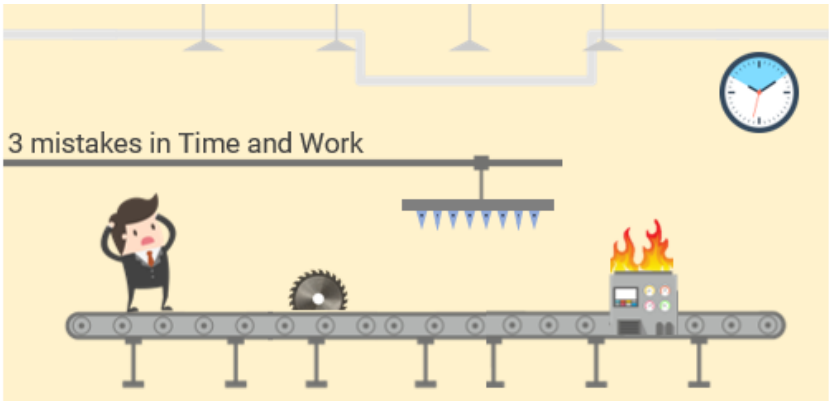
3 **Deadly mistakes** you must avoid in Time and Work problems

By e-gmat.com

3 deadly mistakes you must avoid in Time and Work problems



This is the 2nd and the final article on the topic Time and Work. If you have not read the previous article, we recommend you read that first before going through this article.

To read the article: Solve Time and Work Problems Efficiently using Efficiency Method!

Highlights of the previous article

In the previous article, we discussed:

- ✓ The application of LCM to solve Time and Work questions.
- ✓ A comparative study of methods of solving Time and Work problems – by using fraction and by using LCM method, which one should be used and when?
- ✓ How efficiency is defined and in what ways it can be implemented in problem solving.

Objective of this article

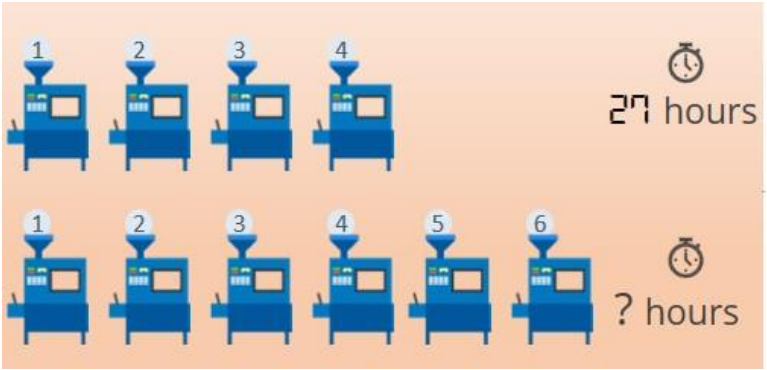
The primary objective of this article is to cite 3 most common mistakes made by students while solving problems on Time and Work. We will demonstrate them with different examples on each type. For students' convenience, we have referred to examples similar to what are provided in the official resources.

Common mistake type 1: Incorrect use of the relationship between efficiency and time taken

Example 1

Six machines at a certain factory operate at the same constant rate. If four of these machines, operating simultaneously, take 27 hours to fill a certain production order, how many fewer hours does it take all six machines, operating simultaneously, to fill the same production order?

Solution:



Common approach used by students:

In this question it is given that

- 4 machines take 27 hours to fill a certain production order
- All machines operate at the same constant rate

We need to find out the difference in time if 6 machines work on the same production order.

Now, it is given that,

- 4 machines take 27 hours to fill the production order
    - Hence, 1 machine will take $\frac{27}{4}$ hours to fill the same production order
    - Therefore, 6 machines will take $\frac{27}{4} \times 6$ hours = 40.5 hours to fill the production order
- Hence, the difference in time taken = (40.5 – 27) hours = 13.5 hours

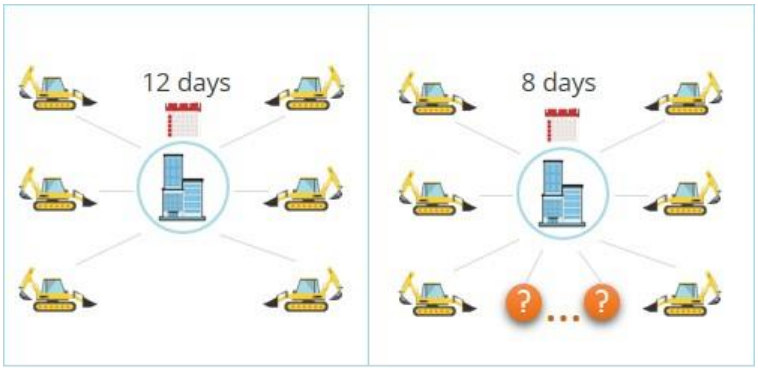
If you have got 13.5 hours as the answer, then you have solved it incorrectly.

Let us take another similar example before discussing the mistake and the solution.

Example 2

Six machines, each working at the same constant rate, together can complete a certain job in 12 days. How many additional machines, each working at the same constant rate, will be needed to complete the job in 8 days?

Solution:



Common approach used by students:

In this question, it is given that:

- 6 machines can complete a certain job in 12 days
- All machines work at the same constant rate

We need to find out the number of additional machines required to complete the same job in 8 days.

Now it is given that,

- 6 machines can complete the job in 12 days
    - Hence, 1 machine can complete the job in $\frac{12}{6}$ days = 2 days
- If 1 machine can complete the job in 2 days, we need $\frac{8}{2}$ = 4 machines to complete the job in 8 days

If you have got 4 as the answer, then you have solved it incorrectly.

The mistakes and the correct approach:

The error made in both the questions are very similar in nature, and the most common type of error that students make while solving. Let us look into the detail of them:

Error in Question 1:

In the first question, when we are calculating the time taken by 1 machine to complete the production order, we have made a logical error.
It is given that all machines operate at the same working rate.

- Therefore, if 4 machines can fill the production order in 27 hours, logically 1 machine should take 4 times the time taken by 4 machines to fill the same production order.
    - This is happening because the efficiency of 1 individual machine is $\frac{1}{4}$ times of the combined efficiency of 4 machines.
    - Because, efficiency and time taken are inversely proportional to each other, the time taken by 1 machine should be 4 times of the time taken by the 4 machines combined.

Error in Question 2:

The solution of this question contains 2 errors.

Error 1: Same as the previous question, if 6 machines can complete the job in 12 days, then 1 machine will be completing it by taking 6 times the original time.

Error 2: Moreover, if 1 machine can complete the job in 2 days, you don't need 4 machines to complete the job in 8 days – as 1 machine itself can complete it in 2 days' time.

Common mistake type 2: The Incorrect Approach

Example 3

At his regular hourly rate, Don had estimated the labor cost of a repair job as $336 and he was paid that amount. However, the job took 4 hours longer than he had estimated and, he earned $2 per hour less than his regular hourly rate. What was the time Don had estimated for the job, in hours?

A. 28

B. 24

C. 16

D. 14

E. 12

Solution:

Information given in the question

This is one of those questions which, if approached incorrectly, can be highly time-consuming and increases the probability of doing it wrong.

In this question, it is mentioned that:

- Don had estimated the labor cost of a repair job as $336, which he was paid
- The job took 4 hours longer than originally estimated
- As a result, Don earned $2 less than his original estimated hourly rate

We need to find out the originally estimated time to finish the job.

Common approach used by students

This question deals with two separate variables:

- Number of hours estimated to complete the job – assuming it to be x
- The payment per hour of job – assuming it to be y

Now, as per the given information, Don got paid $336 for his estimated time and payment rate.

Hence, we can write,

- xy = 336 ..... ①

As he took 4 hours more than the original estimated time, he got paid $2 less in every hour.

Hence, we can write,

- (x + 4) (y – 2) = 336 ..... ②

  Simplifying, we get xy – 2x + 4y – 8 = 336

  Replacing the value of xy from equation 1, we get 336 – 2x + 4y – 8 = 336

  Or, -2x + 4y = 8

  Or, 4y – 2x = 8

  Or, 2y – x = 4

Many students get stuck at this point. As they have already replaced the value of xy from the first equation, they feel that they have only one equation with two unknown variables – hence, the question cannot be solved.

If observed closely, one can still solve the equation by replacing the value of either x or y from the 1st equation into this derived equation to get the answer.

From equation 1 we can write:

- $x = \frac{336}{y}$

replacing the value of x in the derived equation, we get:

- $2y - \frac{336}{y} = 4$

  Or, $2y^2 - 336 = 4y$

  Or, $2y^2 - 4y - 336 = 0$

  Or, $y^2 - 2y - 168 = 0$

  Or, $y^2 - 14y + 12y - 168 = 0$

  Or, $(y - 14)(y + 12) = 0$

  As y cannot be negative, $y = 14$

- Replacing the value of y in the equation $x = \frac{336}{y}$, we get $x = 24$

Now, this whole calculation may take longer time than expected and there exist high chances of making errors in different steps involved in the calculation.

Important Observations

- Primarily this question does not need the usage of two variables. The question asks for the estimated time required for the job and only this can be assumed as the variable for the solving purpose.
- While solving the values for x and y, the value of y should not be solved. The question asks for x only, therefore, x should be solved on the first attempt.

Correct approach and solution

We can assume the estimated time for Don to complete the job is t hours.

- Therefore, the original estimated earnings per hour = $\frac{336}{t}$
- As Don took 4 hours extra, his earnings per hour becomes $\frac{336}{t+4}$

We know, the new earnings per hour is \$2 less than the originally estimated earnings per hour
Hence, we can write:

- $\frac{336}{t} - \frac{336}{t+4} = 2$
  Simplifying, we get $t^2 + 4t - 672 = 0$

- At this point one can also form equation and solving that to get the answer. However, if solving becomes difficult, one can also replace the options in place of t in the equation $\frac{336}{t} - \frac{336}{t+4} = 2$, to get the answer.

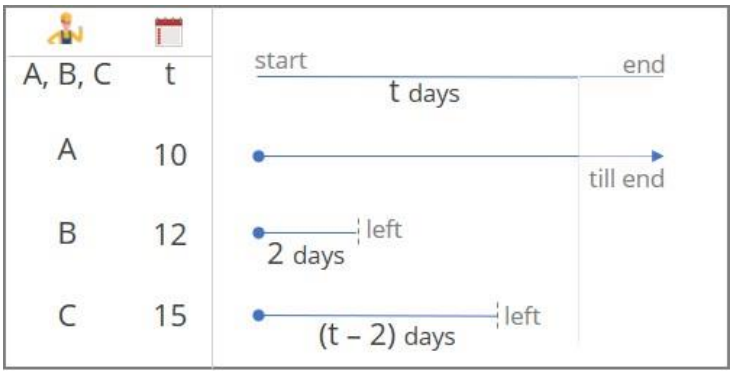
The value of t = 24 will satisfy the equation, hence, the answer is 24 hours.

Common mistake type 3: Incorrect interpretation of the question statement

Example 4

A, B, and C, working individually, can complete a job in 10 days, 12 days, and 15 days respectively. All of them started working on the job together. B left after 2 days. C left 2 days before the originally scheduled number of days at which the job was supposed to be completed. In how many days the job completed?

Solution:



Common approach used by students

In this question, it is given that:

- A, B, and C can complete a certain job in 10, 12, and 15 days respectively, while working individually
- All of them started working on the job together
- B left after working for 2 days
- C left 2 days before the originally scheduled work completion time

We need to find out the total number of days at which the work gets completed.

Now, we can assume the total work to be LCM (10, 12, 15) = 60 units

- A can complete 60 units of work in 10 days
  - Hence, A can complete 6 units of work in 1 day
- B can complete 60 units of work in 12 days
  - Hence, B can complete 5 units of work in 1 day
- C can complete 60 units of work in 15 days
  - Hence, C can complete 4 units of work in 1 day

Now, let's assume the total work gets completed in d days

Hence, we can say

- A works for d days
- B works for 2 days
- C works for (d - 2) days

As we know the work done by them individually in 1 day, we can calculate the total work done by them.

- Work done by A = 6 × d units = 6d units
- Work done by B = 5 × 2 units = 10 units
- Work done by C = 4 × (d – 2) units = 4d – 8 units

Therefore, 6d + 10 + 4d – 8 = 60

Solving, 10d = 58

Or, d = 5.8 days

Hence, the total work gets completed in 5.8 days

If you have got 5.8 days as the answer, then you have solved it incorrectly.

The mistake and the correct approach

Here the mistake lies in the wrong interpretation of the question statement.

In the solution we assumed that the total number of days taken to complete the job is d days. From that, it is inferred that the number of days C worked for is (d – 2) days.

However, in the question it was mentioned that *C left 2 days before the originally scheduled number of days at which the job was supposed to be completed*.

- One need to notice that d days doesn't indicate the originally scheduled work completion time, rather it indicates the modified work completion time
- The originally scheduled work completion time indicates the time in which the work would have been completed if all A, B, and C worked together till last

Now, the total work done by all 3 of them together in 1 day = (6 + 5 + 4) units = 15 units

Hence, the original scheduled work completion time = $\frac{60}{15}$ days = 4 days

Therefore, C worked for (4 – 2) days = 2 days in total

Hence, we can say

- Work done by A = 6 × d units = 6d units
- Work done by B = 5 × 2 units = 10 units
- Work done by C = 4 × 2 units = 8 units

Therefore, 6d + 10 + 8 = 60

Solving, 6d = 42

Or, d = 7 days

The total work gets completed in 7 days' time

Key Takeaways from the article

1. For any person or machine, efficiency and work rate are always inversely proportional to each other. Hence, less number of resources working on a specific job always take more time compared to more number of resources working on the same job

2. While solving, one needs to focus on the exact parameter which the question is asking for. Sometimes assuming more than what required can complicate the calculation and increases the chance of making error.

3. While making inferences from the question stem, one needs to decipher the correct meaning of the given statements, and ensure the details are considered while solving – ignoring which may lead into wrong interpretation and incorrect answer.

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