding n to the nearest ten and then rounding the result to the nearest hundred. In this case, any value of n that *rounds* to _50, _60, _70, _80, or _90 will then be rounded *up* to the nearest hundred; any value of n that rounds to _00, _10, _20, _30, or _40 will be rounded down.

Therefore, with this two-step process, all values of n from _45 to _99 will ultimately be rounded *up* to the next multiple of 100; all values of n from _00 to _44 will ultimately be rounded down.

Consider the same numbers rounded directly to the nearest hundred. In the cases _00 to _49, the number will round down. In the cases _50 to _99, the number will round up. In only one set of cases, the ultimate outcome will be different: the cases _45 through _49, which are rounded up by the two-step process but down by direct rounding to the nearest hundred. Therefore, to satisfy statement (1), the tens digit of n must be 4. The statement is sufficient.

(2) INSUFFICIENT: According to this statement, the hundreds digit times 2 equals the tens digit, and the tens digit times 2 equals the units digit. This statement yields two possibilities, n = 124 and n = 248. These two values of n have different tens digits, so the statement is not sufficient.

The correct answer is A.

8.3039If $an \neq 0$ and n is a positive integer, is n odd?

(1) $a^n + a^{n+1} < 0$

(2) a is an integer.

(1) INSUFFICIENT: Factor a^n out of the left-hand side to yield $a^n(1 + a) < 0$. For this inequality to be true, the individual terms a^n and $1 + a$ must have opposite signs. Consider the two possible cases:
(i) a^n is positive and $1 + a$ is negative: If $1 + a < 0$, then $a < -1$. Because a is negative, it follows that n must be even (since a^n is positive).
(ii) a^n is negative and $1 + a$ is positive: In order for a^n to be negative, a itself must be negative. A positive number will never turn negative when raised to a power (recall that a negative power makes a positive number smaller, but it doesn't change a positive number to a negative one). In this case, n would have to be odd in order to make a^n negative.

Because n may be either even or odd, the statement is insufficient.

(2) INSUFFICIENT: This statement provides no information about n.

(1) AND (2) SUFFICIENT: Examine case (ii) of statement 1 further (the case in which a is negative and n is odd). If $1 + a > 0$, then $a > -1$. At the same time, a itself must be negative, so $-1 < a < 0$.

Statement 2 specifies that a is an integer. There are no integers between –1 and 0, and so case (ii) cannot be valid. Only case (i) is possible. As a result, n must be even, so the two statements together are sufficient.

The correct answer is C.

9.3038If a, b, and c are three positive integers, each greater than 1, what is the remainder when the product abc is divided by 2?

(1) If each of a, b, and c is divided by 2, the product of all three remainders is 0.

(2) If each of a, b, and c is divided by 2, the sum of all three remainders is 2.

At first glance, the question appears to be about remainders, but this language is a disguise. The *remainder upon dividing by 2* is a proxy for whether an integer is odd or even: If the remainder is 0, then the original integer is even; if the remainder is 1, the original integer is odd. Approach the problem from an odd / even point of view.

Rephrase the question:
When three integers are multiplied together, two outcomes are possible: the product could be even or it could be odd. If the product is even, then the remainder when dividing by 2 will be 0. If the product is odd, then the remainder when dividing by 2 will be 1.

If *all* of a, b, and c are odd, then the product will be odd, and the remainder will thus be 1. If at least one of the integers is even, then the product will be even, making the remainder 0. Therefore, the question is this: is at least one of the three integers (a, b, and c) even?

(1) SUFFICIENT: If the product of the three remainders is 0, then at least one of the individual remainders must be 0. In other words, yes, at least one of$a$, b, and c must be an even integer (and, therefore, the remainder when the product abc is divided by 2 must be 0).

(2) SUFFICIENT: Each individual remainder is either 0 or 1. Therefore, if the sum of three such remainders is 2, then exactly two of the remainders must be equal to 1, and the third must be equal to 0. Because one of the remainders is 0, it follows that at least one of a, b,

and *c* is an even integer (and, therefore, the remainder when the product *abc* is divided by 2 must be 0).

The correct answer is D.

10.3035At a particular store, candy bars are normally priced at $1.00 each. Last week, the store offered a promotion under which customers purchasing one candy bar at full price could purchase a second candy bar for $0.50. A third candy bar would cost $1.00, a fourth would cost $0.50, and so on. If, in a single transaction during the promotion, Rajiv spent *D* dollars on *N* candy bars, where *D* and *N* are integers, is *N* even?

(1) *D* is prime.

(2) *D* is divisible by 3.

The nature of the promotion described, with alternating prices for every other candy bar, makes it impractical to write a formula for *D* in terms of *N*. Instead, make a table of different values of *N* and *D*, and look for patterns. Because we're told that *D* must be an integer, we only need to consider the boldface cases listed in the below table:

N	D
1	**1.00**
2	1.50
3	2.50
4	**3.00**
5	**4.00**
6	4.50
7	5.50
8	**6.00**
9	**7.00**
10	7.50
11	8.50
12	**9.00**
13	**10.00**

In looking for patterns, pay attention to the particular number properties that are mentioned in the problem itself – namely, even versus odd (for values of *N*), and multiple of 3 vs. not a multiple of 3 (for values of *D*). The possible values for *N* alternate between even and odd. The values for *D* that are multiples of 3 all correspond to even values of *N*.

(1) INSUFFICIENT: Look in the table for prime values of *D*. When *D* = $3.00, *N* = 4, an even number. When *D* = $7.00, *N* = 9, an odd

number. The answer to the question "Is *N* even?" is sometimes but not always, which is not sufficient to answer the question.

(2) SUFFICIENT: According to the table, when *D* is a multiple of 3, *N* is always even. When *D* is not a multiple of 3, *N* is odd. If we're limited to the values for *D* that are divisible by 3, then *N* must be even. The answer to the question "Is *N* even?" is yes, always.

The correct answer is B.

11.3034If the original price of an item in a retail store is marked up by m percent and the resulting price is then discounted by d percent, where m and d are integers between 0 and 100, is the item's final price (after both changes) greater than its original price?

(1) $m > d$

(2) $m = 1.5d$

Perhaps the most straightforward way to investigate this problem is by testing values. Since the problem deals with inequalities – the prompt question is an inequality, and there is also an inequality in one of the statements – we should pay particular attention to extreme values as test cases. In the following discussion, we will set the starting price to $100.

(1) INSUFFICIENT: Considering extreme cases, if $m = 99$ and $d = 1$, that's a 99% markup followed by a 1% markdown. If the original price is $100, the price after the markup is $199, and the value of the discount is only $1.99. In this case, the final price is much greater than the original, so this is a YES answer to the question.

Can we find a NO answer? Considering the other extreme of d (large values), let $m = 99$ and $d = 90$. (We could investigate d up to 98 if necessary, but it's best to stick to numbers that provide easy calculation.) With these values, the price after the markup is $199. The discount is 90% of $199, which is very close to 90% of 200 (= $180). Therefore, the price will be marked down by almost $180, substantially less than the original $100 price. This is a NO answer to the question.

(2) INSUFFICIENT: Again, consider extreme cases. The smallest possible values of m and d are $m = 3$ and $d = 2$. (We can't use $d = 1$ because then $m$will not be an integer.) In this case, the price after the 3% markup is $103; the discount is 2% of $103, or $2.06. If $2.06 is taken away from $103, the result is greater than the original $100, so this is a YES answer to the question.

Can we find a NO answer? The largest possible values of m and d are $m = 99$ and $d = 66$. (To find these values, set m to the largest possible integer, 99. Then solve for d, given the equation $m = 1.5d$. If you do not receive an integer value for d, then try $m = 98$ and so on.) In this case, the price after the 99% markup is $199. The subsequent discount is 66% of $199, which is very close to 66% of 200 (= $132); this discount is large enough to bring the final price far below the original price of $100. This is a NO answer to the question.

(1) AND (2) INSUFFICIENT: Review the work already done, so as not to duplicate effort needlessly. The case $m = 99$ and $d = 66$ satisfies both statements 1 and 2, so that's a NO answer to the question.

Can we find a YES answer? Consider smaller values that satisfy both statements, say $m = 45$ and $d = 30$. In this case, the markup is 45%, taking the price from \$100 to \$145. The discount is 30% of \$145, or \$43.50. The final price is \$145 - \$43.50 = \$101.50, which is greater than the original price; this is a YES answer to the question.

The correct answer is E.

12.3023Machines A and B each produce tablets at their respective constant rates. Machine A has produced 30 tablets when Machine B is turned on. Both machines continue to run until Machine B's total production catches up to Machine A's total production. How many tablets does Machine A produce in the time that it takes Machine B to catch up?

(1) Machine A's rate is twice the difference between the rates of the two machines.

(2) The sum of Machine A's rate and Machine B's rate is five times the difference between the two rates.

Machine A has an initial head start of 30 tablets. The question asks for the number of tablets produced by Machine A after Machine B is turned on.

Machine B must be working faster and, in particular, the *difference* in the rates of the two machines must allow Machine B to produce the 30 extra tablets to catch up with Machine A:

	Rate	×	Time	=	Work
Machine A	A	×	t	=	
Machine B	B	×	t	=	
Machine B – Machine A	$B – A$	×	t	=	30 tablets

Note that t represents the time that both machines are turned on (which is the time that Machine B takes to catch up to Machine A). The question asks for the number of tablets produced by Machine A during this time, or At. The table also tells us that $(B – A)t = 30$ or $Bt – At = 30$

(1) SUFFICIENT: Translate the statement: $A = 2(B − A)$

Simplify:
$A = 2(B − A)$
$A = 2B − 2A$
$3A = 2B$

Combine with the given equation $Bt – At = 30$ and try to solve for At. Solve the simpler equation, above, for B ($3A/2 = B$) and then substitute into the more complicated equation.
$(3A/2)t – At = 30$
$(3At)/2 – (2At)/2 = 30$
$(At)/2 = 30$

$At = 60$

Note: stop the above math at the point that you can tell it is definitely possible to solve for At. In the requested time frame, Machine A makes 60 tablets.

(2) SUFFICIENT: Translate the statement: $A + B = 5(B - A)$

Simplify:
$A + B = 5B - 5A$
$6A = 4B$
$(3/2)A = B$

Combine with the given equation $Bt - At = 30$ and try to solve for At:
$(3A/2)t - At = 30$ (Note: from this point, the math is the same as for statement 1; don't repeat the work!)
$(3At)/2 - (2At)/2 = 30$
$(At)/2 = 30$
$At = 60$

In the requested time frame, Machine A makes 60 tablets.

The correct answer is D

13.3015If list *S* contains nine distinct integers, at least one of which is negative, is the median of the integers in list *S* positive?

(1) The product of the nine integers in list *S* is equal to the median of list *S*.

(2) The sum of all nine integers in list *S* is equal to the median of list *S*.

Because list *S* has nine integers (an odd number), the median must be the middle integer.

(1) SUFFICIENT: The median is an integer. According to this statement, the nine numbers must collectively multiply to this same integer. None of the numbers is a fraction, and all nine are different numbers. The only way to multiply a bunch of integers together and arrive at the same starting point is to multiply by 0 or 1. In order to use 1, every number on the list except one would have to equal 1; for example, $1 \times 1 \times 1 \times 3 = 3$. In order to use 0, though, only one number has to equal zero; for example, $1 \times 18 \times -3 \times 0 = 0$.

Therefore, the only way in which the product of nine different integers can equal just one of the integers on that list is if the product of the nine integers is zero. (Not convinced? Try to disprove that rule using some real numbers. If the list is -4, -3, -2, -1, 1, 2, 3, 4, 5 then the median is 1 but the product is definitely not 1. If the list is -10, -9, -8, -7, -6, -5, -4, -3, -2, the median is -6 but the product is not -6. If, on the other hand, the list is -4, -3, -2, -1, 0, 1, 2, 3, 4, then the median is 0 and the product is also 0.)

Further, since the product of the nine integers equals the median, the median itself must also be zero. The median, then, is definitely not positive; the statement is sufficient to answer the question.

(2) INSUFFICIENT: The median of list *S* is one of the nine integers in list *S*. Therefore, if the sum of all nine integers equals the median, it follows that the sum of the eight integers on either side of the median must be zero:
Sum of all nine integers = (Median) + (Sum of other eight integers)
Statement 2 says that the Sum of all nine integers equals the Median. Substitute that info into the above equation:
Median = (Median) + (Sum of other eight integers)

0 = Sum of other eight integers

This isn't enough, though, to determine the median; in fact, *any* number can still be the median of the list, as long as the surrounding numbers sum to zero. For example, the list –10, –9, –8, –7, 5, 7, 8, 9, 10 satisfies the criterion and has a median value of 5; the list –10, –9, –8, –7, -1, 6, 8, 9, 11 also satisfies the criterion and has a median value of -1.

The correct answer is A.

14.3013A list contains only integers. Are there more positive than negative integers in the list?

(1) The median of the numbers in the list is positive.

(2) The average (arithmetic mean) of the numbers in the list is positive.

(1) NOT SUFFICIENT: Test numbers and try to disprove the question. For instance, the list 1, 2, 3, 4, 5 has a positive median (3) and contains more positive than negative integers. On the other hand, consider a list with an even number of entries—the median is halfway between two entries in the list, and not necessarily an entry in the list itself. For instance, the list –3, –2, –1, 101, 102, 103 has a positive median (50), but does not contain more positive than negative integers.

(2) NOT SUFFICIENT: Test numbers and try to disprove the question. For instance, the list 1, 2, 3, 4, 5 has a positive median (3) and contains more positive than negative integers. On the other hand, consider a list containing small negative numbers and large positive numbers. For instance, the same list from statement 1 above, –3, –2, –1, 101, 102, 103, has a positive average, but does not contain more positive than negative integers. (In fact, unlike statement 1, this statement can also be satisfied by a list containing *fewer* positive than negative numbers, e.g., –5, –4, –3, –2, –1, 100).

(1) AND (2) NOT SUFFICIENT:
As deduced above, the lists 1, 2, 3, 4, 5 does have more positive integers while the list –3, –2, –1, 101, 102, 103 does not have more positive integers. Both lists satisfy both statements, so even when used together, the statements are still insufficient.

The correct answer is E

15.3011In a certain sport, teams receive 3 points for each win, 1 point for each draw, and no points for losses. In a five-team tournament in this sport, in which each of teams *G*, *H*, *J*, *K*, and *L* played each other team exactly once, did team *L* finish the tournament with the highest point total?

(1) Team *L* finished with 8 points.

(2) The sum of all five teams' point totals for the tournament was 23 points.

Each team plays each other team once, so there are a total of 10 games played. (If you're not sure about that, count it out! GH, GJ, GK, GL, HJ, HK, HL, JK, JL, KL). Each team plays a total of 4 games.

If one team wins, 3 points are awarded (total) for that game. If the teams tie, 2 points are awarded (total) for that game. If one team wins all four of its games, it would earn 12 points.

(1) INSUFFICIENT: We know that team *L* played four games and earned 8 points. The only way to get 8 points in four games is 3 + 3 + 1 + 1, so team *L*must have won two games, drawn two, and lost none.

With this win-loss record, it's possible that team *L* finished first. For instance, say that all of the other games in the tournament were draws. In that case, team *L* would have 8 points, the two teams that lost to *L* would have 0 + 1 + 1 + 1 = 3 points, and the two teams that tied with *L* would have 1 + 1 + 1 + 1 = 4 points.

On the other hand, it's also possible that team *L* finished second. For instance, team *G* could have tied team *L* and beaten all three of the other teams, finishing with 1 + 3 + 3 + 3 = 10 points to team *L*'s 8 points.

(2) INSUFFICIENT: This statement contains no information about the distribution of points among the teams, so we can't tell how any individual team fared.

(1) AND (2) SUFFICIENT: To recap, there are 10 total games, and one game results in either 2 or 3 total points awarded. Say that x of the games were won by one team, and thus the remaining $10 – x$ games were ties. Each "won" game will result in 3 points awarded and each "tie" game will result in two points awarded:
$3(x) + 2(10 – x) = 23$
$3x + 20 – 2x = 23$
$x = 3$

16.3010Four men and three women make up a seven-member committee. The committee has one male captain and one female captain. If all seven committee members are seated in a straight row of seven chairs, does at least one man sit next to another man?

(1) No woman sits next to another woman.

(2) The captains sit in the first and seventh chairs.

Since there are four men out of seven people, there's only one possible way in which *no* two men sit next to each other: namely, the four men sit in chairs 1, 3, 5, and 7 ("MWMWMWM"). Keep this in mind.

(1) INSUFFICIENT: In this statement, the captaincy is irrelevant, and we are concerned only with the relative seating of men and women.

No two women sit in adjacent chairs. The one case in which no two men sit next to each other also fulfills this statement about the women (MWMWMWM). It's also possible, though, to create seating arrangements in which at least two men *do* sit next to each other (e.g., WMMWMMW). This statement isn't sufficient.

(2) SUFFICIENT: The one arrangement in which the men can all be separated is MWMWMWM. Because statement 2 specifies that the one male captain and one female captain must sit in the first and seventh chairs, statement (2) precludes the MWMWMWM arrangement. There may be many remaining possibilities, but all of them will have at least two men seated in adjacent chairs. The statement is sufficient.

The correct answer is B.

17.3009A circle is drawn within the interior of a rectangle. Does the circle occupy more than one-half of the rectangle's area?

(1) The rectangle's length is more than twice its width.

(2) If the rectangle's length and width were each reduced by 25% and the circle unchanged, the circle would still fit into the interior of the new rectangle.

A circle has a constant diameter. If the entire circle is to fit within the rectangle, its diameter must be less than the *shorter* dimension of the rectangle, i.e., less than the rectangle's width. (Remember that the circle must fit *both* length-wise *and* width-wise into the rectangle!)

The circle's diameter is less than *W*, or 2*r* < *W*; thus $r < \frac{W}{2}$. The area of the circle is thus less than $\pi\left(\frac{W}{2}\right)^2 = \frac{\pi}{4}W^2$. The value of π is a little more than 3, so the area is a little more than $\frac{3}{4}W^2$.

The rectangle's area is *LW*. Because *L* > 2*W*, this area is *greater* than (2*W*)(*W*) = $2W^2$.

The circle's area is only a little more than $\frac{3}{4}W^2$, while the rectangle's area is greater than $2W^2$. The circle thus occupies *less* than half the rectangle's area. Statement 1 is sufficient.

(2) SUFFICIENT: Let *L* and *W* stand for the length and width, respectively, of the *original* rectangle (whose area is *LW*). From this statement, then, the circle's diameter is smaller than both $\frac{3}{4}L$ and $\frac{3}{4}W$ (since the circle must fit into the interior both length-wise and width-wise). Because the diameter is twice as long as the radius, the radius must be smaller than either $\frac{3}{8}L$ or $\frac{3}{8}W$.

Use those two dimensions in place of the two *r*'s in the area formula: The circle's area = π(less than $\frac{3}{8}L$)(less than $\frac{3}{8}W$), or < $\frac{9\pi}{64}LW$. The value ofπ is a little more than 3, so $\frac{9\pi}{64} \approx \frac{27}{64}$. The circle's area, then, is about $\frac{27}{64}LW$, or somewhat *less* than half the original rectangle's area of *LW*. Statement 2 is sufficient.

The correct answer is D.

18.3004If *a*, *b*, and *c* are positive numbers such that *a* is *b* percent of *c*, what is the value of *c*?

(1) *a* is *c* percent of *b*.

(2) *b* is *c* percent of *a*.

This looks like a percentage question, but fractions, decimals, and percents are taught together for a reason. A percentage sign can't be used in an equation. Translate the given equation (*a* is *b* percent of *c*) into mathematical notation using either fractions or decimals: $a = \left(\frac{b}{100}\right)c = \frac{bc}{100}$, or *a* = (0.01 *b*) *c* = 0.01 *bc*. (Note: it's your choice whether to use fractions or decimals; use whichever form you find easier.)

(1) NOT SUFFICIENT: This statement can be written as $a = \left(\frac{c}{100}\right)b = \frac{bc}{100}$ or *a* = (0.01 *c*) *b* = 0.01 *bc*. Either way, this statement is identical to the equation already given in the prompt, so statement (1) does nothing to help determine the value of *c*.

(2) SUFFICIENT: This statement can be written as $b = \left(\frac{c}{100}\right)a = \frac{ac}{100}$ or $b = (0.01\ c)\ a = 0.01\ ac$. Use either fractions or decimals, whichever you prefer.

Here's the solution method using fractions: $a = \frac{bc}{100}$ and $b = \frac{ac}{100}$. Substitute:

$$b = \frac{\left(\frac{bc}{100}\right)c}{100}$$
$$b = \frac{bc^2}{10,000}$$

The variable b is nonzero (the question stem says b is positive), so divide it out, giving $1 = \frac{c^2}{10,000}$. Thus $c^2 = 10,000$; since c must be positive, $c = 100$.

Here's the solution method using decimals: $a = 0.01bc$ and $b = 0.01\ ac$. Substitute:

$b = 0.01(0.01bc)\ c$
$b = 0.0001bc^2$

The variable b is nonzero (the question stem says b is positive), so divide it out, giving $1 = 0.0001c^2$, or $10,000 = c^2$. Thus $c^2 = 10,000$; since c must be positive, $c = 100$.

The correct answer is B.

19.3000If the positive integer x is rounded to the nearest ten, will the result be greater than x ?

(1) If x is divided by 10, the remainder is even.

(2) If x is divided by 5, the remainder is odd.

(1): INSUFFICIENT. If a positive integer is divided by 10, the remainder is the same as the integer's units digit. This statement thus implies that the units digit of x is 0, 2, 4, 6, or 8.

If the units digit of x is 6 or 8, then rounding to the nearest ten will result in an increase; otherwise, it will not. The statement is insufficient.

(2): INSUFFICIENT. If a positive integer is divided by 5, the remainder is:

0, if the integer's units digit is 0 or 5
1, if the integer's units digit is 1 or 6
2, if the integer's units digit is 2 or 7
3, if the integer's units digit is 3 or 8
4, if the integer's units digit is 4 or 9

According to this statement, then, the integer's units digit must be 1, 6, 3, or 8. If the units digit of x is 6 or 8, then rounding to the nearest ten will result in an increase; otherwise, it will not. The statement is insufficient.

(1) AND (2): SUFFICIENT. If both statements are true, the units digit of the integer must be 6 or 8 (the only values common to the two statements). In either case, rounding to the nearest ten will result in an increase. The two statements together are sufficient.

The correct answer is C.

20.2999 If $0 < x < 1$, is it possible to write x as a terminating decimal?

(1) $24x$ is an integer.

(2) $28x$ is an integer.

All terminating decimals share one property: the denominator of the decimal (written as a fraction) must contain only 2's and 5's as prime factors. For example, 0.25 written as a fraction is 1/4. The denominator contains only 2's.

(1): INSUFFICIENT. For $24x$ to be an integer, the prime factors of 24 must completely cancel out the factors in the denominator of x. $24 = 2^3 \times 3$. There is no way to know which prime factors the denominator of x actually contains. If it contains a 3, the decimal will not terminate. If the denominator only contains 2's, then it will terminate.

For example, if x is 1/2, $24x = 12$, and the decimal terminates (0.5). If, however, $x = 1/3$, $24x = 8$, but the decimal does not terminate (0.3333….). The statement is insufficient.

(2): INSUFFICIENT. For $28x$ to be an integer, the prime factors of 28 must completely cancel out the factors in the denominator of x. $28 = 2^2 \times 7$. There is no way to know which prime factors the denominator of x actually contains. If it contains a 7, the decimal will not terminate. If the denominator only contains 2's, then it will terminate.

For example, if x is 1/2, $28x = 14$, and the decimal terminates (0.5). If, however, $x = 1/7$, $28x = 4$, but the decimal does not terminate (0.14285….). The statement is insufficient.

(1) & (2): SUFFICIENT. For both $24x$ to be an integer *and* $28x$ to be an integer, the denominator of x can only contain prime factors that both 24 and 28 share. The only prime factors that 24 and 28 share are 2 and $2(2^2)$. Therefore, the denominator of x must only contain 2's, and the decimal must terminate.

The correct answer is C.

PROBLEM SOLVING

1.3068

A rectangular wall is covered entirely with two kinds of decorative tiles: regular and jumbo. 1/3 of the tiles are jumbo tiles, which have a length three times that of regular tiles and have the same ratio of length to width as the regular tiles. If regular tiles cover 80 square feet of the wall, and no tiles overlap, what is the area of the entire wall?

- 160

- 240
- 360
- 440
- 560

The length and the width of a regular tile can be represented as *L* and *w*. The length of a jumbo tile, then, is 3*L*. If the ratio of length to width is the same for all tiles, and the ratio of the regular tile is *L* : *w* then a jumbo tile must have a width of 3*w*.

Represent the area of both types:
Area of one regular tile = *Lw*
Area of one jumbo tile = (3*L*)(3*w*) = 9*Lw*

If 1/3 of the tiles are jumbo, then the other 2/3 must be regular (since only two kinds of tiles are used). There are twice as many regular tiles as jumbo ones. If *J* is the number of jumbo tiles, then there are 2*J* regular tiles.

Area of all jumbo tiles = (the area of one jumbo tile)(the number of jumbo tiles)
Area of all jumbo tiles = (9*Lw*)(*J*)

Area of all regular tiles = (the area of one regular tile)(the number of regular tiles)
Area of all regular tiles = (*Lw*)(2*J*) = 80
Remember that the problem tells you that the area of all regular tiles is 80! Solve to find the value of *LwJ*:
(*Lw*)(2*J*) = 80
LwJ = 40

Find the area of jumbo tiles:
Area of all jumbo tiles = (9*Lw*)(*J*)
Area of all jumbo tiles = (9)(40) = 360

Add them up! Regular + jumbo is 80 + 360, or 440.

Alternatively, use a neat shortcut. If 1/3 of the tiles are jumbo, then 2/3 are regular. In that case, for every two regular tiles, there is one jumbo tile. One jumbo tile has an area 9*Lw* and two normal tiles have an area 2*Lw*, so the tiles will be placed in "sets" of 9*Lw* + 2*Lw* = 11*Lw*. The correct answer, then, must be a multiple of 11, and only answer (D) is a multiple of 11.

The correct answer is D.

2.3059List R contains five numbers that have an average value of 55. If the median of the numbers in the list is equal to the mean and the largest number is equal to 20 more than two times the smallest number, what is the smallest possible value in the list?

- 35

- 30
- 25
- 20
- 15

When a problem provides partial information about the elements contained in a list, organize the information given. In this case, the list contains 5 elements and the median is 55. Therefore, 55 is the middle number:
{*a*, *b*, 55, *c*, *d*}

The variable *a* is the smallest item in the list, *b* is equal to or larger than *a*, *c* is equal to or larger than 55, and *d* is equal to or larger than *c*.

The problem indicates that the largest item, *d*, is equal to 20 more than 2 times the smallest number, *a*:
{*a*, *b*, 55, *c*, 2*a* + 20}

The mean, or average, is equal to 55, so the sum of the 5 elements must be 55 × 5 = 275:
Sum of {*a*, *b*, 55, *c*, 2*a* + 20} = 275

The question asks for the smallest possible value for *a*. How do we minimize this number? Maximize everything else! Note that the problem does not indicate that the 5 numbers have to be different.

The value for *b* has to be less than or equal to 55, so make *b* equal to 55. Likewise, *c* can't be any larger than *d*, so make *c* equal to 2*a* + 20.
Now, the sum of {*a*, 55, 55, 2*a* + 20, 2*a* + 20} = 275. There's only one variable; write it out and solve.

a + 55 + 55 + (2*a* +20) + (2*a* +20) = 275
5*a* + 150 = 275
5*a* = 125
a = 25

Thus, the smallest number in the list is 25.

The correct answer is (C).

You could also try plugging in the answer choices. Ordinarily, you'd start with answer (B) or answer (D), but because the problem asks for the smallest possible value, start with the smallest value given in the answers: 15.
(E) If the smallest number is 15, then the largest number is 2(15) + 20 = 50. The problem states that the median and mean are both 55, though, so the largest number can't be 50. Eliminate (E).

(D) If the smallest number is 20, then the largest number is 2(20) + 20 = 60. If 20 is the smallest possible number, then the other numbers need to be maximized (see discussion above), so the list of numbers would be {20, 55, 55, 60, 60}. In order for the average to be 55, the sum would have to be (5)(55) = 275. Is it?
No. 20 + 55 + 55 + 60 + 60 = 250. Close but not quite! Eliminate answer (D).

(Note: there's a shortcut for calculating that average—if you understand how weighted averages work. If the numbers are 20, 55, 55, 60, and 60, and you want the average to be 55, think of it this way. Two numbers equal the average; ignore them. Two numbers are larger by a total of 10 units (60 is 5 more than 55 and there are two values of 60). The smaller number, 20, is 35 units below the average. The "Over" amount has to equal the "Under" amount in order for the average to be 55, but 10 does not equal 35, so 55 is not the average of these numbers.)

(C) If the smallest number is 25, then the largest number is 2(25) + 20 = 70. The list of numbers would be {25, 55, 55, 70, 70}. Does the sum equal 275?
Yes! 25 + 55 + 55 + 70 + 70 = 275.
(Here's the weighted average shortcut: the two values of 70 are each 15 over the average, for a total of 30 Over. The 25 term is 30 units under the average of 55. 30 Over = 30 Under, so 55 is the average.)

The correct answer is (C).

3.3058The manufacturer's suggested retail price (MSRP) of a certain item is $60. Store A sells the item for 20 percent more than the MSRP. The regular price of the item at Store B is 30 percent more than the MSRP, but the item is currently on sale for 10 percent less than the regular price. If sales tax is 5 percent of the purchase price at both stores, how much more will someone PAY in sales tax to purchase the item at Store A?

- $0
- $0.09
- $0.18
- $0.30
- $1.80

First, determine the price of the item at each store.

The MSRP is $60.

Store A adds 20%. Take 10%, $6, and multiply by 2 to get 20%, or $12. Store A's price is $60 + $12 = $72.

Store B adds 30% and then discounts by 10%. First, find the original price: 10% multiplied by 3 is 6 × 3 = 18 (or 30%). The non-sale price is $60 + $18 = $78. Then subtract 10% of the new number: $78 - $7.80 = $70.20.

The sales price is 5% at both stores. The question asks how much more someone will pay in sales tax at Store A. Don't calculate the full sales tax for the item at each store! The sales tax is identical at the two stores for the first $70.20 spent.

Instead, calculate only the sales tax on the difference between the two prices. The difference is $1.80. 10% of that number is $0.18. 5% is half of that figure, or $0.09.

The correct answer is (B).

4.3057It takes the high-speed train x hours to travel the z miles from Town A to Town B at a constant rate, while it takes the regular train y hours to travel the same distance at a constant rate. If the high-speed train leaves Town A for Town B at the same time that the regular train leaves Town B for Town A, how many more miles will the high-speed train have traveled than the regular train when the two trains pass each other?

$\frac{xy\,(x-y)}{x+y}$

$\frac{z\,(x-y)}{x+y}$

$\frac{z\,(x+y)}{y-x}$

$\frac{xy\,(x-y)}{x+y}$

$\frac{xy\,(y-x)}{x+y}$

This question can be done algebraically or via picking **smart numbers**. The latter solution method is much faster if you have adequately learned how to use it. If you took too much time on this one, learn how to pick smart numbers efficiently and how to visualize the problem; then you'll be able to do these correctly in an appropriate amount of time.

Let's say that the distance between the two towns is z = 30 miles. If the high-speed train takes x = 2 hours to travel that distance, then it travels at a rate of 15 miles per hour. If the regular train takes y = 3 hours to travel that distance, then it travels at a rate of 10 miles per hour. (Note that the regular train should be slower than the high-speed train.)

Now, you have a choice: you can set up an RTD chart or you can visualize what actually happens as the trains move. Again, the latter method is far more efficient as long as you have practiced how to do it!

Both trains leave their respective stations at the same time. You're driving the high-speed train; after an hour, you've traveled 15 miles. Now you're driving the regular train; after an hour, you've traveled 10 miles. The two trains have traveled a total of 25 miles in one hour; in other words, together they travel at a rate of 25 miles per hour.

They need to travel another 5 miles collectively in order to cross paths. Notice that, in the first hour, the high-speed train covered 15 out of the 25 collective miles, or 3/5 of the miles covered by the two trains. The regular train covered 10 out of the 25 miles, or 2/5 of the total miles covered. The same proportion must hold true for the remaining 5 miles.

The high-speed train, then, covers 3 miles and the regular train covers 2 miles.

The high-speed train has traveled 15 + 3 = 18 miles. The regular train has traveled 10 + 2 = 12 miles. The high-speed train has traveled 6 miles more than the regular train.

Check the five answer choices to see which returns a value of 6 when you plug in x = 2, y = 3, and z = 30:

(A) $\frac{30(3-2)}{2+3} = 6$ CORRECT!

(B) $\frac{30(2-3)}{2+3} =$ a negative number

(C) $\frac{30(2+3)}{3-2} = 30(5) =$ too big

(D) $\frac{(2)(3)(2-3)}{2+3} =$ a negative number

(E) $\frac{(2)(3)(3-2)}{2+3} = \frac{6}{5}$

Here's how you would set up an RTD chart. (You do not need to do this if you use the visualization approach above.)

	R	**T**	**D**
High-Speed	15	t	d
Regular	10	t	30 – d

Set-up two equations with two unknowns and solve:

$15t = d$
(+) $10t = 30 - d$

$25t = 30$, so $t = 30/25 = 6/5 = 1.2$

In the 1.2 hours it takes for the two trains to meet, the high speed train will have traveled 15(1.2) = 18 miles, and the regular train will have traveled 10(1.2) = 12 miles. Therefore, the high-speed train will have traveled 6 miles further than the regular train. From here, plug the original values into the answers and check for an answer of 6, as shown earlier.

The algebraic method is shown below; use this *only* if you are completely confident that you can do the math without going down the wrong path or making careless mistakes, and that you can do all of this in 2.5 minutes at the very most.

The high-speed train can travel the z miles in x hours, so its rate is z/x. The regular train can travel the z miles in y hours, so its rate is z/y.

Use an RTD chart to evaluate how far each train travels when they move toward each other starting at opposite ends. Instead of using another variable $d$here, let's express the two distances in terms of their respective rates and times.

	R	T	D
High-Speed	z/x	t	?
Regular	z/y	t	?
Total			z

The distances traveled by the two trains sum to z. Write this as an equation and solve for t:

$$\frac{zt}{x} + \frac{zt}{y} = z$$

$$\frac{t}{x} + \frac{t}{y} = 1$$

$ty + tx = xy$

$t(y + x) = xy$

$$t = \frac{xy}{y + x}$$

To find how much further the high-speed train went in this time:

($rate_{\text{high}}$ × time) – ($rate_{\text{reg}}$ × time)

($rate_{\text{high}}$ – $rate_{\text{reg}}$) × time

$$\left(\frac{z}{x} - \frac{z}{y}\right)\left(\frac{xy}{y + x}\right)$$

$$\left(\frac{zy - zx}{xy}\right)\left(\frac{xy}{y + x}\right)$$

$\dfrac{z(y - x)}{y + x}$ This matches answer (A).

The correct answer is (A).

5.3054Which of the following expressions CANNOT have a negative value?

- $|a + b| - |a - b|$
- $|a + b| - |a|$
- $|2a + b| - |a + b|$
- $a^2 + b^2 - 2|ab|$
- $|a^3 + b^3| - a - b$

As in the case of many—perhaps most—problems involving multiple absolute value signs, it is impractical to try to solve this algebraically. Instead, test some real numbers. Consider the effect of substituting different signs (and, where relevant, different sizes) of *a* and *b* into the expressions.

Because the question asks which choice CANNOT have a negative value, try to find number combinations that will lead to a negative value. One possible way: if the answer consists of one expression minus another expression (as is the case in these answers), try to make the first expression either 0 or as small as possible.

(A) If $a = 1$ and $b = –1$, then this answer choice yields $|0| – |2| = –2$. This choice can be negative so it is incorrect. In general, if *a* and *b* have opposite signs, then $|a – b|$ will have a larger magnitude than $|a + b|$, making the quantity $|a + b| – |a – b|$ negative.

(B) If $a = 1$ and $b = –1$, then this choice yields $|0| – |1| = –1$. This choice can be negative so it is incorrect.

(C) If $a = 1$ and $b = –2$, then this choice yields $|0| – |–1| = 0 – 1 = –1$. This choice can be negative so it is incorrect.

(D) Correct. You might try individual numbers here, as you did for the others, but you would always get a 0 or positive result. After trying a couple but not finding a negative, move on to answer (E) to see whether you can disprove that one (in which case, this answer will be the only one left).

Here's how to prove it definitively:
Note that a^2 must be 0 or positive, as must be b^2. If either variable equals 0, then the value of the expression $2|ab|$ must also be 0, in which case the entire expression $a^2 + b^2 – 2|ab|$ must equal either 0 or positive.

If, on the other hand, neither variable equals zero, then the expression $2|ab|$ must be positive. Note that a^2 can also be written $|a|^2$. Rewrite this expression as:
$|a|^2 + |b|^2 – 2|ab|$
$= |a|^2 – 2|ab| + |b|^2$
$= (|a| – |b|)^2$
This expression is a perfect square (everything inside the parentheses gets squared) and so must be either zero or positive.

(E) This is the toughest one to disprove, because the little shortcut of making the first expression equal 0 doesn't work here. If $a = b = 1/2$, then this choice yields $|1/4| – 1/2 – 1/2 = –3/4$. This choice can be negative so it is incorrect. In general, if *a* and *b* are between 0 and 1, the expression $|a^3 + b^3| –a – b$ will be negative.

The correct answer is (D).

6.3053A_piece of wire is bent so as to form the boundary of a square with area *A*. If the wire is then bent into the shape of an equilateral triangle, what will be the area of the triangle thus bounded in terms of *A*?

- $\frac{\sqrt{3}}{64}A^2$
- $\frac{\sqrt{3}}{4}A$

- $\frac{9}{16}A$
- $\frac{3}{4}A$
- $\frac{4\sqrt{3}}{9}A$

Smart Numbers Solution

A real number for *A* is never given (nor is a real number for anything else), so select your own. Say *A* = 36 square units—a perfect square that is divisible by both 4 and 3, since the problem involves both a square (4 sides) and a triangle (3 sides).

If *A* = 36, then the side length of the square is 6 units, so the total length of the wire (which equals the square's perimeter) is 24 units. If the wire is then bent into the shape of an equilateral triangle, each side of the triangle will be 8 units long.

The base of this triangle is 8 units. From the 30º-60º-90º proportions, the triangle's height is $\frac{\sqrt{3}}{2}(8) = 4\sqrt{3}$ units. The area of the triangle is:

$\frac{1}{2}(\text{base})(\text{height})$

$\frac{1}{2}(8)\left(4\sqrt{3}\right)$

$16\sqrt{3}$ square units

Check the answer choices, substituting *A* = 36 and looking for the answer $16\sqrt{3}$:

(A) $\frac{\sqrt{3}}{64} \times 36 \times 36 = \frac{\sqrt{3}}{4} \times 9 \times 9 = \frac{81\sqrt{3}}{4}$

(B) $\frac{\sqrt{3}}{4}(36) = 9\sqrt{3}$

(C) $\frac{9}{16}(36) = \frac{9}{4}(9) = \frac{81}{4}$ (note that there's no square root in this choice!)

(D) $\frac{3}{4}(36) = 3(9) = 27$ (note that there's no square root in this choice!)

(E) $\frac{4\sqrt{3}}{9}(36) = \left(4\sqrt{3}\right)(4) = 16\sqrt{3}$

Only choice (E) yields $16\sqrt{3}$.

Geometric/Algebraic Solution

The area of the original square is A square units, so the original square's side length is $\sqrt{A}$ units. The full length of the wire is therefore $4\sqrt{A}$ units; if it is reshaped into an equilateral triangle, each side of that triangle will be $\frac{4\sqrt{A}}{3}$ units long.

The base of the resulting equilateral triangle will thus be $\frac{4\sqrt{A}}{3}$ units. Using the 30º-60º-90º proportions, the triangle's height will be

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{4\sqrt{A}}{3}\right)$$
$$=\left(\frac{2\sqrt{3}}{3}\right)\sqrt{A}\text{ units.}$$

The area of the triangle is:

$$\frac{1}{2}bh=\frac{1}{2}\left(\frac{4\sqrt{A}}{3}\right)\left(\frac{2\sqrt{3}}{3}\right)\left(\sqrt{A}\right)$$
$$=\frac{4\sqrt{3}}{9}\left(\sqrt{A}\right)^2$$
$$=\frac{4\sqrt{3}}{9}A\text{ square units.}$$

The correct answer is (E).

7.3044 $\sqrt{\dfrac{1}{3^0+3^{-1}+3^{-2}+3^{-3}+3^{-4}}}=$

- $\frac{1}{11}$
- $\frac{9}{11}$
- $\frac{11}{9}$
- 11
- 3^{10}

Several different approaches are possible; choose the one that is easiest for you.

Approach #1: Get Rid of Negative Exponents
Multiply the top and bottom of the fraction by 3^4, to eliminate all negative exponents:

$$\sqrt{\frac{1}{3^0+3^{-1}+3^{-2}+3^{-3}+3^{-4}}\left(\frac{3^4}{3^4}\right)}$$
$$=\sqrt{\frac{3^4}{3^4+3^3+3^2+3^1+3^0}}$$
$$=\sqrt{\frac{81}{81+27+9+3+1}}$$
$$=\sqrt{\frac{81}{121}}=\frac{9}{11}$$

Approach #2: Get a Common Denominator

Change the expressions with negative exponents into fractions, create a common denominator, and combine them:

$$\sqrt{\frac{1}{3^0+3^{-1}+3^{-2}+3^{-3}+3^{-4}}}$$
$$=\sqrt{\frac{1}{1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}}}$$
$$=\sqrt{\frac{1}{\left(\frac{81+27+9+3+1}{81}\right)}}$$
$$=\sqrt{\frac{81}{81+27+9+3+1}}$$
$$=\sqrt{\frac{81}{121}}=\frac{9}{11}$$

Approach #3: Pull Out a Common Term

Pull out the smallest term (in this case, 3^{-4}) from each term in the denominator:

$$\sqrt{\frac{1}{3^0+3^{-1}+3^{-2}+3^{-3}+3^{-4}}}$$
$$\sqrt{\frac{1}{3^{-4}(3^4+3^3+3^2+3^1+3^0)}}$$
$$\sqrt{\frac{3^4}{(3^4+3^3+3^2+3^1+3^0)}}$$
$$\sqrt{\frac{81}{(81+27+9+3+1)}}$$
$$\sqrt{\frac{81}{121}}=\frac{9}{11}$$

The correct answer is B.

8.3042The Italian size of a suit jacket is N centimeters, where N is the linear distance between the armpits. The American size of a suit jacket is $2.5P$ centimeters, where $2.5P$ is twice the linear distance between the armpits. If, for a particular jacket, $N = P + 10$, which of the following is closest to the value of N?

- 36
- 47
- 58
- 65
- 72

The problem defines N centimeters as the Italian size measurement of a suit jacket and $2.5P$ centimeters as the American size measurement of a suit jacket. Further, the problem describes how N and P are actually measured. It turns out that the two systems do not measure the jackets in the same way. Rather, the P measurement is twice as long as the N measurement.

As a result, it isn't the case that N and $2.5P$ are equivalent. Rather, N and *half* of $2.5P$ are equivalent:

$N = (1/2)(2.5P)$

$N = 1.25P$

The problem also states that $N = P + 10$. Combine the two equations to solve for N. First, isolate P in one of the equations:

$P = N - 10$

Then substitute into the other equation and solve:

$N = 1.25P$

$N = 1.25(N – 10)$

$N = 1.25N – 12.5$

$12.5 = 1.25N – 1N$

$12.5 = 0.25N$ multiply both sides by 4

$50 = N$

The problem asks for the closest value. The closest answer choice to 50 is 47.

The correct answer is B.

9.3037Bill has a set of 6 black cards and a set of 6 red cards. Each card has a number from 1 through 6, such that each of the numbers 1 through 6 appears on 1 black card and 1 red card. Bill likes to play a game in which he shuffles all 12 cards, turns over 4 cards, and looks for pairs of cards that have the same value. What is the chance that Bill finds at least one pair of cards that have the same value?

- 8/33
- 62/165
- 17/33

- 103/165
- 25/33

The chance of getting AT LEAST one pair of cards with the same value out of 4 dealt cards should be computed using the **1-x technique**. That is, you should figure out the probability of getting NO PAIRS in those 4 cards (an easier probability to compute), and then subtract that probability from 1.

First card: The probability of getting NO pairs so far is 1 (since only one card has been dealt).

Second card: There is 1 card left in the deck with the same value as the first card. Thus, there are 10 cards that will NOT form a pair with the first card. there are 11 cards left in the deck.

Probability of NO pairs so far = 10/11.

Third card: Since there have been no pairs so far, there are two cards dealt with different values. There are 2 cards in the deck with the same values as those two cards. Thus, there are 8 cards that will not form a pair with either of those two cards. There are 10 cards left in the deck.

Probability of turning over a third card that does NOT form a pair in any way, GIVEN that there are NO pairs so far = 8/10.

Cumulative probability of avoiding a pair BOTH on the second card AND on the third card = product of the two probabilities above = (10/11) (8/10) = 8/11.

Fourth card: Now there are three cards dealt with different values. There are 3 cards in the deck with the same values; thus, there are 6 cards in the deck that will not form a pair with any of the three dealt cards. There are 9 cards left in the deck.

Probability of turning over a fourth card that does NOT form a pair in any way, GIVEN that there are NO pairs so far = 6/9.

Cumulative probability of avoiding a pair on the second card AND on the third card AND on the fourth card = cumulative product = (10/11) (8/10) (6/9) = 16/33.

Thus, the probability of getting AT LEAST ONE pair in the four cards is 1 − 16/33 = 17/33.

The correct answer is C.

10.3036Each light bulb at HOTEL California is either incandescent or fluorescent. At a certain moment, forty percent of the incandescent bulbs are switched on, and ninety percent of the fluorescent bulbs are switched on. If eighty percent of all the bulbs are switched on at this moment, what percent of the bulbs that are switched on are incandescent?

- 22 (2/9)%

- 16 (2/3)%
- 11 (1/9)%
- 10%
- 5%

Although there are two pairs of mutually exclusive classifications -- incandescent vs. fluorescent, and on vs. off -- this is not best approached as an overlapping set problem. the question is only concerned with the number of light bulbs that are turned on, and be approached as a mixtures problem.

Label the total number of incandescent light bulbs I, and label the total number of fluorescent light bulbs F. Forty percent of the incandescent light bulbs ($0.4I$) are turned on, and ninety percent of the fluorescent light bulbs ($0.9F$) are turned on. Eighty percent of all the light bulbs are turned on. The total number of light bulbs is $I + F$, so the total number that are turned on is $0.8(I + F)$.

Thus, $0.4I + 0.9F = 0.8(I + F)$

Although this equation cannot be used to find the value of either I or F, it can be used to solve for the ratio of I to F:

$0.4I + 0.9F = 0.8(I + F)$
$0.4I + 0.9F = 0.8I + 0.8F$
$0.1F = 0.4I$
$F = 4I$

The percent of light bulbs that are incandescent is $\frac{0.4I}{0.8(I+F)}$. Replace F with ($4I$) to solve for the percent:

$$\frac{0.4I}{0.8(I+4I)} = \frac{0.4I}{0.8(5I)} = \frac{0.4I}{4I} = \frac{1}{10} = \frac{10}{100}$$

Incandescent light bulbs are 10% of the total.

The correct answer is D.

11.3031Which of the following lists has the third largest standard deviation?

- {1, 2, 3, 4, 5}
- {2, 3, 3, 3, 4}
- {2, 2, 3, 3, 5}
- {1, 2, 3, 4, 6}

- {1, 3, 4, 5, 7}

The standard deviation is a measure of how far the data points in a distribution fall from the mean. In general, the more "spread apart" the numbers in the list, the higher the standard deviation.

By inspection, the average of each set is around 3 or 4 (estimate), so compare the answers to determine which are the most "spread out." The values in the lists for answers (D) and (E) are more spread out than the others (they range from 1 to 6 or from 1 to 7), so they have the two largest standard deviations. Furthermore, the values in answer (B) are closely bunched around 3, so it has the smallest standard deviation. Therefore, whichever of data sets (A) and (C) has the larger standard deviation will be the third largest overall. Compare the two directly:

(A) {1, 2, 3, 4, 5}
(C) {2, 2, 3, 3, 5}

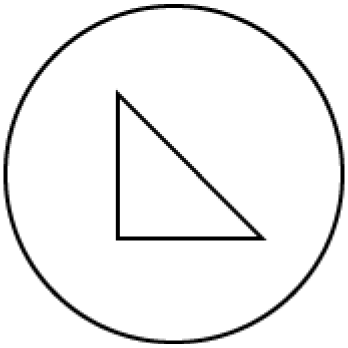
Both sets have an average of 3. Three of the numbers in the list are identical. For both of the "non-identical" numbers, the number in answer (C) is closer to the average (2 vs. 1 and 3 vs. 4). Therefore, the numbers in answer (A)'s list are more spread out and this list has the larger standard deviation. The list {1, 2, 3, 4, 5} therefore has the third-largest standard deviation among the answers.

The correct answer is A.

12.3030A cylindrical tank has a base with a circumference of $4\sqrt{3\pi}$ meters and an isosceles right triangle painted on the interior side of the base. A grain of sand is dropped into the tank, and has an equal probability of landing on any particular point on the base. If the probability of the grain of sand landing on the portion of the base outside the triangle is 3/4, what is the length of a side of the triangle?

- 3
- 12
- $\sqrt{2}$
- $\sqrt{3}$
- $\sqrt{6}$

First, visualize what the problem describes. You can ignore the cylinder; the important portion is the circular base with the triangle inside. Note that an “isosceles right triangle” is a 45-45-90 triangle:



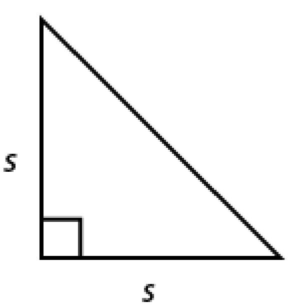
The circumference of the circle is $4\sqrt{3\pi}$.
Use this information to find the area of the circle. First, find the radius. The circumference equals $2\pi r$.

$$4\sqrt{3\pi} = 2\pi r$$
$$\left(4\sqrt{3\pi}\right)^2 = (2\pi r)^2$$
$$16(3\pi) = 4\pi^2 r^2$$
$$\frac{16(3\pi)}{4\pi^2} = r^2$$
$$\frac{12}{\pi} = r^2$$

Don't solve for *r*! Remember that the goal is to find the area of the circle, and the area equals πr^2.

$$A = \pi\left(\frac{12}{\pi}\right)$$
$$A = 12$$

Because the probability of the grain of sand landing outside the triangle is 3/4 , the triangle must comprise 1/4 of the area of the circle. The area of the triangle, then, is 3.
Call the two shorter legs of the 45-45-90 triangle *s*.



The base is *s* and the height is also *s*.

The area of a triangle = 1/2 × base × height, so the area of this triangle can be expressed as: $\frac{1}{2}s^2$.

This triangle has an area of 3, so:

$$3 = \frac{1}{2}s^2$$

$$6 = s^2$$

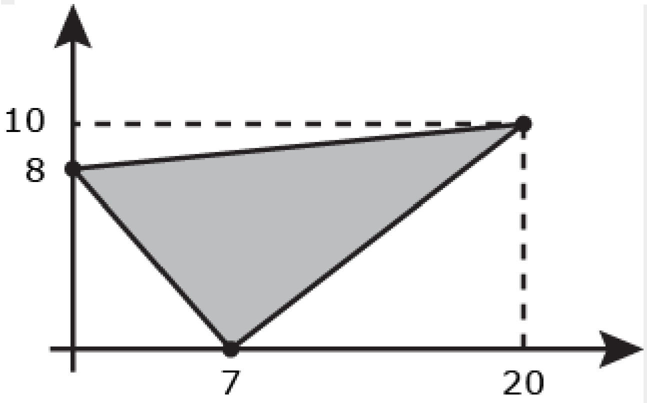
$$\sqrt{6} = s$$

The correct answer is E.

13.3014A triangle in the *xy*-coordinate plane has vertices with coordinates (7, 0), (0, 8), and (20, 10). What is the area of this triangle?

- 72
- 80
- 87
- 96
- 100

It's not immediately possible to use the standard formula $A = bh / 2$ for the area of this triangle, because none of the triangle's sides is horizontal or vertical. However, if the triangle is circumscribed by a rectangle, as depicted below, the areas of the three *surrounding* triangles can readily be found and then subtracted from the rectangle's area to yield the desired result.



First, the triangle to the lower left has $b = 7$ and $h = 8$, so its area is (7)(8) / 2 = 28.

Second, the triangle to the lower right has $b = 13$ and $h = 10$, so its area is (13)(10) / 2 = 65.

Third, the topmost triangle has $b = 20$ and $h = 2$, so its area is (20)(2) / 2 = 20.

The total area of the surrounding triangles—that is, the triangles that are *not* part of the desired area—is 28 + 65 + 20 = 113 square units.

The total area of the rectangle is 20 x 10 = 200 square units, so the triangle's area is 200 – 113 = 87 square units.

The correct answer is C.

14.3005Two teams are distributing information booklets. Team A distributes 60% more boxes of booklets than Team B, but each box of Team A's has 60% fewer booklets than each box of Team B's. Which of the following could be the total number of booklets distributed by the two groups?

- 2,000
- 3,200
- 4,100
- 4,800
- 4,900

This problem is disguising several things, but a few key clues will help to decipher the best solution. First, the question asks which of the answers*could* be the total. More than one number, then, will satisfy the described conditions, but all of the numbers that do fulfill the conditions will need to share some characteristic.

What is that characteristic? Try to find the simplest number that will actually satisfy the details of the problem; use that to help decipher the needed characteristic. Team A distributes 60% more boxes than Team B. If Team B distributes 10 boxes, then Team A distributes 16. Simplify those numbers even more: if team B distributes 5 boxes, then Team A distributes 8.

Are you getting any ideas about what the problem is really testing? Ratios! The question looks like a percentage problem to start, but it's actually a ratio problem. If you pick this up from the start, then you'll know the ratio for 60% is 5 : 8. The ratio of Team A boxes to Team B boxes is 8 : 5. (Remember that A has more boxes.)

Next, one box from Team A contains 60% *fewer* booklets than one box from Team B. That's the same as saying that Team A's boxes contain 40% of the number contained in Team B's boxes. 40% is equivalent to a ratio of 2/5, or 2 : 5. For booklets per box, the ratio of Team to Team B is 2 : 5.

At the lowest level in the ratio, then, Team A distributes 8 boxes containing 2 booklets each, for a total of 16 booklets distributed. Team B distributes 5 boxes containing 5 booklets each, for a total of 25 booklets distributed. Together, the two teams distribute 16 + 25 = 41 booklets. This is the smallest possible value for the number of booklets distributed.

What would happen if we increased the numbers? Because everything has to happen according to the given ratios, and because the number of boxes and the number of booklets must be integers, the total number of booklets must be a multiple of 41.

Not sure why this always works? Combine the ratios. To get the total number of booklets, multiply the number of boxes by the number of booklets for each Team and then add the totals.

Team A boxes : Team B boxes = 8 : 5

Team A booklets per box : Team B booklets per box = 2 : 5
Total Team A booklets : Total Team B booklets = (8)(2) : (5)(5) = 16 : 25
The sum of that last ratio is 16 + 25 = 41.

Test the answers. Only 4,100 is a multiple of 41.

The correct answer is C.

15.3003A driver paid *n* dollars for auto insurance for the year 1997. This annual premium was raised by *p* percent for the year 1998; for each of the years 1999 and 2000, the premium was decreased by 1/6 from the previous year's figure. If the driver's insurance premium for the year 2000 was again *n* dollars, what is the value of *p*?

- 12
- $33\frac{1}{3}$
- 36
- 44
- 50

Note that you can pick a smart number for *n*, but not for *p*. The value of *p* must match one of the answers (since the answer choices represent *p*).

One approach, then, is to plug those answers into the problem. Let the 1997 premium be *n* = \$100. Ordinarily, you'd start with answer B or D, but the numbers are pretty annoying. Make your life easy; start with 50%.

If *p* = 50, then the 1998 premium is 50% more than \$100, or \$150. One-sixth of \$150 is \$25, so the 1999 premium is \$150 – \$25 = \$125. One-sixth of that amount is \$20 plus some change, so the 2000 premium is \$125 – about \$20 = about \$104.

This number doesn't equal the starting point, \$100, but it's not off by very much, so try choice (D) next.

If *p* = 44, then the 1998 premium is 44% more than \$100, or \$144. One-sixth of \$144 is \$24, so the 1999 premium is \$144 – \$24 = \$120. One-sixth of that amount is \$20, so the 2000 premium is \$120 – \$20 = \$100. Bingo!

The correct answer is D.

Alternatively, you can use algebra (although it isn't easy). In 1997, the premium is *n*. In 1998, the premium is $n+\left(\frac{p}{100}\right)n$. Decreasing a quantity by one-sixth is equivalent to multiplying the quantity by 5/6, so the 1999 premium is (5/6) of the 1998 premium and the 2000 premium is (5/6)(5/6) of the 1998 premium. Remember that the 2000 premium also equals *n*:

$$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(n+\left(\frac{p}{100}\right)n\right)=n$$ divide both sides by *n*

$$\left(\frac{25}{36}\right)\left(1+\frac{p}{100}\right)=1$$

$$\frac{25}{36}+\frac{25p}{3600}=1$$ multiply all terms by 3600 to get rid of fractions

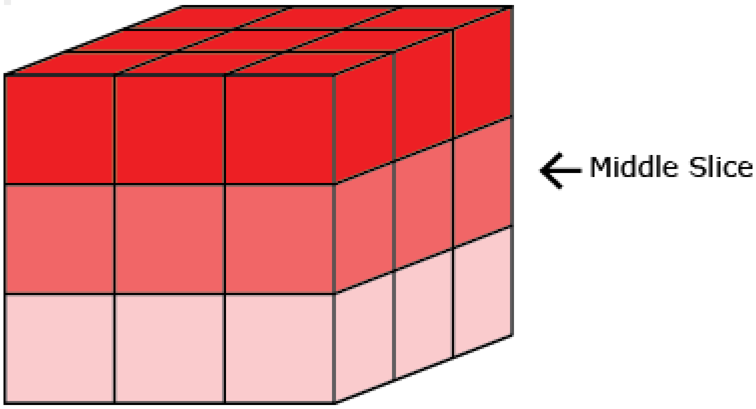
$$2500+25p=3600$$
$$25p=1100$$
$$p=\frac{1100}{25}$$
$$p=44$$

The correct answer is D.

16.3002The entire exterior of a large wooden cube is painted red, and then the cube is sliced into n^3 smaller cubes (where $n > 2$). Each of the smaller cubes is identical. In terms of *n*, how many of these smaller cubes have been painted red on at least one of their faces?

- $6n^2$
- $6n^2 – 12n + 8$
- $6n^2 – 16n + 24$
- $4n^2$
- $24n – 24$

The problem never specifies a real number for any of the steps, so select a smart number for *n*, say *n* = 3. In this case, the original cube is painted and then cut into a 3 × 3 × 3 assemblage of 27 smaller cubes. Imagine the top face of the original (large) cube (if you know what a Rubik's Cube is, picture one!). Every cube on that face has been painted on at least one side. The same is true for the bottom face. Now think about the middle "slice" of the cube. This "slice" contains a total of 9 cubes, but only the one in the very middle has not been painted on any of its sides.



Therefore, only one of those 27 cubes—the one in the middle of the structure—remains unpainted. When $n = 3$, the answer is 26.

Plug $n = 3$ into the answers and look for 26:

(A) $6(3^2) = 6(9) = 54$
(B) $6(3^2) - 12(3) + 8 = 54 - 36 + 8 = 26$
(C) $6(3^2) - 16(3) + 24 = 54 - 48 + 24 = 30$
(D) $4(3^2) = 4(9) = 36$
(E) $24(3) - 24 = 72 - 24 = 48$

Only choice (B) gives the desired answer.

The correct answer is B.

17.2997 $6, y, y + 6, 6 - 4y, 6y, 2y$

For the set of terms shown above, if the mean of the set equals 10 then, in terms of y, the median must be

- $\frac{y+6}{2}$
- $y + 3$
- y
- $\frac{3y}{2}$
- $y + 2$

The mean of a set is equal to the sum of terms divided by the number of terms in the set. Therefore,

$$\frac{6 + y + 6 + y + 6 - 4y + 6y + 2y}{6} = 10$$
$$\frac{6y + 18}{6} = 10$$
$y + 3 = 10$
$y = 7$

Given that $y = 7$, the terms of the set can now be ordered from least to greatest:

$6 - 4y, 6, y, y + 6, 2y, 6y$ à $-22, 6, 7, 13, 14, 42$

The median of a set of six terms is the mean of the third and fourth terms (the two middle terms). The mean of the terms 7 and 13 is 10. Plug 7 for y into the answer choices and look for a match. Only ($y + 3$) gives a value of 10.

The correct answer is B.

18.2996A farmer has an apple orchard consisting of Fuji and Gala apple trees. Due to high winds this year 10% of his trees cross pollinated, creating trees that are part Fuji and part Gala. The number of his trees that are pure Fuji plus the number that are part Fuji and part Gala totals 187, while 3/4 of all his trees are pure Fuji. How many of his trees are pure Gala?

- 22
- 33
- 55
- 77
- 88

This problem can be solved using a set of four equations with four unknowns. The fourth variable will represent the *total* number of apple trees. We'll add two equations together to solve for the number of Gala apples relatively simply:

Let F = the number of Fuji trees
Let C = the number of cross pollinated trees
Let G = the number of Gala trees
Let T = the total number of trees

Equation (1): By definition, $F + C + G = T$

Equation (2): The pure Fuji trees plus the cross pollinated trees total 187, so $F + C = 187$

Equation (3): 75% of his trees are Fuji, so $F = 0.75T$
Equation (4): 10% of his trees cross pollinated, so $C = 0.1T$

We can add these two equations together to show that 85% of the trees are Fuji or cross pollinated: $F + C = 0.85T$

Substituting from Equation (2), we get $0.85T = 187$, or $T = 220$

Plugging back into the original "total" equation, we get the following:

$F + C + G = T$
$(F + C) + G = 220$
$187 + G = 220$
$G = 33$

So the farmer has 33 trees that are pure Gala.

The correct answer is B.

19.2993Two musicians, Maria and Perry, work at independent constant rates to tune a warehouse full of instruments. If both musicians start at the same time and work at their normal rates, they will complete the JOB IN 45 minutes. However, if Perry were to work at twice Maria's rate, they would take only 20 minutes. How long would it take Perry, working alone at his normal rate, to tune the warehouse full of instruments?

- 1 hr 20 min
- 1 hr 45 min
- 2 hr
- 2 hr 20 min
- 3 hr

We can use an algebraic approach or a "real-world" approach. Both methods are shown below.

Algebraic Approach
Organize the given information in an RTW chart, using *m* and *p* to represent Maria's and Perry's normal rates, and *w* to represent work (to tune 1 warehouse full of instruments):

	Rate	**Time**	**Work**
Maria	m		w
Perry	p	?	w
Maria & Perry	$m + p$	45 minutes	w

The question mark denotes the desired value: Perry's time. Note that Perry's time also equals w/p.

How do we represent the hypothetical situation in which Perry is twice as fast as Maria? We can create an additional row for this situation:

Maria & Perry (hyp)	$m + 2m$	20 minutes	w

Note that we would *not* want to label the combined rate $p + (1/2)p$. Remember that p represents Perry's *normal* rate, not the hypothetical "twice as fast as Maria" rate.

From the hypothetical situation:
$RT = W$
$3m(20) = w$
$m = w/60$

Fill this into the original RTW chart. Is there any way to solve for w/p?

	Rate	**Time**	**Work**
Maria	$w/60$		w
Perry	p	?	w
Maria & Perry (hyp)	$w/60 + p$	45 minutes	w

(w/60 + p)45 = w
(3/4)w + 45p = w
45p = (1/4)w
180p = w
180 = w/p

Perry's time = w/p = 180 minutes = 3 hours

The correct answer is E.

"Real-world" approach
Start with the hypothetical case. If Perry were working at twice Maria's rate (m), that would be like having three Marias on the job: 2m + m. If the three Marias could finish the job in 20 minutes, then each is doing 1/3 of the job in that 20 minutes. At that rate, a single person working at Maria's rate could finish the entire job in 1 hour. Maria's rate, then, is 1 job per hour.

When Perry is working at his real rate, he and Maria together complete the job in 45 minutes. If Maria works for 45 minutes at her rate of 1 job per 60 minutes, then calculate the portion of the work that she completes. RT = W. (45)(1/60) = W. Maria completes 45/60 or 3/4 of the job in that 45 minute timeframe.

Perry, then, must complete the other 1/4 of the job in 45 minutes. How long does it take him to complete the entire job? He can do 1/2 of the job in 1.5 hours and the entire job in 3 hours.

The correct answer is E.

20.2992Pascal has 96 miles remaining to complete his cycling trip. If he reduced his current speed by 4 miles per hour, the remainder of the trip would take him 16 hours longer than it would if he increased his speed by 50%. What is his current speed?

- 6
- 8
- 10
- 12
- 16

The best way to solve this problem is to ***Work Backwards from the Answer Choices***, testing each answer choice in turn as follows:

(A) 6mph

	Speed	**Time (T = D/R)**	**Difference**
4 mph slower	6 - 4 = 2	96 ÷ 2 = 48	
50% faster	6 × 1.5 = 9	96 ÷ 9 = $10\frac{2}{3}$	More than 16!

(B) 8 mph

	Speed	**Time (T = D/R)**	**Difference**
4 mph slower	8 - 4 = 4	96 ÷ 4 = 24	
50% faster	8 × 1.5 = 12	96 ÷ 12 = 8	16

Once you test B, you can stop. C yields too small a difference in speed.

(C) 10 mph

	Speed	**Time (T = D/R)**	**Difference**
4 mph slower	10 - 4 = 6	96 ÷ 6 = 16	
50% faster	10 × 1.5 = 15	96 ÷ 15 = 6.4	9.6

To get a larger difference, look for a smaller initial speed, because a fixed change (4 miles per hour slower) makes more of an impact when the numbers are lower. As Pascal's speed increases, 4 miles per hour becomes smaller and smaller in relation to his overall speed, and therefore makes less of a difference.

There is also an algebraic way to answer this question, although it is far more time consuming.

The problem presents two hypothetical situations and compares them in terms of time. Therefore, begin by writing expressions to show the time each hypothetical trip would take.

Label Pascal's current speed s, and express the time it would take him at the reduced speed as follows:

Time = Distance/Rate = $\frac{96}{s-4}$

Similarly, if he increases his speed by 50%, then his time 150% of the original speed. Multiply s by 1.5:

Time = Distance/Rate = $\frac{96}{1.5s}$

Because we know that at reduced speed he would take 16 hours longer than at increased speed, we can relate the two times as follows:

Slow time = Fast time + 16

$\frac{96}{s-4} = \frac{96}{1.5s} + 16$

$\frac{96}{s-4} = \frac{64}{s} + 16$

$\frac{96}{s-4} = \frac{64+16s}{s}$

$96s = (s - 4)(64 + 16s)$

$96s = 64s + 16s^2 - 256 - 64s$

$0 = 16s^2 - 96s - 256$

$0 = s^2 - 6s - 16$

$0 = (s - 8)(s + 2)$

$s = +8$ or -2

Because Pascal's current speed cannot be negative, the only possible value is 8 miles per hour.

The correct answer is B.