Chapter 1: BASIC EQUATIONS

- **Simultaneous Equations: Three Equations**: Keep careful track of your work to avoid care- less errors.

Solve the following for *w*, *x*, and *y*.
$x + w = y$ $2y + w = 3x - 2$ $13 - 2w = x + y$

→ Substitute for y in the 2nd and 3rd equation, then combine to solve.

- **Mismatch Problem**: Do not assume that you need 3 equations to solve for 3 variables.

What is *x*?	
(1) $y = x^3 - 1$	(2) $y = x - 1$

→(x-1)(x)(x+1)=0

- **RULE** for determining whether 2 equations involving 2 variables will be sufficient to solve for the variables is:
  - If both of the equations are linear – that is, if there are no squared terms (such as x^2 or y^2) and no xy terms – the equations will be sufficient UNLESS the two equations are mathematically identical (e.g., x + y = 10 is identical to 2x + 2y = 20).
- **Absolute Value Equations**: It is important to consider this rule when thinking about GMAT questions that involve absolute value. The following three-step method should be used when solving for a variable expression inside absolute value brackets:
  (1) Isolate the expression within the absolute value brackets
  (2) Once we have an equation of the form $|x| = a$ with $a>0$, we know that $x = \pm a$.
  Remove the absolute value brackets and solve the equation for 2 different cases
  (3) Check to see whether each solution is valid. The possibility of a failed solution is possible
  Example: Solve for n, given that $|n + 9| - 3n = 3$.

Chapter 2: EQUATIONS WITH EXPONENTS

- **Even Exponent Equations: 2 Solutions** – for any x, SQRT(x^2) = $|x|$
  Examples: $a^2 - 5 = 12$ → Two solutions | $x^2 + 3 = 3$ → One Solution | $x^2 + 9 = 0$ → No solution
- **Odd Exponents: 1 Solution**: Example: $243 = y^5$
- **Same Base or Same Exponent**: REWRITE the bases so that either the same base/exponent appears on both sides
  Example: $(4^w)^3 = 32^{w-1}$ – You must be careful if 0, 1, or -1 is the base (or could be the base), since the outcome of raising those bases to powers is not unique. if $(0/1)^x = (0/1)^y$, we cannot claim that x =y. Also note -1^{even} or -1^{odd}.
- **Eliminating Roots: Square Both Sides –** After you have solved for the variable, check that the solution works in the original equa- tion. Squaring both sides can actually introduce an extraneous solution.

Solve the following equation for *x*: $\sqrt{x} = x - 2$

→ $x = (x-2)^2$ → x = 4 or 1. However, 1 does not work as a solution!
Cubing a number preserves the sign, so no extraneous solution can be introduced when you cube an equation.

Chapter 3: QUADRATIC EQUATIONS

- **Factoring Quadratic Equations**: $ax^2 + bx + c$; If a is not equal to 1, simply divide the equation through by a.
  The trick to factoring is to find two integers whose product equals c and whose sum equals b.
- **Disguised Quadratics**: forms that do not quite look like the traditional form of $ax^2 + bx + c = 0$
  Example: $3w^2 = 6w$ | $36/b = b - 5$ | $x^3 + 2x^2 - 3x = 0$ → Be careful not to just divide both sides by x, This division improperly eliminates the solution x = 0
- **One-Solution Quadratics**: Be careful not to assume that a quadratic equation always has two solutions.
  Example: $x^2 - 6x + 9 = 0$ → $(x-3)^2 = 0$
- **The Three Special Products**: Three quadratic expressions called special products come up so frequently on the GMAT that it pays to memorize them. $a^2 - b^2 = (a - b)(a + b)$ | $(a - b)^2 = a^2 - 2ab + b^2$ | $(a + b)^2 = a^2 + 2ab + b^2$
- Avoid the following common mistakes with special products: Wrong: $(x + y)^2 = x^2 + y^2$

Chapter 4: FORMULAS

- **Strange Symbol Formulas**: In these problems, the GMAT introduces an arbitrary symbol, which defines a certain procedure. The symbol is IRRELEVANT; Carefully follow each step in the procedure that the symbol indicates.
  - It can be helpful to refer to the variables as "the first number," "the second number," and so on. In this way, you use the physical position of the numbers to keep them straight in relation to the strange symbol.
  - Watch out for symbols that INVERT the order of an operation.
  - More challenging strange-symbol problems require you to use the given procedure more than once.
  - Always perform the procedure inside the parentheses first.

> $W \Phi F = (\sqrt{F})^W$ for all integers W and F. What is $4 \Phi (3 \Phi 16)$?

- **Formulas with Unspecified Amounts**: Some of the most challenging formula problems on the GMAT are those that involve unspecified amounts. Just as with other GMAT problems with unspecified amounts, solve these problems by PICKING NUMBERS!

> If the length of the side of a cube decreases by two-thirds, by what percentage will the volume of the cube decrease?

→ Formula for Percentage Change: (new-old)/old

- **Sequence Formulas**: A sequence is a collection of numbers in a set order. The order is determined by a RULE.
  - An important thing to remember about sequence problems is that you MUST be given the rule in order to find a particular number. For example, If S7=5 and S8=6, you must be given a rule to compute for S9.
  - A common sequence definition is a **linear** (or arithmetic) sequence in which $S_n = kn + x$.
  - Another common sequence definition is an exponential (or geometric) sequence. These sequences are of the form $S_n = x(k^n)$.

> The first four terms of a sequence are 20, 200, 2,000, and 20,000, in that order. If each term is equal to the previous term times a constant number, what is the rule for this sequence?

Each term is 10 times the previous term, so this sequence must be in the form $x(10^n)$.

- **Sequence Problems: Alternate Method**:

> If each number in a sequence is three more than the previous number, and the sixth number is 32, what is the 100th number?

From the sixth to the one hundredth term, there are 94 "jumps" of 3. Hence, 32 + 94 x 3 = 282.

- **Sequences and Patterns**: Some sequences are easier to look at in terms of patterns, rather than rules.

> If $S_n = 3^n$, what is the units digit of S_{65}?

→ when n is a multiple of 4, unit digit is equal to 1

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2,187$
$3^8 = 6,561$

Chapter 5: FUNCTIONS

> You put a 2 into the magic box, and a 7 comes out. You put a 3 into the magic box, and a 9 comes out. You put a 4 into the magic box, and an 11 comes out. What is the magic box doing to your number?

→ f(x) = 2x + 3

> If $f(x) = x^3 + 1$, and $g(x) = 2x$, for what value of x does $f(g(x)) = g(f(x))$?

→ Working from the inside out, $8x^2 + 1 = 2x^3 + 2$

- **Common Function Types**: This section explores some of the common types of functions.
  - Direct Proportionality: means that the two quantities always change by the same factor and in the same direction. Relationships are of the form $Y = k*x$, where x is the input value and Y is the output value.

> The maximum height reached by an object thrown directly upward is directly proportional to the square of the velocity with which the object is thrown. If an object thrown upward at 16 feet per second reaches a maximum height of 4 feet, with what speed must the object be thrown upward to reach a maximum height of 9 feet?

Example:

$f(v) = kv^2$, where v is the velocity and f(v) the height.

$f(16) = k16^2 = 4 \rightarrow k = 4^{-3} \rightarrow f(v) = 4^{-3}v^2$

$f(x) = 4^{-3}(x^2) = 9 \rightarrow x = 24$

Alternatively, set up ratios (since both ratios are equal to the same constant k) to solve the problem:

$\frac{h}{v^2} : \frac{4}{16^2} : \frac{9}{x^2}$

- Inverse proportionality means that the two quantities change by RECIPROCAL factors. Relationships are of the form $Y = k/x$. This equation can also be written as $xy = k$, which means that the product of the output and input values is always constant.
  As with other proportion problems, you will typically be given "before" and "after" values. However, this time you set up products, not ratios, to solve the problem-for example, y_1x_1 can be used for the "before" values and y_2x_2 can be used for the "after" values. Next, we write $y_1x_1=y_2x_2$, since each product equals the same constant k.
  Example:
  $i_1o_1=i_2o_2$ $4*1=i_2*1/3$

> The amount of electrical current that flows through a wire is inversely proportional to the resistance in that wire. If a wire currently carries 4 amperes of electrical current, but the resistance is then cut to one-third of its original value, how many amperes of electrical current will flow through the wire?

- Linear Growth: function: $Y = mx + b$.
  Example: $y = mx + 54$
  $$\frac{3m+54}{m+54} = \frac{5}{4}$$
  $54 + 3m = 1.2(54 + m)$

> Jake was 4½ feet tall on his 12th birthday, when he began to have a growth spurt. Between his 12th and 15th birthdays, he grew at a constant rate. If Jake was 20% taller on his 15th birthday than on his 13th birthday, how many inches per year did Jake grow during his growth spurt? (12 inches = 1 foot)

Chapter 6: INEQUALITIES

- **Much Like Equations, With One Big Exception**: When you multiply or divide an inequality by a negative number, the inequality sign flips! A corollary of this is that you cannot multiply or divide an inequality by a variable, unless you know the sign of the number that the variable stands for.
- **Combining Inequalities: Line 'Em Up!** – Example: Given that u<t, b>r, f<t, and r>t, is b>u?

> $u < t$
> $r < b$
> $f < t$
> $t < r$

- **Manipulating Compound Inequalities:** $1 > 1 - ab > 0 \rightarrow 0 > -ab > -1$
  Adding inequalities together is a powerful technique on the GMAT. However, note that we should never subtract or divide two inequalities. Moreover, you can only multiply inequalities together under certain circumstances.
  Example: if m = -2 and n = - 10, then m*n = 20

> Is $mn < 10$?
> (1) $m < 2$
> (2) $n < 5$

- **Using Extreme Values**: Whenever a question asks about the possible range of values for a problem, consider using extreme values.
  - Plug extreme values using "LT" for "less than" or "GT" for "greater than."
    Ex: 8 x LT3 = LT24 | g = 15 - LT6 → g = GT9 | -7 x LT2 = GT-14 | 8 / LT2 = GT4

> Given that $-1 \leq x \leq 3$, and $y < 8$, what is the possible range of values for xy?

→ Any number

- **Optimization Problem**

> If $-4 \leq m \leq 7$ and $-3 < n < 10$, what is the maximum possible integer value for $m - n$?

→ m - n = LESS THAN 10. The largest integer less than 10 is 9.

m		*n*		*m* − *n*
Min	−4	Min	GT(−3)	(−4) − GT(−3) = LT(−1)
Min	−4	Max	LT10	(−4) − LT10 = GT(−14)
Max	7	Min	**GT(−3)**	**7 − GT(−3) = LT10**
Max	7	Max	LT10	7 − LT10 = GT(−3)

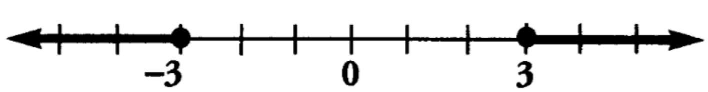
- **Testing Inequalities**: There are other common situations in which you need to consider multiple cases.

> Is $d > 0$?
> (1) $bc < 0$
> (2) $cd > 0$

b	*c*	*d*	*bc* < 0?	*cd* > 0?
+	−	−	YES	YES
−	+	+	YES	YES

STATEMENT	IMPLICATION
$xy > 0$	x and y are both positive OR both negative
$xy < 0$	x and y have different signs (one positive, one negative)
$x^2 - x < 0$	$x^2 < x$, so $0 < x < 1$

- **Inequalities and Absolute Value**: if $|x + 2| < 5$, then $-5 < x+2 < 5$. The center point of the graph is –2
  The inequality $|x + b| < c$, the center point of the graph is -b, and the "less than" symbol tells us that x must be LESS THAN c units away from -b.
- **Square Rooting Inequalities**: $\sqrt{x^2} = |x|$; Hence, If $10 + x^2 > 19$, $|x| > 3$
  x must be MORE THAN (or exactly) 3 units away from 0 on the number line.



Chapter 7: VICs (Variable Expressions in the Answer Choice)

- **Categories of VIC problems**: Word Problems, Algebra, Percents, or Geometry.
- **Three Strategies for Solving VICs**: Direct Algebra, Picking Numbers, and The Hybrid Method.

  Example:

  WORD TRANSLATIONS VIC EXAMPLE

  Jack bought x pounds of candy at d dollars per pound. If he ate w pounds of his candy and sold the rest to Jill for m dollars per pound, how much money did Jack spend, in dollars, net of Jill's payment?

  (A) $xd - wm$ (B) $xm - wd$ (C) $xd - xm + wm$
  (D) $xd + xm - xw$ (E) $2xd - xm$

- **Pick Numbers and Calculate a Target**
  - First Step: pick numbers for each variable. It can be helpful to use a chart. There are several guidelines to keep in mind when picking numbers:
    1. Never pick 0/1, avoid 100 (easy to confuse with 100%), and avoid picking numbers that appear as a coefficient in answer choices.
    2. Make sure the numbers are different.
    3. Pick Small numbers.
    4. Pick Prime Numbers.

Variable	Number	Description
x	11	pounds Jack bought
d	3	price per pound in store
w	7	pounds Jack ate
m	2	price Jill paid per pound

  - Second Step: answer the question using the numbers to find the target.
  - Third Step: Test each answer choice for the target.
- **Direct Algebra**: The best thing to do in this situation is to break the problem down into manageable parts.
  - The primary problem with using the Direct Algebra approach is that if you make a mistake along the way, you may not catch it.
- **The Hybrid Method**: Pick numbers to help you think through the problem. However, rather than plug these numbers into the answer choices, use the numbers to think through the computations, and therefore the matching algebra, step by step.
- PERCENT VIC EXAMPLE

  If x percent of y is equal to y percent less than z, what is y in terms of x and z?

  (A) $\frac{100(x+z)}{x}$ (B) $\frac{100x}{x+z}$ (C) $\frac{100(x+z)}{x-z}$ (D) $\frac{100z}{x-z}$ (E) $\frac{100z}{x+z}$

  Direct Algebra: $\frac{x}{100} * y = z\left(\frac{100-y}{100}\right) \rightarrow xy = 100z - zy$ (z minus y percent of z)

  Pick Numbers: x=5, y=80, 4 is equal to 80% less than 20 (z).
- GEOMETRY VIC EXAMPLE

  A circular swimming pool is enclosed in a square fence, such that each side of the fence touches the circumference of the pool. What is the area of the swimming pool if the perimeter of the fence is q meters?

  (A) $q^2\pi$ (B) $(q^2 - q)\pi$ (C) $\frac{q^2\pi}{4}$ (D) $\frac{q^2\pi}{16}$ (E) $\frac{q^2\pi}{64}$

  Direct Algebra: $Side = \frac{p}{4}; Radius = \frac{p}{8};\ Area = \pi * (\frac{p}{8})^2$ (z minus y percent of z)

  Pick Numbers: p = 24, r = 3, Area = 9π.

- **RULE**: That said, if you are trying to figure out the algebraic manipulation but you get stuck, you should immediately switch to a number-picking strategy! When in doubt, you can almost always use the Pick Number and Calculate a Target approach. Never give up on a VIC problem before picking numbers.

Chapter 8: STRATEGIES FOR DATA SUFFICIENCY (Paste problems that you find tricky.)

- **Rephrasing**: In some cases, you will need to manipulate the original question; in others, you will need to manipulate the statements. Sometimes, you will need to manipulate both.

  What is the value of ab^2?

  (1) $a = b - 1$
  (2) $a = b^2 - 1$

  b = {0,1}. If b = 0, a = -1, and ab = 0. If b = 1, a = 0, and ab = 0. ANSWER: C

Chapter 9: OFFICIAL GUIDE PROBLEMS: PART I (Paste problems that you find tricky.)

Chapter 10: EQUATIONS: ADVANCED

- **Complex Absolute Value Equations**:
  There are two primary types of these complex absolute value equations:
  - The equation contains TWO or more variables in more than one absolute value expression. NOT easy to solve with algebra. Instead, a conceptual approach is preferable. Review: "+ & - Strategy"
  - The equation contains ONE variable and at least one CONSTANT in more than one absolute value expression.

If $\lvert x-2\rvert = \lvert 2x-3\rvert$, what are the possible values for x?

(1) The positive/positive case:	$(x-2)=(2x-3)$
(2) The positive/negative case:	$(x-2)=-(2x-3)$
(3) The negative/positive case:	$-(x-2)=(2x-3)$
(4) The negative/negative case:	$-(x-2)=-(2x-3)$

- **Integer Constraints**: only one integer solution among all answer choices. Solve for x and test using A, B, C, D, & E

> $2y - x = 2xy$ and $x \neq 0$. If x and y are integers, which of the following could equal y?
>
> (A) 2
> (B) 1
> (C) 0
> (D) −1
> (E) −2

→ x = 2y / (2y+1)

> If x and y are nonnegative integers and $x + y = 25$, what is x?
>
> (1) $20x + 10y < 300$
> (2) $20x + 10y > 280$

→ substitute 25 – x for y.

- **Advanced Algebraic Techniques**: Multiply or divide two equations when it seems that you can caned a lot of variables in one move.

If $xy^2 = -96$ and $\frac{1}{xy} = \frac{1}{24}$, what is y?	If $\frac{a}{b} = 16$ and $\frac{a}{b^2} = 8$, what is ab?

→ multiply/divide is faster.

- **Advanced Factoring & Distributing**: A general rule of thumb is that when you encounter any expression or equation in which two or more terms include the same variable, you should consider factoring as an approach. Similarly, when an expression is given in factored form, consider distributing it.
- **Advanced Quadratic Techniques:** Any quadratic mat can be solved by taking the square root of both sides SHOULD be solved mat way.
  Example: If $(Z+3)^2 = 25$, what is z? → $\lvert z+3\rvert = 5$ → $z+3 = \pm 5$ → $z = -3 \pm 5$
  Substituting: make the following substitution: $z = x^2$.

> Given that $x^4 - 5x^2 + 4 = 0$, what is NOT a possible value of x?
> (A) −2 (B) −1 (C) 0 (D) 1 (E) 2

QUADRATIC FORMULA: very occasionally you might encounter a problem difficult to solve with factoring. Use $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ If the discriminant is greater than zero, there will be two solutions. If the discriminant is equal to zero, there will be one solution. If the discriminant is less than zero, there will be no solutions.

Example:

> Which of the following equations has no solution for x?
>
> (A) $x^2 - 8x - 11 = 0$
> (B) $x^2 + 8x + 11 = 0$
> (C) $x^2 + 7x + 11 = 0$
> (D) $x^2 - 6x + 11 = 0$
> (E) $x^2 - 6x - 11 = 0$

Chapter 11: FORMULAS & FUNCTIONS: ADVANCED

- **Recursive Formulas**: You should learn the direct counterparts for some common recursive formulas. The general method is to solve for the unknown constants k and x and write the direct formula, S_n.
  1. **Linear / Arithmetic Sequence:** **$k = S_n - S_{n-1}$** **$x = S_n - n*k$** **$S_n = k_n + x$**
     $2k = S_n - S_{n-2}$ | $3k = S_n - S_{n-3}$ | Etc.

     Example: if $S_1 = 8$ and $S_2 = 6$, $k = S_2 - S_1 = -2$ $x = S_1 - 1*-2 = 10.$ $S_n = -2n + 10$

  2. **Exponential (or geometric) Sequence:** **$k = S_n / S_{n-1}$** **$x = S_n / k^n$** **$S_n = x*k^n$**
     $k^2 = S_n / S_{n-2}$ | $k^3 = S_n / S_{n-3}$ | Etc.

     Example:

     > In sequence C_n, we are given that $C_3 = 12$ and $C_5 = 3$. If each term is equal to the previous term times a constant number, and if all the terms in the sequence are positive, what is the recursive rule for this sequence? What is the value of C_{10}?

     $k^2 = C_5 / C_3 \rightarrow k = ½$ $x = C_5 / ½^5 = 96$ $S_n = 96*½^n$

- **Advanced Function Types:**
  - **Exponential Growth Sequence**: Any exponential growth can be written as $y_t = y_0*k^t$ (Similar to **2.**)

    Example:

    > A quantity increases in a manner such that the ratio of its values in any two consecutive years is constant. If the quantity doubles every 6 years, by what factor does it increase in two years?

    $2y_0 = y_0*k^6$ or $k^6 = S^6 / S^1 = 2 \rightarrow k^2 = 2^{1/3} = 1/8$

  - **Symmetry**: In most cases, picking numbers will probably be easier.

    > For which of the following functions does $f(x) = f\left(\frac{1}{x}\right)$, given that $x \neq -2, -1,$ 0, or 1?
    >
    > (A) $f(x) = \left|\frac{x+1}{x}\right|$ (C) $f(x) = \left|\frac{x-1}{x}\right|$ (E) $f(x) = \left|\frac{x+1}{x+2}\right|$
    >
    > (B) $f(x) = \left|\frac{x+1}{x-1}\right|$ (D) $f(x) = \left|\frac{x}{x+1}\right|$

    $$f\left(\frac{1}{x}\right) = \left|\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right| = \left|\frac{\frac{x+1}{x}}{\frac{1-x}{x}}\right| = \left|\frac{x+1}{1-x}\right| = \left|\frac{x+1}{-(1-x)}\right| = \left|\frac{x+1}{x-1}\right|$$

  - **Properties**: An effective approach is simply to pick numbers and see which function gives the desired result.

    > For which of the following functions does $f(x - y)$ NOT EQUAL $f(x) - f(y)$?

- **Optimization Problem**:
  - **Linear Functions**: The extremes of linear functions occur at the smallest possible x and the largest possible x.
  - **Quadratic Functions**: Unlike linear functions, quadratic functions do not grow continuously or decline continuously. Rather, they form parabolas.

    The key to deciphering the function is to make the squared expression equal to 0.

    Example:

    > Consider the quadratic function $f(x) = 7 - (x + 1)^2$.
    >
    > (a) Does this function have a minimum value or a maximum value?
    > (b) At what value of x does this minimum or maximum occur?
    > (c) What is the minimum or maximum value?

    Note: $f(x) = 2x^2 + 1$ is shifted upward by 1 unit and squeezed (it has steeper slopes than $x^2 + 1$)

Chapter 12: INEQUALITIES: ADVANCED

- **Working with Advanced Inequalities**: Set up a Negative/Positive cases table
  Because we do not know whether y is positive or negative, we are NOT ALLOWED to multiply both sides of the equation by y without considering two separate cases.

  > Is $x < y$?
  >
  > (1) $\frac{x}{y} < 1$
  >
  > (2) $y > 0$

- **Reciprocals of Inequalities**: If we do not know the sign of x or y, we cannot take reciprocals.
  - if $a > b$, then $\frac{1}{a} < \frac{1}{b}$ *unless a and b have different signs, in which case* $\frac{1}{a} > \frac{1}{b}$

Example: Set up a table

Given that $ab < 0$ and $a > b$, which of the following must be true?

I. $a > 0$
II. $b > 0$
III. $\frac{1}{a} > \frac{1}{b}$

Is $x^2 > y^2$?
(1) $x > y$
(2) $x > 0$

Is $a > 0$?
(1) $a^3 - a < 0$
(2) $1 - a^2 > 0$

- **Squaring Inequalities**: $a<b$
  - If both sides are known to be negative, then flip the inequality sign when you square. Ex: $x < -3$
  - If both sides are known to be positive, then do not flip the inequality sign when you square. Ex: $x > 3$
  - If one side is positive and one side is negative, then you cannot square. The result is not warranted.
  - If the signs are unclear, then you cannot square. The sign of x can be either + or -. Ex: $x < 3$
- **A Challenging Problem**: Use overlapping table (Lycée).

Chapter 13: ADDITIONAL VIC PROBLEMS (Paste problems that you find tricky)

Chapter 14: OFFICIAL GUIDE PROBLEMS: PART II (Paste problems that you find tricky.)