Two very basic principles of combinatorics:

* "AND" --> Multiply
* "OR" --> Add (provided that the different situations are mutually exclusive)

... can't happen together -- i.e., only ONE of them can happen

Basic illustrations:

* I have 4 shirts and 5 pairs of pants. How many outfits?

Basic idea: You have to choose a shirt $\boxed{\text{AND}}$ a pair of pants.

(multiply)

$4 \times 5 = 20$

* A woman has 4 long dresses and 5 short dresses. How many outfits?

Basic idea: You have to choose a long dress $\boxed{\text{OR}}$ a short dress.

mutually exclusive

ADD

$4 + 5 = 9$

Quick test of your understanding of "ORDER MATTERS"

An advertiser wants to pick five pages in a symphony program, in which to place advertisements. (The pages in the program are numbered 1-16.)

DOES ORDER MATTER?

The operative question is: IF I SCRAMBLE THE ELEMENTS, DO I GET A DIFFERENT OUTCOME?

* The advertiser picks pages 1, 3, 7, 8, and 11
* The advertiser pickes pages 3, 11, 7, 1, and 8

these are the same outcome!

It's also possible for order NOT to matter WITHIN SOME SUBSETS.

An advertiser wants to pick five pages from pages 1-16 in a symphony program.
On three of the pages, there will be half-page ads.
On two, there will be quarter-page ads.

In this case:

The order WITHIN THE 3 PAGES CHOSEN FOR 1/2 ADS doesn't matter.
The order WITHIN THE 2 PAGES CHOSEN FOR 1/4 ADS doesn't matter.
But -- this is NOT a set of 5 elements in which order doesn't matter at all.

ORDER MATTERS. (codes FLRIA and IFALR are different)

A 5-letter code must be chosen from the letters in the word FLORIDA.

No repetition of letters is allowed.

How many different such codes are possible?

1 * Create SLOTS for each decision that must be made.

5 slots -- because 5 decisions (each letter of the code)

___ ___ ___ ___ ___

2 * FILL the slots with the numbers of options

$\underline{7}$ $\underline{6}$ $\underline{5}$ $\underline{4}$ $\underline{3}$

or, $\underline{5}$ (etc.) $\underline{3}$ $\underline{6}$ (pick next) $\underline{7}$ (pick first) $\underline{4}$

IMPORTANT TAKEAWAY:

You don't have to pick things in the given order.

In many problems, you are better off picking in a DIFFERENT order.

3 * MULTIPLY the numbers.

5 x 4 = 20
x 3 = 60
x 6 = 360
x 7 (in longhand) = $\boxed{2520}$

Be sure to multiply in the easiest possible order.

A 5-letter code must be chosen from the letters in the word FLORIDA.

No repetition of letters is allowed.

The first and last letters of the code must be consonants (non-vowels)

How many different such codes are possible?

* solve the problem

* wait to type your answers until i'm back

1 * Create a SLOT for EACH decision.

4	6			???	<-- this is WRONG.

we don't know whether this is a vowel or a consonant ... so it's now impossible to fill in the last blank.

2 * Fill in the slots with the numbers of options.

4	5	4	3	3	<--- correct approach
C 1st PICK				C 2nd PICK	PICK THE RESTRICTED SLOTS FIRST, so that you'll know how many options you have.

3 * MULTIPLY: 4 x 5 x 3 = 60 ... 60 x 12 = $\boxed{720}$

TAKEAWAY:

You MUST pick the elements in an order such that the numbers of options are ALWAYS THE SAME.

Now, let's look at this: the default meaning of the word "set" is that ORDER DOES NOT MATTER. (E.G. F, L, R, I, A L, F, R, A, I ...same set)

You are to choose a SET of 5 letters from the letters in FLORIDA (again, no repetition).

How many such sets are possible?

Consider the following:

FLORI
FLOIR ...if these are CODES, then they're all different!
FLRIO (there are 5! = 120 of them)
...
etc ...if these are SETS, then ALL of them are the same 1 set.
... so, there are WAY more codes.
IROLF

IMPORTANT FACT:
There are n! different ways to scramble the elements in a set of n things.

In fact: for every SET, there are exactly 5! different CODES (since you can scramble the letters).

so: # of codes (in which ORDER MATTERS) $7 \times 6 \times 5 \times 4 \times 3$

therefore

# of sets (in which ORDER DOESN'T MATTER) $\dfrac{7 \times 6 \times 5 \times 4 \times 3}{\boxed{5!}}$

since there are 5 interchangeable items.

$$\frac{7 \times 6 \times \cancel{5 \times 4 \times 3}}{\cancel{5 \times 4 \times 3} \times 2 \times 1} = 7 \times 3 = 21$$

Now, let's look at this:

You are to choose a SET of 3 consonants and 2 vowels from the letters in FLORIDA (again, no repetition).

How many such sets are possible?

let's use the slot method:

1 * make slots for each decision

2 * fill in with #s of possibilities

3 * multiply

4 * divide by factorials of the #s of interchangeable elements.

$$\left(\frac{\overset{c}{\underline{4}}\ \ \overset{c}{\underline{3}}\ \ \overset{c}{\underline{2}}}{3!}\right)\left(\frac{\overset{v}{\underline{3}}\ \ \overset{v}{\underline{2}}}{2!}\right)$$

A committee of three people is to be chosen from four married couples. What is the number of different committees that can be chosen if two people who are married to each other cannot both serve on the committee?

A 16

B 24

C 26

D 30

E 32

If you normally use the "C" or "P" formulas, BE AWARE THAT THOSE FORMULAS ONLY WORK ON PROBLEMS IN WHICH YOUR CHOICES ARE NOT RESTRICTED.
Also, the slot method will solve ALL problems that can be solved by the formulas.

A committee of three people is to be chosen from four married couples. What is the number of different committees that can be chosen if two people who are married to each other cannot both serve on the committee?

Because of this RESTRICTION, you can't use the formulas.
Each time you pick someone, THAT person is disqualified ... as is his/her SPOUSE

1 * make SLOTS for the individual decisions.

2 * fill in the slots with NUMBERS OF OPTIONS.

3 * multiply

4 * if ORDER DOESN'T MATTER, then DIVIDE BY THE FACTORIAL(S) of the INTERCHANGEABLE SET(S).

$$\frac{8 \times \cancel{6} \times 4}{\cancel{3!}} = 32$$

another way to conceptualize:
"How many IDENTICAL POSITIONS TO FILL?"

A 16

B 24

C 26

D 30

$\boxed{\text{E } 32}$

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another way to conceptualize:
"How many IDENTICAL POSITIONS TO FILL?"

What's the answer to this problem if the restrictions are the same, but ONE member of the committee is now chosen as president?

$$\frac{8 \times 6 \times 4}{2!} \quad \text{(or, } 2!\ 1!\text{)}$$

there are only 2 positions that can be "scrambled" now.

you can think of this as "choose president, THEn choose the other two)

$$\left(8\right)\left(\frac{6 \times 4}{2!}\right)$$

...or you could conceive of this as "pick the non-presidential members and THEN the president"

$$\left(\frac{8 \times 6}{2!}\right)\left(4\right)$$

$\boxed{96}$

A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)

A 48

B 100

C 120

D 288

E 600

A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)

this problem has MULTIPLE POSSIBILITIES:

1 senior, 2 juniors; $\boxed{\text{OR}}$ 2 seniors, 1 junior; $\boxed{\text{OR}}$ 3 seniors

ORDER DOESN'T MATTER BETWEEN THE 2 JUNIORS

$$\underset{S}{\cancel{4}\,2} \cdot \underset{J}{6} \cdot \underset{J}{5} \,/\, \cancel{2!}$$

(OR 2! 1! if you want)

10 x 6

60

ORDER DOESN'T MATTER BETWEEN THE 2 SENIORS

$$\underset{S}{\cancel{4}\,2} \cdot \underset{S}{3} \cdot \underset{J}{6} \,/\, \cancel{2!}$$

6 x 6

36

ORDER DOESN'T MATTER AT ALL

$$\frac{4 \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3!}}$$

4

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B 100

C 120

D 288

E 600

60 + 36 + 4 = 100

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ORDER DOESN'T MATTER BETWEEN THE 2 JUNIORS | ORDER DOESN'T MATTER BETWEEN THE 2 SENIORS | ORDER DOESN'T MATTER AT ALL

$$\frac{\overset{S}{\cancel{4}\,2} \quad \overset{J}{6} \quad \overset{J}{5}}{\cancel{2!}} \qquad \frac{\overset{S}{\cancel{4}\,2} \quad \overset{S}{3} \quad \overset{J}{6}}{\cancel{2!}} \qquad \frac{4 \quad \cancel{3} \quad \cancel{2}}{\cancel{3!}}$$

(OR 2! 1! if you want)

10 x 6 | 6 x 6 | 4

60 | 36

A 48

B 100

C 120

D 288

E 600

60 + 36 + 4 = 100

with PROBABILITY, you can SUBTRACT FROM 1 to find the probability of the OPPOSITE EVENT.

with COMBINATORICS, you can SUBTRACT FROM ...

TOTAL NUMBER OF POSSIBILITIES! minus possibilities that you DON'T want: ALL JUNIORS

$$\frac{10 \times 9 \times 8}{3!} \quad \text{minus} \quad \frac{6 \times 5 \times 4}{3!} = 120 - 20 = 100$$

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(OR 2! 1! if you want)

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$\frac{\overset{S}{\cancel{4}}\,2 \times \overset{S}{3} \times \overset{J}{6}}{\cancel{2!}}$

6 x 6

36

ORDER DOESN'T MATTER AT ALL

$\frac{4 \times \cancel{3} \times \cancel{2}}{\cancel{3!}}$

4

WRONG solution:

Sr	Anyone	Anyone
4	9	8

?????

(2! in some situations; 3! in others)

A 48

B 100

C 120

D 288

E 600

60 + 36 + 4 = 100

with PROBABILITY, you can SUBTRACT FROM 1 to find the probability of the OPPOSITE EVENT.

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$\frac{10 \times 9 \times 8}{3!}$ minus $\frac{6 \times 5 \times 4}{3!}$ = 120 - 20 = 100