e-GMAT Number Properties Knockout Challenge

Summary

e-GMAT Number Properties Knockout Challenge tests your clarity of the following concepts:

a) Even/Odd
b) Primes
c) LCM-GCD

It also tests your clarity of thought which you need to solve 700+ level questions. Given is the summary and details of the challenge. Try it out, if you haven't till now.

Best of Luck!!

Question No:	Concept
Q1	Primes, Even/Odd
Q2	Even/Odd
Q3	Primes, Even/Odd
Q4	LCM-GCD, Primes
Q5	LCM-GCD
Q6	Primes
Q7	Primes, Even/Odd
Q8	Primes
Q9	Primes
Q10	Even/Odd

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Takeaways

Even/Odd

1. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even/odd combinations.
2. Know the properties of even/odd combinations to save the time spent deriving them in the test
3. The even/odd nature of some expressions can be determined without knowing the exact even/odd nature of the variables of the expressions by using the even/odd combination property.

Primes

1. The number of factors of a perfect square would always be odd
2. There is only 1 even prime number i.e. 2.
3. If 2 is a prime factor of the number, the number would be even else it would be odd.
4. If the total number of factors of a number is given to be T, evaluate the possible ways in which T can be expressed as a product of 2 or more numbers.
5. The prime factors of the GCD of a set of numbers would also be the prime factors of the numbers themselves.
6. Prime factorize a number to understand the ways in which a number can be represented.

LCM-GCD

1. Familiarize yourself with all the names by which the test makers can call the GCD and the LCM. For example,
   a) GCD is also known as the HCF
   b) GCD can also be described as 'the largest number which divides all the numbers of a set'
   c) LCM of a set of numbers can also be described as 'the lowest number that has all the numbers of that set as it factors'.

2. If a set contains the factorials of different positive integers, the GCD of the set will be equal to the factorial of the smallest integer

Next Steps after trying these questions

Questions

Q1: Is the product of two integers A and B odd?

(1) A is the number of factors of N, where N is a perfect square and $B = A^3 - 1$
(2) A is a product of two consecutive prime numbers and when $B + 3^{11}$ is added to A, the sum is an odd number.

View Solution

Q2: If a, b and n are positive integers such that $n = 3a - b^3$, is $n^2 + 3$ divisible by 2?

(1) $a^2 - 4b^3 - 5 = 0$
(2) $3b^3 - a^2 + 6 = 0$

View Solution

Q3: If positive integer N = A – B, where A is a positive integer and B is a prime number, is N odd?

(1) B and X are the only prime factors of A, and B – X = 4
(2) A is divisible by 9 numbers in total, one of which is B^2

View Solution

Q4: Two positive integers a and b are divisible by 5, which is their largest common factor. What is the value of a and b?

(1) The lowest number that has both integers a and b as its factors is the product of one of the integers and the greatest common divisor of the two integers.
(2) The smaller integer is divisible by 4 numbers and has the smallest odd prime number as its factor.

View Solution

Q5: For any integer n greater than 1, factorial denotes the product of all the integers from 1 to n, inclusive. It's given that a and b are two positive integers such that b > a. What is the total number of factors of the largest number that divides the factorials of both a and b?

(1) a is the greatest integer for which 3^a is a factor of product of integers from 1 to 20, inclusive.
(2) b is the largest possible number that divides positive integer n, where n^3 is divisible by 96

View Solution

Q6: The Prime Sum of an integer X greater than 1 is the sum of all prime factors of X including repetitions. A positive integer n can be expressed as a product of two natural numbers at least one of which is even. What is the Prime Sum of n?

(1) n has only 3 prime factors and the smallest prime factor of n is raised to a power equal to the next biggest prime factor of n.
(2) The prime factors of n are consecutive and the total number of divisors of n is 16

View Solution

Q7: Is the product of integers M and N even?

(1) N can be expressed as a difference of squares of two consecutive prime numbers at least one of which is odd. M can be expressed as a product of two natural numbers P and Q, where 2P + 1= Q.
(1) N can be expressed as a difference of squares of two consecutive prime numbers which lie at a distance of 2 units. M is the sum of all the numbers from 1 to Z where (Z+1) is a multiple of 4.

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Q8: P and Q are prime numbers less than 70. What is the units digit of P*Q?

(1) Units digit of $(P^{4k+2} - Q)$ is equal to 7, where k is a positive integer.
(2) Units digit of the expression $[PQ + Q*(Q+1) - Q^2]$ is a perfect cube.

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Q9: Abe, Beth, Carl and Duncan are four siblings, among which Abe and Carl are twins and Beth and Duncan are also twins. When the present ages of the four siblings are multiplied, the product is 900. If Beth is older than Abe, what is the age of Duncan? Assume the ages of all siblings to be integers

(1) The difference between Beth's age and Abe's age is a prime number.
(2) If Carl had been born four years earlier, the difference between Duncan's age and Carl's age would have been a prime number

View Solution

Q10: If x, y and z are positive integers such that $x^4 y^3 = z^2$, is $x^9 - y^6$ odd?

(1) $\frac{x^4 y^3}{x^2 + y^2}$ can be written in the form 4k + 3, where k is a positive integer.
(2) z = x + y

View Solution

Correct Answers

Ques. No.	Correct Answer	Solution Link
1	**A**	View Solution
2	**B**	View Solution
3	**A**	View Solution
4	**E**	View Solution
5	**A**	View Solution
6	**C**	View Solution
7	**B**	View Solution
8	**E**	View Solution
9	**C**	View Solution
10	**D**	View Solution

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Next Steps after trying these questions

Detailed Solutions & Takeaways

Question 1

Q1: Is the product of two integers A and B odd?

(1) A is the number of factors of N, where N is a perfect square and B = A^3 -1
(2) A is a product of two consecutive prime numbers and when $B + 3^{11}$ is added to A, the sum is an odd number.

Correct Answer

A

Solution

Step-I: Understand the question

The question tells us about two integers A & B and asks us if their product is odd.

Step-II: Draw Inferences from the question statement

To find if the product of two numbers is even/odd, we need to establish if either of the number is even or not. In case either of the number is even, the product would be even else the product would be odd.

Step-III: Analyze Statement 1 independently

Statement 1tells us that A is the number of factors of a perfect square, since the no. of factors of a perfect square is odd, we can deduce that A is odd. Since A is not even, to find the nature of product of A & B, we need to find if B is odd/even.

It's given that $B = A^3 - 1$, since we have established that A is odd, A^3 will also be odd. Subtracting 1 from an odd number will give us an even number, hence we can deduce that B is even.

Since we know now that B is even it is sufficient for us to deduce that the product of A & B would be even.

Thus statement 1 is sufficient to get the answer.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that A is a product of two consecutive prime numbers, so A may be even if one of the prime number is 2 and may be odd if none of the prime number is 2. So, we can't establish with certainty the nature of A.

The statement also tells us that $A + B + 3^{11} = odd$, since 3 is an odd number, 3^{11} would also be odd and subtracting an odd from an odd number would give us an even number. So, we can rewrite

A+ B = even which would imply that either both A, B are even or both are odd. In both the cases the nature of product of A & B can't be established with certainty, it will be even if both A & B are even and will be odd if both A & B are odd.

So, statement 2 is not sufficient to answer the question.

Step-V: Analyze both statements together (if needed)

Since we have a unique answer from Statement 1we don't need to be combine Statements 1 & 2

Hence, the correct answer is **Option A**

Takeaways

1. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even/odd combinations.
2. The number of factors of a perfect square would always be odd.
3. Know the properties of even/odd combinations to save the time spent deriving them in the test

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Question 2

Q2: If a, b and n are positive integers such that $n = 3a - b^3$, is $n^2 + 3$ divisible by 2?

(1) $a^2 - 4b^3 - 5 = 0$
(2) $3b^3 - a^2 + 6 = 0$

Correct Answer

B

Solution

Step-I: Understand the question

We are given that a, b, n >0 and are integers. Also $n = 3a - b^3$ and we are asked to find if $(n^2 + 3)$ is divisible by 2.

Step-II: Draw Inferences from the question statement

Let's start from our expression i.e. $(n^2 + 3)$, this expression is divisible by 2 only if it's even, since 3 is odd, for $n^2 + 3$ to be even n^2 has to be odd (*as odd + odd =even*) and n^2 can be odd only when n is odd.

Now, we know that $n = 3a - b^3$, for n to be odd, one of the 3a or b^3 has to be odd and other has to be even as the difference of an even and an odd number will always be odd. The even/odd nature of 3a would depend on the nature of a and similarly the even odd nature of b^3 would depend on the even/odd nature of b. So, if we can establish that the even/odd nature of a and b are either similar or opposite, we will find our answer.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that $a^2 - 4b^3 = 5$, it tells us that difference of two numbers is odd. Since $4b^3$ would always be even, for the difference of a^2 and $4b^3$ to be odd, a^2 would have to be odd. For a^2 to be odd, a has to be odd. But St-I does not tell us anything about the even/odd nature of b.

So, statement 1 alone is insufficient.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that $a^2 - 3b^3 = 6$, it tells us that difference of two numbers is even. This is only possible in two cases:

a) When both a^2 and $3b^3$ are odd, for this to happen both a and b have to be odd.
   or
b) When both a^2 and $3b^3$ are even, for this to happen both a and b have to be even

But, we know that for n to be odd, both a and b have to of opposite even/odd natures. We see that in Statement 2, in both the cases, a and b are of the same nature; thus in both the cases, n would be even.

Hence, statement 2 is sufficient to answer our question.

Step-V: Analyze both statements together (if needed)

Since we have a unique answer from Statement 2 alone, we don't need to combine the information from statements 1 and 2. Thus, the answer is **Option B**.

Takeaways

1. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even/odd combinations.
2. Know the properties of even/odd combinations to save the time spent deriving them in the test
3. The even/odd nature of some expressions can be determined without knowing the exact even/odd nature of the variables of the expressions by using the even/odd combination property

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Question 3

Q3: If positive integer N = A – B, where A is a positive integer and B is a prime number, is N odd?

(1) B and X are the only prime factors of A, and B – X = 4
(2) A is divisible by 9 numbers in total, one of which is B^2

Correct Answer

A

Solution

Step-I: Understand the question

The statement tells us about a positive integer N = A – B and we are told that A is a positive integer and B is a prime number. We are asked to find if N is odd

Step-II: Draw Inferences from the question statement from the question statement

The expression N is expressed as a difference of two numbers. For N to be odd, the nature of these two numbers would have to be opposite i.e. one has to be odd and other has to be even. We are given that B is prime number; we know that except for 2, all the prime numbers are odd. So, if we can establish if B is greater than 2, we would be able to say with certainty that B is odd.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that B and X are prime numbers such that B-X = 4. The difference of two prime numbers is even, which would mean that either both are odd or both are even. Since there is only even prime number possible (i.e. 2), we can say with certainty that both B and X are odd.

Since A has only B and X as its prime factors, this would imply that X is a product of two odd numbers. Thus A would also be odd.

We now know the even/odd nature of both A and B. Thus we can determine with certainty the even/odd nature of N.

Hence, statement 1 is sufficient to answer the question.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that A has 9 factors. Since 9 can factorized only as (9*1) or (3*3), this would imply that A can be written as:

A = P_1^8 or $P_1^2*P_2^2$, where P_1*P_2 are the prime factors of A.

Since we are told that A is divisible by B^2, this would mean that P_1 = B. Please note here that B^2 is a divisor of A in both the cases where A = P_1^8 or $P_1^2*P_2^2$. So, we can't say with certainty that A can be expressed in one of the ways. Let's evaluate both the cases now:

- Case-I: A= B^8
  In this case A will have the same even/odd nature as that of B. We know that the difference of two even numbers or two odd numbers would always be even. Hence, we can say that N will be even.

- Case-II: $A= B^2*P_2^2$
  In this case, if B is an even prime number (i.e. 2), then A would also be even and subsequently N would also be even. However if B is odd, the nature of A would depend on the nature of P_2.

  - ✓ If P_2 is even, then A would be even and N would be odd (since B is odd)
  - ✓ If P_2 is odd, then A would be odd and N would be even (since B is odd)

  So, we see here that we can't predict with certainty the exact even/odd nature of N.

Hence, statement 2 is insufficient to answer the question.

Step-V: Analyze both statements together (if needed)

Since we have received our unique answer from statement 1, we don't need to combine the inferences from statements 1 & 2.

Hence, the correct Answer is **Option A.**

Takeaways

1. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even/odd combinations.
2. Know the properties of even/odd combinations to save the time spent deriving them in the test
3. Remember that there is only 1 even prime number i.e. 2.
4. If 2 is a prime factor of the number, the number would be even else it would be odd.
5. If the total number of factors of a number is given to be T, evaluate the possible ways in which T can be expressed as a product of 2 or more numbers.

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Question 4

Q4: Two positive integers a and b are divisible by 5, which is their largest common factor. What is the value of a and b?

(1) The lowest number that has both integers a and b as its factors is the product of one of the integers and the greatest common divisor of the two integers.
(2) The smaller integer is divisible by 4 numbers and has the smallest odd prime number as its factor

Correct Answer

E

Solution

Step-I: Understand the question

We are given two positive integers a and b such that their highest common factor is 5. We are asked to find the values of a and b.

Step-II: Draw Inferences from the question statement

Since the highest common factor of a and b is 5, it implies that the GCD of (a, b) = 5. So, we can deduce that both a and b definitely have 5 as one of its prime factors. Let's use the information given in the statements to see if we can find out the values of a and b.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that the LCM (a, b) is the product of one of the integers of a and b with GCD (a, b) i.e. 5. Since we know that the product of the numbers is equal to the product of their LCM & GCD, we can write

LCM * GCD = a * b

⇨ 5 * (a or b) * 5 = a * b

Simplifying this statement would give us the value of one of the number as 25. Since we do not have any information about the value of the second number, statement 1 is insufficient.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that the smaller of the two integers a and b is divisible by 4 factors and has the smallest odd prime number (i.e. 3) as its factor. So, the integer can be represented as:

$x * y$ or x^3.

Since we know that both the integers have 5 as one of its prime factors (since 5 is the GCD), we can represent the integer as product of two prime factors i.e. x * y = 3 * 5= 15.

Now, we know that one of the integers has the value of 15, but we are not given any information about the value of the second integer.

Hence, statement 2 is insufficient to answer the question.

Step-V: Analyze both statements together (if needed)

Statement 1 tells us that one of the integers of a and b has the value as 25. Statement 2 tells us that the other integer has the value as 15. This would mean that:

Either a= 25 and b =15 or

a = 15 and b =25.

Hence, we can't say with certainty the specific values of a and b.

Thus, combining statements 1 & 2 is also not sufficient to answer the question.

Hence, the answer is **Option E**.

Takeaways

1. The prime factors of the GCD of a set of numbers would also be the prime factors of the numbers themselves.
2. Familiarize yourself with all the names by which the test makers can call the GCD and the LCM. For example,
    - GCD is also known as the HCF
    - GCD can also be described as 'the largest number which divides all the numbers of a set'
    - LCM of a set of numbers can also be described as 'the lowest number that has all the numbers of that set as it factors'
3. Read the question carefully.

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Question 5

Q5: For any integer n greater than 1, factorial denotes the product of all the integers from 1 to n, inclusive. It's given that a and b are two positive integers such that b > a. What is the total number of factors of the largest number that divides the factorials of both a and b?

(1) a is the greatest integer for which 3^a is a factor of product of integers from 1 to 20, inclusive.
(2) b is the largest possible number that divides positive integer n, where n^3 is divisible by 96

Correct Answer

A

Solution

Step-I: Understand the question

We are given two positive integers a and b such that b>a. We are asked to find the total number of factors of the largest number which divides the factorials of both a and b?

Step-II: Draw Inferences from the question statement

Since factorial is the product of all integers from 1 to n inclusive:

i. factorial of b would consist of product of all the numbers from 1 to b
ii. factorial of a would consist of product of all the numbers from 1 to a

As b > a, this would imply that factorial of b would consist of all the numbers present in factorial of a. For example, factorial of 30 would consist of all the numbers present in the factorial of 20.

So, the largest number which divides the factorial of both b and a, i.e. the GCD of factorial of b and a, would be the factorial of a itself. So, if we can calculate the value of a, we would get to our answer.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that a is the greatest integer for which 3^a is a factor of factorial of 20. Since we can calculate the number of times 3 comes as a factor of numbers between 1 to 20, we can find the value of a.

Thus statement 1 is sufficient to answer the question.

Please note that we do not need to actually calculate the value of a. Just the knowledge, that we can calculate the unique value of a is sufficient for us to get to our answer.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that b is the largest possible number that divides n, where n^3 is divisible by 96.

Note here that the statement talks only about b and nothing about a. Since we do not have any relation between b and a which would give us the value of a, if we find b, we can say with certainty that this statement is insufficient to answer the question.

Again, note here that we did not solve the statement as we could infer that it's not going to give us the value of a, which is our requirement.

Step-V: Analyze both statements together (if needed)

Since we have received our unique answer from statement 1, we don't need to combine the inferences from statements 1 & 2.

Hence, the correct answer is **Option A**

Takeaways

1. Familiarize yourself with all the names by which the test makers can call the GCD and the LCM. For example,
   - GCD is also known as the HCF
   - GCD can also be described as 'the largest number which divides all the numbers of a set'
   - LCM of a set of numbers can also be described as 'the lowest number that has all the numbers of that set as it factors
2. If a set contains the factorials of different positive integers, the GCD of the set will be equal to the factorial of the smallest integer

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Question 6

Q6: The Prime Sum of an integer X greater than 1 is the sum of all prime factors of X including repetitions. A positive integer n can be expressed as a product of two natural numbers at least one of which is even. What is the Prime Sum of n?

(1) n has only 3 prime factors and the smallest prime factor of n is raised to a power equal to the next biggest prime factor of n.
(2) The prime factors of n are consecutive and the total number of divisors of n is 16

Correct Answer

C

Solution

Step-I: Understand the question

We are given that a positive integer n can be expressed as a product of two natural numbers, at least one of which is even. We are asked to find the Prime Sum of n.

Step-II: Draw Inferences from the question statement

For finding the prime sum of n, we need to know all the prime factors of n along with the no. of times they are repeated in the prime factorization of n.

For example, if $n= P_1^a*P_2^b$, we need to find the values of P_1, P_2 along with the values of a and b to calculate the prime sum of the number.

We are told that n has an even number as it factor, which implies that n is even. Hence, we can say with certainty that 2 is one of the prime factors of n. Now, we need to find the other prime factors of n along with the no. of times they are repeated.

Step-III: Analyze Statement 1 independently

We know from statement 1 that n has 3 prime factors. So, we can write n as:

$n= P_1^a* P_2^b * P_3^c$ *where P_1, P_2 and P_3 are prime numbers.*

The statement also tells us that the smallest prime factor of n, which is 2 (as 2 is a prime factor of n and there is no smaller prime number than 2) is raised to a power equal to the next biggest prime factor of n. So n can be rephrased as:

$n= 2^{P_2}* P_2^b * P_3^c$

But, we don't have any information about the values of other prime factors of n as well as about the no. of times they are repeated.

So, statement 1 is insufficient to answer the question.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that the total number of divisors of n is 16, so n can be written as:

- ✓ $n= P_1* P_2 * P_3 * P_4$ (as 16 = 2*2*2*2) or

- ✓ n= $P_1^a * P_2^b * P_3^c$ (as 16 = 4*2*2) where either of a, b or c can be equal to 3 and rest as 1 or
- ✓ n= $P_1^3 * P_2^3$ (as 16 = 4*4)

We know from the question statement that $P_1 = 2$. Since the statement tells us that the prime factors of n are consecutive, the prime factors may be:

- ✓ 2,3,5,7 or
- ✓ 2,3,5 or
- ✓ 2,3

We can only know from case III the required values of the prime factors and the number of times they are repeated. The other two cases do not provide us with a unique answer.

Thus statement 2 is insufficient to answer the question.

Step-V: Analyze both statements together (if needed)

If we combine statements-1 & 2, we can notice that n has 3 prime factors, so n can be written as:

n= $2^{P_2} * P_2^b * P_3^c$. Since the prime factors of n are consecutive that would imply $P_2 = 3$ and $P_3 = 5$.

Also, statement 1 tells us that 2 is raised to the power equal to the next biggest prime factor of n i.e. 3, so, n can be written as:

n= $2^3 * 3^1 * 5^1$

Now, we know the prime factors of n and the no. of times they are repeated, we can definitely find the prime sum of n.

Thus combination of statements 1 & 2 is sufficient to answer the question.

Hence, the correct answer is **Option C**

Takeaways

1. 2 is the smallest prime number
2. If the total number of factors of a number is given to be T, evaluate the possible ways in which T can be expressed as a product of 2 or more numbers

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Question 7

Q7: Is the product of integers M and N even?

(1) N can be expressed as a difference of squares of two consecutive prime numbers at least one of which is odd. M can be expressed as a product of two natural numbers P and Q, where 2P + 1= Q.
(2) N can be expressed as a difference of squares of two consecutive prime numbers which lie at a distance of 2 units. M is the sum of all the numbers from 1 to Z where (Z+1) is a multiple of 4.

Correct Answer

B

Solution

Step-I: Understand the question

We are given two integers M and N and are asked to find if their product is even.

Step-II: Draw Inference

The product of two numbers would be even if at least one of them is even. So, we need to find if either of M and N is even.

Step-III: Analyze Statement 1 independently

The statement tells us that M is expressed as a difference of two consecutive prime numbers, of which at least one is odd. Two cases are possible:

- We know that there is only one even prime number i.e. 2, so, if one of the prime numbers is 2, the other would be 3, which is odd. Squaring them would not change their even/odd nature. The difference of an even and an odd number would be odd, so N would be odd.

- If both the prime numbers are odd, then the difference of their squares would be even (as odd-odd= even). So, N would be even.

From the above two cases, we can't say with certainty whether N is odd or even.

The statement also tells us that M is a product of P & Q where Q= 2P + 1. We can infer from this that Q is an odd number, but we do not have any information about the even/odd nature of P. So, if P is odd, M would be odd and if P is even, M would be even.

Hence, we can't say with certainty whether M is odd or even.

Since we don't know with certainty that either of M, N is even or not, statement 1 is insufficient to answer the question.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that N can be expressed as difference of squares of two consecutive prime numbers which lie at a distance of 2 units. We know that all the prime numbers except 2 are odd. Since the next prime number after 2 is 3, we can say that 2, 3 are not the consecutive prime numbers (as they lie at a distance of 1 unit). Thus we can conclude that N can be expressed as difference of two odd prime numbers. The difference of two odd numbers will be even.

So, N would be even. Note here that we don't need to find the even/odd nature of M because irrespective of the nature of M, the product of M & N would always be even as N is even.

Hence, statement 2 is sufficient to answer the question.

Step-V: Analyze both statements together (if needed)

Since we have a unique answer from statement 2, we don't need to be combine statements 1 & 2

Hence, the correct answer is **Option B**

Takeaways

1. Know the properties of even/odd combinations to save the time spent deriving them in the test.
2. There is only 1 even prime number i.e. 2.
3. Odd/Even number raised to any power would not change its even/odd nature.

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Question 8

Q8: P and Q are prime numbers less than 70. What is the units digit of P*Q?

(1) Units digit of (P^{4k+2} – Q) is equal to 7, where k is a positive integer.
(2) Units digit of the expression [PQ + Q*(Q+1) – Q^2] is a perfect cube.

Correct Answer

E

Solution

Step-I: Understand the question

We are given two prime numbers P, Q < 70. We are asked to find the units digit of P*Q.

Step-II: Draw Inferences from the question statement

The unit's digit of the product of P&Q would depend on the units digit of P & Q. Hence, our endeavour would be to find the units digit of P & Q.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that the difference between two numbers is odd which would imply opposite even-odd natures of the numbers i.e. one is even and the other is odd. Since there is only on even number (i.e. 2), there are two possible scenarios:

- P=2, so P^{4k+2} will have the units digit as 4. As a result the units digit of Q would be 7. Hence units digit of product of P & Q would be 4

- Q=2, in such a case units digit of P^{4k+2} will be 9, which implies that units digit of P is either 7 or 3. Hence, units digit of product of P & Q would be either 4 or 6.

Since we do not have a unique answer, statement 1 is not sufficient to answer the question.

Step-IV: Analyze Statement 2 independently

The expression in statement 2 can be simplified to Q(P+1), since its unit digit is a perfect cube, the possible values are 0, 1 and 8.

If unit digit is 0,

- Units digit of P=9 (as P can be 19, 29, 59) and units digit of Q = {1, 3, 5, 7, 9} as Q can be any other possible prime number as product of 0 with any other digit will always be 0. So units digit of product of P&Q would be {9, 7, 5, 3, 1}.

If unit digit is 1,

- Units digit of P= 2 and units digit of Q =7, so the units digit of product of P&Q would be 4

If unit digit is 8,

Two cases are possible:

- Units digit of Q= 2 and units digit of P =3, so the units digit of product of P&Q would be 6
- Units digit of Q= 1 and units digit of P= 7, so the units digit of product of P&Q would be 7

Since we do not have a unique answer, statement 2 is not sufficient to answer the question

Step-V: Analyze both statements together (if needed)

Statement 1 tells us that the units digit of P & Q can be 4 or 6. Statement 2 tells us that the units digit of P & Q can be { 1,3,4,5,6,7,9}.

By combining statements 1 & 2, we still do not have a unique answer.

Thus combination of statements 1 & 2 is also insufficient to answer the question.

Hence, the correct answer is **Option E**

Takeaways

1. Know the properties of even/odd combinations to save the time spent deriving them in the test
2. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even/odd combinations
3. Know the cyclicity of the numbers to arrive at their units digit

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Question 9

Q9: Abe, Beth, Carl and Duncan are four siblings, among which Abe and Carl are twins and Beth and Duncan are also twins. When the present ages of the four siblings are multiplied, the product is 900. If Beth is older than Abe, what is the age of Duncan? Assume the ages of all siblings to be integers

(1) The difference between Beth's age and Abe's age is a prime number.
(2) If Carl had been born four years earlier, the difference between Duncan's age and Carl's age would have been a prime number

Correct Answer

C

Solution

Step-I: Understand the question

We are told about four siblings Abe, Beth, Carl and Duncan such that Abe and Carl are twins and Beth and Duncan are also twins. We are also given that the product of the present ages of the four siblings is 900. Further we are told that Beth is older than Abe and we are asked to find the age of Duncan

Step-II: Draw Inferences from the question statement

Since Abe and Carl are twins, their ages would be same, let's assume it to be x. Similarly, since Beth and Duncan are twins, their ages would be same, let's assume it to be y.

We are told that Beth is older than Abe, i.e. y > x and the product of the ages of the siblings is 900, so we can write $x^2 * y^2 = 900$.

We can observe here that 900 is written as product of two squares, since 900 can be prime factorized as 900 =$2^2 * 3^2 * 5^2$, the possible set of values of (x, y) can be:

- (1, 30) or
- (2, 15) or
- (3, 10) or
- (5,6)

Let's proceed to the solutions to see if we can get a unique value of x with this understanding.

Step-III: Analyze Statement 1 independently

Statement 1 tells us that y –x is a prime number. Let's evaluate our possible cases to see if we can find a unique value for x:

- (1, 30) –>= 29-> Prime
- (2, 15) –>= 13 -> Prime
- (3, 10) – > = 7 -> Prime
- (5,6) – > = 1 -> Not Prime

We observe here that, there are three possible values for x, hence statement 1 is not sufficient to arrive at the answer.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that had Carl been born four years earlier, the difference between Duncan's age and Carl's age would have been a prime number. Since Carl's present age is x, had he been born four years earlier, his present age would be (x +4).

The statement tells us that y – (x +4) is a prime number. Let's evaluate our possible cases to see if we can find a unique value for x.

- (1, 30) –>= 25-> Not Prime
- (2, 15) –>= 9 -> Not Prime
- (3, 10) – > = 3 -> Prime
- (5,6) – > = 3 -> Prime

We observe here that there are two possible values for x, hence statement 2 is not sufficient to arrive at the answer.

Step-V: Analyze both statements together (if needed)

Statement 1 gives us the possible values of (x, y) as (1, 30), (2, 15) and (3, 10). Statement 2 gives us the possible values of (x, y) as (3, 10) and (5,6).

Combining statements 1 & 2 give us only possible option for values of (x, y) which is (3, 10).

Thus combination of statements 1 & 2 is sufficient to answer the question.

Hence, the correct answer is **Option C**

Takeaways

1. Prime factorize a number to understand the ways in which a number can be represented

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Question 10

Q10: If x, y and z are positive integers such that $x^4 y^3 = z^2$, is $x^9 - y^6$ odd?

1. $\frac{x^4\, y^3}{x^2 + y^2}$ can be written in the form 4k + 3, where k is a positive integer.
2. z = x + y

Correct Answer

D

Solution

Step-I: Understand the question

We are given three positive integers x, y and z such that $x^4y^3 = z^2$, and we are asked to find if $x^9 - y^6$ is odd

Step-II: Draw Inferences from the question statement

For the expression $x^9 - y^6$ to be odd, both the expressions need to be of opposite even/odd nature i.e. one has to be odd and another has to be even (as even- odd = odd or odd-even = odd). Since the even/odd nature of x^9 would have the same even/odd nature as x and even/odd nature of y^6 would be the same even/odd nature as y, if we can determine the similar or opposite nature of x, y, we can determine the even/odd nature of $x^9 - y^6$.

Step-III: Analyze Statement 1 independently

We are told that $\frac{x^4\, y^3}{x^2 + y^2}$ can be written in the form 4k + 3, where k is a positive integer. We observe here that a fraction has been simplified to an odd number (as 4k is always even and even + odd = odd) which would imply that both the numerator and the denominator are either even or odd.

Hence, we can say that the product of x and y and the sum of x and y have the same even/odd nature.

This is possible only if x, y are both even. Since we have determined the similar nature of x, y, we can say with certainty that expression $x^9 - y^6$ is always even.

Hence, statement 1 is sufficient to answer the question.

Step-IV: Analyze Statement 2 independently

Statement 2 tells us that z = x + y, we know that z is expressed as a product of x and y. Since the product of x, y have the same even/odd nature as that of the sum of x and y, we can say with certainty that x, y are both even.

Hence, Statement 2 is sufficient to answer our question.

Step-V: Analyze both statements together (if needed)

Since we have a unique answer from statements 1& 2 we don't need to be combine statements 1 & 2

Hence, the correct answer is **Option D**

Takeaways

1. In even/odd questions, simplify complex expressions into simpler expressions using the properties of even-odd combinations.
2. Know the properties of even/odd combinations to save the time spent deriving them in the test
3. The even/odd nature of some expressions can be determined without knowing the exact even/odd nature of the variables of the expressions by using the even/odd combination property

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Next Steps

What Next?

We hope you enjoyed the challenge! The questions demanded both **Clarity of Concepts** and **Clarity of Thought**.

You must do a thorough review of the questions that you couldn't answer correctly. Make a note of the mistakes that you did, and devise the strategies to avoid those mistakes in the future.

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