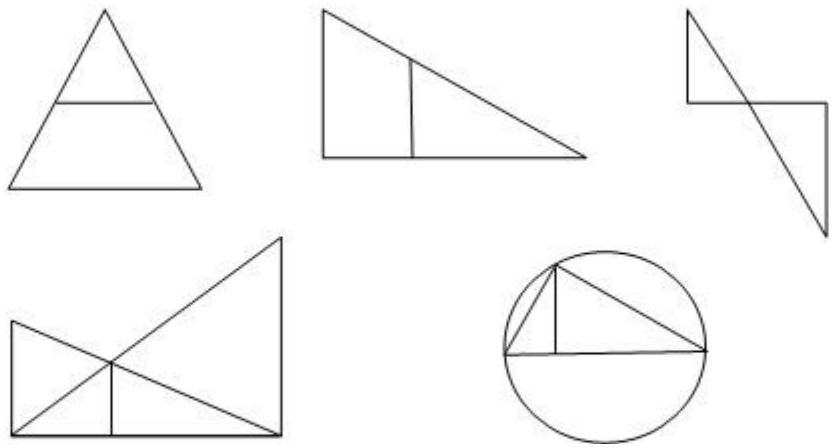
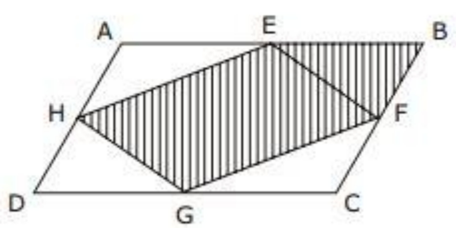
Quarter Wit_Quarter Wisdom- Part-3

1. Looking for Similar Triangles

Our Geometry book discusses the various rules we use to recognize similar triangles such as SSS, AA, SAS and RHS so we are assuming that we needn't take those up here.
We are also assuming that you are comfortable with the figures that beg you to think about similar triangles such as

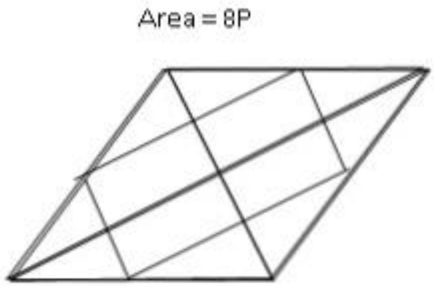


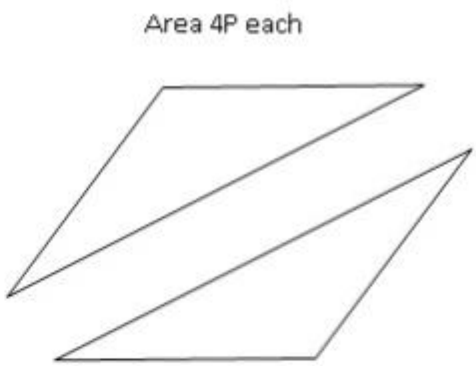
Try to figure out the similar triangles and the reason they are similar in each one of these cases. (Angles that look 90 are 90). Most of the figures have right angles/parallel lines.
This topic was also discussed by David Newland in a rather engaging post last week. You must check it out for its content as well as its context!
What we would like to discuss today are situations where most people do not think about similar triangles but if they do, it would make the question very easy for them. But before we do that, we would like to discuss a concept related to similar triangles which is very useful but not discussed often.
We already know that sides of similar triangles are in the same ratio. Say two triangles have sides a, b, c and A, B, C respectively. Then, a/A = b/B = c/C = k
Note that the altitudes of the two triangles will also be in the same ratio, 'k', since all lengths have the ratio 'k'.
Then what is the relation between the areas of the two triangles? Since the ratio of the bases is k and the ratio of the altitudes is also k, the ratio of the areas will be k*k = k^2.
So if there are two similar triangles such that their sides are in the ratio 1:2, their areas will be in the ratio 1:4.
Now we are all ready to tackle the question we have in mind.
Question: In the given figure, ABCD is a parallelogram and E, F, G and H are midpoints of its respective sides. What is the ratio of the shaded area to that of the un-shaded area?

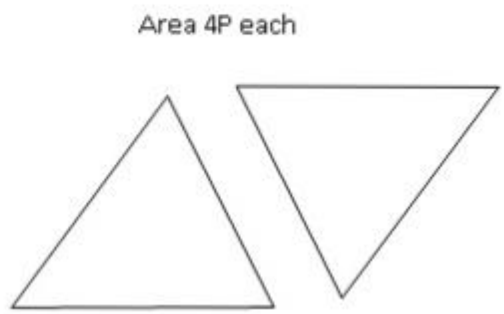


(A) 3:8
(B) 3:5
(C) 5:8
(D) 8:5
(E) 5:3

Solution: There are many ways to do this question but we will look at the method using similar triangles (obviously!). Assume the area of the parallelogram is 8P. In a parallelogram, the lengths of opposite sides are the same. The two triangles formed by the diagonal and two sides are similar by SSS and the ratio of their sides is 1. So they will have equal areas of 4P each (look at the figures in second row below)







Now look at the original figure.
HE is formed by joining the mid-points of AD and AB. So AH/AD = AE/AB = 1/2 and included angle A is common. Hence by SAS rule, triangle AHE is similar to triangle ADB. If the ratio of sides is 1/2, ratio of areas will be 1/4.
Since area of triangle ADB is 4P, area of AHE is P. We have 3 such triangles, AHE, DHG and CGF which are not shaded so the area of these three triangles together will be 3P.
The total area of parallelogram is 8P and the unshaded region is 3P. So the shaded region must be 5P.
Hence, area of shaded region : Area of unshaded region = 5:3
Answer (E)
Try to think of other ways in which you can solve this question.

2. <u>Determining Area of Similar Triangles</u>

Recall the important property that we discussed about the relation between the areas of the two similar triangles last week – if the ratio of their sides is 'k', the ratio of their areas will be k^2. As mentioned last week, it's an important property and helps you easily solve otherwise difficult questions. The question I have in mind today also brings in focus the Pythagorean triplets. There are some triplets that you should know out cold: (3, 4, 5), (5, 12, 13) and (8, 15, 17). Usually you will find one of these three or their multiples on GMAT. Given a right triangle and two sides, say the two legs, of length 20 and 48, we need to immediately bring them down to the lowest form 20:48 = 5:12. So we know that we are talking about the 5, 12, 13 triplet and the hypotenuse will be 13*4 = 52. These little things help us save a lot of time. Why is it that some people get done with the

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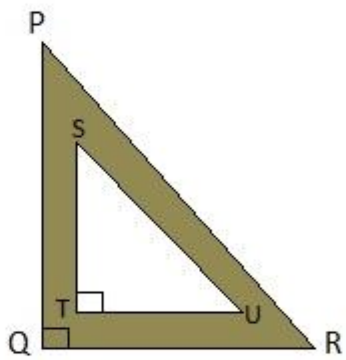
Quant section in less than an hour while others fall short on time? It is these little things that an adept test taker has mastered which make all the difference.

Anyway, let us go on to the question we have in mind.

Question: In the figure given below, the length of PQ is 12 and the length of PR is 15. The area of right triangle STU is equal to the area of the shaded region. If the ratio of the length of ST to the length of TU is equal to the ratio of the length of PQ to the length of QR, what is the length of TU?



(A) (9?2)/4

(B) 9/2

(C) (9?2)/2

(D) 6?2

(E) 12

Solution: The information given in the question seems to overwhelm us but let's take it a bit at a time.

"length of PQ is 12 and the length of PR is 15"

PQR is a right triangle such that PQ = 12 and PR = 15. So PQ:PR = 4:5. Recall the 3-4-5 triplet. A multiple triplet of 3-4-5 is 9-12-15. This means QR = 9.

"ratio of the length of ST to the length of TU is equal to the ratio of the length of PQ to the length of QR"

ST/TU = PQ/QR

The ratio of two sides of PQR is equal to the ratio of two sides of STU and the included angle between the sides is same (= 90). Using SAS, triangles PQR and STU are similar.

"The area of right triangle STU is equal to the area of the shaded region"

Area of triangle PQR = Area of triangle STU + Area of Shaded Region

Since area of triangle STU = area of shaded region, (area of triangle PQR) = 2*(area of triangle STU)

In similar triangles, if the sides are in the ratio k, the areas of the triangles are in the ratio k^2. If the ratio of the areas is given as 2 (i.e. k^2 is 2), the sides must be in the ratio ?2 (i.e. k must be ?2).

Since QR = 9, TU must be 9/?2. But there is no 9/?2 in the options – in the options the denominators are rationalized. TU = 9/?2 = (9*?2)/(?2*?2) = (9*?2)/2.

Answer (C)

The question could take a long time if we do not remember the Pythagorean triplets and the area of similar triangles property.

Takeaways:

1. Pythagorean triplets you should know: (3, 4, 5), (5, 12, 13) and (8, 15, 17) and their multiples.
2. In similar triangles, if the sides are in the ratio k, the areas of the triangles are in the ratio k^2.

3. A GMAT Formula to remember: Profit on One & Loss on Another

I am no fan of formulas, especially the un-intuitive ones but the one we are going to discuss today has proved quite useful. It is for a concept tested on GMAT Prep so it might be worth your while to remember this little formula.

When two items are sold at the same selling price, one at a profit of x% and the other at a loss of x%, there is an overall loss. The loss% = (x^2/100)%

We will see how this formula is derived but the algebra involved is tedious. You can skip it if you wish.

Say two items are sold at $S each. On one, a profit of x% is made and on the other a loss of x% is made.

Say, cost price of the article on which profit was made = Ct

Ct (1 + x/100) = S

Ct = S/(1 + x/100)

Cost Price of the article on which loss was made = Cs

Cs (1 – x/100) = S

Cs = S/(1 – x/100)

Total Cost Price of both articles together = Ct + Cs = S/(1 + x/100) + S/(1 – x/100)

Ct + Cs = S[1/(1 + x/100) + 1/(1 - x/100)]

Ct + Cs = 2S/(1 – (x/100)^2)

Total Selling Price of both articles together = 2S

Overall Profit/Loss = 2S – (Ct + Cs)

Overall Profit/Loss % = [2S – (Ct + Cs)]/[Ct + Cs] * 100

= [2S/(Ct + Cs) – 1] * 100

= [2S/[2S/(1 – (x/100)^2)] – 1] * 100

= (x/100)^2 * 100

= x^2/100

Overall there is a loss of (x^2/100)%.

Let' see how this formula works on a GMAT Prep question.

Question: John bought 2 shares and sold them for $96 each. If he had a profit of 20% on the sale of one of the shares but a loss of 20% on the sale of the other share, then on the sale of both shares John had

(A) a profit of $10

(B) a profit of $8

(C) a loss of $8

(D) a loss of $10

(E) neither a profit nor a loss

Solution:

Note that the question would have been straight forward had the COST price been the same, say $100. A 20% profit would mean a gain of $20 and a 20% loss would mean a loss of $20. Overall, there would have been no profit no loss.

Here the two shares are sold at the same SALE price. One at a profit of 20% on cost price which must be lower than the sale price (to get a profit) and the other at a loss of 20% on cost price which must be higher than the sale price (to get a loss). 20% of a lower amount will be less in dollar terms and hence overall, there will be a loss.

The loss % = (20)^2/100 % = 4%.

But we need the amount of loss, not the percentage of loss.

Total Sale price of the two shares = 2*96 = $192

Since there is a loss of 4%, the 96% of the total cost price must be the total sale price

(96/100)*Cost Price = Sale Price
Cost Price = $200
Loss = $200 – $192 = $8
Answer (C)

4. Advance Number Properties-1

Don't worry, we are not going to discuss (Even + Even = Even) and (Odd + Odd = Even) type of basic number properties in this post. What we have in mind for today is something based on this but far more advanced. Often, people complain that they thoroughly understand the theory but have difficulties applying it and hence are stuck at a score of 600. They look for practice questions and tend to ignore concepts since they already "know" them. We often ask them to go back to concepts since we believe that a strong foundation of concepts is necessary for 'score increase'. Mind you, when we do that, we don't mean to ask them to review the basic concepts again, we mean to ask them to deduce and work on advanced concepts. Let's show you with the help of a question.

Question: If two integers are chosen at random out of first 5 positive integers, what is the probability that their product will be of the form a^2 – b^2, where a and b are both positive integers?

A. 2/5
B. 3/5
C. 7/10
D. 4/5
E. 9/10

Solution: This might look like a probability question but isn't. Questions like these are the reason we ask you to go through basics of every topic including probability. If you do not know probability at all, you may skip this question even though it needs very basic knowledge of probability.

Probability will tell you that

Required probability = Favorable cases/Total cases

Total cases are very easy to find: 5C2 = 10 or 5*4/2 = 10 whatever you prefer. This is the number of ways in which you select any 2 distinct numbers out of the given 5 distinct numbers.

Number of favorable cases is the challenge here. That is why it is a number properties question and not so much a probability question. Let's focus on the main part of the question:

First five positive integers: 1, 2, 3, 4, 5

We need to select two integers such that their product is of the form a^2 – b^2. What does a^2 – b^2 remind you of? It reminds me of (a + b)(a – b). So the product needs to be of the form (a + b)(a – b). So is it necessary that of the two numbers we pick, one must be of the form (a + b) and the other must be (a – b)? No. Note that we should be able to write the product in this form. It is not necessary that the numbers must be of this form only.

But first let's focus on numbers which are already of the form (a + b) and (a – b).

Say you pick two numbers, 2 and 5. Are they of the form (a + b) and (a – b) such that a and b are integers? No.

5 = 3.5 + 1.5
2 = 3.5 – 1.5
So a = 3.5, b = 1.5.
a and b are not integers.

What about numbers such as 3 and 5? Are they of the form (a + b) and (a – b) such that a and b are integers? Yes.

5 = 4 + 1
3 = 4 – 1

Note that whenever the average of the numbers will be an integer, we will be able to write them as a+b and a – b because one number will be some number more than the average and the other will be the same number less than average. So a will be the average and the amount more or less will be b.

When will the average of two numbers (Number1 + Number2)/2 be an integer? When the sum of the two numbers is even! When is the sum of two numbers even? It is when both the numbers are even or when both are odd. So then does the question boil down to "favorable cases are when we select both numbers even or both numbers odd?" Yes and No. When we select both even numbers or both odd numbers, the product can be written as a^2 – b^2. But are those the only cases when the product can be written as a^2 – b^2?

The question is not so much as whether both the numbers are even or both are odd as whether the product of the numbers can be written as product of two even numbers or two odd numbers. We need to be able to write the product (whatever we obtain) as product of two even or two odd numbers.

To explain this, let's say we pick two numbers 4 and 5

4*5 = 20

Can we write 20 as product of two even numbers? Yes 2*10.

So even though, 4 is even and 5 is odd, their product can be written as product of two even numbers. So in which all cases will this happen?

- Whenever you have at least 4 in the product, you can write it as product of two even numbers: give one 2 to one number and the other 2 to the other number to make both even.

If the product is even but not a multiple of 4, it cannot be written as product of two even numbers or product of two odd numbers. It can only be written as product of one even and one odd number.

If the product is odd, it can always be written as product of two odd numbers.

Let's go back to our question:

We have 5 numbers: 1, 2, 3, 4, 5

Our favorable cases constitute those in which either both numbers are odd or the product has 4 as a factor.

3 Odd numbers: 1, 3, 5

2 Even numbers: 2, 4

Number of cases when both numbers are odd = 3C2 = 3 (select 2 of the 3 odd numbers)

Number of cases when 4 is a factor of the product = Number of cases such that we select 4 and any other number = 1*4C1 = 4

Total number of favorable cases = 3 + 4 = 7

Note that this includes the case where we take both even numbers. Had there been more even numbers such as 6, we would have included more cases where we pick both even numbers such as 2 and 6 since their product would have 4 as a factor.

Required Probability = 7/10

Answer (C)

Takeaway:

When can we write a number as difference of squares?

- When the number is odd

or

- When the number has 4 as a factor

5. Advance Number Properties-2

Before we get started, be sure to take a look at Part I of this article. Number properties concepts come across as pretty easy, theoretically, but they have some of the toughest questions. Today let's take a look at some properties of prime numbers and their sum. Note that don't try to "learn" all the takeaways you come across for number properties – it will be very stressful. Instead, try to understand why the properties are such so that if you get a question related to some such properties, you can replicate the results effortlessly.

To start off, we would like to take up a simple question and then using the takeaway derived from it, we will look at a harder problem.

Question 1: Which of the following CANNOT be the sum of two prime numbers?

(A) 19

(B) 45

(C) 68

(D) 79

(E) 88

Solution: What do we know about sum of two prime numbers?

e.g. 3 + 5 = 8

5 + 11 = 16

5 + 17 = 22

23 + 41 = 64

Do you notice something? The sum is even in all these cases. Why? Because most prime numbers are odd. When we add two odd numbers, we get an even sum.

We have only 1 even prime number and that is 2. Hence to obtain an odd sum, one number must be 2 and the other must be odd.

2 + 3 = 5

2 + 7 = 9

2 + 17 = 19

Look at the options given in the question. Three of them are odd which means they must be of the form 2 + Another Prime Number.

Let's check the odd options first:

(A) 19 = 2 + 17 (Both Prime. Can be written as sum of two prime numbers.)

(B) 45 = 2 + 43 (Both Prime. Can be written as sum of two prime numbers.)

(D) 79 = 2 + 77 (77 is not prime.)

79 cannot be written as sum of two prime numbers. Note that 79 cannot be written as sum of two primes in any other way. One prime number has to be 2 to get a sum of 79. Hence there is no way in which we can obtain 79 by adding two prime numbers.

(D) is the answer.

Now think what happens if instead of 79, we had 81?

81 = 2 + 79

Both numbers are prime hence all three odd options can be written as sum of two prime numbers. Then we would have had to check the even options too (at least one of which would be different from the given even options). Think, how would we find which even numbers can be written as sum of two primes? We will give the solution of that next week. So the takeaway here is that if you get an odd sum on adding two prime numbers, one of the numbers must be 2.

Question 2: If m, n and p are positive integers such that m < n < p, is m a factor of the odd integer p?

Statement 1: m and n are prime numbers such that (m + n) is a factor of 119.

Statement 2: p is a factor of 119.

Solution: First of all, we are dealing with positive integers here – good. No negative numbers, 0 or fraction complications. Let's move on.

The question stem tells us that p is an odd integer. Also, m < n < p.

Question: Is m a factor of p?

There isn't much information in the question stem for us to process so let's jump on to the statements directly.

Statement 1: m and n are prime numbers such that (m + n) is a factor of 119.

Write down the factors of 119 first to get the feasible range of values.

119 = 1, 7, 17, 119

All factors of 119 are odd numbers. So (m + n), a sum of two primes must be odd. This means one of m and n is 2. There are many possible values of m and n e.g. 2 and 5 (to give sum 7) or 2 and 15 (to give sum 17) or 2 and 117 (to give sum 119).

Note that we also have m < n. This means that in each case, m must be 2 and n must be the other number of the pair.

So now we know that m is 2. We also know that p is an odd integer. Is m a factor of p? No. Odd integers are those which do not have 2 as a factor. Since m is 2, p does not have m as a factor.

This statement alone is sufficient to answer the question!

Statement 2: p is a factor of 119

This tells us that p is one of 7, 17 and 119. p cannot be 1 because m < n < p where all are positive integers.

But it tells us nothing about m. All we know is that it is less than p. For example, if p is 7, m could be 1 and hence a factor of p or it could be 5 and not a factor of p. Hence this statement alone is not sufficient.

Answer (A)

Something to think about: In this question, if you are given that m is not 1, does it change our answer?

Key Takeaways:

- When two distinct prime numbers are added, their sum is usually even. If their sum is odd, one of the numbers must be 2.
- Think what happens in case you add three distinct prime numbers. The sum will be usually odd. In case the sum is even, one number must be 2.
- Remember the special position 2 occupies among prime numbers – it is the only even prime number.

6. Advance Number Properties -3

Continuing our discussion on number properties, today we will discuss how factorials affect the behavior of odd and even integers. Since we are going to deal with factorials, positive integers will be our concern. Using a question, we will see how factorials are divided.

Question: If x and y are positive integers, is y odd?

Statement 1: (y+2)!/x! = odd

Statement 2: (y+2)!/x! is greater than 2

Solution: The question stem doesn't give us much information – just that x and y are positive integers.

Question: Is y odd?

Statement 1: (y+2)!/x! = odd

Note that odd and even are identified only for integers. Since (y+2)!/x! is odd, it must be a positive integer. This means that x! must be equal to or less than (y+2)!

Now think, how are y and y+2 related? If y+2 is odd, y+1 is even and hence y is odd. If y+2 is even, by the same logic, y is even.

(y+2)! = 1*2*3*4*…*y*(y+1)*(y+2)

x! = 1*2*3*4*…*x

Note that (y+2)! and x! have common factors starting from 1. Since x! is less than or equal to (y+2)!, x will be less than or equal to (y+2). So all factors in the denominator, from 1 to x will be there in the numerator too and will get canceled leaving us with the last few factors of (y+2)!

To explain this, let us take a few examples:

Example 1: Say, y+2 = 6, x = 6

(y+2)!/x! = 6!/6! = 1

Example 2: Say, y+2 = 7, x = 6

(y+2)!/x! = 7!/6! = (1*2*3*4*5*6*7)/(1*2*3*4*5*6) = 7 (only one leftover factor)

Example 3: Say, y+2 = 6, x = 4

(y+2)!/x! = 6!/4! = (1*2*3*4*5*6)/(1*2*3*4) = 5*6 (two leftover factors)

If the division of two factorials is an integer, the factorial in the numerator must be larger than or equal to the factorial in the denominator.

So what does (y+2)!/x! is odd imply? It means that the leftover factors must be all odd. But the leftover factors will be consecutive integers. So after one odd factor, there will be an even factor. If we want (y+2)!/x! to be odd, we must have either no leftover factors (such that (y+2)!/x! = 1) or only one leftover factor and that too odd.

If we have no leftover factor, it doesn't matter what y+2 is as long as it is equal to x. It could be odd or even. If there is one leftover factor, then y+2 must be odd and hence y must be odd. Hence y could be odd or even. This statement alone is not sufficient.

Statement 2: (y+2)!/x! is greater than 2

This tells us that y+2 is not equal to x since (y+2)!/x! is not 1. But all we know is that it is greater than 2. It could be anything as seen in examples 2 and 3 above. This statement alone is not sufficient.

Both statements together tell us that y+2 is greater than x such that (y+2)!/x! is odd. So there must be only one leftover factor and it must be odd. The leftover factor will be the last factor i.e. y+2. This tells us that y+2 must be odd. Hence y must be odd too.

Answer (C)

Takeaways: Assuming a and b are positive integers,

- a!/b! will be an integer only if a >= b
- a!/b! will be an odd integer whenever a = b or a is odd and a = b+1
- a!/b! will be an even integer whenever a is even and a = b+1 or a > b+1

Think about this: what happens when we put 0 in the mix?

7. A take on GMAT takeaways

Once you have covered your fundamentals, we suggest you to practice advanced questions and jot down your takeaways from them. Sometimes students wonder how to find that all important "takeaway". Today, let's discuss how to elicit a takeaway from a question which seems to have none.

What is a takeaway? It is a small note to yourself which you would do well to remember while going for the exam. Even if you don't remember the exact property you jotted down, knowing that such a property exists is enough. You can always try it on a couple of numbers in the test to recall the exact content.

The question we will discuss today serves another purpose – it discusses properties of squares of odd and even integers so in a sense is a continuation of our advanced number properties discussion.

Question: Given x and y are positive integers such that y is odd, is x divisible by 4?

Statement 1: When (x^2 + y^2) is divided by 8, the remainder is 5.

Statement 2: x – y = 3

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Solution: As of now, we don't know any specific properties of squares of odd and even integers. However, we do have a good (presumably!) understanding of divisibility. To recap quickly, divisibility is nothing but grouping. To take an example, if we divide 10 by 2, out of 10 marbles, we make groups of 2 marbles each. We can make 5 such groups and nothing will be left over. So quotient is 5 and remainder is 0. Similarly if you divide 11 by 2, you make 5 groups of 2 marbles each and 1 marble is left over. So 5 is the quotient and 1 is the remainder. For more on these concepts, check out our previous posts on divisibility.

Coming back to our question,

First thing that comes to mind is that if y is odd, $y = (2k + 1)$.

We have no information on x so let's proceed to the two statements.

Statement 1: When $(x^2 + y^2)$ is divided by 8, the remainder is 5.

The statement tell us something about y^2 so let's get that.

If $y = (2k + 1)$

$y^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$

Since one of k and (k+1) will definitely be even (out of any two consecutive integers, one is always even, the other is always odd), k(k+1) will be even. So 4k(k+1) will be divisible by 4*2 i.e. by 8. So when y^2 is divided by 8, it will leave a remainder 1.

When y^2 is divided by 8, remainder is 1. To get a remainder of 5 when $x^2 + y^2$ is divided by 8, we should get a remainder of 4 when x^2 is divided by 8. So x must be even. If x were odd, the remainder when x^2 were divided by 8 would have been 1. So we know that x is divisible by 2 but we don't know whether it is divisible by 4 yet.

$x^2 = 8a + 4$ (when x^2 is divided by 8, it leaves remainder 4)

$x^2 = 4(2a + 1)$

So x = 2*?Odd Number

Square root of an odd number will be an odd number so we can see that x is even but not divisible by 4. This statement alone is sufficient to say that x is NOT divisible by 4.

Statement 2: $x – y = 3$

Since y is odd, we can say that x will be even (Since Even – Odd = Odd). But whether x is divisible by 2 only or by 4 as well, we cannot say. This statement alone is not sufficient.

Answer (A)

So could you point out the takeaway from this question?

Note that when we were analyzing y, we used no information other than that it is odd. We found out that the square of any odd number when divided by 8 will always yield a remainder of 1.

Now what can you say about the square of an even number? Say you have an even number x.

$x = 2a$

$x^2 = 4a^2$

This tells us that x^2 will be divisible by 4 i.e. we can make groups of 4 with nothing leftover. What happens when we try to make groups of 8? We join two groups of 4 each to make groups of 8. If the number of groups of 4 is even, we will have no remainder leftover. If the number of groups of 4 is odd, we will have 1 group leftover i.e. 4 leftover. So when the square of an even number is divided by 8, the remainder is either 0 or 4.

Looking at it in another way, we can say that if a is odd, x^2 will be divisible by 4 and will leave a remainder of 4 when divided by 8. If a is even, x^2 will be divisible by 16 and will leave a remainder of 0 when divided by 8.

Takeaways

- The square of any odd number when divided by 8 will always yield a remainder of 1.
- The square of any even number will be either divisible by 4 but not by 8 or it will be divisible by 16 (obvious from the fact that squares have even powers of prime factors so 2 will have a power of 2 or 4 or 6 etc). In the first case, the remainder when it is divided by 8 will be 4; in the second case the remainder will be 0.

8. A Remainders Shortcut for GMAT

We firmly believe that teaching someone is a most productive learning for oneself and every now and then, something happens that strengthens this belief of ours. It's the questions people ask – knowingly or unknowingly – that connect strings in our mind such that we feel we have gained more from the discussion than even our students!

The other day, we came across this common GMAT question on remainders and many people had solved it the way we would expect them to solve. One person, perhaps erroneously, used a shortcut which upon reflection made perfect sense. Let me give you that question and the shortcut and the problem with the shortcut. We would like you to reflect on why the shortcut actually does make sense and is worth noting down in your log book.

Question: Positive integer n leaves a remainder of 4 after division by 6 and a remainder of 3 after division by 5. If n is greater than 30, what is the remainder that n leaves after division by 30?

(A) 3

(B) 12

(C) 18

(D) 22

(E) 28

Solution: We are assuming you know how people do the question usually:

The logic it uses is discussed here and the solution is given below as Method I.

Method I:

Positive integer n leaves a remainder of 4 after division by 6. So n = 6a + 4

n can take various values depending on the values of a (which can be any non negative integer).

Some values n can take are: 4, 10, 16, 22, 28, …

Positive integer n leaves a remainder of 3 after division by 5. So n = 5b + 3

n can take various values depending on the values of a (which can be any non negative integer).

Some values n can take are: 3, 8, 13, 18, 23, 28, …

The first common value is 28. So n = 30k + 28

Hence remainder when positive integer n is divided by 30 is 28.

Answer: E.

Perfect! But one fine gentleman came up with the following solution wondering whether he had made a mistake since it seemed to be "super simple Math".

Method II:

Given in question: "n leaves a remainder of 4 after division by 6 and a remainder of 3 after division by 5."

Divide the options by 6 and 5. The one that gives a remainder of 4 and 3 respectively will be correct.

(A) 3 / 6 gives Remainder = 3 -> INCORRECT

(B) 12 / 6 gives Remainder = 0 -> INCORRECT

(C) 18 / 6 gives Remainder = 0 -> INCORRECT

(D) 22 / 6 gives Remainder = 4 but 22 / 5 gives Remainder = 2 -> INCORRECT

*(E) 28 / 6 gives Remainder = 4 and 28 / 5 gives Remainder = 3 -> **CORRECT***

Now let us point out that the options are not the values of n; they are the values of remainder that is leftover after you divide n by 30. The question says that n must give a remainder of 4 upon division by 6 and a remainder of 3 upon division by 5. This solution divided the options (which are not the values of n) by 6 and 5 and got the remainder as 4 and 3 respectively. So the premise that when you divide the correct option by 6 and 5, you should get a remainder of 4 and 3 respectively is faulty, right?

This is where we want you to take a moment and think: Is this premise actually faulty?

The fun part is that method II is perfectly correct too. Method I seems a little complicated when compared with Method II, doesn't it? Let us give you the logic of why method II is correct:

Recall that division is nothing but grouping. When you divide n by 30, you make complete groups of 30 each. The number of groups you get is called the quotient (not relevant here) and the leftover is called the remainder. If this is not clear, check this post first.

When n is divided by 30, groups of 30 are made. Whatever is leftover is given in the options. 30 is completely divisible by 6 and by 5 hence the groups of 30 can be evenly divided into groups of 6 as well as groups of 5. Now, whatever is leftover (given in the options) after division by 30, we need to split that into further groups of 6 and 5. When we split it into groups of 6 (i.e. divide the option by 6), we must have remainder 4 since n leaves remainder 4. When we split it into groups of 5 (i.e. divide the option by 5), we must have remainder 3 since n leaves remainder 3. And, that is the reason we can divide the options by 6 and 5, check their remainders and get the answer!

Now, isn't that neat!

9. Medians, Altitudes, & Angle Bisectors in Special Triangles

We are assuming you know the terms median, angle bisector and altitude but still, just to be sure, we will start our discussion today by defining them:

Median – A line segment joining a vertex of a triangle with the mid-point of the opposite side.

Angle Bisector – A line segment joining a vertex of a triangle with the opposite side such that the angle at the vertex is split into two equal parts.

Altitude – A line segment joining a vertex of a triangle with the opposite side such that the segment is perpendicular to the opposite side.

Usually, medians, angle bisectors and altitudes drawn from the same vertex of a triangle are different line segments. But in special triangles such as isosceles and equilateral, they can overlap. We will now give you some properties which can be very useful.

I.

In an **isosceles triangle** (where base is the side which is not equal to any other side):

- the altitude drawn to the base is the median and the angle bisector;
- the median drawn to the base is the altitude and the angle bisector;
- the bisector of the angle opposite to the base is the altitude and the median.

II.

The reverse is also true. Consider a triangle ABC:

- If angle bisector of vertex A is also the median, the triangle is isosceles such that AB = AC and BC is the base. Hence this angle bisector is also the altitude.
- If altitude drawn from vertex A is also the median, the triangle is isosceles such that AB = AC and BC is the base. Hence this altitude is also the angle bisector.
- If median drawn from vertex A is also the angle bisector, the triangle is isosceles such that AB = AC and BC is the base. Hence this median is also the altitude.

and so on…

III.

In an **equilateral triangle**, each altitude, median and angle bisector drawn from the same vertex, overlap.

Try to prove all these properties on your own. That way, you will not forget them.

A few things this implies:

- Should an angle bisector in a triangle which is also a median be perpendicular to the opposite side? Yes.

- Can we have an angle bisector which is also a median which is not perpendicular? No. Angle bisector which is also a median implies isosceles triangle which implies it is also the altitude.
- Can we have a median from vertex A which is perpendicular to BC but does not bisect the angle A? No. A median which is an altitude implies the triangle is isosceles which implies it is also the angle bisector.

and so on…

Let's take a quick question on these concepts:

Question: What is ?A in triangle ABC?

Statement 1: The bisector of ?A is a median in triangle ABC.

Statement 2: The altitude of B to AC is a median in triangle ABC.

Solution: We are given a triangle ABC but we don't know what kind of a triangle it is.

Jump on to the statements directly.

Statement 1: The bisector of ?A is a median in triangle ABC.

The angle bisector is also a median. This means triangle ABC must be an isosceles triangle such that AB = AC. But we have no idea about the measure of angle A. This statement alone is not sufficient.

Statement 2: The altitude of ?B to AC is a median in triangle ABC.

The altitude is also a median. This means triangle ABC must be an isosceles triangle such that AB = BC (Note that the altitude is drawn from vertex B here). But we have no idea about the measure of angle A. This statement alone is not sufficient.

Using both statements together, we see that AB = AC = BC. So the triangle is equilateral! So angle A must be 60 degrees. Sufficient!

Answer (C)

10. When Permutations, Combinations & DS Come together

While discussing Permutations and Combinations many months back, we worked through several examples of arranging people in seats. Today we bring you an interesting question based on those concepts. It brings to the fore the tricky nature of both Data Sufficiency and Combinatorics – so much so that when the two get together, it is unlimited fun! In some circumstances, we suggest you to travel the whole nine yards – i.e. solve for the answer in Data Sufficiency questions too even if you feel that sufficiency has already been established. This is especially true for quadratic equations which we assume will give us two values of x but might actually give just a single unique value (such that both roots are the same). In Combinatorics too, sometimes what may look like two distinct cases could actually give the same answer. Let's jump on to the question.

Question 1: There are x children and y chairs in a room where x and y are prime numbers. In how many ways can the x children be seated in the y chairs (assuming that each chair can seat exactly one child)?

Statement 1: x + y = 12

Statement 2: There are more chairs than children.

Solution:

There are x children and y chairs.

x and y are prime numbers.

Statement 1: x + y = 12

Since x and y are prime numbers, a quick run on 2, 3, 5 shows that there are two possible cases:

Case 1: x=5 and y=7

There are 5 children and 7 chairs.

Case 2: x=7 and y=5

There are 7 children and 5 chairs

At first glance, they might look like two different cases and you might feel that statement 1 is not sufficient alone. But note that the question doesn't ask you for number of children or number of chairs. It asks you about the number of arrangements.

Case 1: x=5 and y=7

If there are 5 children and 7 chairs, we select 5 chairs out of the 7 in 7C5 ways. We can now arrange 5 children in 5 seats in 5! ways.

Total number of arrangements would be 7C5 * 5!

Case 2: x = 7 and y = 5

If there are 7 children and 5 chairs, we select 5 children out of the 7 in 7C5 ways. We can now arrange 5 children in 5 seats in 5! ways.

Total number of arrangements would be 7C5 * 5!

Note that in both cases the number of arrangements is 7C5*5!. Combinatorics does not distinguish between people and things. 7 children on 5 seats is the same as 5 children on 7 seats because in each case you have to select 5 out of 7 (either seats or children) and then arrange 5 children in 5! ways.

So actually this statement alone is sufficient! Most people would not have seen that coming!

Statement 2: There are more chairs than people.

We don't know how many children or chairs there are. This statement alone is not sufficient.

Answer: A

We were tempted to answer the question as (C) but it was way too easy. Statement 1 gave 2 cases and statement 2 narrowed it down to 1. Be aware that if it looks too easy, you are probably missing something!

Now, what if we alter the question slightly and make it:

Question 2: There are x children and y chairs *arranged in a circle* in a room where x and y are prime numbers. In how many ways can the x children be seated in the y chairs (assuming that each chair can seat exactly one child)?

Statement 1: x + y = 12

Statement 2: There are more chairs than children.

11. Rounding Rules: Slip to the side; look for five

The famous rounding song by Joe Crone is pretty much all you need to solve the trickiest of rounding questions on GMAT:

You just slip to the side, and you look for a five.
Well if the number that you see is a five or more,
You gotta round up now, that's for sure.
If the number that you see is a four or less,
You gotta round down to avoid a mess.

To put it in our own words, when we round a decimal, we drop the extra decimal places and apply certain rules:

- If the first dropped digit is 5 or greater, we round up the last digit that we keep.
- If the first dropped digit is 4 or smaller, we keep the last digit that we keep, the same.

For Example, we need to round the following decimals to two digits after decimal:

(a) 3.857

We drop 7. Since 7 is '5 or greater', we are left with 3.86

(b) 12.983

We drop 3. Since 3 is '4 or smaller', we are left with 12.98

(c) 26.75463

We drop 463. Since 4 is '4 or smaller', we are left with 26.75

(d) 8.9675

We drop 75. Since 7 is '5 or greater', we are left with 8.97

Note example (c) carefully:

When we round 26.75463 to two decimal places, we do not start rounding from the rightmost digit i.e. this is incorrect: 26.75463 becomes 26.7546 which becomes 26.755 which further becomes 26.76 – this is not correct. .00463 is less than .005 and hence should be ignored. You only need to worry about the digit right next to the digit you are keeping. Just slip to the side, and look for a five!

A logical question arises: what happens when we have, say, 2.5 and we need to round it to the nearest integer? 2.5 is midway between 2 and 3. In that case, why do we round the number up, as the rule suggests? Note that a 2.5 is a tie and we have many tie breaking rules that can be used. They are 'Round half to odd', 'Round half to even', 'Round up', 'Round down', 'Round towards 0', 'Round away from 0' etc. We don't need to worry about all these since GMAT uses only Round up i.e. 2.5 will be rounded up to 3.

Let's take a look at a question now which uses these fundamentals.

Question: The exact cost price to make each unit of a widget is $7.6xy7, where x and y represent single digits. What is the value of y?

Statement 1: When the cost is rounded to the nearest cent, it becomes $7.65.

Statement 2: When the cost is rounded to the nearest tenth of a cent, it becomes $7.65.

Solution: The question is based on rounding. We need to figure out the value of y given some rounding scenarios. Let's look at them one by one.

Statement 1: When the cost is rounded to the nearest cent, it becomes $7.65.

When rounded to the nearest cent, the cost becomes 7 dollars and 65 cents. 6xy7 cents got rounded to 65 cents. When will .6xy7 get rounded to .65? When .6xy7 lies anywhere in the range .6457 to .6547. Note that in all these cases, when you round the number to 2 digits, it will become .65.

Say price is 7.6468. We need to drop 68 but since 6 is '5 or greater', 4 gets rounded up to 5.

Similarly, say the price is 7.6543. We need to drop 43. Since 4 is '4 or smaller', 5 stays as it is.

So x and y can take various different values. This statement alone is not sufficient.

Statement 2: When the cost is rounded to the nearest tenth of a cent, it becomes $7.65

Now the cost is rounded to the tenth of a cent which means 3 places after the decimal. But the cost is given to us as $7.65. Since we need 3 places, the cost must be $7.650 (which will be written as $7.65)

When will 7.6xy7 get rounded to 7.650? Now this is the tricky part of the question – from 7.6xy7, you need to drop the 7 and round up y. When you do that, you get 7.650. This means 7.6xy7 must have been 7.6497. Only in this case, when we drop the 7, we round up the 9 to make 10, carry the 1 over to 4 and make it 5. This is the only way to get 7.650 on rounding 7.6xy7 to the tenth of a cent. Hence x must be 4 and y must be 9. This statement alone is sufficient to answer the question.

Answer (B)

Hope you see that a few simple rules can make rounding questions quite easy.

12. Rounding Up some Official Questions

Last week we looked at some rounding rules. Today, let's go over some official questions on rounding. They are quite simple and if we just keep the "Slip to the side and look for a 5" rule in mind, they can be easily solved.

Question 1: If n = 2.0453 and n* is the decimal obtained by rounding n to the nearest hundredth, what is the value of n* – n?

(A) -0.0053
(B) -0.0003
(C) 0.0007
(D) 0.0047
(E) 0.0153

Solution: A quick note on place value nomenclature:

Given a decimal 345.789, we know that 5 represents the units digit, 4 the tens digit and 3 the hundreds digit. Also, 7 represents the tenths digit, 8 the hundredths digit and 9 the thousandths digit and so on…

Now let's go back to this question:

n = 2.0453

We need to round n to the nearest hundredth which means we will retain 2 digits after the decimal. The third digit after the decimal is 5 so 2.0453 rounded to the nearest hundredth is 2.05.

Thus n* – n = 2.05 – 2.0453 = 0.0047

Answer (D)

Question 2: If digit h is the hundredths digit in the decimal n = 0.2h6, what is the value of n, rounded to the nearest tenth?

Statement 1: n < 1/4

Statement 2: h < 5

Solution: Given that n = 0.2h6

We need to find the value of n rounded to the nearest tenth i.e. we need to keep only one digit after the decimal.

Statement 1: n < 1/4

In decimal form, it means n < 0.25

If h were 5 or greater, n would become 0.256 or 0.266 or higher. All these values would be more than 0.25 so h must be less than 5 such as 0.246 or 0.236 etc. In all such cases, n would be rounded to 0.2

This statement alone is sufficient.

Statement 2: h < 5

This is even simpler. Since we have been given that h is less than 5, when we round n to the tenths digit, we will get 0.2

This statement alone is also sufficient.

Answer (D)

Question 3: If d denotes a decimal number, is d >= 0.5?

Statement 1: When d is rounded to the nearest tenth, the result is 0.5.

Statement 2: When d is rounded to the nearest integer, the result is 1.

Solution: Again, a simple question!

We need to find whether d is greater than or equal to 0.5 or not.

Statement 1: When d is rounded to the nearest tenth, the result is 0.5.

This means that whatever d is, when we round it to the nearest tenth, we get 0.5. What are the possible values of d? If d is anywhere from 0.450 to 0.5499999…, it will be rounded to 0.5

Some of these numbers are less than 0.5 and others are greater than 0.5 so this statement alone is not sufficient.

Statement 2: When d is rounded to the nearest integer, the result is 1.

In this case d must be at least 0.5; only then can it be rounded to 1.

d can be anything from 0.50 to 1.499999… In any case, d will be greater than or equal to 0.5.

This statement alone is sufficient to answer the question.

Answer (B)

We hope you see that if we just remember the rules, we can solve most rounding questions very quickly and efficiently.

13. Easy A/B Trap in Data Sufficiency Questions

We know that 'Easy C' is a common trap of DS questions – have you wondered whether there could be trap called 'Easy A/B' such that the answer would actually be (C)? Such questions also exist! The point is that whenever you feel that the question was way too simple, you might want to take a step back and review. GMAT will try every trick in the trade to delineate you.

Let us show you a question which looks like an easy (A) but isn't:

Question: 25 integers are written on a board. Are there at least two consecutive integers among them?

Statement 1: For every value in the list, if the value is increased by 1, the number of distinct values in the list does not change.

Statement 2: At least one value occurs more than once in the list.

Solution: Let's first review the information given to us here:

25 integers are written on the board – we don't know whether they are all distinct. We want to know if there is any pair of consecutive integers among them.

Let's look at the statements:

Statement 1: For every value in the list, if the value is increased by 1, the number of distinct values in the list does not change.

It is easy to fall for statement 1 and think that it is sufficient alone. Say, if any single value is increased by 1 and it doesn't match any other value already there in the list, it means that there are no consecutive integers, doesn't it? Well, no! But we will talk about that in a minute. Let's first look at why we might think that statement 1 is sufficient.

Say, the numbers are: 1, 5, 8, 10, 35, 76 …

If you increase 1 by 1, you get 2 and the list looks like this:

Now the numbers are 2, 5, 8, 10, 35, 76 …

Note that the number of distinct integers is the same.

Had there been two consecutive integers such as 1, 2, 8, 10, 35, 76 …

If we increase 1 by 1, the list would have become 2, 2, 8, 10, 35, 76 … – this would have decreased the number of distinct integers.

You might be tempted to say here that statement 1 alone is sufficient. What you might forget is that when you increase a number by 1, one distinct integer could be getting wiped out and another taking its place! It may not occur to you that the case might be different when one value occurs more than once, but statement 2 should give you a hint. Obviously, statement 2 alone is not sufficient but let's analyze what happens when we take both statements together.

Since statement 1 doesn't tell you that all values are distinct, statement 2 should make you think that you need to consider the case where one value occurs more than once in the list. In that case, is it possible that number of different values in the list does not change even though there is a pair of consecutive integers?

Say the numbers are 1, 1, 2, 8, 10, 35, 76 …

Now if you increase 1 by 1, the list would look like 1, 2, 2, 8, 10, 35, 76 …

Here, the number of distinct integers stays the same even when you increase a number by 1 and you have consecutive integers! In this case, if there were no consecutive integers, the number of distinct integers would have increased. Hence if the numbers are not all distinct and the number of distinct numbers needs to stay the same, there must be a pair of consecutive integers.

This tells you that statement 1 is not sufficient alone but both statements together answer the question with a 'Yes'.

Answer (C)

14. The Reason behind Absolute Value Questions

Even after working extensively on absolute value questions, sometimes students come up with “why?” i.e. why do we have to take positive and negative values? Why do we have to consider ranges etc. They know the process but they do not understand the reason they need to follow the process. So here today, in this post, we will try to explain the reason. You know how to solve an equation such as $x + 2x = 4$. Simple enough, right? Just add x with 2x to get 3x and separate out the x on one side. But what do you do when you have an equation with absolute values? How will you handle that equation? Say, you have $|x| + 2x = 4$. Is this your regular equation? No! You CANNOT say that $x + 2x = 4 \Rightarrow 3x = 4 \Rightarrow x = 4/3$. You have an absolute value and that complicates matters. You need to get rid of it to get a solution for x. How do you get rid of absolute values? The **definition of absolute value** helps us here:

$|x| = x$ if $x >= 0$

$|x| = -x$ if $x < 0$

So you can substitute x for |x| to make it a regular equation but only if x is non negative. If x is negative, then you put -x instead of |x| to convert it into a simple equation. And that is the reason you need to take positive and negative values of what is inside the absolute value sign.

Similarly,

$|x-5| = (x-5)$ if $(x-5) >= 0$ i.e. if $x >= 5$

$|x-5| = -(x-5)$ if $(x-5) < 0$ i.e. if $x < 5$

Let’s go back to the previous example and see how we can get rid of the absolute value to make it a regular equation:

Question 1: What is the value of x given $|x| + 2x = 4$?

We don’t know whether x is positive or negative so we will look at what happens in both cases:

Case 1: x is positive or 0

If $x >= 0$ then equation becomes $x + 2x = 4 \Rightarrow x = 4/3$

Our initial condition is that x is non negative. We get a positive solution on solving it and hence 4/3 is a valid solution.

Case 2: x is negative

If $x < 0$ then equation becomes $-x + 2x = 4 \Rightarrow x = 4$

Our initial condition is that x is negative. We get a positive solution on solving it and hence $x = 4$ is not a valid solution. Had we obtained a negative solution, it would have been valid.

So there is only one solution $x = 4/3$.

We hope the entire process makes more sense now. Let’s follow it up with a complex question from our algebra book.

Question 2: If x and y are integers and $y = |x+3| + |4-x|$, does y equal 7?

Statement 1: $x < 4$

Statement 2: $x > -3$

Solution: Now what do you do when you have $y = |x+3| + |4-x|$? How do you convert this into a regular equation? You don’t know whether whatever is in the absolute value sign is positive or negative. How will you get rid of the sign then? You will work on all the cases (messy algebra coming up!).

Now, we see the same logic in this question:

$y = |x+3| + |4-x|$

$|x+3| = (x+3)$ if $(x+3) >= 0$. In other words, if $x >= -3$

$|x+3| = -(x+3)$ if $(x+3) < 0$. In other words, if $x < -3$

$|4-x| = (4-x)$ if $(4-x) >= 0$. In other words, if $x <= 4$

$|4-x| = -(4-x)$ if $(4-x) < 0$. In other words, if $x > 4$

So our absolute values behave differently when $x < -3$, between -3 and 4 and when $x > 4$. We say that -3 and 4 are our transition points.

Case 1:

When x < -3, |x+3| = -(x+3) and |4-x| = (4-x).
So the equation becomes y = -(x+3) + (4-x)
y = 1 – 2x
For different values of x, y will take different values. Recall that x must be less than -3. Say x = -4, then y = 9. If x = -5, y = 11.
Case 2:
When -3 <= x <= 4, |x+3| = (x+3) and |4-x| = (4-x).
So the equation becomes y = (x+3) + (4-x)
y = 7
In this range, y will always be 7.
Case 3:
When x > 4, |x+3| = (x+3) and |4-x| = -(4-x)
So the equation becomes y = (x+3) – (4-x)
y = 2x – 1
For different values of x, y will take different values. Recall that x must be more than 4. Say x = 5, then y is 9. If x = 6, then y is 11.
Note that y equals 7 only when x is between -3 and 4. Both statements together tell us that x is between -3 and 4. No statement alone gives us this information. Hence, using both statements, we know that y must be 7.
Answer (C)

15.Find Correct Answers for Diagonals of a Polygon

In today's post, we will give you a question with two solutions and two different answers. You have to find out the correct answer and explain why the other is wrong. But before we do that, let's give you some background.
Given an n sided polygon, how many diagonals will it have?
An n sided polygon has n vertices. If you join every distinct pair of vertices you will get nC2 lines. These nC2 lines account for the n sides of the polygon as well as for the diagonals.
So the number of diagonals is given by nC2 – n.
nC2 – n = n(n-1)/2 – n = **n(n – 3)/2**
Taking quick examples:
Example 1: How many diagonals does a polygon with 25 sides have?
No. of diagonals = n(n – 3)/2 = 25*(25 – 3)/2 = 275
Example 2: How many diagonals does a polygon with 20 sides have, if one of its vertices does not send any diagonal?
The number of diagonals of a 20 sided figure = 20*(20 – 3)/2 = 170
But one vertex does not send any diagonals. Each vertex makes a diagonal with (n-3) other vertices – it makes no diagonal with 3 vertices: itself, the vertex immediately to its left, and the vertex immediately to its right. With all other vertices, it makes a diagonal. So we need to remove 20 – 3 = 17 diagonals from the total.
Total number of diagonals if one vertex does not make any diagonals = 170 – 17 = 153 diagonals.
We hope everything done till now makes sense. Now let's go on to the part which seems to make no sense at all!
Question: How many diagonals does a polygon with 18 sides have if three of its vertices, which are adjacent to each other, do not send any diagonals?
Answer: We will use two different methods to solve this question:
Method 1: Using the formula discussed above
Number of diagonals in a polygon of 18 sides = 18*(18 – 3)/2 = 135 diagonals
Each vertex makes a diagonal with n-3 other vertices – as discussed before.

So each vertex will make 15 diagonals.
Total number of diagonals if 3 vertices do not send any diagonals = 135 – 15*3 = 90 diagonals.
Method 2:
The polygon has a total of 18 vertices. 3 vertices do not participate so we need to make all diagonals that we can with 15 vertices.
Number of lines you can make with 15 vertices = 15C2 = 15*14/2 = 105
But this 105 includes the sides as well. A polygon with 18 vertices has 18 sides. Since 3 adjacent vertices do not participate, 4 sides will not be formed. 15 vertices will have 14 sides which will be a part of the 105 we calculated before.
Total number of diagonals if 3 vertices do not send any diagonals = 105 – 14 = 91
Note that the two answers do not match. Method 1 gives us 90 and method 2 gives us 91. Both methods look correct but only one is actually correct. Your job is to tell us which method is correct and why the other method is incorrect.

16. How to go from 48 to 51 in Quant- Part-1

People often ask – how do we go from 48 to 51 in Quant? This question is very hard to answer since we don't have a step by step plan – do theory from here – do questions from there – take a test from here – read posts from there etc. Today and in the next few weeks, we will discuss how to go from 48 to 51 in Quant.
Above Q48, the waters are pretty choppy! Questions are hard less because of the content and more because they look so unique – even though they're testing the same concepts. Training yourself to see familiarity in the obscure is difficult, and that happens from seeing a lot of problems. There is barely any scope for making silly mistakes – you must run through all simple questions quickly and neatly, leaving you plenty of time to think through the tougher ones. It's important to have enough time and keep your cool, which is easier to do if you have more time.
The question for today is: how do you handle simple questions quickly? We have mentioned many times that most GMAT Quant questions do not need Algebra. We can easily solve them by just analyzing while reading the question stem!
Here is how we can do that:
Question: School A is 40% girls and school B is 60% girls. The ratio of the number of girls at school A to the number of girls at school B is 4:3. if 20 boys transferred from school A to school B and no other changes took place at the two schools, the new ratio of the number of boys at school A to the number of boys at school B would be 5:3. What would the difference between the number of boys at school A and at school B be after the transfer?
(A) 20
(B) 40
(C) 60
(D) 80
(E) 100
Solution: This is a pretty simple non-tricky PS question. To solve it, most people use an algebraic method which looks something like this.
Girls in school A : Girls in school B = 4 : 3
So number of girls in school A = 4n and number of girls in school B = 3n
Since in school A, 40% students are girls and 60% are boys, number of boys is 6n.
Since in school B, 60% students are girls and 40% students are boys, number of boys is 2n.
If we transfer 20 boys from school A, we are left with 6n – 20 and when 20 boys are added to school B, we get 2n + 20 boys in school B.
(6n – 20)/(2n + 20) = 5/3
You get n = 20

Boys at school A after transfer = 6*20 – 20 = 100
Boys at school B after transfer = 2*20 + 20 = 60
Difference = 40
Answer (B)
This method gives you the correct answer, obviously, but it does take quite a bit of time. On the other hand, this is what should go through your mind while reading the question if you are focused on using logic:
"School A is 40% girls and school B is 60% girls."
School A – 40% girls
School B – 60% girls
"The ratio of the number of girls at school A to the number of girls at school B is 4:3"
When we read this line, we should take a step back to the previous line with the % figures. We see that school A has more girls than school B (4:3) but its % of girls is lower (only 40% compared to 60% in B). This means that school A has more students than school B. Say something like school A has 200 students while school B has 100 (use easy numbers). So school A has 80 girls while school B has 60 girls. This gives us a ratio of 4:3. (If you do not get 4:3 on your first try, you should tweak the assumed numbers a bit but you should stick to simple numbers.) Then verify the rest of the data against these numbers and get your answer.
School A has 120 boys and school B has 40 boys. Transfer 20 boys from school A to school B to get 100 boys in school A and 60 boys in school B giving us a difference of 40 boys.
This takes lesser time but requires some ingenuity. That could be the difference between Q48 and Q51.
Hope this gave you some ideas. Try the reasoning approach on other simple questions. With practice, you can save a ton of time!

17. How to go from 48 to 51- Part-2

This post is continuation of last week's post which you can check here.
Another method of saving time on simple questions – use data given in one statement to examine the other!
Now you might think we have lost it! After all, you know very well that in Data Sufficiency questions of GMAT, you must examine each statement independently. You CANNOT use data from one to analyze the other – absolutely correct. So you should ignore the other statement completely while examining one – hmm, not necessarily!
Sometimes, one statement could give us ideas about the next one such that we could save time while examining it. Needless to say, we need to be very careful but it certainly is a useful strategy. Also, it could help us verify that our calculations are correct. Here is why…
When we say DS question, think of a puzzle. The question stem gives you the statement of a puzzle ending with something like "What is the value of x?" or "Is x 7?" etc. You have to answer the question asked in the puzzle. Think of the two statements that come with the question as clues to the puzzle. So the puzzlemaster gives you the first clue (statement 1) and asks you: can you answer the question now? If you are able to, your answer is either (A) or (D). Then he tells you to ignore the first clue and gives you another clue (statement 2). Again he asks you: can you answer the question now? Again, you may or may not able to. If you are able to, your answer will be (B) or (D) depending on how you fared in statement 1. If you are unable to answer the question, he tells you to consider both statements together and then try to answer. If you are able to, your answer is (C).
The point to note here is that both clues lead you to answer the same puzzle. Say if the puzzle is: What is x? If clue 1 tells you that x is 6, clue 2 cannot tell you that x is 9. They both must lead you to the same value of x. Clue 1 could tell you that x is either 6 or 8 and clue 2 could tell you that x is either 8 or 9. In this case, when we use both clues together, we find that x must be 8 to satisfy both. Hence the statements never contradict each other. This means, if we get possible values of x from statement 1, we know that statement 2 will also give us at least one of those values.

This is how one statement could give us a starting point for the next one. Now that you understand the "why", let's go on to "how", using a question.

Question: If K is a positive integer less than 10 and N = 4,321 + K, what is the value of K?

Statement 1: N is divisible by 3

Statement 2: N is divisible by 7

Solution:

Given: N = 4321 + K

1 <= K <= 10

So N could range from 4322 (when K = 1) to 4331 (when K = 10). To find the value of K, we need to find the unique value of N.

Statement 1 tells us that N is divisible by 3.

4321 is not divisible by 3 since the sum of its digits is 4+3+2+1 = 10. It is 1 more than a multiple of 3. So the next multiple of 3 will be 4323. Hence N could be 4323. But there are some other multiples of 3 which could be the value of N. After 4323, 4326 and 4329 could also be the values of N since they are multiples of 3 too. We know this because if A is a multiple of 3, A+3, A+6, A+9, A-3, A-6 etc are also multiples of 3. So since 4323 is a multiple of 3, 4326 and 4329 will also be multiples of 3. We did not get a unique value for N so statement 1 alone is not sufficient.

Now let's go on to statement 2. This tells us that N must be a multiple of 7. In 10 consecutive numbers, there will be either one multiple of 7 or two multiples of 7. If there is only one multiple of 7 in the range 4322 to 4331, statement 2 alone will be sufficient to give us the value of N. If there are two multiples of 7 in this range, then statement 2 alone will not be sufficient.

Recall that from statement 1, we already know that N will take one of three values: 4323, 4326 or 4329.

Let's check for 4326 because it is in the middle. If 4326 is divisible by 7, there will be no other multiple of 7 in the range 4322 to 4331 because the closest multiples of 7 to 4326 will be 4326 – 7 and 4326 + 7. When we divide 4326 by 7, we find that it is divisible. This means that statement 2 gives us a single value of N. Hence statement 2 alone is sufficient.

Hypothetically, what if we had found that 4326 is not divisible by 7? Then we would have known that either 4323 or 4329 must be a multiple of 7. In both cases, statement 2 would have given us 2 multiples of 7 because both 4330 (7 more than 4323) and 4322 (7 less than 4329) are in the possible range. Then we would have known that the answer will be (C) i.e. we will need both statements to answer the question since the possible values from the two statements will have only one overlap in either case.

Note that what we gleaned from statement 1 helped us quickly examine statement 2 and get to the answer right away. But this is an advanced technique and you should use it only if you understand it very well. Else, it is best to stick to completely ignoring one statement while working on the other.

18.How to go from 48 to 51- Part-3

Let's get back to strategies that will help us reach the coveted 51 in Quant. First, take a look at Part I and Part II of this blog series. Since the Quant section is not a Math test, you need conceptual understanding and then some ingenuity for the hard questions (since they look unique). Today we look at a Quant problem which is very easy if the method "strikes". Else, it can be a little daunting. What we will do is look at a "brute force" method for times when the textbook method is not easily identifiable.

Question: What is 0.99999999/1.0001 – 0.99999991/1.0003?

(A) 10^(-8)

(B) 3*10^(-8)

(C) 3*10^(-4)

(D) 2*10^(-4)

(E) 10^(-4)

Solution: Usually, when we have decimals such as .99999999 or 1.0001, we round them off to 1 without thinking twice. The issue here is that all numbers are very close to 1 so if we round them off to 1, we will get 1/1 – 1/1 = 0. This, obviously, doesn't work and we need to work with the complicated numbers only.

Now here is the official method, something a Math professor will give us:

Method 1:

For simplification, you will need to use a^2 – b^2 = (a – b)(a + b)

Note that 0.99999999 is .00000001 less than 1 and 1.0001 is .0001 more than 1.

.00000001 is the square of .0001.

0.99999999/1.0001 – 0.99999991/1.0003

{1 – .00000001}/{1 + .0001} – {1 – .00000009}/{1 + .0003}

{1^2 – .0001^2}/{1 + .0001} – {1^2 – 0.0003^2}/{1 + .0003}

{(1 – .0001)(1 + .0001)}/{1 + .0001} – {(1 – .0003)(1 + .0003)}/{1 + .0003}

(1 – .0001) – (1 – .0003)

.0002 = 2*10^{-4}

All in all, the question only required us to recall something we learnt in 7th standard: a^2 – b^2 = (a – b)(a + b)

Does it mean it is a very simple question? Not really. The problem is that it is hard to identify that all you need is this formula and that you need to bring the terms in this format.

So here is a "brute force" method that people came up with and that we can use when Math fails us:

Method 2:

The fractions are quite complicated but the options are not fractions. This means that we are able to get rid of the denominator somehow. This brings an idea to mind: 0.99999999 might be a multiple of 1.0001. But how do we find 'which multiple'?

For that, we need to use some pattern recognition.

9*1.0001 = 9.0009

99*1.0001 = 99.0099

and so on till 9999*1.0001 = 9999.9999 (to get eight 9s)

Now since the decimal is 4 digits to the left i.e. the number is actually 0.99999999,

0.9999 * 1.0001 = 0.99999999

This means 0.99999999/1.0001 = 0.9999

On the same lines, we might guess that 0.99999991 is a multiple of 1.0003. To find 'which multiple', we might need to think even harder now. Note that something needs to multiply 1.0003 to give something ending in 1. Perhaps this multiple ends with a 7 because 3*7 ends in 1.

And sure, 9997*1.0003 = 9999.9991 which gives us 0.9997 * 1.0003 = 0.99999991

This gives us 0.99999991/1.0003 = 0.9997

Thus, the problem boils down to

0.9999 – 0.9997 = .0002

We encourage you to look for some more brute force methods.

19. Advance Number properties- Part-4

As pointed out by a reader, we need to complete the discussion on a question discussed in our previous 'Advanced Number Properties' posts so let's do that today. Note that the discussion that follows doesn't fall in the purview of GMAT and you needn't know it. You will be able to solve any question without taking this post into account but that has never stopped us from letting loose our curiosity so here goes…

Question 1: Which of the following CANNOT be the sum of two prime numbers?

(A) 19

(B) 45

(C) 58

(D) 79

(E) 88

Solution: We discussed in that post that the sum of two prime numbers is usually even because prime numbers are usually odd. We also discussed that if the sum of two prime numbers is odd, it means one of the prime numbers is certainly 2 – the only even prime number.

For example:

2 + 3 = 5

2 + 7 = 9

2 + 17 = 19

Then it makes perfect sense to first look at the options which are odd. To be sum of two prime numbers, the sum must be of the form 2 + Another Prime Number.

We saw that (D) 79 = 2 + 77 (77 is not prime.) and hence we got (D) as our answer.

Now the question we raised there was: What happens if instead of 79, we had 81?

81 = 2 + 79

Then all three odd options would have been sum of two prime numbers and we would have needed to check the even options too. How do you figure whether an even number can be written as the sum of two prime numbers?

This is where Goldbach's Conjecture comes into play (you don't really need to know it. We are doing it for intellectual purposes. GMAC will never put you in this fix).

It says "Every even integer greater than 2 can be expressed as the sum of two primes."

Mind you, it's a conjecture i.e. it hasn't been proven for all even numbers (only for even numbers till 4 * 10^{18}) but it does seem to hold.

For example:

4 = 2 + 2

6 = 3 + 3

8 = 3 + 5

10 = 3 + 7 = 5 + 5

12 = 5 + 7

and so on…

So given any even sum greater than 2, you can say that it CAN be written as sum of two prime numbers, for all practical purposes.

In fact, and here we are going into really geeky territory, we expect that every large even integer has not just one representation as the sum of two primes, but in fact has very many such representations. For all we know, 6 may be the only even number greater than 2 which cannot be written as the sum of two distinct prime numbers.

Coming back to our original question, we will actually check only odd numbers to see whether they can be written as sum of two primes. One of them has to be such that it cannot be written as sum of two primes and finding that is very simple! (as discussed in the previous post)

So all in all, the question that seemed very tedious turned out to be very simple!

20. How to go from 48 to 51- Part-4

To take a look at the previous posts of this thread, check: Part I, Part II and Part III.

Another point to keep in mind while targeting Q50+ in GMAT: don't buy complex official solutions. Most GMAT questions can be solved in a few steps. The point is that sometimes it is hard to identify those "few steps" and we keep going round and round in circles for a while till we arrive at the answer. The way to hit 51 is to look for simple solutions for difficult questions. The best example of this would be question number 148 of Official Guide 12. The question is tough, no doubt about it but just because it is tough, don't think that the solution needs to be tough too – you don't have to live with the solution provided.

If, even after reading the solution a couple of times, you know that if you try the question again in a week, you won't be able to solve it on your own, this means you need to review either the concept or the solution. If the given solution is too complex and you almost have to learn it up step by step, it means you need a better solution. The next step of the solution should be apparent to you – you should be able to solve it on your own within two minutes.

Also, even if one method looks good, try to find other ways of solving the question. Often, there are multiple good ways of solving a particular question.

Here is a question similar to question number 148 of OG12. Let me give a few good methods of solving it:

Question: If x, y, and k are positive numbers such that {x/(x+y)}*20 + {y/(x+y)}*40 = k, and if x < y, which of the following could be the value of k?

(A) 15

(B) 20

(C) 25

(D) 35

(E) 40

Solution: One solution you have in the OG. Three more are provided here:

Method 2: Algebra

Note the "could be" in the question. This means that k can take multiple values and one of them is provided here.

20*x/(x+y) + 40*y/(x+y) = k

20(x+2y)/(x+y) = k

20*{(x + y)/(x + y) + y/(x + y)} = k

20*{ 1 + y/(x + y)} = k

Now since y is greater than x, y/x+y will be more than 1/2 but definitely less than 1 (x and y are positive numbers).

So the value of k will lie in the range 20*{1 + 1/2} < k < 20*{1 + 1}

i.e. 30 < k < 40

Only option (D) falls in this range.

Answer (D)

Method 3: Weighted Average

Does this equation remind you of something: 20*x/(x+y) + 40*y/(x+y) = k?

If you are a weighted average fan like me, you will notice that this is just the weighted average formula applied:

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

Where Cavg = k

C1 = 20

C2 = 40

w1 = x

w2 = y

k = (20*x + 40*y)/(x + y)

It might be hard to see this on your own but the point is that if you do see it, the return is very high.
We know that the average of two quantities will lie in between them. So k must lie between 20 and 40. Also, we are given that x is less than y i.e. weight given to 20 is less than the weight given to 40. So the weighted average will lie toward 40. Between 30 and 40, there is only option (D)
Hence, answer (D).

Method 4: Plugging Numbers

Now, what if neither of the above given methods work for you during the test and your mind goes blank? Then you can pick some numbers to get an idea of the kind of values you will get. This is absolute brute force and may not always work out but it will give you a fighting chance of getting the correct answer.

20*x/(x+y) + 40*y/(x+y) = k

- Say, x = 1, y = 3 (x and y are positive numbers and x < y)

Then 20*1/(1+3) + 40*3/(1+3) = k = 35

- Say, x = 2, y = 3 (when you assume numbers, assume those which make the denominator a factor of 20 and 40 for ease of calculations. So assume numbers such that x+y is 4 or 5 or 10 etc)

Then 20*2/(2+3) + 40*3/(2+3) = k = 32

- Say, x = 1, y = 4

Then 20*1/(1+4) + 40*4/(1+4) = k = 36

Even if you do not get 35, note that the other values of k lie in the 30s. So your best bet would be to mark answer as (D).
Hope you see that there are many different ways of solving a given question, so you don't usually require complex solutions. Practice on!

21.2 simple quant questions that will help score higher

Let's discuss races today. It is a very simple concept but questions on it tend to be tricky. But if you understand how to handle them, most questions can be done easily.

A few points to remember in races:

1. Make a diagram. Draw a straight line to show the track and assume all racers are at start at 12:00. Then according to headstart, place the participants.

2. There are two types of head starts: Time and distance

Say there is a 1000 feet race between A and B which starts at 12:00.

Time – A gives B a headstart of 1 min means B starts running at 12:00 and A waits at the start point. Then A starts running from the start point at 12:01.

Distance – A gives B a headstart of 10 feet means A starts from the start point but B starts from the point 10 feet ahead (and hence runs only 990 feet to complete the race)

3. A dead heat is a race in which both the participants finish exactly at the same time. Most races in race questions end in a dead heat!

4. There are two ways in which a participant can beat another: Time and distance

Say A beats B in the 1000 feet race in which both start from the start point at 12:00.

Distance – If A beats B by 20 feet, it means A finishes the race (full 1000 feet) and at that time, B is 20 feet away from the finish line.

Time – If A beats B by 2 mins, it means that if A finished at 12:10, B is still 2 mins away from the finish line i.e. at his/her speed, B takes 2 mins to reach the finish line.

That is all! Now let's look at some questions:

Question 1: A's speed is 20/17 times that of B. If A and B run a race, what part of the length of the race should A give B as a head start, so that the race ends in a dead heat?

(A) 1/17
(B) 3/17
(C) 1/10
(D) 3/20
(E) 3/10

Solution: We have the ratio of A's speed and B's speed. This means, we know how much distance A covers compared with B in the same time.

This is what the beginning of the race will look like:

(Start) A_________B______________________________

If A covers 20 meters, B covers 17 meters in that time. So if the race is 20 meters long, when A reaches the finish line, B would be 3 meters behind him. If we want the race to end in a dead heat, we want B to be at the finish line too at the same time. This means B should get a head start of 3 meters so that he doesn't need to cover that. In that case, the time required by A (to cover 20 meters) would be the same as the time required by B (to cover 17 meters) to reach the finish line.

So B should get a head start of 3/20th of the race.

Answer (D)

This question was relatively very straight forward and we gave it only to help you apply the concepts discussed above. Let's make it slightly tricky now.

Question 2: A's speed is 20/17 times that of B. If A and B run a race, what part of the length of the race should A give B as a head start, so that B beats A by 20% of the length of the race?

(A) 15%
(B) 20%
(C) 28%
(D) 32%
(E) 35%

Solution: Again, we have ratio of A's speed and B's speed given as 20:17. If A covers 20 meters, B covers 17 meters in that time. This time, let's assume that the length of the race is 25 meters.

At the beginning, this is what the 25 meter track will look like with a head start to B:

(Start) A_________B________________________________

Since A will give B a head start so A must start from the start line while B will start from ahead.

Since A should cover only 80% of the length of the race, when B reaches the finish line, A should still have 20% of the track leftover.

20% of the track will be (20/100)*25 = 5 meters. So A should be at 20 meters when B is at the finish line.

So this is what the finish of the race will look like:

____________20_________________A____5_____B (Finish)

A will cover a total of 20 meters when B should be at the finish line. In this time, B will cover only 17 meters. But the total track is of 25 meters. So the rest of the 25-17 = 8 meters, B should get as a head start.

Head start will be 8/25 *100 = 32% of the race.

Answer (D)

If you found it tricky, we would suggest you to practice some more races questions. It is usually easy to "figure out" the answer logically and the calculations required are minimum.

Now try this official question. We will solve it for you next week.

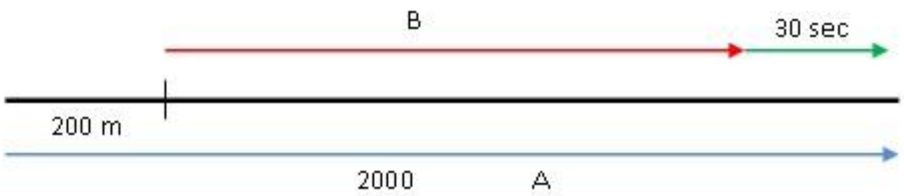
Question 3: A and B run a race of 2000 m. First, A gives B a start of 200m and beats him by 30 seconds. Next, A gives B a start of 3mins and is beaten by 1000m. Find the time in minutes in which A and B can run the race separately.

22.A 700+ Question on Races

This week we will look at the question on races that we gave you last week.
Question 3: A and B run a race of 2000 m. First, A gives B a head start of 200 m and beats him by 30 seconds. Next, A gives B a head start of 3 mins and is beaten by 1000 m. Find the time in minutes in which A and B can run the race separately?

(A) 8, 10
(B) 4, 5
(C) 5, 9
(D) 6, 9
(E) 7, 10

Solution: Now this question is a little tougher than the previous ones we saw last week.
There are two scenarios given:
1 – A gives B a head start of 200 m and beats him by 30 seconds.
2 – A gives B a head start of 3 mins and is beaten by 1000m.
Let's study both of them and see what we can derive from them.
Scenario 1: A gives B a start of 200m and beats him by 30 seconds.
As we suggested before, we will start by making a diagram.



A runs from the Start line till the finish line i.e. a total distance of 2000 m.
A gives B a head start of 200 m so B starts, not from the starting point, but from 200 m ahead. A still beats him by 30 sec which means that A completes the race while B takes another 30 sec to complete it. So obviously A is much faster than B.
In this race, A covers 2000m. In the same time, B covers the distance shown by the red line. Since B needs another 30 sec (i.e. 1/2 min) to cover the distance, he has not covered the green line distance. The green line distance is given by (1/2)*s where s is the speed of B in meters per minute. The distance B has actually covered in the same time as A is the distance shown by the red line. This distance will be (1/2)*s less than 1800 i.e. it will be [1800 - (1/2)*s].
Scenario 2: A gives B a head start of 3mins and is beaten by 1000m.

A gives B a head start of 3 mins means B starts running first while A sits at the starting point. After 3 mins, B covers the distance shown by the red line which we do not know yet. Now, A starts running too. B beats A by 1000 m which means that B reaches the end point while A is still 1000 m away from the end i.e. at the mid point of the 2000 m track.

In this race, A covers a distance of 1000 m only. In that time, B covers the distance shown by the green line. The distance shown by the red line was covered by B in his first 3 mins i.e. this distance is 3*s. This distance shown by the green line is given by (2000 – 3s).
Now you see that in the first race, A covers 2000m while in the second race, he covers only 1000m. So in the second race, he must have run for only half the time. Therefore, in half the time, B would also have covered half the previous distance.
Distance covered by B in first race = 2*Distance covered by B in second race
1800 – (1/2)*s = 2*(2000 – 3s) (where s is the speed of B in meters/min)
s = 400 meters/min
Time taken by B to run a 2000 m race = Distance/Speed = 2000/400 = 5 min
Only one option has time taken by B as 5 mins and that must be the answer.
If required, you can easily calculate the time required by A too.
Distance covered by B in scenario 1 = 1800 – (1/2)*s = 1600 m
In the same time, A covers 2000 m which is a ratio of A:B = 5:4. Hence time taken by A:B will be 4:5.
Answer (B)

23. 95% students find this question tough

Today we continue to look at ways to achieve that much desired score of 51 in Quant. Obviously, we don't need Sheldon Cooper's smarts to realize that for that revered high score, we must do well on the high level questions but the actual question is – how to do well on the high level questions?
We will illustrate that with the help of a supremely beguiling official question today. We are sure you wouldn't call an academician's work exactly thrilling but questions like these do add a decent bit of joie de vivre to our lives. It's hard to explain the gratification we get when it all falls into place in your mind and you light up with – "shoot, so simple, and yet, it seemed like a monster a few minutes back!" – we basically live for those moments!
Let us first give you some stats which indicate the difficulty level of this question:
95% of people find this question hard. Only 1/3rd of respondents answer it correctly (which includes the ton of people who had tried it before and hence knew the correct answer).
Let us give you the question now:
Question: Each piglet in a litter is fed exactly one-half pound of a mixture of oats and barley. The ratio of the amount of barley to that of oats varies from piglet to piglet, but each piglet is fed some of both grains. How many piglets are there in the litter?
Statement 1: Piglet A was fed exactly 1/4 of the oats today.
Statement 2: Piglet A was fed exactly 1/6 of the barley today.
First think, what concept does it test? Fractions? Ratios? Or is it just a word problem requiring algebraic manipulation? Actually, none of these. We can look at the question and say straight away that the answer is (C). It needs no manipulation and no calculation. Of course, what it does need is a solid understanding of the weighted averages principle!
For now, forget the data given in the question.
Consider this:
Say, 10% of total Oats and 20% of total Barley was fed to a piglet.
The question now is – Of the total food (Oats + Barley) what percentage was fed to this piglet?
We hope you agree that it will depend on the ratio of Oats and Barley. If the mixture was only oats, the piglet was fed 10% of the total food. If the mixture was only Barley, the piglet was fed 20% of the total mixture. If the mixture was half oats and half barley, the piglet was fed 15% of the total mixture. If the mixture was 1 part Oats for every 4 parts of Barley,

the piglet was fed 18% of the mixture (it is just weighted average with weights being the amount of initial quantity of Oats and Barley). Whatever the case, the piglet was fed more than 10% of total food and less than 20% of total food if the mixture consisted of both Oats and Barley.

If this is not clear, look at this example:

Say a meal consists of a sandwich and a milkshake. You eat 1/2 of the sandwich and drink 1/2 of the milkshake. Can we say that you have had 1/2 of the meal? Sure.

If you eat only 1/4 of the sandwich and drink 1/4 of the milkshake, then you would have had only 1/4 of the meal.

What happens in case you eat 1/2 of the sandwich but drink only 1/4 of the milkshake? In that case, you have had less than 1/2 of the meal but certainly more than 1/4 of the meal, right?

Go through this again till you are satisfied with this logic.

If this sounds good, consider data given in the question – piglet A was fed 25% Oats (1/4 Oats) and 16.66% Barley (1/6 Barley). So definitely, the piglet was fed more than 16.66% (which is 1/6) of the total mixture and less than 25% (which is 1/4) of the total mixture (as reasoned above). Stay with this idea.

Another piece of information from the question stem: the total food mixture was split equally among all the piglets. Since all piglets got the same quantity of food, we can say that all piglets were fed more than 1/6 of the total mixture but less than 1/4 of the total mixture. Number of piglets has to be an integer, say n. Then, each piglet gets the same amount of food i.e. 1/n of the total mixture. This 1/n must lie between 1/4 and 1/6. Note that the number of pigs i.e. n, must be a positive integer. What integer value can n take? Can it be 7? Will 1/7 lie between 1/6 and 1/4? No. 1/7 will be less than 1/6. Can n be 3? Will 1/3 lie between 1/4 and 1/6? No, because 1/3 will be greater than 1/4. n cannot be greater than 6 or less than 4 because it goes out of range. Only 1/5 lies between 1/4 and 1/6 (such that n is a positive integer). Hence n must be 5.

Notice that we did not need to do any calculations – just looking at the two statements, we can say that 1/n must lie between 1/4 and 1/6 and hence n must be 5.

Questions such as this one set GMAT apart from other tests. It tests you on basic concepts but how!!!

24.How to expect the unexpected on GMAT

Most of us know that GMAT is a shrew, (euphemism for a more choice adjective that comes to mind!) and is very hard to tame. It is well established that it is able to give a pretty accurate estimate of aptitude with just a few questions, and that the only way to “deceive” it is by actually improving your aptitude! It has numerous tricks up its sleeves to uncloak a rather basic player.

Let’s discuss one such trick today – a trick in which you need to realize that the situation calls for a complete U-turn of the usual.

Let’s take an example:

Question: Two cars run in opposite directions on a circular track. Car A travels at a rate of 6? miles per hour and Car B runs at a rate of 8? miles per hour. If the track has a radius of 6 miles and the cars both start from Point S at the same time, how long, in hours, after the cars depart will they again meet at Point S?

(A) 6/7 hrs

(B) 12/7 hrs

(C) 4 hrs

(D) 6 hrs

(E) 12 hrs

Solution: What would we usually do in such a question? Two cars start from the same point and run in opposite directions – their speeds are given. This would remind us of relative speed. When two objects move in opposite directions, their relative speed is the sum of their speeds. So we might be tempted to do something like this:

Perimeter of the circle = 2?r = 2?*6 = 12? miles

Time taken to meet = Distance/Relative Speed = 12?/(6? + 8?) = 6/7 hrs

But take a step back and think – what does 6/7 hrs give us? It gives us the time taken by the two of them to complete one circle together. In this much time, they will meet somewhere on the circle but not at the starting point. So this is definitely not our answer.

The actual time taken to meet at point S will be given by 12?/(8? – 6?) = 6 hrs

This is what we mean by unexpected! The relative speed should be the sum of their speeds. Why did we divide the distance by the difference of their speeds? Here is why:

For the two objects to meet again at the starting point, obviously they both must be at the starting point. So the faster object must complete at least one full round more than the slower object. In every hour, car B – the one that runs at a speed of 8? mph covers 2? miles more compared with the distance covered by car A in that time (which runs at a speed of 6? mph). We want car B to complete one full circle more than car A. In how much time will car B cover 12? miles (a full circle) more than car A? In 12?/2? hrs = 6 hrs.

Now we will keep the question the same but will change the figures a bit:

Modified Question: Two cars run in opposite directions on a circular track. Car A travels at a rate of 3? miles per hour and Car B runs at a rate of 5? miles per hour. If the track has a radius of 7.5 miles and the cars both start from Point S at the same time, how long, in hours, after the cars depart will they again meet at Point S?

So following the same logic as above,

Perimeter of the circle = 2?r = 2?*7.5 = 15? miles

The time taken to meet at point S will be given by 15?/(5? – 3?) = 7.5 hrs

But note that the two cars will not even be at the starting point, S, in 7.5 hrs. So this answer is wrong. Why? It has something to do with the word "at least" used in the explanation above i.e. "So the faster object must complete <u>at least</u> one full round more than the slower object. "

Try to put it all together.

Meanwhile, let's give you **another method**. This will not fail you no matter what the figures.

Using the original question:

Time taken by car A to complete one full circle = 12?/6? = 2 hrs

Time taken by car B to complete one full circle 12?/8? = 1.5 hrs

So every 2 hrs car A is at S and every 1.5 hrs, car B is at S. When will they both be together at S?

Car A at S -> 2 hrs, 4 hrs, 6 hrs, 8 hrs …

Car B at S -> 1.5 hrs, 3 hrs, 4.5 hrs, 6 hrs …

In 6 hrs – the first common time, both cars will be at the point S together. So answer is 6 hours.

Using the same method on the Modified Question,

Time taken by car A to complete one full circle = 15?/3? = 5 hrs

Time taken by car B to complete one full circle = 15?/5? = 3 hrs

So every 5 hrs, car A is at S and every 3 hrs, car B is at S. When will they both be together at S?

Car A at S -> 5 hrs, 10 hrs, 15 hrs, 20 hrs

Car B at S -> 3 hrs, 6 hrs, 9 hrs, 12 hrs, 15 hrs

In 15 hrs – the first common time (LCM of 3 and 5), both cars will be at the point S together.

This all makes sense now.

25.First do what you know on Quant question

We have read a lot about one way of handling complex questions – simplify them to a question you know how to solve. Here is another way – first do what you do know, and then figure out the rest!

We know that basic concepts are twisted to make advanced questions. Our aim is to break down the question into two parts – 'the basic concept' and 'the complexity'. You can either deal with the complexity first and then glide through the basic concept or you can glide through the basic concept first and then face the complexity. The method you use will depend on the question. If the question seems too complex at the outset, it means you will have to deal with the complexity first. If the question seems familiar but has some extra not-so-familiar elements, it means you should get the familiar out of the way first. Let's take a question today to see how to do that.

Question: During a sale of 20% on everything in a store, a kid is successful in convincing the store manager to give him 20 candies for the discounted price of 14 candies. The store still makes a profit of 12% on this sale. What is the mark up percentage on each candy?

(A) 100%

(B) 80%

(C) 75%

(D) 66+2/3%

(E) 55%

Solution:

This question can get very messy if you let it! We have seen people working on this question with multiple variables: C for cost price, S for sale price, M for marked price etc. That can get very confusing because there are two types of mark up – the actual mark up (the store marks up the price of every candy by this percentage and lists it on the candy) and the effective mark up (because the kid takes 6 extra candies, this is the effective mark up). So let's not go the algebra way. Instead, let's focus on what we can do without much effort. As a first step, let's do what we know already (and hope that the rest will work out!).

We already know the relation between mark-up, discount and profit. The problem is that this question has another aspect – the kid takes 20 candies but pays the price of only 14 candies (which is the price obtained by reducing the marked price by 20% of discount). But let's worry about it later.

Let's first deal with the mark-up, discount and profit aspect of the question.

We know that (1 + m/100)(1 – d/100) = (1 + p/100) (already discussed in detail in this post)

Since p is the effective profit that the store got, m must be the effective mark up here.

(1 + m/100)(1 – 20/100) = (1 + 12/100)

(1 + m/100) = (5/4)*(28/25)

(1 + m/100) = 7/5

m = 40

So effective mark up was 40% – i.e. 40% was the mark up in a situation where 14 articles were sold and charged for. This tells us this – effective mark up turned out to be 40% though his actual mark up must have been higher since he gave away 20 articles for the cost of 14.

Now what we already know is done. We get to the really tricky part – the thing that makes this question different – how do we find the actual mark up?

Let's say the cost price of each of the 20 candies was $1. Then total cost price for the 20 candies was $20. This is the cost of the candies to the store. The effective markup was 40% i.e. the articles were effectively marked at 20 + (40/100)*20 = $28. The store gave a discount of 20% on this amount and made a profit of 12%. But this amount of $28 actually represents the mark up on 14 candies only. The cost price of 14 candies is $14 to the store. So the actual mark up percentage on the 14 candies is (28 – 14)/14 * 100 = 100%

Answer (A)

Obviously, there are many other ways of solving this question. See if you can figure out another one on your own!

26.A 750+ Question on Statistics

Today, we have a very interesting statistics question for you. We have already discussed statistics concepts such as mean, median, range etc in our QWQW series. Check them out here if you haven't already done so:
The Meaning of Arithmetic Mean
Can You Solve these Mean GMAT Questions?
Finding Arithmetic Mean Using Deviations
Application of Arithmetic Means
Mean Questions on Median
A Range of Questions
This question needs you to apply all these concepts but can still be easily done in under two minutes. Now, without further ado, let's go on to the question – there is a lot to discuss there.
Question: An automated manufacturing unit employs N experts such that the range of their monthly salaries is $10,000. Their average monthly salary is $7000 above the lowest salary while the median monthly salary is only $5000 above the lowest salary. What is the minimum value of N?
(A)10
(B)12
(C)14
(D)15
(E)20
Solution: Let's first assimilate the information we have. We need to find the minimum number of experts that must be there. Why should there be a minimum number of people satisfying these statistics? Let's try to understand that with some numbers.
Say, N cannot be 1 i.e. there cannot be a single expert in the unit because then you cannot have the range of $10,000. You need at least two people to have a range – the difference of their salaries would be the range in that case.
So there are at least 2 people – say one with salary 0 and the other with 10,000. No salary will lie outside this range.
Median is $5000 – i.e. when all salaries are listed in increasing order, the middle salary (or average of middle two) is $5000. With 2 people, one at 0 and the other at 10,000, the median will be the average of the two i.e. (0 + 10,000)/2 = $5000. Since there are at least 10 people, there is probably someone earning $5000. Let's put in 5000 there for reference.
0 … 5000 … 10,000
Arithmetic mean of all the salaries is $7000. Now, mean of 0, 5000 and 10,000 is $5000, not $7000 so this means that we need to add some more people. We need to add them more toward 10,000 than toward 0 to get a higher mean. So we will try to get a mean of $7000.
Let's use deviations from the mean method to find where we need to add more people.
0 is 7000 less than 7000 and 5000 is 2000 less than 7000 which means we have a total of $9000 less than 7000. On the other hand, 10,000 is 3000 more than 7000. The deviations on the two sides of mean do not balance out. To balance, we need to add two more people at a salary of $10,000 so that the total deviation on the right of 7000 is also $9000.
Note that since we need the minimum number of experts, we should add new people at 10,000 so that they quickly make up the deficit in the deviation. If we add them at 8000 or 9000 etc, we will need to add more people to make up the deficit at the right.
Now we have
0 … 5000 … 10000, 10000, 10000

Now the mean is 7000 but note that the median has gone awry. It is 10,000 now instead of the 5000 that is required. So we will need to add more people at 5000 to bring the median back to 5000. But that will disturb our mean again! So when we add some people at 5000, we will need to add some at 10,000 too to keep the mean at 7000.
5000 is 2000 less than 7000 and 10,000 is 3000 more than 7000. We don't want to disturb the total deviation from 7000. So every time we add 3 people at 5000 (which will be a total deviation of 6000 less than 7000), we will need to add 2 people at 10,000 (which will be a total deviation of 6000 more than 7000), to keep the mean at 7000 – this is the most important step. Ensure that you have understood this before moving ahead.
When we add 3 people at 5000 and 2 at 10,000, we are in effect adding an extra person at 5000 and hence it moves our median a bit to the left.
Let's try one such set of addition:
0 … 5000, *5000, 5000, 5000* … 10000, 10000, 10000, *10000, 10000*
The median is not $5000 yet. Let's try one more set of addition.
0 … 5000, 5000, 5000, 5000, *5000, 5000, 5000* … 10000, 10000, 10000, 10000, 10000, *10000, 10000*
The median now is $5000 and we have maintained the mean at $7000.
This gives us a total of 15 people.
Answer (D)
Granted, the question is tough but note that it uses very basic concepts and that is the hallmark of a good GMAT question!
Try to come up with some other methods of solving this.

27.How to go from 48 to 51 in Quant- part-5

First, let us give you the link to the last post of this series: Post IV. It contains links to previous parts too.
Today, we bring another tip for you to help get that dream score of 51 – if you must write down the data given, write down all of it! Let us explain.
If you think that you will need to jot down the data given in the question and then solve it on your scratch pad (instead of in your mind), you must jot down every single detail. It is easy to overlook small things which are difficult to express algebraically such as 'x is an integer'. These details are often critical and could make all the difference between an 'unsolvable' question and a 'solvable within 2 minutes' one. Once you start solving the question on your scratch pad, you will not refer back to the original question again and again and hence might forget these details. Have them along with the rest of the data. Read every word of the question carefully, and ensure that it is consolidated on your scratch pad. For example, look at this question:
A set of five positive integers has an arithmetic mean of 150. A particular number among the five exceeds another by 100. The rest of the three numbers lie between these two numbers and are equal. How many different values can the largest number among the five take?
It is a difficult question because it incorporates statistics as well as max-min – both tricky topics. On top of it, people often overlook the 'are equal' part of the question here. The reason for that is that they are actively looking for implications of the sentences and the moment they read "The rest three numbers lie between these two numbers", they go back to the previous sentence which tells us "A particular number among the five exceeds another by 100". They then make a note of the fact that 100 is the range of the five positive integers. In all this excitement, they miss the three critical words "and are equal". Ensure that when you go to the sentence above, you pick the next sentence from the point where you left it. Another thing to note here is that all numbers are positive integers. This information will be critical to us.
Let's demonstrate how you will solve this question after incorporating all the information given.

Quarter Wit_Quarter Wisdom- Part-3

Question: A set of five positive integers has an arithmetic mean of 150. A particular number among the five exceeds another by 100. The rest of the three numbers lie between these two numbers and are equal. How many different values can the largest number among the five take?

(A) 18

(B) 19

(C) 21

(D) 42

(E) 59

Solution:

Let's assume that the 5 natural numbers in increasing order are: a, b, b, b, a+100

We are given that $a < b < a+100$.

Also, we are given that a and b are positive integers. This information is critical – we will see later why.

The average of the 5 numbers is $(a+b+b+b+a+100)/5 = 150$

$(a+b+b+b+a+100) = 5*150$

$2a+3b = 650$

We need to find the number of distinct values that a can take because a+100 will also take the same number of distinct values.

Now there are two methods to proceed. Let's discuss both of them.

Method 1: Pure Algebra – Write b in terms of a and plug it in the inequality

$b = (650 - 2a)/3$

$a < (650 - 2a)/3 < a+100$

$3a < 650 - 2a < 3a + 300$

Now split it into two inequalities: $3a < 650 - 2a$ and $650 - 2a < 3a + 300$

Inequality 1: $3a < 650 - 2a$

$5a < 650$

$a < 130$

Inequality 2: $650 - 2a < 3a + 300$

$5a > 350$

$a > 70$

So we get that $70 < a < 130$. Since a is an integer, can we say that a can take all values from 71 to 129? No. What we are forgetting is that b is also an integer. We know that

$b = (650 - 2a)/3$

For which values will be get b as an integer? Note that 650 is not divisible by 3. You need to add 1 to it or subtract 2 out of it to make it divisible by 3. So a should be of the form 3x+1.

$b = (650 - 2*(3x+1))/3 = (648 - 6x)/3 = 216 - 2x$

Here, for any positive integer x, b will be an integer.

From 71 to 129, we have the following numbers which are of the form 3x+1:

73, 76, 79, 82, 85, … 127

This is an Arithmetic Progression. How many terms are there here?

Last term = First term + (n – 1)*Common Difference

$127 = 73 + (n - 1)*3$

$n = 19$

a will take 19 distinct values so the last term i.e. (a+100) will also take 19 distinct values.

Method 2: Using Transition Points

Note that $a < b < a+100$

Since a < b, let's find the point where a = b, i.e. the transition point

2a + 3a = 650

a = 130 = b

But b must be greater than a. If we increase b by 1, we need to decrease a by 3 to keep the average same. But decreasing a by 3 decreases the largest number i.e. a+100 by 3 too; so we need to increase b by another 1.

We get a = 127 and b = 132. This give us the numbers as 127, 132, 132, 132, 227. Here the average is 150

Since b < a+100, let's find the point where b = a+100

2a + 3(a+100) = 650

a = 70, b = 170

But b must be less than a+100. If we decrease b by 1, we need to increase a by 3 to keep the average same. But increasing a by 3 increases the largest number, i.e. a+100 by 3 too, so we need to decrease b by another 1.

We get a = 73 and b = 168. This gives us the numbers as 73, 168, 168, 168, 173. Here the average is 150

Values of a will be: 73, 76, 79, ….127 (Difference of 3 to make b an integer)

This is an Arithmetic Progression.

Last term = First term + (n – 1)*Common difference

127 = 73 + (n – 1)*3

n = 19

a will take 19 distinct values so the last term i.e. (a+100) will also take 19 distinct values.

Answer (B)

28.What are the Weights in Weighted Average

We have discussed weighted averages in detail here but one thing we are yet to talk about is how you decide what the weights will be in weighted average problems. It is not always straight forward to identify the weights. For example, in a question such as this one,

While traveling from Detroit to Novi, a car averaged 10 miles per gallon, and while traveling from Novi to Lapeer, it averaged 18 miles per gallon. If the distance between Detroit and Novi is half the distance between Novi and Lapeer, what is the average miles per gallon for the entire journey?

We have two figures for mileage given here – 10 miles per gallon and 18 miles per gallon. We need to find the average mileage. So we can use the weighted average formula but what will the weights be? Will they be 1:2 since the distance between the two cities is given to be in the ratio 1:2? If you think that taking the distance to be the weights in this problem is correct, then you fell for the trap in this question.

To explain the concept, let us use a simpler example first:

When talking about average speed, what are the weights? We know that the weight given to each speed is the time for which that speed was maintained, right? Yes! But why?

Let's review our weighted average formula:

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

Average Speed = (Speed1*Time1 + Speed2*Time2)/(Time1 + Time2)

Average Speed = (Distance1 + Distance2)/(Time 1 + Time2)

Average Speed = Total Distance/Total Time

This is an accurate representation of average speed.

Now see what happens when you use distance as the weights.

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

Average Speed = (Speed1*Distance1 + Speed2*Distance2)/(Distance1 + Distance2)

Speed*Distance doesn't represent any physical quantity. So this doesn't make sense. The units of the quantities will help you see the relation clearly.

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

Average Speed = (Speed1*Time1 + Speed2*Time2)/(Time1 + Time2)

Average Speed = (miles/hour * hour + miles/hour * hour)/(hour + hour)

Average Speed = (miles + miles)/(hour + hour)

Average Speed = Total miles/Total hours

What happens when you take distance as the weights?

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

Average Speed = (Speed1*Distance1 + Speed2*Distance2)/(Distance1 + Distance2)

Average Speed = (miles/hour * miles + miles/hour * miles)/(miles + miles)

miles^2/hour doesn't represent a physical quantity and hence doesn't make sense here. Therefore, whenever you are confused what the weights should be, look at the units.

Let's go back to the original question now. Average required is miles per gallon. So you are trying to find the weighted average of two quantities whose units must be miles/gallon.

Cavg = (C1*w1 + C2*w2)/(w1 + w2)

The unit of Cavg, C1 and C2 is miles/gallon so w1 and w2 should be in gallons to get

miles/gallon = (miles/gallon * gallon + miles/gallon * gallon)/(gallon + gallon)

miles/gallon = Total miles/Total gallons

So how will we actually solve this question?

Question: While traveling from Detroit to Novi, a car averaged 10 miles per gallon while traveling from Novi to Lapeer, it averaged 18 miles per gallon. If the distance between Detroit and Novi is half the distance between Novi and Lapeer, what is the average miles per gallon for the entire journey?

Solution:

Let the distance between Detroit and Novi be D. So the distance between Novi and Lapeer must be 2D.

Amount of fuel used to cover distance D = D/10

Amount of fuel used to cover distance 2D = 2D/18 = D/9

So the two weights used must be D/10 and D/9

Average miles/gallon = (10*D/10 + 18*D/9)/(D/10 + D/9) = 3D*90/19D = 270/19 = 14.2 miles/gallon

Or simply, Average miles/gallon = Total miles/Total gallons = 3D/(D/10 + D/9) = 14.2 miles/gallon

Food for thought: Which one of the following can you solve?

- If a vendor sold 10 apples at a profit of 10% and 15 oranges at a profit of 20%, what was his overall profit%?
- If a vendor sold apples at a profit of 10% and oranges at a profit of 20%, what was his overall profit% if cost price of each apple was $0.20 and the cost price of each orange was $.06?

29. Figure out the topic of discussion in GMAT

You must have come across questions which you thought tested one concept but later found out could be easily dealt with using another concept. Often, crafty little mixture problems belong to this category. For example:

Mark is playing poker at a casino. Mark starts playing with 140 chips, 20% of which are $100 chips and 80% of which are $20 chips. For his first bet, Mark places chips, 10% of which are $100 chips, in the center of the table. If 70% of Mark's remaining chips are $20 chips, how much money did Mark bet?

You can view this as a word problem where you assume the number of chips and then go splitting them up or you can view this as a mixtures problem even though it doesn't use words such as 'mixture', 'solution', 'combined' etc. As we have seen enough number of times, our mixture problems are solved in seconds using the weighted average concept.

Quarter Wit_Quarter Wisdom- Part-3

The question discussed here also belongs to the same category – looks super tricky but can be easily solved with weighted averages formula. But we have seen plenty and more of such questions in our blog posts. Today we will take a look at a different type of sinister question and I suggest you to think about the concept being tested in that before trying to solve it.
Question: Mark owns four low quality watches. Watch1 loses 15 minutes every hour. Watch2 gains 15 minutes every hour relative to watch1 (that is, as watch1 moves from 12:00 to 1:00, watch2 moves from 12:00 to 1:15). Watch3 loses 20 minutes every hour relative to watch2. Finally, watch4 gains 20 minutes every hour relative to watch3. If Mark resets all four watches to the correct time at 12 noon, what time will watch4 show at 12 midnight that day?

(A)10:00

(B)10:34

(C)11:02

(D)11:48

(E)12:20

Before we look at the solution, think about the concept being tested here – clocks? Circular motion?

Neither!

Solution: Note that when giving data about watch1, you are told how it varies with the actual time. Data about all other watches tells us about the time they show relative to the incorrect watches. The concept being tested here is Relative Speed! What do we mean by "gains 15 mins" or "loses 20 mins" etc? When a watch gains 15 mins every hour, it means that even though it should show that one hour has passed, it shows that 1 hr 15 mins have passed. So the watch runs faster than it should. Hence the speed of the watch is more than the speed of a correct watch. Now the question is how much more? The minute hand of the correct watch travels one full circle in one hour. The minute hand of the incorrect watch travels one full circle and then a quarter circle in one hour (to show that 1 hour 15 mins have passed even when only an hour has passed). So it is 5/4 times the speed of a correct watch. On the same lines, let's analyze each watch.

Say the speed of a correct watch is s.

- "Watch1 loses 15 minutes every hour. "

Watch1 covers only three quarters of the circle in an hour.

Speed of watch1 = (3/4)*s

- "Watch2 gains 15 minutes every hour relative to watch1 (that is, as watch1 moves from 12:00 to 1:00, watch2 moves from 12:00 to 1:15)."

Now we have the speed of watch2 relative to speed of watch1. Speed of watch2 is (5/4) times the speed of watch1.

Speed of watch2 = (5/4)*(3/4)s = (15/16)*s

- "Watch3 loses 20 minutes every hour relative to watch2."

Watch3 loses 20 mins every hour means its speed is (2/3)rd the speed of watch2

Speed of watch3 = (2/3)*(15/16)*s = (5/8)*s

- "Finally, watch4 gains 20 minutes every hour relative to watch3."

Speed of watch4 = (4/3)*Speed of watch3 = (4/3)*(5/8)*s = (5/6)*s

So the speed of watch4 is (5/6)th the speed of a correct watch. So if a correct watch shows that 6 hours have passed, watch4 will show that 5 hours have passed. If a correct watch shows that 12 hours have passed, watch4 will show that 10 hours have passed. From 12 noon to 12 midnight, a correct watch would have covered 12 hours. Watch4 will cover 10 hours and will show the time as 10:00.

Answer (A)

30.Finding last two digits on Quant- Part-1

We all know how to find the last digit using cyclicity when we are given a number raised to a power. Last digit of a number depends only on the last digit of the base. You must be quite familiar with something like this -

Last Digit of Base:

0 – Last digit of expression with any power will be 0.

1 – Last digit of expression with any power will be 1.

2 – 2, 4, 8, 6, 2, 4, 8, 6… Cyclicity is 4.

3 – 3, 9, 7, 1, 3, 9, 7, 1… Cyclicity is 4.

4 – 4, 6, 4, 6, 4, 6, 4, 6… Cyclicity is 2.

5 - Last digit of expression with any power will be 5.

6 – Last digit of expression with any power will be 6.

7 – 7, 9, 3, 1, 7, 9, 3, 1… Cyclicity is 4.

8 – 8, 4, 2, 6, 8, 4, 2, 6… Cyclicity is 4.

9 – 9, 1, 9, 1, 9, 1, 9, 1… Cyclicity is 2.

Cyclicity is nothing but pattern recognition. You see that when you multiply 2 by itself, there is a pattern of last digit which goes 2, 4, 8, 6, 2, 4, 8, 6 and so on. We can use the same principle for when a question asks us for the last two digits of the expression. Let me remind you first that here at QWQW, we sometimes flirt with the lines that define GMAT scope. Obviously, we do point out whenever we are indulging and that's exactly what we are going to do in this post. We are carrying on for the love of Math and the Q51 score.

The last two digits of the base decide the last two digits of the expression. For example,

Example 1: Let's look at powers of 11.

11^1 = 11

11^2 = 121

11^3 = 1331

11^4 = …41 (we should just multiply the last two digits together and ignore the rest)

11^5 = …51

11^6 = …61

11^7 = …71

Note that the last two digits are displaying a pattern depending on the power. So we expect the cyclicity here to be 10.

11^8 = …81

11^9 = …91

11^(10) = …01

11^(11) = …11

11^(12) = …21

and so on. So the last two digits should go from 11, 21 to 91, 01 and then go to 11 again. The cycle of 10 starts from power of 1, 11, 21 etc. This means that 11^(46) should have last two digits as 61, 11^(92) should have last two digits as 21 and 11^(168) should have last two digits as 81.

Let's look at some other numbers now:

Example 2: Say, we need the last two digits of 6^{58}

6^1 = 6 (No second last digit)

6^2 = 36

6^3 = 216

6^4 = …96 (Just multiply the last two digits)

6^5 = …76

6^6 = …56
6^7 = …36
and hence starts the cycle again:
3, 1, 9, 7, 5, 3, 1, 9, 7, 5 and so on.
The new cycle with tens digit of 3 begins at the powers of 2, 7, 12, 17, 22, 27 etc. So the new cycle will also begin at power of 57 and 6^58 will have 1 as the tens digit.
Example 3: How about the last two digits of 7^102?
7^1 = 7 (No second last digit)
7^2 = 49
7^3 = 343
7^4 = …01
7^5 = …07
7^6 = …49
7^7 = …43
We see a cyclicity of 4 here: 49, 43, 01, 07, 49, 43, 01, 07 … and so on. The new cycle begins at 2, 6, 10, 14 i.e. even powers which are not multiples of 4. So a new cycle will begin at 102 too. So the last two digits of 7^(102) will be 49.
Now there can be many variations in the questions asking us to find the last two digits. We will use different concepts for different question types. Today we saw how to use pattern recognition. We will look at some other methods next week.

31. Finding Last 2 digits- Part-2

Let's continue the discussion of last two digits we started last week. We discussed the concept of pattern recognition and how it can help us determine the last two digits in case of numbers raised to some powers. Today we look at what happens when there is no pattern to determine! What if we are asked to determine the last two digits of the product of a bunch of numbers. We know that getting the last digit in this case is very easy – just multiply the last digits of the numbers together. But last TWO digits would seem much more complicated.
Actually, we can find the last two digits quite easily in most such cases by using the concepts of remainders.
There are two concepts you need to understand before we go on to see how to solve such questions:
I. **When you divide a number by 100, the remainder is formed by the last two digits of the number.** Say, you divide 138 by 100, the remainder will be 38 (last two digits). Take another example – divide 1275 by 100, the remainder will be 75 and so on.
II. **When you divide (px + a)(qx + b)*…*(tx + e) by x, the remainder will be the remainder obtained by dividing a*b*…*e by x.** This should remind you of the binomial theorem we discussed many weeks ago. When we multiply all these terms together (px + a), (qx + b) etc, each term obtained will have at least one x except the last term which is obtained by multiplying the remainders together. To get a better idea, let's take some numbers:
Let's say we need to find the remainder when we divide 12*23*52*81 by 10.
K = 12*23*52*81 = (10 + 2)*(20 + 3)*(50 +2)*(80 + 1)
When you multiply these four terms together, you will get many terms such as 10*20*50*80, 10*20*50*1, 10*20*2*80 etc. All these will have a multiple of 10 except the last one. The last one will be 2*3*2*1 = 12. That doesn't have a multiple of 10. Now divide 12 by 10 to get the remainder 2. So when you divide K by 10, the remainder will be 2.
Now, let's look at a question:
Question 1: What are the last two digits of 63*35*37*82*71*41?
(A) 10
(B) 30
(C) 40

(D) 70

(E) 80

Solution: Using concept 1, we know that to find the last two digits, we need to find the remainder we get when we divide the product by 100.

Remainder of (63*35*37*82*71*41)/ 100

Note that we can simplify this expression by canceling out the 5 and 2 in the numerator and denominator. But before we do that, here is an important note:

Note: We cannot just cancel off the common terms in the numerator and denominator to get the remainder. But, if we want to cancel off to simplify the question, we can do it, provided we remember to multiply it back again.

*So say, we want to find the remainder when 14 is divided by 10 i.e. 14/10 (remainder 4). But we cancel off the common 2 to get 7/5. The remainder here will be 2 which is not the same as the remainder obtained by dividing 14 by 10. But if we multiply 2 back by 2 (the number we canceled off), the remainder will become 2*2 = 4 which is correct.*

Take another example to reinforce this – what is the remainder when 85 is divided by 20? It is 5.

We might rephrase it as – what is the remainder when 17 is divided by 4 (cancel off 5 from the numerator and the denominator). The remainder in this case is 1. We multiply the 5 back to 1 to get the remainder as 5 which is correct.

So keeping this very important point in mind, let's go ahead and cancel the common 5 and 2.

We need the

Remainder of (63*7*37*41*71*41*5*2)/10*5*2

Remainder of (63*7*37*41*71*41)/10

Now using concept 2, let's write the numbers in form of multiples of 10

Remainder of (60+3)*7*(30+7)*(40+1)*(70+1)*(40+1)/10

Remainder of 3*7*7*1*1*1/10

Remainder of 147/10 = 7

Now remember, we had canceled off 10 so to get the actual remainder so we need to multiply by 10: 7*10 = 70.

When 63*35*37*82*71*41 is divided by 100, the remainder is 70. So the last two digits of 63*35*37*82*71*41 must be 70.

Answer (D)

Next week, we will see some more complicated questions using these and other fundamentals.

32.Finding last 2 digits- part-3

As promised last week, we will look at another question which involves finding the last two digits of the product of some random numbers. In this question, along with the concepts discussed last week, we will assimilate the concept of negative remainders too discussed some weeks ago.

Let's recap the concepts before we see the question:

I. When you divide a number by 100, the remainder is formed by the last two digits of the number.

II. When you divide (px + a)(qx + b)*…*(tx + e) by x, the remainder will be the remainder obtained by dividing a*b*…*e by x.

III. We cannot just cancel off the common terms in the numerator and denominator to get the remainder. But, if we want to cancel off to simplify the question, we can do it, provided we remember to multiply it back again.

These three were discussed with examples last week.

IV. When m is divided by n and a negative remainder (–r) is obtained, we can find the actual remainder simply as (n – r). This is discussed with examples in this post.

Now, let's solve a question involving all these concepts.

Question: What are the last two digits of (301*402*503*604*646*547*448*349)^2

(A) 96

(B) 76

(C) 56
(D) 36
(E) 16

Solution: We need to find the last two digits of the product. It means we need to find the remainder when we divide the product by 100.

Find remainder of (301*402*503*604*646*547*448*349)^2/100

Note that 301 = 300 + 1 which gives us a small remainder 1 to work with but 349 = 300 + 49, a large remainder with which calculations will become cumbersome. But note that 349 is close to 350. All the numbers in the product are quite close to a multiple of 50, if not to a multiple of 100.

We need to find the remainder of:

(301*402*503*604*646*547*448*349)*(301*402*503*604*646*547*448*349)/100

This implies we need to find the remainder of:

(301*201*503*604*646*547*448*349)*(301*402*503*604*646*547*448*349)/50

We cancel off a 2 (of 402) from the numerator with a 2 of the denominator to make the divisor 50. We will multiply the remainder we obtain by 2 back at the end.

We need the remainder of:

(300 + 1)*(200+1)*(500+3)*(600+4)*(650-4)*(550-3)*(450-2)*(350-1)* (300+ 1)*(400+2)*(500+3)*(600+4)*(650-4)*(550-3)*(450-2)*(350-1)/50

Note that in this product, all terms obtained will have a multiple of 50 except the last term obtained by multiplying the remainders together:

We need:

the remainder of 1*1*3*4*(-4)*(-3)*(-2)*(-1)*1*2*3*4*(-4)*(-3)*(-2)*(-1) / 50

the remainder of (12)*(24)*(24)*(24) / 50

Notice now that the remainders are far too great but they are close to 25. So let's bring the divisor down to 25 by canceling off another 2.

We need:

the remainder of (6)*(24)*(24)*(24) / 25

the remainder of (6)*(25-1)*(25-1)*(25-1) / 25

Again, the only product we need to worry about is the last one obtained by multiplying the remainders together:

the remainder of 6*(-1)*(-1)*(-1) / 25

The remainder is -6 which is negative. To get the positive remainder, 25 – 6 = 19. But remember that we had divided the divisor twice by 2 so to get the actual remainder, we must multiply the remainder obtained back by 4: the actual remainder is 4*19 = 76

Answer (B)

33.Bringing back the "Lazy Genius" to solve

Those of you who have seen the previous version of our curriculum would know that we had tips and tricks under the heading of 'Lazy Genius'. These used to discuss innovative shortcuts for various questions – the way very smart people would solve the question – without putting in too much effort!

Today, let's bring back the beloved lazy genius through a question. Try to solve it lazily i.e. try to do minimum work on paper. This means making equations and solving them is a big no-no and doing too many calculations is cumbersome.

Question: A tank has two water pumps Alpha and Beta and one drain Gamma. Pump Alpha alone can fill the whole tank in x hours, and pump Beta alone can fill the whole tank in y hours. The drain can empty the whole tank in z hours such that z>x. When the tank was empty, pumps Alpha and Beta started pumping water in the tank and the drain Gamma simultaneously was draining water from the tank. When finally the tank is full, which of the following represents the amount of water in terms of the fraction of the tank which pump Alpha pumped into the tank?

(A) yz/(x+y+z)

(B) yz/(yz + xz – xy)

(C) yz/(yz + xz + xy)

(D) xyz/(yz + xz – xy)

(E) (yz + xz – xy)/yz

Note that you have variables in the question and the options. Since we are looking for a lazy solution, making equations out of the variables is not acceptable. So then, should we plug in numbers? With three variables to take care of, that might involve a lot of calculations too. Then what else?

Here is our minimum-work-solution to this problem; try to think one of your own and don't forget to share it with us.

Plugging in numbers for the variables can be troublesome but you can give some very convenient values to the variables so that the effect of a pump and a drain will cancel off.

There are no constraints on the values of x, y and z except z > x (drain Gamma empties slower than pipe Alpha fills)

Let's say, x = 2 hrs, y = 4 hrs, z = 4 hrs

What did we do here? We made the rate of Beta same as the rate of Gamma i.e. 1/4 of the tank each. This means, whenever both of them are working together, drain Gamma cancels out the work of pump Beta. Every hour, pump Beta fills 1/4th of the tank and every hour drain Gamma empties 1/4th of the tank. So the entire tank will be filled by pump Alpha alone. Hence, if y = z, pump Alpha fills the entire tank i.e. the amount of water in terms of fraction of the tank pumped by Alpha will be 1.

In the options, put y = z and see which option gives you 1. Note that you don't have to put in the values of 2, 4 and 4. We gave those values only for illustration purpose.

If y = z, xy = xz.

So in option (B), xz cancels xy in the denominator giving yz/yz = 1

Again, in option (E), xz cancels xy in the numerator giving yz/yz = 1

The other options will not simplify to 1 even though when we put y = z, the answer should be 1 irrespective of the value of x, y and z. The other options will depend on the values of x and/or y. Hence the only possible options are (B) and (E). But we still need to pick one out of these two.

Now let's say, x = 4, y = 2, z = 4.00001 (z should be greater than x but let's assume it is infinitesimally greater than x such that we can approximate it to 4 only)

Rate of work of Gamma (1/4th of the tank per hour) is half the rate of work of Beta (1/2 the tank per hour). Rate of work of Gamma is same as rate of work of Alpha. Half the work done by pump Beta is removed by drain Gamma. So if pump Beta fills the tank, drain Gamma empties half of it in that time – the other half would be filled by pump Alpha i.e. the amount of water in terms of fraction of the tank pumped by Alpha will be 1/2.

Put x = z in the options (B) and (E). The one that gives you 1/2 with these values should be the answer. Again, you don't need to plug in the actual values till the end.

If x = z, yx = yz

(B) yz/(yz + xz – xy)

yz cancels xy in the denominator giving us yz/xz = y/x = 2/4 = 1/2

(E) (yz + xz – xy)/yz

yz cancels xy in the numerator giving us xz/yz = x/y = 4/2 = 2

Only option (B) gives 1/2. Answer (B)

Even if you end up feeling that this method is complicated, try and wrap your head around it. If you do, you are on your way to becoming a lazy genius yourself!

34.The Speed and Accuracy Trade off :

We know that speed is important in GMAT. We have about 2 mins per question and we always have questions in which we get stuck, waste 3-4 mins and probably still answer incorrectly. So we are always trying to go faster, rush, complete the easy ones in less time! In our bid to save time, sometimes we sacrifice accuracy. We should know that accuracy is most important. No point running through questions and completing all of them before time if at the end of it all, most of our answers are incorrect – there are no bonus points for completing the test before time, after all!

In your haste to complete the test on time, don't overlook the important details. Getting too many easy questions wrong is certainly disastrous. Take a step back and ensure that what they asked is what you have found and that your logic is solid. To illustrate the problem, let's give you a question – people gloss over it, consider it an easy remainders problem, answer it incorrectly and move on. But guess what, it isn't as easy as it looks!

Question: If m and n are positive integers such that m > n, what is the remainder when m^2 – n^2 is divided by 21?

Statement 1: The remainder when (m + n) is divided by 7 is 1.

Statement 2: The remainder when (m – n) is divided by 3 is 1.

First let's give you the **incorrect** solution provided by many.

Question: What is the remainder when (m^2 – n^2) is divided by 21?

Statement 1: The remainder when (m + n) is divided by 7 is 1.

(m + n) = 7a + 1

Statement 2: The remainder when (m – n) is divided by 3 is 1.

(m – n) = 3b + 1

Therefore, remainder of product (m^2 – n^2) = (m + n)*(m – n) = (7a + 1)(3b + 1) when it is divided by 21 is 1.

Answer (C)

This would have been correct had the statements been:

Statement 1: The remainder when (m + n) is divided by 21 is 1.

Statement 2: The remainder when (m – n) is divided by 21 is 1.

Statement 1: (m + n) = 21a + 1

Statement 2: (m – n) = 21b + 1

(m^2 – n^2) = (m + n)*(m – n) = (21a + 1)*(21b + 1) = 21*21ab + 21a + 21b + 1

Here, every term is divisible by 21 except the last term 1. So when we divide (m^2 – n^2) by 21, the remainder will be 1.

But let's go back to our original question. If you solved it the way given above and got the answer as (C), you are not the only one who jumped the gun. Many people end up doing just that. But here is the correct solution:

The statements given are:

Statement 1: The remainder when (m + n) is divided by 7 is 1.

(m + n) = 7a + 1

Statement 2: The remainder when (m – n) is divided by 3 is 1.

(m – n) = 3b + 1

This gives us (m^2 – n^2) = (m + n)*(m – n) = (7a + 1)(3b + 1) = 21ab + 7a + 3b + 1

Here only the first term is divisible by 21. We have no clue about the other terms. We cannot say that 7a is divisible by 21. It may or may not be depending on the value of a. Similarly, 3b may or may not be divisible by 21 depending on the value of b. So how can we say here that the remainder must be 1? We cannot. We do not know what the remainder will be in this case even with both statements together.

Say, if a = 1 and b = 1,

m^2 – n^2 = 21*1*1 + 7*1 + 3*1 + 1 = 21 + 11

The remainder when you divide m^2 – n^2 by 21 will be 11.

Say, if a = 2 and b = 1,

m^2 – n^2 = 21*2*1 + 7*2 + 3*1 + 1 = 21*2 + 18
The remainder when you divide m^2 – n^2 by 21 will be 18.
Hence, both statements together are not sufficient to answer the question.
Answer (E)

35.Intelligent Guessing

We often tell you that if you are short on time, you can guess intelligently on a few questions and move on. Today we will discuss what we mean by "intelligent guessing". There are many techniques – most of them involving your reasoning skills to eliminate some options and hence generating a higher probability of an accurate guess. Let's look at one such method to get values in the ballpark.

A few months back, we had discussed a 700 level 'Races' question.

Question 1: A and B run a race of 2000 m. First, A gives B a head start of 200 m and beats him by 30 seconds. Next, A gives B a head start of 3 mins and is beaten by 1000 m. Find the time in minutes in which A and B can run the race separately.

(A) 8, 10
(B) 4, 5
(C) 5, 9
(D) 6, 9
(E) 7, 10

Check out its complete solution here.

Now, what if we had only 30 seconds to guess on it and move on? Then we could have easily guessed (B) here and moved on. Actually, the question implies that the only possible options are those in which the time taken by B is somewhere between 3 mins and 6 mins (excluding) – we would guess 4 mins or 5 mins. Since only option (B) has time taken by (B) as 5 mins, that must be the answer – no chances of error here – perfect! Had there been 2 options with 4 mins/5 mins, we would have increased the probability of getting the correct answer to 50% from a mere 20% within 30 seconds.

Now you are probably curious as to how we got the 3 min to 6 min range. Here is the logic:

Read one sentence of the question at a time -

"*A and B run a race of 2000 m. First, A gives B a head start of 200 m and beats him by 30 seconds.*"

So first, A gives B a head start of 1/10th of the race but still beats him. This means B is certainly quite a bit slower than A. This should run through your mind on reading this sentence.

"*Next, A gives B a head start of 3 mins and is beaten by 1000 m.*"

Next, A gives B a head start of 3 mins and B beats him by 1000 m i.e. half of the race. What does this imply? It implies that B ran more than half the race in 3 mins. To understand this, say B covers x meters in 3 mins. Once A, who is faster, starts running, he starts reducing the distance between them since he is covering more distance than B every second. At the end, the distance between them is still 1000 m. This means the initial distance that B created between them by running for 3 mins was certainly more than 1000 m (This was intuitively shown in the diagram in this post). Since B covered more than 1000 m in 3 mins, he would have taken less than 6 mins to cover the length of the race i.e. 2000 m. A must be even faster and hence would take even lesser time.

Only option (B) has time taken by B as 5 mins (less than 6 mins) and hence satisfies our range! So the answer has to be (B).

Let's try the same technique on another question.

Question 2: If 12 men and 16 women can do a piece of work in 5 days and 13 men and 24 women can do it in 4 days, how long will 7 men and 10 women take to do it?

(A) 4.2 days
(B) 6.8 days

(C) 8.3 days
(D) 9.8 days
(E) 10.2 days

Solution: If we try to use algebra here, the calculations involved will be quite complicated. The options are not very close together so we can try to get a ballpark value and move forward. Let's take each sentence at a time:

"*If 12 men and 16 women can do a piece of work in 5 days*"

Say rate of work of each man is M and that of each woman is W. This statement gives us that

12M + 16W = 1/5 (Combined rate done per day)

In lowest terms, it is 3M + 4W = 1/20

"*13 men and 24 women can do it in 4 days*,"

This gives us 13M + 24W = 1/4

"*how long will 7 men and 10 women take to do it?"*

Required: 7M + 10W = ?

Solving the two equations above will be tedious so let's estimate:

(3M + 4W = 1/20) * 6 gives 6M + 8W = 1/10

So 6 men and 8 women working together will take 10 days. Hence, 7 men and 10 women will certainly take fewer than 10 days.

(13M + 24W = 1/4) / 2 gives 6.5M + 12W = 1/8

So 7 men and 10 women might take about 8 or perhaps a little bit more than 8 days to complete the work. There is only 0.5 additional man (hypothetically) but 2 fewer women to complete it. So we would guess that the number of days would lie between 8 to 10 and closer to 8 days.

Answer (C) fits.

Note that it seems like there are many equations here but all you have actually done is made two equations. Once you write them down, you don't even need to actually multiply them with some integer to get them close to the required equation. Just looking at the first one, you can say that 6 men and 8 women will take 10 days. It takes but a couple of seconds to arrive at these conclusions.

36.Pre-thinking in Quant Questions

We all know about the role of pre-thinking in Critical Reasoning and how anticipating the answer can be supremely beneficial in not just the physical aspect of saving time in analyzing options but also the psychological aspect of promoting our self-confidence – we were thinking that the answer should look like this and that is exactly what we found! Pre-thinking puts us in the driver's seat and we feel energized without consuming any red bull!

The exciting thing is that pre-thinking is useful in Quant too. If you take a step back to review what the question asks and think about what you are going to do and what you expect to get, it is highly likely that you will not get distracted mid-way during your solution. Let's show you with the help of an example:

Question: Superfast train A leaves Newcastle for Birmingham at 3 PM and travels at the constant speed of 100 km/hr. An hour later, it passes superfast train B, which is making the trip from Birmingham to Newcastle on the same route at a constant speed. If train B left Birmingham at 3:50 PM and if the sum of the total travel time of the two trains is 2 hours, at what time did train B arrive at Newcastle?

Statement I: Train B arrived at Newcastle before train A arrived at Birmingham.

Statement II: The distance between Newcastle and Birmingham is greater than 140 km.

Following are the things that would ideally constitute pre-thinking on this question:

- Quite a bit of data is given in the question stem with some speed and time taken.
- Distance traveled by both the trains is the same since they travel along the same route.

- We could possibly make an equation by equating the two distances and come up with multiple answers for the time at which train B arrived at Newcastle.
- The statements do not provide any concrete data. We cannot make any equation using them but they might help us choose one of the answers we get from the equation of the question stem.

Mind you, the thinking about the statements helping us to arrive at the answer is just speculation. The answer may well be (E). But all we wanted to do at this point was find a direction.

Train A starts from Newcastle toward Birmingham at 3:00 and meets train B at 4:00. Train B starts from Birmingham toward Newcastle at 3:50 and meets train A at 4:00. Let x be the distance from Birmingham to the meeting point.

Speed of train A = 100 km/hr

Speed of train B = Distance/Time = x/(10 min) = x/(1/6) km/hr = 6x km/hr (converted min to hour)

If we get the value of x, we get the value of speed of train B and that tells us the time it takes to travel from the meeting point to Newcastle (a distance of 100 km). So all we need to figure out is whether the statements can give us a unique value of x.

By 4:00, train A has already travelled for 1 hour and train B has already travelled for 10 mins i.e. 1/6 hour. Total time taken by both is 2 hrs. The remaining (5/6) hrs is the time needed by both together to reach their respective destinations.

Time taken by train A to reach Birmingham + Time taken by train B to reach Newcastle = 5/6

Distance(x)/Speed of train A + 100/Speed of train B = 5/6

$x/100 + 100/6x = 5/6$

$3x^2 – 250x + 5000 = 0$

$3x^2 – 150x – 100x + 5000 = 0$

$3x(x – 50) – 100(x – 50) = 0$

$(3x – 100)(x – 50) = 0$

x = 100/3 or 50

So speed of train B = 6x = 200 km/hr or 300 km/hr

Statement 1: Train B arrived at Newcastle before Train A arrived at Birmingham.

If x = 50, time taken by train A to reach Birmingham = 50/100 = 1/2 hour and time taken by train B to reach Newcastle = 100/300 = 1/3 hour. Train B takes lesser time so it arrives first.

If x = 33.33, time taken by train A to reach Birmingham = (100/3)/100 = 1/3 hour and time taken by train B to reach Newcastle = 100/200 = 1/2 hour. Here, train A takes lesser time so it arrives first at its destination.

Since train B arrived first, x must be 50 and train B must have taken 1/3 hour i.e. 20 mins to arrive at Newcastle. So train B must have arrived at 4:20.

This statement is sufficient alone.

Statement 2: The distance between Newcastle and Birmingham is greater than 140 km.

Total distance between Newcastle and Birmingham = (100 + x) km. x must be 50 to make total distance more than 140.

Time taken by train B must be 1/3 hr (as calculated above) and it must have arrived at 4:20.

This statement is sufficient alone.

Answer (D)

So our speculation was right. Each of the statements provided us relevant information to choose one of the two values that the quadratic gave us.

37.Advance Number properties- part-5

Today, let’s look in detail at a relation between arithmetic mean and geometric mean of two numbers. It is one of those properties which make sense the moment someone explains to us but are very hard to arrive on our own.

When two positive numbers are equal, their Arithmetic Mean = Geometric Mean = The number itself

Say, the two numbers are m and n (and are equal). Their arithmetic mean = (m+n)/2 = 2m/2 = m

Their geometric mean = $\sqrt{m*n} = \sqrt{m^2} = m$ (the numbers are positive so $|m| = m$)
We also know that Arithmetic Mean >= Geometric Mean
So when arithmetic mean is equal to geometric mean, it means the arithmetic mean is taking its minimum value. So when $(m+n)/2$ is minimum, it implies $(m+n)$ is minimum. Therefore, sum of numbers takes its minimum value when the numbers are equal.
When geometric mean is equal to arithmetic mean, it means the geometric mean is taking its maximum value. So when $\sqrt{m*n}$ is maximum, it means $m*n$ is maximum. Therefore, product of numbers takes its maximum value when the numbers are equal.
Let's see how to solve a difficult question using this concept.
Question: If x and y are positive, is $x^2 + y^2 > 100$?
Statement 1: $2xy < 100$
Statement 2: $(x + y)^2 > 200$
Solution:
We need to find whether $x^2 + y^2$ must be greater than 100.
Statement 1: $2xy < 100$
Plug in some easy values to see that this is not sufficient alone.
If $x = 0$ and $y = 0$, $2xy < 100$ and $x^2 + y^2 < 100$
If $x = 40$ and $y = 1$, $2xy < 100$ but $x^2 + y^2 > 100$
So $x^2 + y^2$ may be less than or greater than 100.
Statement 2: $(x + y)^2 > 200$
There are two ways to deal with this statement. One is the algebra way which is easier to understand but far less intuitive. Another is using the concept we discussed above. Let's look at both:
<u>Algebra solution</u>:
We know that $(x - y)^2 >= 0$ because a square is never negative.
So $x^2 + y^2 - 2xy >= 0$
$x^2 + y^2 >= 2xy$
This will be true for all values of x and y.
Now, statement 2 gives us $x^2 + y^2 + 2xy > 200$. The left hand side is greater than 200. If on the left we substitute 2xy with $(x^2 + y^2)$, the left hand side will either become greater than or same as before. So in any case, the left hand side will remain greater than 200.
$x^2 + y^2 + (x^2 + y^2) > 200$
$2(x^2 + y^2) > 200$
$x^2 + y^2 > 100$
This statement alone is sufficient to say that $x^2 + y^2$ will be greater than 100. But, we agree that the first step where we start with $(x - y)^2$ is not intuitive. It may not hit you at all. Hence, here is another way to analyze this statement.
<u>Logical solution</u>:
Let's try to find the minimum value of $x^2 + y^2$. It will take minimum value when $x^2 = y^2$ i.e. when $x = y$ (x and y are both positive)
We are given that $(x+y)^2 > 200$
$(x+x)^2 > 200$
$x > \sqrt{50}$
So $x^2 + y^2$ will take a value greater than $[\sqrt{50}]^2 + [\sqrt{50}]^2 = 100$.
So in any case, $x^2 + y^2$ will be greater than 100. This statement alone is sufficient to answer the question.
Answer (B)

38. Using Symmetry in Probability

We know that Combinatorics and Probability are tricky topics. It is easy to misinterpret questions of these topics and get the incorrect answer – which, unfortunately, we often find in the options, giving us a false sense of accomplishment.
In many questions, we need to account for different cases one by one but we don't really see such questions on the GMAT since we have limited time. Also, we don't tire of repeating this again and again – GMAT questions are more reasoning based than calculation intensive. Usually, there will be an intellectual method to solve every GMAT question – a method that will help you solve it in seconds.
We have discussed using symmetry in Combinatorics before. It can be used in many questions though most people don't realize that. In our ongoing endeavor to expose you to intellectual methods, here we present how most people tackle a question and how you can tackle it instead to be in the top 1%ile.
Question: Let S be the set of permutations of the sequence 2, 3, 4, 5, 6 for which the first term is not 2. A permutation is chosen randomly from S. The probability that the second term is 5 is given by a/b (in lowest terms). What is a+b?
(A) 5
(B) 6
(C) 11
(D) 16
(E) 19
Solution:
Most Common Solution:
What are the permutations of sequence S? They are the different ways in which we can arrange the elements of S. For example, 3, 2, 4, 5, 6 or 4, 2, 3, 6, 5 or 6, 3, 4, 5, 2 etc
In how many different ways can we make the sequence? The first element can be chosen in 4 ways – one of 3, 4, 5 and 6. (You are given that 2 cannot be the first element).
The second element can be chosen in 4 ways (2 and the leftover 3 numbers).
The third element can be chosen in 3 ways.
The fourth element can be chosen in 2 ways.
And finally there will be only 1 element left for the last spot.
Number of ways of making set S = 4*4*3*2*1 = 96
In how many of these sets will 5 be in the second spot?
If 5 is reserved for the second spot, there are only 3 ways of filling the first spot (3 or 4 or 6).
The second spot has to be taken by 5.
The third element will be chosen in 3 ways (ignoring 5 and the first spot)
The fourth element can be chosen in 2 ways.
And finally there will be only 1 element left for the last spot.
Number of favorable cases = 3*1*3*2*1 = 18
Required Probability = Favorable Cases/Total Cases = 18/96 = 3/16 = a/b
a+b = 3 + 16 = 19
Answer (E)
Intellectual Approach:
Use a bit of logic of symmetry to solve this question without any calculations.
Set S would include all such sequences as 3, 2, 4, 5, 6 or 4, 2, 3, 6, 5 or 6, 3, 4, 5, 2 etc – starting with 3, with 4, with 5 or with 6 with equal probability.
By symmetry, note that 1/4th of them will start with 5 – which we need to ignore – so we are left with the rest of the 3/4th sequences.

Now, in these 3/4th sequences which start with either 3 or 4 or 6, 5 could occupy any one of the 4 positions – second, third, fourth or fifth with equal probability. So we need 1/4th of these sequences i.e. only those sequences in which 5 is in the second spot.
Probability that 5 is the second element of the sequence = (3/4)*(1/4) = 3/16
Therefore, a+b = 3+16 = 19
Answer (E)

39. 4 Average Formula you need to know for GMAT:

Many people have asked us to clear the confusion surrounding the various formulas of average speed. We will start with the bottom line – There is a single versatile formula for ALL average speed questions and that is

Average Speed = Total Distance/Total Time

No matter which formula you choose to use, it will always boil down to this one. Keeping this in mind, let's discuss the various formulas we come across:

1. **Average Speed = (a + b)/2**

Applicable when one travels at speed a for half the time and speed b for other half of the time. In this case, average speed is the arithmetic mean of the two speeds.

2. **Average Speed = 2ab/(a + b)**

Applicable when one travels at speed a for half the distance and speed b for other half of the distance. In this case, average speed is the harmonic mean of the two speeds. On similar lines, you can modify this formula for one-third distance.

3. **Average Speed = 3abc/(ab + bc + ca)**

Applicable when one travels at speed a for one-third of the distance, at speed b for another one-third of the distance and speed c for rest of the one-third of the distance.

Note that the generic Harmonic mean formula for n numbers is

Harmonic Mean = n/(1/a + 1/b + 1/c + …)

4. You can also use weighted averages. Note that in case of average speed, the weight is always 'time'. So in case you are given the average speed, you can find the ratio of time as

t1/t2 = (a – Avg)/(Avg – b)

As you already know, this is just our weighted average formula.

Now, let's look at some simple questions where you can use these formulas.

Question 1: Myra drove at an average speed of 30 miles per hour for T hours and then at an average speed of 60 miles/hr for the next T hours. If she made no stops during the trip and reached her destination in 2T hours, what was her average speed in miles per hour for the entire trip?

(A) 40

(B) 45

(C) 48

(D) 50

(E) 55

Solution: Here, time for which Myra traveled at the two speeds is same.

Average Speed = (a + b)/2 = (30 + 60)/2 = 45 miles per hour

Answer (B)

Question 2: Myra drove at an average speed of 30 miles per hour for the first 30 miles of a trip & then at an average speed of 60 miles/hr for the remaining 30 miles of the trip. If she made no stops during the trip what was her average speed in miles/hr for the entire trip?

(A) 35

(B) 40

(C) 45

(D) 50

(E) 55

Solution: Here, distance for which Myra traveled at the two speeds is same.

Average Speed = 2ab/(a+b) = 2*30*60/(30 + 60) = 40 mph

Answer (B)

Question 3: Myra drove at an average speed of 30 miles per hour for the first 30 miles of a trip, at an average speed of 60 miles per hour for the next 30 miles and at a average speed of 90 miles/hr for the remaining 30 miles of the trip. If she made no stops during the trip, Myra's average speed in miles/hr for the entire trip was closest to

(A) 35

(B) 40

(C) 45

(D) 50

(E) 55

Solution: Here, Myra traveled at three speeds for one-third distance each.

Average Speed = 3abc/(ab + bc + ca) = 3*30*60*90/(30*60 + 60*90 + 30*90)

Average Speed = 3*2*90/(2 + 6 + 3) = 540/11

This is a bit less than 50 so answer (D).

Quarter Wit_Quarter Wisdom- Part-3

Question 4: Myra drove at an average speed of 30 miles per hour for some time and then at an average speed of 60 miles/hr for the rest of the journey. If she made no stops during the trip and her average speed for the entire journey was 50 miles per hour, for what fraction of the total time did she drive at 30 miles/hour?

(A) 1/5

(B) 1/3

(C) 2/5

(D) 2/3

(E) 3/5

Solution: We know the average speed and must find the fraction of time taken at a particular speed.

t1/t2 = (A2 – Aavg)/(Aavg – A1)

t1/t2 = (60 – 50)/(50 – 30) = 1/2

So out of a total of 3 parts of the journey time, she drove at 30 mph for 1 part and at 60 mph for 2 parts of the time. Fraction of the total time for which she drove at 30 mph is 1/3.

Answer (B)

Hope this sorts out some of your average speed formula confusion.