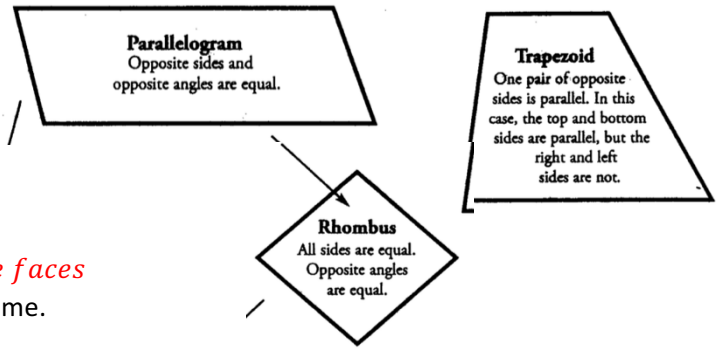
Chapter 1: POLYGONS

Polygon	# of Sides	Sum of Interior Angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°

- **Polygons**: A polygon is a closed shape formed by line segments.
  Sum of Interior Angles of a Polygon = (n - 2) * 180
- **Polygons and Area**
  Area of any PARALLELOGRAM: $Base * Height$
  Area of a TRIANGLE: $\frac{Base * Height}{2}$
  Area of a TRAPEZOID: $\frac{(Base\ 1 + Base\ 2) * Height}{2}$
  Area of a RHOMBUS: $\frac{Diagonal\ 1 * Diagonal\ 2}{2}$
  All of the above shapes can actually be divided into some combination of rectangles and right triangles.



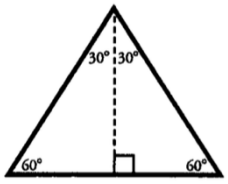
- **3 Dimensions: Surface Area**
  Surface Area = $The\ SUM\ of\ the\ areas\ of\ ALL\ of\ the\ faces$
- **3 Dimensions: Volume** "Capacity" is another word for volume.
  Volume = $Lenght * Width * Height$
  - Beware of a GMAT volume trick:

Ex: How many books, each with a volume of 100 in3, can be packed into a crate with a volume of 5,000 in^3? Without knowing the exact shapes of all the books, we cannot tell whether they would all fit into the crate.

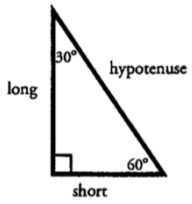
Chapter 2: TRIANGLES & DIAGONALS

- **The Angles of a Triangle**:
  - Angles correspond to their opposite sides.
  - Additionally, if two sides are equal, their opposite angles are also equal.
- **The Sides of a Triangle**:
  - If you are given two sides of a triangle, the length of the third side must lie between the difference and the sum of the two given sides. For instance, if you are told that two sides are of length 3 and 4, then the length of the third side must be between 1 and 7.
- **Common Right Triangle**: Appear frequently on the GMAT

Common Combinations	Key Multiples
3–4–5 The most popular of all right triangles $3^2 + 4^2 = 5^2$ (9 + 16 = 25)	6–8–10 9–12–15 12–16–20
5–12–13 Also quite popular on the GMAT $5^2 + 12^2 = 13^2$ (25 + 144 = 169)	10–24–26
8–15–17 This one appears less frequently $8^2 + 15^2 = 17^2$ (64 + 225 = 289)	None



EQUILATERAL TRIANGLE



30–60–90 TRIANGLE

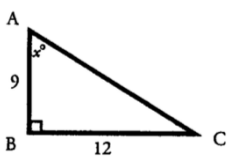
Watch out for impostor triangles! A random triangle with one side equal to 3 and another side equal to 4 does not necessarily have a third side of length 5.

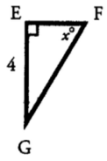
- **The 45-45-90 Triangle**
  - The lengths of the legs of every 45-45-90 triangle have a specific ratio: $1:1:\sqrt{2}$
- **The 45-45-90 Triangle**
  - The lengths of the legs of every 30-60-90 triangle have the following ratio: $1:\sqrt{3}:2$
  - Notice that two of these triangles, when put together, form an equilateral triangle

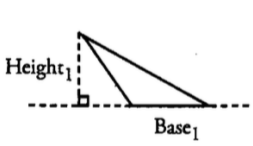
Ex: Given that an equilateral triangle has a side of 10, what is its height? ➔ $5\sqrt{3}$ since 10 is the hypotenuse.

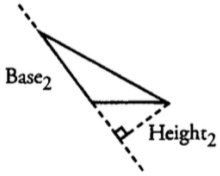
- **Diagonals of Other Polygons**:
  - The main diagonal of a cube can be found using this formula: $d = s\sqrt{3}$, where s is an edge of the cube.
  - To find the diagonal of a rectangle, you must know EITHER the length and the width OR one dimension and the proportion of one to the other.
  - **"Deluxe" Pythagorean Theorem**: $d^2 = x^2 + y^2 + z^2$, where x, y, and z are the sides of the rectangular solid and d is the main diagonal.

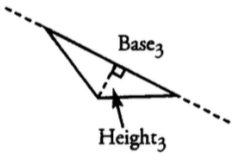
- **Similar Triangles**: Triangles are defined as similar if all their corresponding angles are equal and their corresponding sides are in proportion.
  - If two similar triangles have corresponding side lengths in ratio a:b, then their areas will be in ratio $a^2:b^2$.
  - For similar solids with corresponding sides in ratio a:b, their volumes will be in ratio $a^3:b^3$.
- **Triangles and Area, Revisited**:
  - Be able to see any side of a triangle as the base, not just the side that happens to be drawn horizontally. Also be able to draw the height from that base.

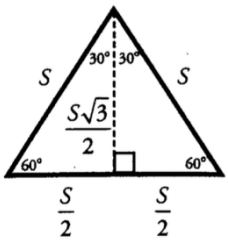










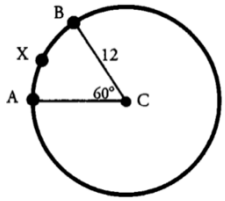


  - The area of an equilateral triangle with a side of length S is equal to $\frac{S^2\sqrt{3}}{4}$

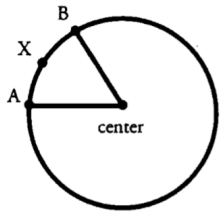
Chapter 3: CIRCLES AND CYLINDERS

- Know the difference between Radius, Chord and Diameter.
- $C = 2\pi r$, $A = \pi r^2$
- Note that a full revolution, or turn, of a spinning wheel is equivalent to the wheel going around once.
  Ex: If a wheel spins at 3 revolutions per second and has a diameter of 10 units, then the point travels at a rate of $3*10\pi$ units per second.

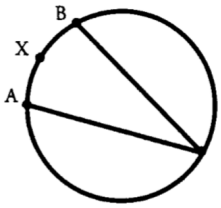
- **Circumference and Arc Length**:
  - Arc length can be found by determining what fraction the arc is of the entire circumference. This can be done by looking at the central angle that defines the arc.
    Ex: What is the length of Arc AXB? → 4π



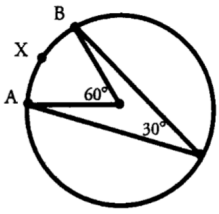
- **Perimeter of a Sector**: Think of a sector as a slice of pizza. Here, the perimeter is $12+12+4\pi$
- **Area of a Sector**: You can find the area of a sector by determining the fraction of the entire area that the sector occupies. Here, the area of the sector is 24π.
- **Inscribed vs. Central Angles:** A central angle is as an angle whose vertex lies at the center point of a circle.
  - An inscribed angle is equal to half of the arc it intercepts.

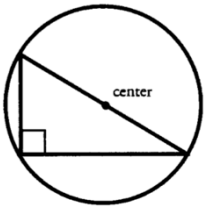


CENTRAL ANGLE

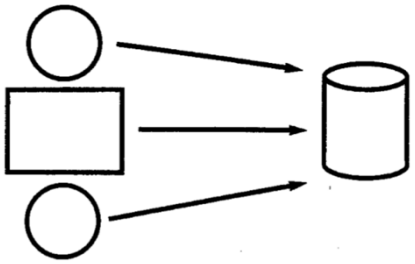


INSCRIBED ANGLE





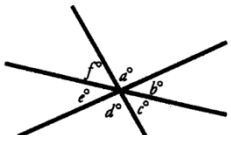
- **Inscribed Triangle**: A triangle is said to be inscribed in a circle if all of its vertices are points on the circle.
  - Any right triangle inscribed in a circle must have the diameter of the circle as one of its sides.
- **Cylinders and Surface Area**: sum the areas of the 3 surfaces.
  - A = 2circles + rectangle $=2(\pi r^2) +2\pi rh$
- **Cylinders and Volume**: measures how much "stuff" it can hold inside.
  - $V = \pi r^2 h$



Chapter 4: LINES & ANGLES

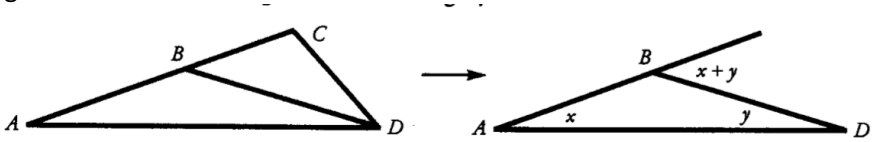
There are two major line-angle relationships that you must know for the GMAT.

- **The angles formed by any intersecting lines.**

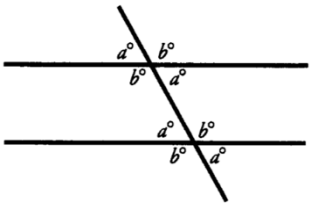


  - a + b + c + d + e + f = 360
  - **Supplementary Angles**: a + b + c = 180
  - **Vertical Angles**: a = d, b = e, and c = f

- **Exterior Angles of a Triangle**: An exterior angle of a triangle is equal in measure to the sum of the two non-adjacent (opposite) interior angles of the triangle.

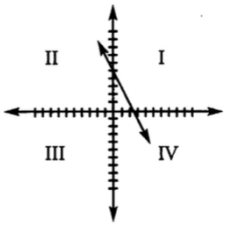


- **The angles formed by parallel lines cut by a transversal.**
  - All the acute angles (a< 90°) in this diagram are equal. Likewise, all the obtuse angles (90°<a<180°) are equal
  - Any acute angle is supplementary to any obtuse angle. Thus, a + o = 180°.
  - When you see a transversal cutting two parallel lines, fill in all the acute and obtuse angles.



Chapter 5: COORDINATE PLAN

- **The Coordinate Plan**: x-coordinate, y-coordinate, origin
- **The Slope of a Line**: $\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
- **The 4 Types of Slopes**: A line with positive slope rises upward from left to right.
  A line with negative slope falls downward from left to right.
  A horizontal line has zero slope. A vertical line has undefined slope.
- **Slope-Intercept Equation**: All lines can be written as equations in the form y = mx + b, where m represents the slope of the line and b represents the y-intercept of the line.
  Horizontal lines are expressed in the form: y = some number, such as y = 3 or y = 5.
  Vertical lines are expressed in the form: x =some number, such as x = 4 or x =7.
- **Step by Step: From 2 Points to a Line**:
  To find the equation of a line, you should start by finding its slope. Then, plug the slope in for m in the slope-intercept equation. Finally, solve for b by plugging the coordinates of one point into the equation.
- **The Distance Between 2 Points**:
  Ex: What is the distance between the points (1, 3) and (7, -5)?
  - First, draw a right triangle connecting the points.
  - Then, find the lengths of the two legs of the triangle by calculating the rise and the run.
  - Finally, use the Pythagorean Theron to find the length of the third leg.



- **Positive and Negative Quadrants**:
  Ex: Which quadrants does the line 2x + Y = 5 pass through?
  Find two points on the line by setting x and y equal to zero in the original equation.
- **Perpendicular Bisectors**: The perpendicular bisector of a line segment forms a 90° angle with the segment and divides the segment exactly in half.
  The key to solving perpendicular bisector problems is remembering this property: the perpendicular bisector has the negative reciprocal slope of the line segment it bisects. That is, the product of the two slopes is -1.

If the coordinates of point *A* are (2, 2) and the coordinates of point *B* are (0, –2), what is the equation of the perpendicular bisector of line segment *AB*?

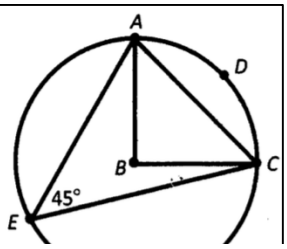
Slope: -½.
Midpoint: (1,0)
Equation: y = -½(x) + ½

- **The Intersection of Two Lines:** If two lines in a plane intersect in a single point, the coordinates of that point solve the equations of both lines.
- Ex: At what point does the line represented by y = 4x - 10 intersect the line represented by 2x + 3y = 26?
  Replace y in the second equation with 4x - 10 and solve for x.

Chapter 6: STRATEGIES FOR DATA SUFFICIENCY

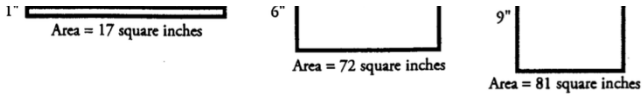
If *B* is the center of the circle to the right, what is the length of line segment *AC*?

(1) The area of sector *ABCD* is 4π
(2) The circumference of the circle is 8π



→ Answer D

Chapter 7: OFFICIAL GUIDE PROBLEMS: PART I



Chapter 8: ADVANCED GEOMETRY

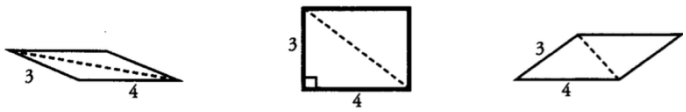
- **Maximum Area of Polygons**:

  Maximum Area of a Quadrilateral

  Of all quadrilaterals with a given perimeter, the SQUARE has the largest area.

  Ex: For instance, of all quadrilaterals with a perimeter of 25 feet, the one with the largest area is a square with 25/4 = 6.25 feet per side.

  Of all quadrilaterals with a given area, the SQUARE has the minimum perimeter.



  Maximum Area of a Parallelogram or Triangles

  If you are given two sides of a triangle or parallelogram and you want to maximize the area, establish those sides as the base and height, and make the angle between them 90°.

- **Function Graphs and Quadratics**: The graph of any quadratic function is a parabola opening upward or downward, depending on the sign of a, the x2 coefficient.

Positive value for a	Curve opens upward
Negative value for a	Curve opens downward
Large $\|a\|$ (absolute value)	Narrow curve
Small $\|a\|$	Wide curve

  <u>The most important questions you will be asked about the parabola are these:</u>

  (1) How many times does the parabola touch the x-axis?

  (2) If the parabola does touch the x-axis, where does it touch?

  The x-axis is the line representing y = 0. In other words, the parabola touches the x-axis at those values of x that make f(x) = 0. Therefore, these values solve the quadratic equation given by $f(x) = ax^2 + bx + c = 0$.

  LOOK FOR DELTA:

  1. If $b^2 - 4ac > 0$, the quadratic formula produces two roots of the quadratic equation. This means that the parabola crosses the x-axis twice and has two x-intercepts.
  2. If $b^2 - 4ac = 0$, then the square root operation yields zero. The quadratic formula only produces one root of the quadratic equation.
  3. If $b^2 - 4ac < 0$, then the square root operation cannot be performed. This means that the quadratic formula produces no roots of the quadratic equation, and the parabola never touches the x-axis.

  <u>The following points lie at the heart of all problems involving graphs of other non-linear functions, as well as lines and parabolas:</u>

  (1) If a point lies on the graph, then you can plug its coordinates into the equation y =f(x). Conversely, if a value of x and a value of y satisfy the equation y =f(x), then the point (x, y) lies on the graph of f(x).

  (2) To find x-intercepts, find the values of x for which y =f(x) = 0.

  (3) To find y-intercepts, set x = 0 and find y =f(0).

Chapter 9: OFFICIAL GUIDE PROBLEMS: PART II