

## Chapter 1: DIGITS & DECIMALS

- **Using Place Value on the GMAT:** In general, for unknown digits problems, be ready to create variables (such as  $x$ ,  $y$ , and  $z$ ) to represent the unknown digits.

$A$  and  $B$  are both two-digit numbers, with  $A > B$ . If  $A$  and  $B$  contain the same digits, but in reverse order, what integer must be a factor of  $(A - B)$ ?

- (A) 4      (B) 5      (C) 6      (D) 8      (E) 9

$$\rightarrow A - B = 10x + Y - (10y + x) = 9x - 9y = 9(x - y)$$

- **Last Digit Shortcut:** To find the units digit of a product or a sum of integers, only pay attention to the units digits of the numbers you are working with. Drop any other digits.

What is the units digit of  $(7)^2(9)^2(3)^3$ ?

- **The Heavy Division Shortcut:**

What is  $1,530,794 \div (31.49 \times 10^4)$  to the nearest whole number?

$$\rightarrow 15.3/3.1 \text{ can be rounded to } 15/3$$

If this answer is not precise enough, keep one more decimal place and do long division (eg.,  $153 \div 31 = 4.9$ ).

- **Decimal Operations:**

- $0.002 \times 1.4 = 2 \times 10^{-3} \times 14 \times 10^{-1}$
- $12.42 \div 0.3 = 124.2 \div 3$
- $\text{CBRT}(0.000027) = \text{CBRT}(27 \times 10^{-6}) = 3 \times 10^{-2}$

## Chapter 2: FRACTIONS

Fractional answers on the GMAT will always be presented in fully simplified form.

- **Fractions:** Proper Fractions are those that fall between 0 and 1. Ex:  $3/4$   
Improper Fractions are those that are greater than 1. Ex:  $6/5 \rightarrow 1 \frac{1}{5}$
- **Numerator and Denominator Rules:** Adding the same number to BOTH the numerator and the denominator brings the fraction CLOSER TO 1, regardless of the fraction's value. Ex:  $1/2 \rightarrow 1001/1002$
- **The Multiplication Shortcut:**

$$\frac{8}{15} \times \frac{35}{72} = \frac{8(35)}{15(72)} = \frac{1(7)}{3(9)} = \frac{7}{27}$$

- **Dividing Fractions:** Use the Reciprocal. Ex:  $(3/4) \div (12/3) = (3/4) \times (3/12)$
- **Comparing Fractions: Cross-Multiply.** Ex: Which fraction is greater,  $7/9$  or  $4/5$ ?  $\rightarrow 7 \times 5 < 4 \times 9$   
Cross-multiply the fractions and put each answer by the corresponding numerator (NOT denominator!)
- **Benchmark Values:**

Which is greater:  $\frac{127}{255}$  or  $\frac{162}{320}$ ?

If you recognize that 127 is less than half of 255, and 162 is more than half of 320, you will save yourself a lot of cumbersome computation.

What is  $\frac{10}{22}$  of  $\frac{5}{18}$  of 2000?

If you recognize that these fractions are very close to the Benchmark Values  $\frac{1}{2}$  and  $\frac{1}{4}$ , you can estimate  $1/8$  of 2,000, which is 250.

Try to make your rounding errors cancel by rounding some numbers up and others down.

Notice that the rounding errors compensated for each other.

$$\frac{10}{22} \approx \frac{10}{20} = \frac{1}{2} \quad \text{You decreased the denominator, so you rounded up: } \frac{10}{22} < \frac{1}{2}$$
$$\frac{5}{18} \approx \frac{5}{20} = \frac{1}{4} \quad \text{You increased the denominator, so you rounded down: } \frac{5}{18} > \frac{1}{4}$$

- **Smart Numbers: Multiples of the Denominators**

The Crandalls' hot tub is half filled. Their swimming pool, which has a capacity four times that of the tub, is filled to four-fifths of its capacity. If the hot tub is drained into the swimming pool, to what fraction of its capacity will the pool be filled?

The denominators in this problem are 2 and 5. The Smart Number is the least common denominator, which is 10. Therefore, assign the hot tub a capacity of 10 units.

- **When Not to Use Smart Numbers:** Do pick smart numbers when no amounts are given in the problem, but do not pick smart numbers when any amount or total is given!

Mark's comic book collection contains  $\frac{1}{3}$  Killer Fish comics and  $\frac{3}{8}$  Shazaam Woman comics. The remainder of his collection consists of Boom! comics. If Mark has 70 Boom! comics, how many comics does he have in his entire collection?

### Chapter 3: PERCENTS

- **The Percent Table:** A simple but useful way of structuring basic percent problems on the GMAT is by relating percents to fractions through a percent table.

Example 3: 90 is what percent of 40?

Note that the "part" is BIGGER than the "whole"

PART	90	x
WHOLE	40	100

CHANGE	4	x
ORIGINAL	80	100

- **Percent Change vs. Percent of Original:**

- **Percent change:** The price of a cup of coffee increased from 80c to 84c. By what % did the price change?  
*The price change (84 - 80 = 4c) is considered the part, while the original price (80 c) is considered the whole.*
- **Percent of Original:** what percent OF the original price is represented by the new price.
- **RULE:** If a quantity is increased by x percent, then the new quantity is (100 + x) % OF the original.

Example 5: What number is 150% greater than 60?

$$\text{ORIGINAL} \times \left( 1 + \frac{\text{Percent Increase}}{100} \right) = \text{NEW}$$

→ The answer is NOT 90!

- **Successive Percents:** A great way to solve successive percent problems is to choose real numbers and see what happens.
- **Smart Numbers:** The fastest way to success with GMAT % problems with unspecified amounts is to pick 100.

A shirt that initially cost  $d$  dollars was on sale for 20% off. If  $s$  represents the sale price of the shirt,  $d$  is what percentage of  $s$ ?

PART	100	x
WHOLE	80	100

- **Interest Formulas**

	Formula	Example
SIMPLE INTEREST	Principal $\times$ Rate $\times$ Time	\$5,000 invested for 6 months at an annual rate of 7% will earn \$175 in simple interest. Principal = \$5,000, Rate = 7% or 0.07, Time = 6 months or 0.5 years. $Prt = \$5,000(0.07)(0.5) = \$175$
COMPOUND INTEREST	$P\left(1 + \frac{r}{n}\right)^n$ , where $P$ = principal, $r$ = rate (decimal) $n$ = number of times per year $t$ = number of years	\$5,000 invested for 1 year at a rate of 8% compounded quarterly will earn approximately \$412: $\$5,000\left(1 + \frac{0.08}{4}\right)^{4(1)} = \$5,412$

- **Chemical Mixtures:** Set up a mixture chart with the substance labels in rows and "Original," "Change" and "New" in the columns. In this way, you can keep careful track of the various components and their changes.  
Ex: A 500 mL solution is 20% alcohol by volume. If 100 mL of water is added, what is the new concentration of alcohol, as a percent of volume?

Volume (mL)	ORIGINAL	CHANGE	NEW
Alcohol		0	
Water		+100	
Total Solution	500	+100	

## Chapter 4: FRACTIONS, DECIMALS, & PERCENTS

- Common FDP Equivalents should be memorized

1/100	0.01	1/6	0.166...	3/5	0.6	7/8	0.875
1/50	0.02	1/5	0.2	5/8	0.625	9/10	0.9
1/25	0.04	1/4	0.25	2/3	0.66...	1/1	1
1/20	0.05	3/10	0.3	7/10	0.7	5/4	1.2
1/10	0.1	1/3	0.33...	3/4	0.75	4/3	1.33...
1/9	0.11...	3/8	0.375	4/5	0.8	3/2	1.5
1/8	0.125	2/5	0.4	5/6	0.833	7/4	1.75

Ex: What is 37.5% of 240?

If you simply convert the percent to a decimal and multiply, you will have to do a fair bit of arithmetic.

Alternatively, use 3/8 for 0.375.  $3/8$  of 240 =  $3 \times 30 = 90$

Ex: A dress is marked up 16 2/3 % to a final price of \$140. What is the original price of the dress?

16 2/3 % is 1/6. Hence the price of the dress is  $140 / (7/6)$

**General Rule:** Prefer fractions for doing multiplication or division, but prefer decimals and percents for doing addition or subtraction, for estimating numbers, or for comparing numbers.

- FDPs and Word Translations

X percent	X/100	E is 10% greater than F	E=1.1F
Of	Multiply	G is 30% less than H	G=0.7H
Y is X percent of Z	Y=(X/100) * Z Part/Whole = Percent/100	The dress cost \$J. Then it was marked up 25% and sold. What's the profit?	Profit=Revenue-Cost 1.25J-J=0.25J

## Chapter 5: STRATEGIES FOR DATA SUFFICIENCY

If a brokerage firm charged a commission of 2% of the total dollar amount of a certain trade, what was the total dollar amount of that trade?

- The dollar amount of the trade minus the brokerage firm's commission was \$88,000.
- The brokerage firm's commission decreased the profit earned on the trade by 20%.

$$C=0.02d$$

Statement (1):  $d - c = 88,000 \rightarrow 0.98d = 88,000$

Sufficient: Solve for d

Statement (2):  $p - c = 0.80p$

p = the profit before the commission

Since we don't know p, we can't solve for c.

## Chapter 6: OFFICIAL GUIDE PROBLEM SETS: PART I

### Chapter 7: FDPs: ADVANCED

- Repeating Decimals:**  $4/9 = 0.4...$      $23/99 = 0.23...$      $9/99 = 0.09...$      $27/99 = 0.27...$   
If the denominator is 9, 99, 999 or another number equal to a power of 10 minus 1, then the numerator gives you the repeating digits.
- Terminating Decimals:** If, after being fully reduced, the denominator has any prime factors besides 2 or 5, then its decimal will not terminate. If the denominator only has factors of 2 and/or 5, then the decimal will terminate.
- Unknown Digit Problems:** Occasionally, the GMAT asks tough problems involving unknown digits. These problems look like "brainteasers"; it seems it could take all day to test the possible digits.

Principles:

- Look at the answer choices first, to limit your search.
- Use other given constraints to rule out additional possibilities.
- Focus on the units' digit in the product or sum. This unit's digit is affected by the fewest other digits.
- Test the remaining answer choices.

- Formulas That Act on Decimals:** Follow the formula's instructions precisely.

Example:

$$\begin{array}{r} AB \\ \times CA \\ \hline DEBC \end{array}$$

In the multiplication above, each letter stands for a different non-zero digit, with  $A \times B < 10$ . What is the two-digit number AB?

- (A) 23 (B) 24 (C) 25 (D) 32 (E) 42

Ex:  $[x]$  represents the largest integer less than or equal to  $x$ .  $[-3.6] = 4$ .

- **Fractions and Exponents & Roots:** When you raise a fraction to a power, pay attention to both the sign and the size (relative to +1 or -1) of the fraction.
- **% and Weighted Averages:** A mixture chart can be used to solve weighted average problems that involve %.

Kris-P cereal is 10% sugar by weight, whereas healthier but less delicious Bran-O cereal is 2% sugar by weight. To make a delicious *and* healthy mixture that is 4% sugar, what should be the ratio of Kris-P cereal to Bran-O cereal, by weight?

Pounds (lbs)	Kris-P	Bran-O	Total
Sugar	10	(0.02)x	(0.04)(100 + x)
Other stuff			
Total Cereal	100	x	100 + x

- **Percent Change and Weighted Averages**

A company sells only pens and pencils. The revenue from pen sales in 2007 was up 5% from 2006, but the revenue from pencil sales declined 13% over the same period. If overall revenue was down 1% from 2006 to 2007, what was the ratio of pencil revenues to pen revenues in 2006?

Dollars (\$)	2006	Change	2007
Pen Revenue	100	+5	105
Pencil Revenue	x	-0.13x	0.87x
Total Revenue	100 + x	-0.01(100 + x)	0.99(100 + x)

Last, do not forget that on any real GMAT problem, you can: plug in answer choices. You will always be given the correct ratio in one of the answer choices. Simply pick an answer choice (say, a ratio of 1:3) and invent revenues based on that ratio (say, \$100: \$300). Then work forward from there, finding the changes in revenue per product and overall revenue.

- **Other Percent Changes**

In the first quarter of 2008, Harry's Hardware Store sold 300 hammers and 12,000 nails. In the second quarter of 2008, the store sold 375 hammers and 18,000 nails. By approximately what percent did the ratio of hammers sold to nails sold decrease from the first quarter to the second quarter?

- (A) 0.4%    (B) 2.1%    (C) 2.5%    (D) 17%    (E) 25%

→ USE:  $\frac{\text{New Ratio} - \text{Old Ratio}}{\text{Old Ratio}}$  to find the percent change

- **Estimating Decimals Equivalent:**

$$\frac{100,000}{96} \approx \frac{100,000}{100} = 1,000 < \text{real value}$$

100,000/96 is greater than 100,000/100

Use with Caution: You increase the denominator (from 96 to 100) by approximately 4%, increase the result (1000) by 4%. 1,040 is very close to the real result that is 1,041.6

- Any positive proper fraction raised to a power greater than 1 will result in a number smaller than the original fraction. Any positive proper fraction raised to a power between 0 and 1 will result in a number larger than the original fraction.

A professional gambler has won 40% of his 25 poker games for the week so far. If, all of a sudden, his luck changes and he begins winning 80% of the time, how many more games must he play to end up winning 60% of all his games for the week?

Poker Games	First 25 Games	Remaining Games	Total
Wins	(0.4)25 = 10	(0.8)x	(0.6)(25 + x)
Losses			
TOTAL	25	x	25 + x

A feed store sells two varieties of birdseed: Brand A, which is 40% millet and 60% sunflower, and Brand B, which is 65% millet and 35% safflower. If a customer purchases a mix of the two types of birdseed that is 50% millet, what percent of the mix is Brand A?

Pounds (lbs)	Brand A	Brand B	Total
Millet	40	0.65x	(0.5)(100 + x)
Other stuff	60	0.35x	(0.5)(100 + x)
Total birdseed	100	x	100 + x

Last year, all registered voters in Kumannia voted either for the Revolutionary Party or for the Status Quo Party. This year, the number of Revolutionary voters increased 10%, while the number of Status Quo voters increased 5%. No other votes were cast. If the number of total voters increased 8%, what fraction of voters voted Revolutionary this year?

Voters	Last Year	This Year	Total
Revolutionary	100	+10	100(1.1) = 110
Status Quo	x	+0.05x	x(1.05) = 1.05x
Total Voters	100 + x	+0.08(100 + x)	110 + 1.05x

22. X is what percent greater than Y percent of Z?

$$10. \sqrt[4]{\left(\frac{3}{4}\right)^3} > \frac{3}{4}$$