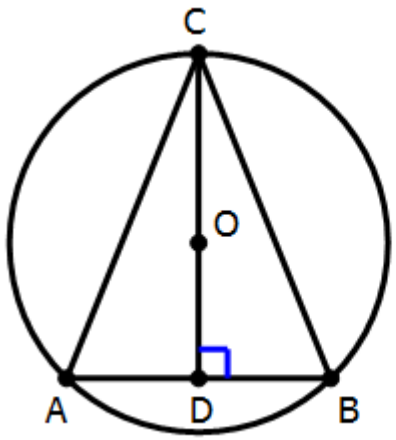


Note: Diagram not drawn to scale

In the figure above, triangle ABC is inscribed in the circle with center O, such that CD is perpendicular to AB. If the length of side AC is 5 and arc AB has a length of $\frac{10\sqrt{3}}{9}\pi$, find the perimeter of the triangle.

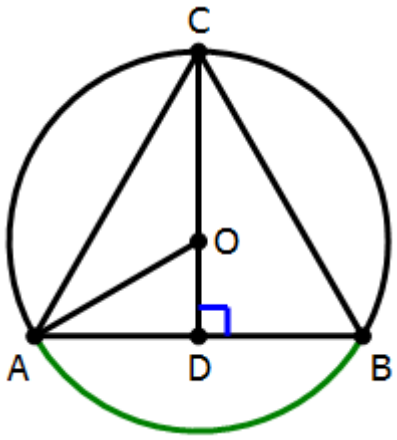
(A) 14
(B) 15
(C) 16
(D) 20
(E) none of the above

This problem is too hard for the GMAT. Solving it in a direct way would involve advanced trigonometry, much harder than the GMAT would expect. The best we can do using only GMAT-level math is to *guess* the right answer and then verify that the right answer works with the numbers given. The GMAT never expects you do to this in a geometric situation such as this.

Because we have a $\sqrt{3}$ floating around, we suspect that 30-60-90 triangles may be involved. If ACD and BCD were 30-60-90 triangles, that would make ABC an equilateral triangle. I want to emphasize: right now, this is little more than a shot-in-the-dark guess. Let's see what happens.

Suppose ABC is equilateral, so that AC = CB = AB = 5, and so that triangles ACD and BCD were 30-60-90 triangles. Then, AD = BD = 2.5 = 5/2.

Draw a radius, for example, AO.



This diagram is drawn rigorously to scale. Triangle AOD is a 30-60-90. If we were to draw a line from O to the midpoint of AC, that would create two more congruent 30-60-90 triangles. We can always subdivide a larger 30-60-90 triangle in three smaller ones, and we have to find at least one of these smaller ones when we construct the radius of a circle surrounding an equilateral.

From the 30-60-90 ratios:

$AD = \frac{5}{2}$

$OD = \frac{5\sqrt{3}}{6}$

$AO = r = \frac{5\sqrt{3}}{3}$

That last value is the radius of the circle. From this we can find the circumference:

$c = 2\pi r = \frac{10\sqrt{3}}{3}\pi$

The equilateral triangle divides the circle into three equal arcs, so arc AB is a third of the circumference.

$\text{arc } AB = \frac{10\sqrt{3}}{9}\pi$

This is the length of the arc given in the problem, so we know that we guessed correctly. Triangle ABC is equilateral, so each side is 5, and the perimeter is 15. Answer = **(B)**.

Once again, we solved this by guessing and then verifying that our guess worked. The GMAT would NOT give you a problem that only could be solved in this way.

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