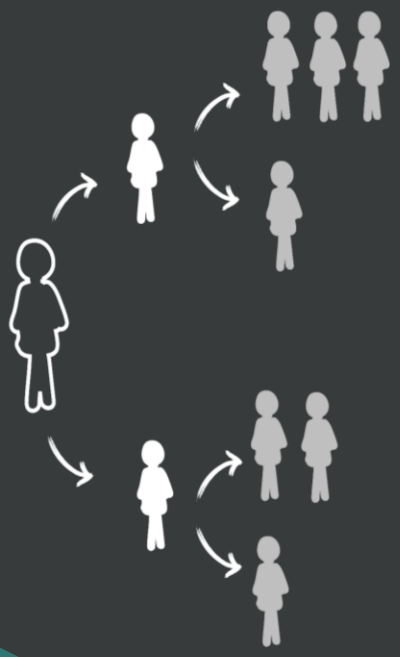
Fool-proof method to Differentiate between Permutation & Combination Questions



By e-gmat.com

Highlights of the previous article

In the previous article, we discussed:

- When to add and multiply by the keyword approach in Permutation and combination question.
- How to solve the permutation and combination question when keywords are not present.

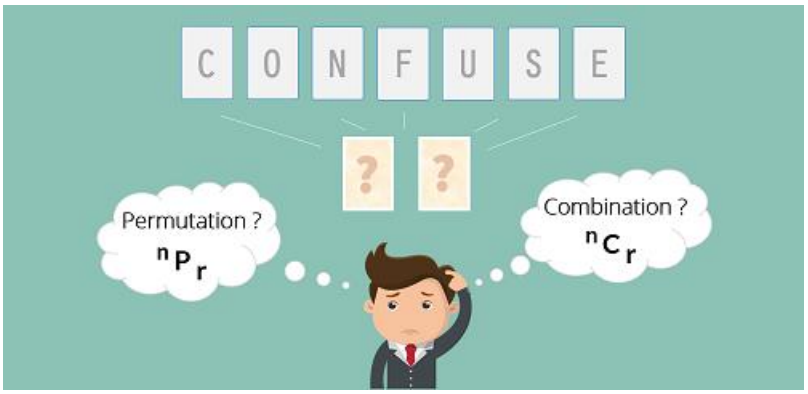
You can read the previous article here: Link

With the basic understanding of AND-OR keywords, let us dive into the advanced concept of-combination and permutation. Then we will apply the learnings to solve few GMAT-like questions.

Agenda of the article

In this article, we will discuss:

- How to figure out when to apply combination and when to apply permutation.
- Keyword approach to identify the combination or permutation type of questions.
- How to visualize a permutation and combination question if keywords are not given.



We will also provide few GMAT like practice questions to test the understanding.

A general case

In most of the p and c questions, we arrive at a point where we need to select or arrange a few things and many students fall prey to the same mistake of applying selection in place of arrangement and vice-versa.

To clarify this confusion, let us discuss two simple cases:

From 3 players, A, B, and C, how many doubles team can be formed?

From 3 letters, A, B, and C, how many 2-digit words can be formed?

Do both the examples look same to you?

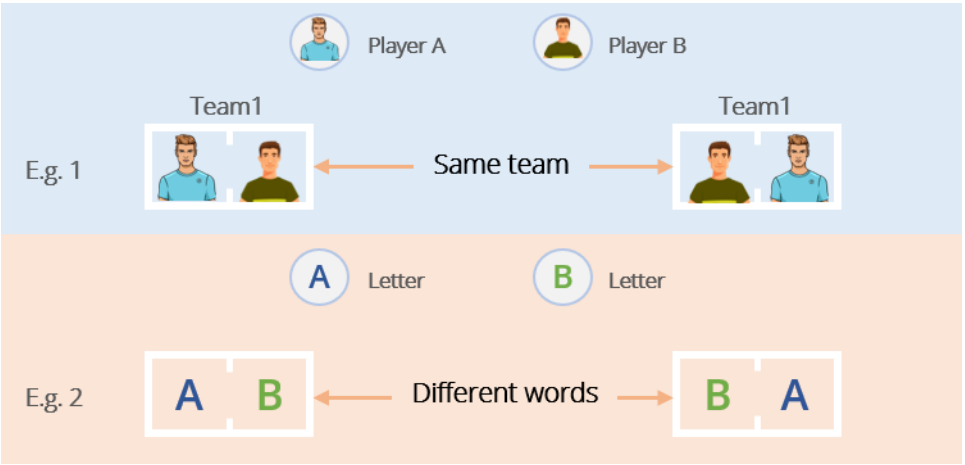
Well, the examples are not same.

From 3 players, A, B, and C, how many doubles team can be formed?

- The team (A B) is same as the team (B A).
- arrangement of the "team members" does not affect the team composition.

From 3 letters, A, B, and C, how many 2-digit words can be formed?

- the word AB is not same as word BA.
- the arrangement of the letters can give us two different words.



This simple example clearly shows that the understanding of combination and permutation can help to decide when arrangement matters and when selection matters.

Combination

Let us understand the concept of combination by solving example 1-

From 3 players, A, B, and C, how many doubles team can be formed?

Solution:

From 3 players A, B, and C, the teams of 2-players can be:

1. Team AB
2. Team AC
3. Team BC

Thus, we can have only 3 doubles teams from 3 players.

Now, instead of solving this manually, let us apply the keyword approach to solve this question.

Keyword Approach by e-GMAT

Let us list all the cases in which a doubles team can be formed.

- Select A and select B
- Select A and select C
- Select B and select C

Can you notice the keyword- "Select", in all the cases?

In all the above cases, the selection of 2 players is same as combination of 2 players only.

Therefore, we can infer that the keyword select is used for a combination question.

Per our understanding, the formula to select 'r' things from 'n' things, is ${}^{n}C_{r}$, which is equal to $= \frac{n!}{[(n-r)! \times r!]}$.

Thus, going by the above formula, we can conclude that the number of ways to select 2 players from 3 players is ${}^{3}c_{2} = \frac{3!}{[(3-2)! \times 2!]} = 3$ ways.

Can you see that getting answer by ${}^{n}C_{r}$ formula is actually easier than manually counting all the cases?

Let us look at some frequently used keywords that imply a combination question.

Important keywords to identify a combination question

Some of the important keywords are:

- Select
- Choose
- Pick
- Combination

Whenever you read a question, look for the above keywords as these are the useful indicators that clearly tells us that the question is a combination question.
Let us see the application of the above keywords in 2 practice questions.

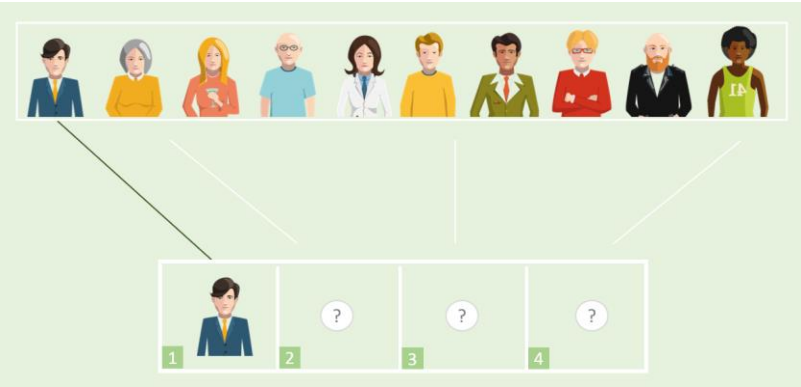
e-GMAT Example 1

In a society of 10 members, we have to select a committee of 4 members. As the owner of the society, John, is already a member of the committee. In how many ways the committee can be formed.

Solution:

Notice the highlighted keyword- select in the question.
Thus, this is a combination question. And for selection, we apply the ${}^{n}C_{r}$ formula to arrive at the answer.

- Now, in the question, we are asked to select a committee of 4 members from 10 members and John is already a part of the committee.
  - Thus, we have to select 3 members among 9 members.



- By the application of ${}^{n}C_{r}$ formula, we can select 3 members from 9 members in ${}^{9}c_{3}$ ways which is equal to $\frac{9!}{6! \times 3!}$ = 84 ways

Now, let us solve a slightly difficult question.

e-GMAT Example 2

An analyst will recommend a combination of 3 industrial stocks, 2 transportation stocks, and 2 utility stocks. If the analyst can choose from 5 industrial stocks, 4 transportation stocks, and 3 utility stocks, how many different combinations of 7 stocks are possible?

Solution:

Notice the highlighted keywords- Choose, Combinations.

Now, we can easily identify that this is selection question, right??

The analyst needs to form different combination of 7 different stocks. Can you visualize how can he do that?

Approach:

- He needs to select 3 industrial stocks out of 5 industrial stock **AND,**
- He needs to select 2 transportation stocks out of 4 transportation stocks **AND,**
- He needs to select 2 utility stocks out of 3 utility stocks.
    - Notice the keyword 'AND' which indicates all the above 3 events have to occur simultaneously.
    - Thus,

Total combinations	=	Ways to select 3 industrial stocks out of 5 industrial stock	×	Ways to select 2 transportation stocks out of 4 transportation stocks	×	Ways to select 2 utility stocks out of 3 utility stocks

By the application of ${}^{n}C_{r}$ formula, we can write:

- 3 industrial stocks out of 5 industrial stock can be selected in ${}^{5}C_{3} = 10$ ways.
- 2 transportation stocks out of 4 transportation stocks can be selected in ${}^{4}C_{2} = 6$ ways.
- 2 utility stocks out of 3 utility stocks can be selected in ${}^{3}C_{2} = 3$ ways.

Thus, the total ways to select 7 stocks = $10 \times 6 \times 3 = 180$ ways.

Key Takeaways

1. Keep an eye on the important keywords like- select, choose, combination in the question stem.
2. The number of ways to select 'r' things among 'n' things = ${}^{n}C_{r}$.

Now, let us see how permutation works.

Permutation

Let us understand the concept of permutation by solving example 2-

From 3 letters, A, B, and C, how many 2-letter words can be formed?

Solution:

The 2-letter words that can be formed from 3 letters A, B, and C are:

- AB
- BA
- AC
- CA
- BC
- CB

Thus, we can form 6 different words.

Can you observe that in combination, the selection of A and B gives only 1 team i.e. AB?

However, the selection of A and B gives 2 different words i.e. AB and BA.

This happens because the order of arrangement in case of words matters. But while creating teams, the team composition does not change whether we say AB or BA.

This arrangement is known as permutation.

Can you notice the usage of keyword- arrangement in permutation??

- If not, then keep note: The word arrangement in a question implies a permutation question.

Now, instead of solving this manually, let us apply the keyword approach to solve this question.

Keyword Approach by e-GMAT

Let us form all the cases in a different way.

In this way, we will first select the two letters and then we will arrange the selected letters.

- Select A and Select B
  - We now have two letters, A and B and we can arrange them in two different ways.
    - A then B
    - B then A
- Select A and Select C
  - We now have two letters, A and C and we can arrange them in two different ways.
    - A then C
    - C then A

- Select B and Select C
  - We now have two letters, B and C and we can arrange them in two different ways.
    - B then C
    - C then B

By counting all the cases, total 2 letter words= 6

Per our understanding, the formula to arrange 'r' things from 'n' things, is ${}^{n}P_{r}$ which is equal to

$= \frac{n!}{(n-r)!}$

Thus, going by the above formula, we can conclude that different 2-letter word = ${}^{4}P_{2} = \frac{4!}{(4-2)!}$

= 6 words.

Interesting facts

- From the above example, can you see that permutation is same as doing selection first **AND** then doing arrangement? (Notice- keyword: AND)
- Let us understand this mathematically
  - ${}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{n!}{(n-r)! \times r!} =$
  - Thus, we can simply write:

Permutation of 'r' things from 'n' things	=	Selection of 'r' things from 'n' things	×	Arrangement of all the selected 'r' things

- Hence, Permutation= ${}^{n}C_{r} \times r!$

Let us look at some frequently used keywords that imply a permutation question.

Important keywords to identify a combination question

Some of the important keywords are:

- Arrangements
- Ordered Ways
- Unique

Keep an eye on the above keywords in a question.

Whenever you get a question having the above three keywords, it will imply a permutation question.

Let us solve 1 question to understand the application of keywords.

e-GMAT Example 1

Each signal that a certain ship can make is comprised of 3 different flags hanging vertically in a particular order. How many unique signals can be made by using 4 different flags?

Solution:

Method 1:

Keyword Approach by e-GMAT

Notice the keyword, **unique,** in the question.

- Thus, we only have to apply ${}^{n}P_{r}$ formula to arrive at the answer.
- Hence, number of unique signals = ${}^{4}P_{3} = 24$

Method 2:

We can first select 3 different flags and then we can arrange them.

- Number of unique signals= ${}^{4}C_{3} \times 3! = 4 \times 3! = 24$

Key Takeaways

1. Look for the important keyword- arrangements, ordered ways, and unique to identify the permutation question.
2. The number of ways to arrange 'r' things from 'n' things = ${}^{n}P_{r}$.
3. An arrangement question can also be solved by first choosing 'r' things among 'n' things and then arranging all the 'r' things.

Visualizing the scenario when keywords are not present

At times, you can get a question that implicitly uses the application of permutation and combination. So, how do we determine whether the question is a combination question or a permutation question??

Let us understand this with the help of some examples:

e-GMAT Example 1

There are 8 teams in a certain league and each team plays with the other teams exactly once. What is the total number of games played in the league?

Solution:

This question does not include the important keywords then how should we solve this question????

- When we cannot find any keyword to identify whether the question is combination type or permutation type then we need to visualize the information provided to us in the question stem.

Let us visualize the information given in the question and see if we can identify the type of the question.

We are given:

- There are 8 teams in a league.

We know that each game is played between two teams.

- The match between team A and team B is same as the match between team B and team A.
- Thus, for each match to happen we must select 2 teams only and in this case, the arrangement will not matter.

Can you observe we arrived at the keyword **SELECT** by dissecting the given information carefully and making meaningful inferences.

Now, we only have to find the number of ways of selecting 2 teams from 8 teams.

- Hence, total number of games played = ${}^{8}C_{2}$ = 28 matches

Let us now increase the difficulty a bit and solve the next question.

e-GMAT Example 2

In a board meeting of the company, there are 10 members. In how ways 2 members can get the mandate for the post of CEO and COO of the company?

Solution:

We do not have any keyword in the question to directly identify the type of question and apply **^nC_r and ^nP_r** formula.

Thus, the next step to solve such type of questions is to visualize the scenario presented in the question.

- The question is about getting mandate for the post of CEO and COO of the company.
  - Let us suppose A and B are the top 2 vote-getters and hence, can get the mandate for the post of either CEO or COO.
  - Now, there can be 2 cases in which A and B can get the mandate.
    - **A-> CEO and B-> COO**
    - **B-> CEO and A-> COO**

- Can you see we have 2 different arrangements for the selection of 2 members only?
  - Thus, the arrangement after selection of 2 members implies a permutation question.

Thus, we can find the answer in 2 ways:

Method 1:

By applying the formula ^nP_r

- The number of ways in which 2 members from 10 members can get mandate for the post of CEO and COO of the company = $^{10}P_2 = 90$

Method 2:

By first selecting the 2 members from 10 members and then arranging the 2 members:

- Hence, total ways = Total ways of combination of 2 members × arranging the 2 members
- Total ways = $^{10}C_2 \times r! = 45 \times 2 = 90$

Takeaways from this article

1. Always keep an eye on the keywords used in the question. The keywords can help you get the answer easily.
2. The keywords like- selection, choose, pick, and combination- indicates that it is a combination question.
3. The keywords like- arrangement, ordered, unique- indicates that it is a permutation question.
4. If keywords are not given, then visualize the scenario presented in the question and then think in terms of combination and arrangement.

Upcoming e-GMAT articles on Permutation & Combination

Next week we will come up with the 3rd and final article in permutation and combination series.

You will learn how to avoid double counting: A common mistake made by a majority of student!

Stay tuned....

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- Click here to read the first article - Learn when to “Add” and “Multiply” in Permutation & Combination questions

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