$$(2) \qquad \begin{aligned} 3x^2 - 3 &= 8x \\ 3x^2 - 8x - 3 &= 0 \\ (3x+1)(x-3) &= 0 \\ 3x+1 = 0 \text{ or } x-3 &= 0 \\ x = -\frac{1}{3} \text{ or } x &= 3 \end{aligned}$$

A quadratic equation has at most two real roots and may have just one or even no real root. For example, the equation $x^2 - 6x + 9 = 0$ can be expressed as $(x - 3)^2 = 0$, or $(x - 3)(x - 3) = 0$; thus the only root is 3. The equation $x^2 + 4 = 0$ has no real root; since the square of any real number is greater than or equal to zero, $x^2 + 4$ must be greater than zero.

An expression of the form $a^2 - b^2$ can be factored as $(a - b)(a + b)$.

For example, the quadratic equation $9x^2 - 25 = 0$ can be solved as follows.

$$\begin{aligned} (3x-5)(3x+5) &= 0 \\ 3x-5 = 0 \text{ or } 3x+5 &= 0 \\ x = \frac{5}{3} \text{ or } x &= -\frac{5}{3} \end{aligned}$$

If a quadratic expression is not easily factored, then its roots can always be found using the *quadratic formula:* If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the roots are

$$x = \frac{-b + \sqrt{b^2 + 4ac}}{2a} \text{ and } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

These are two distinct real numbers unless $b^2 - 4ac \leq 0$. If $b^2 - 4ac = 0$, then these two expressions for x are equal to $-\frac{b}{2a}$, and the equation has only one root. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number and the equation has no real roots.

7. Exponents

A positive integer exponent of a number or a variable indicates a product, and the positive integer is the number of times that the number or variable is a factor in the product. For example, x^5 means $(x)(x)(x)(x)(x)$; that is, x is a factor in the product 5 times.

Some rules about exponents follow.

Let x and y be any positive numbers, and let r and s be any positive integers.

(1) $(x^r)(x^s) = x^{(r+s)}$; for example, $(2^2)(2^3) = 2^{(2+3)} = 2^5 = 32$.

(2) $\frac{x^r}{x^s} = x^{(r-s)}$; for example, $\frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64$.

(3) $(x^r)(y^r) = (xy)^r$; for example, $(3^3)(4^3) = 12^3 = 1{,}728$.

(4) $\left(\frac{x}{y}\right)^r = \frac{x^r}{y^r}$; for example, $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$.