1. If x is an integer, what is the value of x?

(1) $|23x|$ is a prime number. From this statement it follows that x=1 or x=-1. Not sufficient.

(2) $2\sqrt{x^2}$ is a prime number. The same here: x=1 or x=-1. Not sufficient.

(1)+(2) x could be 1 or -1. Not sufficient.

Answer: E.

2. If a positive integer n has exactly two positive factors what is the value of n?

Notice that, *n has exactly two positive factors* simply means that n is a prime number, so its factors are 1 and n itself.

(1) n/2 is one of the factors of n. Since n/2 cannot equal to n, then n/2=1, thus n=2. Sufficient.

(2) The lowest common multiple of n and n + 10 is an even number. If n is an odd prime, then n+10 is also odd. The LCM of two odd numbers cannot be even, therefore n is an even prime, so 2. Sufficient.

Answer: D.

3. If 0 < x < y and x and y are consecutive perfect squares, what is the remainder when y is divided by x?

Notice that since x and y are consecutive perfect squares, then $\sqrt{x}$ and $\sqrt{y}$ are consecutive integers.

(1) Both x and y have 3 positive factors. This statement implies that $x=(prime_1)^2$ and $y=(prime_2)^2$. From above we have that $\sqrt{x}=prime_1$ and $\sqrt{y}=prime_2$ are consecutive integers. The only two consecutive integers which are primes are 2 and 3. Thus, $x=(prime_1)^2=4$ and $y=(prime_2)^2=9$. The remainder when 9 is divided by 4 is 1. Sufficient.

(2) Both $\sqrt{x}$ and $\sqrt{y}$ are prime numbers. The same here: $\sqrt{x}=2$ and $\sqrt{y}=3$. Sufficient.

Answer: D.

4. Each digit of the three-digit integer K is a positive multiple of 4, what is the value of K?

(1) The units digit of K is the least common multiple of the tens and hundreds digit of K. K could be 444, <u>488</u>, <u>848</u>, or 888. Not sufficient.

(2) K is NOT a multiple of 3. K could be 448, 484, <u>488</u>, 844, <u>848</u>, or 884. Not sufficient.

(1)+(2) From above K could be 488 or 848. Not sufficient.

Answer: E.

5. If a, b, and c are integers and a < b < c, are a, b, and c consecutive integers?

Note that:
A. The factorial of a negative number is undefined.
B. 0!=1.
C. Only two factorials are odd: 0!=1 and 1!=1.
D. Factorial of a number which is prime is 2!=2.

(1) The median of {a!, b!, c!} is an odd number. This implies that b!=odd. Thus b is 0 or 1. But if b=0, then a is a negative number, so in this case a! is not defined. Therefore a=0 and b=1, so the set is {0!, 1!, c!}={1, 1, c!}. Now, if c=2, then the answer is YES but if c is any other number then the answer is NO. Not sufficient.

(2) c! is a prime number. This implies that c=2. Not sufficient.

(1)+(2) From above we have that a=0, b=1 and c=2, thus the answer to the question is YES. Sufficient.

Answer: C.

6. Set S consists of more than two integers. Are all the integers in set S negative?

(1) The product of any three integers in the set is negative. If the set consists of only 3 terms, then the set could be either {negative, negative, negative} or

{negative, positive, positive}. If the set consists of more than 3 terms, then the set can only have negative numbers. Not sufficient.

(2) The product of the smallest and largest integers in the set is a prime number. Since only positive numbers can be primes, then the smallest and largest integers in the set must be of the same sign. Thus the set consists of only negative or only positive integers. Not sufficient.

(1)+(2) Since the second statement rules out {negative, positive, positive} case which we had from (1), then we have that the set must have only negative integers. Sufficient.

Answer: C.

7. Is x the square of an integer?

The question basically asks whether x is a perfect square (a perfect square, is an integer that is the square of an integer. For example 16=4^2, is a perfect square).

Perfect square always has **even powers of its prime factors**. The reverse is also true: *if a number has even powers of its prime factors then it's a perfect square*. For example: $36=2^2*3^2$, powers of prime factors 2 and 3 are even.

(1) When x is divided by 12 the remainder is 6. Given that $x=12q+6=6(2q+1)=2*3*(2q+1)$. Now, since 2q+1 is an odd number then the power of 2 in x will be odd (1), thus x cannot be a perfect square. Sufficient.

(2) When x is divided by 14 the remainder is 2. Given that $x=14p+2$. So, x could be 2, **16**, 30, ... Thus, x may or may not be a perfect square. Not sufficient.

Answer: A.

8. Set A consist of 10 terms, each of which is a reciprocal of a prime number, is the median of the set less than 1/5?

(1) Reciprocal of the median is a prime number. If all the terms equal 1/2, then the median=1/2 and the answer is NO but if all the terms equal 1/7, then the median=1/7 and the answer is YES. Not sufficient.

(2) The product of any two terms of the set is a terminating decimal. This statement implies that the set must consists of 1/2 or/and 1/5. Thus the median could be 1/2, 1/5 or (1/5+1/2)/2=7/20. None of the possible values is less than 1/5. Sufficient.

Answer: B.

Theory:

Reduced fraction $\frac{a}{b}$ (meaning that fraction is already reduced to its lowest term) can be expressed as terminating decimal *if and only* b (denominator) is of the form 2^n5^m, where m and n are non-negative integers. For example: $\frac{7}{250}$ is a terminating decimal 0.028, as 250 (denominator) equals to $2*5^2$. Fraction $\frac{3}{30}$ is also a terminating decimal, as $\frac{3}{30}=\frac{1}{10}$ and denominator $10=2*5$.

Note that if denominator already has only 2-s and/or 5-s then it doesn't matter whether the fraction is reduced or not.

For example $\frac{x}{2^n5^m}$, (where x, n and m are integers) will always be the terminating decimal.

We need reducing in case when we have the prime in denominator other then 2 or 5 to see whether it could be reduced. For example fraction $\frac{6}{15}$ has 3 as prime in denominator and we need to know if it can be reduced.

Questions testing this concept:
does-the-decimal-equivalent-of-p-q-where-p-and-q-are-89566.html
any-decimal-that-has-only-a-finite-number-of-nonzero-digits-101964.html
if-a-b-c-d-and-e-are-integers-and-p-2-a3-b-and-q-2-c3-d5-e-is-p-q-a-terminating-decimal-125789.html
700-question-94641.html
is-r-s2-is-a-terminating-decimal-91360.html
pl-explain-89566.html
which-of-the-following-fractions-88937.html

9. If [x] denotes the greatest integer less than or equal to x for any number x, is [a] + [b] = 1 ?

Given that some function [] rounds DOWN a number to the nearest integer. For example [1.5]=1, [2]=2, [-1.5]=-2, ...

(1) ab = 2. First of all this means that a and b are of the same sign.

If both are negative, then the maximum value of [a] + [b] is -2, for any negative a and b. So, this case is out.

If both are positive, then in order [a] + [b] = 1 to hold true, must be true that [a]=0 and [b]=1 (or vise-versa). Which means that $0\leq a<1$ and $1\leq b<2$ (or vise-versa). But in this case ab cannot be equal to 2. So, this case is also out.

We have that the answer to the question is NO. Sufficient.

(2) 0 < a < b < 2. If a=1/2 and b=1, then [a] + [b] = 0 + 1 = 1 but if a=1/4 and b=1/2, then [a] + [b] = 0 + 0 = 0. Not sufficient.

Answer: A.

10. If N = 3^x*5^y, where x and y are positive integers, and N has 12 positive factors, what is the value of N?

$N=3^x*5^y$ has 12 positive factors means that (x+1)(y+1)=12=2*6=6*2=3*4=4*3. We can have the following cases:

$N=3^1*5^5$;
$N=3^5*5^1$;
$N=3^2*5^3$;
$N=3^3*5^2$.

(1) 9 is NOT a factor of N. This implies that the power of 3 is less than 2, thus N could only be 3^1*5^5. Sufficient.

(2) 125 is a factor of N. This implies that the power of 5 is more than or equal to 3, thus N could be 3^1*5^5 or 3^2*5^3. Not sufficient.

Answer: A.

THEORY.
Finding the Number of Factors of an Integer

First make prime factorization of an integer $n=a^p*b^q*c^r$, where a, b, and c are prime factors of n and p, q, and r are their powers.

The number of factors of n will be expressed by the formula $(p+1)(q+1)(r+1)$. **NOTE:** this will include 1 and n itself.

Example: Finding the number of all factors of 450: $450=2^1*3^2*5^2$

Total number of factors of 450 including 1 and 450 itself is $(1+1)*(2+1)*(2+1)=2*3*3=18$ factors.

11. If x and y are positive integers, is x a prime number?

(1) |x - 2| < 2 - y . The left hand side of the inequality is an absolute value, so the least value of LHS is zero, thus 0 < 2 - y, thus y < 2 (if y is more than or equal to 2, then $y-2\leq 0$ and it cannot be greater than |x - 2|). Next, since given that y is a positive integer, then y=1.

So, we have that: $|x-2|<1$, which implies that $-1<x-2<1$, or $1<x<3$, thus $x=2=prime$. Sufficient.

(2) x + y - 3 = |1-y|. Since y is a positive integer, then $1-y\leq 0$, thus $|1-y|=-(1-y)$. So, we have that $x+y-3=-(1-y)$, which gives $x=2=prime$. Sufficient.

Answer: D.