PERCENTAGES- PROFIT AND LOSS

1. If the value of an item goes **up/down** by x%, the percentage **reduction/increment** to be now made to bring it back to the original level is $\frac{100x}{100+x}\%$ / $\frac{100x}{100-x}\%$ respectively.

2. If A is x% **more/less** than B, then B is $\frac{100x}{100+x}\%$ / $\frac{100x}{100-x}\%$ less/more than A respectively.

3. If the price of an item goes **up/down** by x% then the quantity consumed should be reduced/increased by $\frac{100x}{100+x}\%$ / $\frac{100x}{100-x}\%$ respectively so that the total expenditure remains the same.

4. S.P. = C.P * $\frac{100+P}{100}$, where S.P.= Selling Price, C.P.= Cost Price, P=Profit percentage

5. S.P. = C.P. * $\frac{100-L}{100}$, where S.P.= Selling Price, C.P.= Cost Price, L=Loss percentage

6. C.P. = S.P* $\frac{100}{100+P}$,where S.P.= Selling Price, C.P.= Cost Price, P=Profit percentage

7. C.P. = S.P. * $\frac{100}{100-L}$,where S.P.= Selling Price, C.P.= Cost Price, L=Loss percentage

8. When two articles are sold at the same price such that there is a profit P% on one article and a loss of P% on the other, the net result of the transaction is LOSS.
   Loss % = $\frac{(Common\ profit \vee loss)^2}{100}$
   = $\frac{P^2}{100}$

9. If A makes a profit of x% by selling an item to B and B makes a profit of y% by selling it to C, then the resultant profit% is (x + y + $\frac{xy}{100}$)%

10. Selling Price = Marked Price – Discount

11. Discount% = $\frac{(Marked\ Price - Selling\ Price)}{Marked\ Price}$ * 100

TIME AND DISTANCE

1. In general, if a person travelling between two points reaches 'p' hours late travelling at a speed of 'u' kmph and reaches 'q' hours early travelling at 'v' kmph, the distance between two points is given by $\frac{(vu)(p+q)}{v-u}$
2. If a body travels from point A to point B with a speed of 'p' and back to point A (from B) with a speed of 'q', then the average speed of the body can be calculated as $\frac{2pq}{p+q}$. This does not depend upon the distance between A and B.
3. If a body covers part of the journey at speed 'p' and the remaining part of the journey at speed 'q' and the distance of the two parts of the journey are in the ratio m:n, then the average speed for the entire journey is: $\frac{(m+n)pq}{mq+np}$
4. Time taken by a train to overtake another train =
$$\frac{(Length\,of\ faster\,train+Length\,of\ slower\,train)}{Their\ relative\ speed}$$

BOATS AND STREAMS

1. Speed of the boat against stream= Stream of the boat in still water – Speed of the stream
2. Speed of the boat with stream= Stream of the boat in still water +Speed of the stream
3. Speed of the boat in still water = $\frac{u+v}{2}$ where, u = Speed of the boat upstream, v = Speed of the boat downstream.
4. Speed of the water current = $\frac{v-u}{2}$ where, u = Speed of the boat upstream, v = Speed of the boat downstream.

When 2 people are running around a circular track

	SAME DIRECTION	OPPOSITE DIRECTION
Time taken to meet for the first time ever	$\frac{L}{a-b}$	$\frac{L}{a+b}$
Time taken to meet for the first time at the starting time	LCM($\frac{L}{a}$, $\frac{L}{b}$)	LCM($\frac{L}{a}$, $\frac{L}{b}$)

When 3 people are running around a circular track

	SAME DIRECTION	OPPOSITE DIRECTION
Time taken to meet for the first time ever	LCM { $\frac{L}{a-b}$, $\frac{L}{b-c}$ }	LCM { $\frac{L}{a-b}$, $\frac{L}{b-c}$ }
Time taken to meet for the first time at the starting time	LCM ($\frac{L}{a},\frac{L}{b},\frac{L}{c}$)	LCM ($\frac{L}{a},\frac{L}{b},\frac{L}{c}$)

PROGRESSIONS

1. If both progressions are AP's, then the common difference is the L.CM. of the common difference of the 2 progressions.
   Nth common term of the 2 progression = First common term of the 2 progressions + (N-1)* L.C.M (d1,d2)
   Where, d1 = common difference of the first arithmetic progression
   And d2 = common difference of the second arithmetic progression

2. Sum of the terms of an arithmetic progression = (number of terms)*(the middle term of the A.P.), if number of terms is odd.
3. Difference between sum of odd numbered terms and even numbered terms is $\frac{n*d}{2}$
   if n i.e., number of terms is even.

4. Sum of n terms of an A.P. = $\frac{(a+l)}{2}*n$
5. The Arithmetic Mean (AM) between two numbers a and b = $\frac{a+b}{2}$
6. To solve most of the problems related to AP, the terms can be conveniently taken as
   3 terms: (a – d), a, (a +d)
   4 terms: (a – 3d), (a – d), (a + d), (a +3d)
   5 terms: (a – 2d), (a – d), a, (a + d), (a +2d)
7. $T_n = S_n - S_{n-1}$
8. The Harmonic Mean(HM) between two numbers a and b = $\frac{2ab}{a+b}$
9. If a, b, c are in HP, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

10. To solve most of the problems related to GP, the terms of the GP can be conveniently taken as
    3 terms: $\frac{a}{r}$, a, ar
    5 terms: $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2

INEQUALITIES AND MODULUS

1. A.M ≥ G.M. ≥ HM. The equality occurs only when the number are all equal.
2. If the sum of two positive quantities is given, their product is the greatest when they are equal. If the product of two positive quantities is given, their sum is the least when they are equal.
3. For any positive number x ≥ 1,

$$2 < \left(1+\frac{1}{x}\right)^n \leq 2.8$$

   The equality in the first part will occur only if x = 1.

4. When the expression (ax+by) is constant, the maximum value of $x^m \quad y^n$ is realized when

$$\frac{ax}{m} = \frac{by}{n}$$

5. When the expression $x^m \quad y^n$ is constant, the minimum value of (ax+by) is realized when
   $\frac{ax}{m} = \frac{by}{n}$

6. If the product of several numbers is constant, their sum is minimum when they are all equal.

7. $(n!)^2 \geq n^n$

PERMUTATIONS AND COMBINATIONS

1. A group of 4n distinct items can be divided equally.
   a. Among 4 boys in $\frac{4n!}{(n!)^4}$ ways.
   b. Into 4 parcels in $\frac{4n!}{4!(n!)^4}$ ways.
2. Number of rectangles that can be formed in a n*n chess board

$$= \sum i^3 = [\frac{n(n+1)}{2}]^2$$

3. Number of squares that can be formed in a n*n chess board

$$= \sum i^2 = [\frac{n(n+1)(2n+1)}{6}]$$

4. If p things are alike of one kind, q things are alike of second kind and r things are alike of third kind, then one or more things can be selected in (p+1) (q+1) (r+1) – 1 ways.

5. $^nC_r = {}^nC_{n-r}$

6. Number of ways of selecting one or more items from n given items is 2^n - 1.
7. Number of ways of dividing (p+q) items into two groups of p and q items respectively is

$$= \frac{(p+q)!}{p!q!}$$

8. Number of ways of dividing 2p items into 2 equal groups of p each is

$$= \frac{(2p)!}{(p!)^2}$$, where 2 groups have distinct identity.

9. Number of ways of dividing 2p items into 2 equal groups of p each is

$$= \frac{(2p)!}{(2!)(p!)^2}$$, where 2 groups do not have distinct identity.

10. Number of ways in which (p+q+r) things can be divided into 3 groups containing p,q and r things respectively is : $\frac{(p+q+r)!}{p!q!r!}$.

11. Number of circular arrangements of n distinct items is:
    a. (n-1)! , if there is a difference between clockwise and anticlockwise arrangements.
    b. $\frac{(n-1)!}{2}$, if there is NO difference between clockwise and anticlockwise arrangements.

12. Number of ways of distributing n identical things into r groups = ${}^{n+r-1}C_{r-1}$

13. If there are m objects that are to be equally distributed among n people such that each person gets p objects, total number of ways in which this is done is $\frac{m!}{(p!)^m}$

14. If all the possible n – digit numbers using n distinct digits are formed, the sum of all the numbers so formed = (n-1)! * (Sum of the n digits) * {1111…. n times}

15. For x1+x2+x3+….+xn = S, where S ≥ 0,

    a. the number of positive integral solutions (where S ≥ n) is ${}^{S-1}C_{n-1}$

    b. the number of non-negative integral solutions is ${}^{S+n-1}C_{n-1}$.

QUADRATIC EQUATIONS

1. $ax^2 + bx + c = 0$

   Sum of the roots = $\frac{-b}{a}$

   Product of the roots = $\frac{c}{a}$

2. When
   a. Discriminant = $b^2 - 4ac < 0$, roots are complex and unequal
   b. Discriminant = $b^2 - 4ac = 0$, roots are real and equal
   c. Discriminant = $b^2 - 4ac > 0$, roots are real and unequal

3. For a polynomial of degree 3 of the form, $ax^3 + bx^2 + cx + d = 0$,
   a. Product of the roots = $\frac{-d}{a}$
   b. Sum of the roots = $\frac{-b}{a}$
   c. Sum of the product of roots taken two at a time = $\frac{c}{a}$

4. If p is the sum of the roots of the quadratic equation and q is the product of the roots of the quadratic equation, then the equation can be written as $x^2 - px + q = 0$.

5. The quadratic expression $ax^2 + bx + c$:
   a. Has a minimum value whenever a> 0 , the minimum value of the quadratic expression is $\frac{4ac-b^2}{4a}$ and it occurs at x = $\frac{-b}{2a}$.
   b. Has a maximum value whenever a < 0, the maximum value of the quadratic expression is $\frac{4ac-b^2}{4a}$ and it occurs at x = $\frac{-b}{2a}$.

6. Number of roots = degree

7. Equation whose roots are reciprocals of the roots of the equation ax^2 + bx + c = 0 is given by : c x^2 + bx + a = 0.

8. To determine a 1st degree expression, 2 conditions are needed, for a quadratic expression, 3 degrees are needed and for a cubic expression, 4 conditions are needed.

9. The difference of the roots of a x^2 + bx +c = 0 is $\frac{\sqrt{b^2-4ac}}{a}$

General formulae:

1. If a1x+b1y+c = 0 and a2x+b2y+c = 0 have no solutions,
   Then, $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$

2. If a1x+b1y+c = 0 and a2x+b2y+c = 0 have infinite solutions,
   Then, $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

3. If a1x+b1y+c = 0 and a2x+b2y+c = 0 have unique solution,
   Then, $\frac{a1}{a2} \neq \frac{b1}{b2}$

GEOMETRY AND MENSURATION

1. Circumradius of a right angled triangle = $\frac{Hypotenuse}{2}$

2. Inradius of a triangle must always be less than $\frac{1}{2}$ (Smallest altitude in it).

3. Inradius of an equilateral triangle = $\frac{1}{2\sqrt{3}}$ (Side of the triangle).

4. Circumradius of an equilateral triangle = $\frac{1}{\sqrt{3}}$ (Side of the triangle).

5. Area of an isosceles triangle = $\frac{b\sqrt{4a^2-b^2}}{4}$, where b is the third side of the isosceles triangle.

6. Sum of the interior angles of a polygon = (n-2) 180.

7. Each interior angle = $\frac{(n-2)180}{n}$

8. An isosceles triangle whose perimeter is fixed will have the maximum area when it is equilateral.

9. If medians are equal in a triangle, then their corresponding sides are equal.

10. For any right - angled triangle whose inradius is r and whose circumradius is R ,

    a. Perimeter = (2r+4R)

    b. Area = (r^2 + 2Rr)

11. Inradius of a right angled triangle = $\frac{a+b-c}{2}$, where c = Hypotenuse.

12. Longest diagonal of cuboid = $\sqrt{l^2+b^2+h^2}$

13. Face diagonal of a cube = $\sqrt{2}$ side

14. Body diagonal of a cube = $\sqrt{3}$ side

15. Total surface area of cuboid = 2(lb+bh+lh)

16. Top surface area = lb; Front surface area = lh

17. Diagonal of a square = $\sqrt{2}$ s

18. Number of revolutions = $\frac{Distance\ travelled\ by\ the\ wheel}{Circumference\ of\ the\ wheel}$

19. Area of the field = (Curved surface area of the roller) * Number of revolutions

20. Area of a regular hexagon = $\frac{3\sqrt{3}}{2}$ $Side^2$

21. Area of the 4 walls of a room = 2h(l+b)

22. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ $Side^2$

23. Circumradius of a triangle = $\frac{a*b*c}{4*(Area\ of\ triangle)}$, where a, b and c are the sides of the triangle.

TIME AND WORK

1. If A can do a piece of work in n days, work done by A in 1 day = 1/n

2. If A does 1/n work in a day, A can finish the work in n days

3. If M1 men can do W1 work in D1 days working H1 hours per day and M2 men can do W2 work in D2 days working H2 hours per day (where all men work at the same rate), then

$$\frac{M1D1H1}{W1} = \frac{M2D2H2}{W2}$$

4. If A can do a piece of work in p days and B can do the same in q days, A and B together can finish it in pq / (p+q) days.

5. Number of days = Total work / work done in 1 day