3 **Must Learn** Applications of Percentages

By e-gmat.com

3 Must Learn Applications of Percentages

This is the 2nd and the final article on the topic of percentages and its applications. If you have not read the previous article, we recommend you read that first before going through this article.

To read the article: 3 Major Concerns Students Face in Solving Percentage Problems

Highlights of the Previous Article

In the previous article, we discussed:

- How to make a correct choice of base, while calculating a percentage or change of percentage.
- The effect of simultaneously percentage increase and decrease on the value of a certain element – and how to calculate that.
- How to optimise the solution of a percentage question, by using the fractional equivalent of percentage.

Objective of this article

The primary objective of this article is to:

- cite 3 specific cases, where the use of percentages is different from the mostly used conventional approaches.

In the explanation, we have considered cases from two different topics:

- 2 specific cases in the context of sales and purchases
- 1 specific case in the context of savings and interest

to demonstrate the important takeaways.

Case 1: Profit/Loss Percentage Calculation with Respect to a Different Base

While finding the percentage increase or decrease in a question, most of the students are confused while writing the formula for percentage change. A mistake while writing the wrong value of the numerator and denominator will lead an incorrect answer and we must learn to avoid this at all cost!

Let us take a simple example and understand what are the different mistake that we end up making and how should we avoid them.

Example 1

e-GMAT example with explanation

A shopkeeper bought an article at a wholesale price of $80. He sold the article at certain profit after providing discount on the original retail price. If both the discount percentage as well as his profit percentage over the original retail price is 16.67%, find the selling price of the article.

Solution:

Understanding the Question:

This question provides us all the necessary information in a very short and crisp manner. As per the context, this question provides the change in price of an article from the wholesale price to the original retail price to finally the selling price.

The provided information:

- *Wholesale price* of the article: $80
- Final selling price is obtained after providing a *discount of 16.67%* over the *original retail price*
- The *profit percentage is 16.67%*, when calculated based on *the original retail price*

We need to find out the selling price of the article.

Important Observation:

In general, when we calculate the profit or loss percentage of any item, it is always based on the cost price by default. However, if we read the question carefully, we can see the profit of 16.67% is calculated on the basis of the original retail price, which is nothing but the marked price or list price of the article.

In such cases, we need to change the conventional formula of calculating the profit or loss percentage, as the base of the percentage calculation becomes different.

Let's look at the *approach* of solving the question:

The Correct Approach and Solution:

Let us assume the original retail price of the article is x
It is given that the *wholesale price of the article is $80*
As the *discount percentage is 16.67%* of the original retail price,

- The amount of discount = $\frac{1}{6}$ of original retail price

  Or, the amount of discount = $\frac{1}{6} \times x = \frac{x}{6}$

We know, *selling price is equal to original retail price – discount amount*

- Therefore, selling price of the article = $x - \frac{x}{6} = \frac{5x}{6}$

It is also given that, the profit percentage, when calculated over the original retail price, is 16.67%
As we have already assumed the value of the original retail price to be x, we can say

- Profit amount = 16.67% of the original retail price

  Or, profit amount = $\frac{1}{6} \times x = \frac{x}{6}$

Now, as the wholesale purchase price of the article is 80 and the profit amount is $\frac{x}{6}$, we can write

- Wholesale purchase price + profit amount = selling price

  Or, selling price = $80 + \frac{x}{6}$

From the two obtained equations of selling price,

- $\frac{5x}{6} = 80 + \frac{x}{6}$

OR, $\frac{5x}{6} - \frac{x}{6} = 80$

OR, $\frac{4x}{6} = 80$

OR, x = 120

Therefore, we can say the original retail price of the article is $120

As we already know the discount is $\frac{1}{6}^{th}$ of the original retail price,

Discount amount = $120 \times \frac{1}{6} = 20$

Therefore, the selling price = 120 – 20 = 100

[we can find the selling price from the wholesale price also. As we know the value of x, we can say

The selling price $= 80 + \frac{x}{6} = 80 + \frac{120}{6} = 80 + 20 = 100$*]*

Also, the profit amount $= \frac{x}{6} = \frac{120}{6} = 20$

Key Takeaways

- By default, the percentage of profit or loss is always calculated based on the cost price. However, if the question statement mentions otherwise, one needs to change the calculation base as per that.
    - Like in this case, the profit percent of 16.67% has been calculated, taking the original retail price as the base of calculation.

      Or, $\frac{\text{Profit Amount}}{\text{Original Retail Price}} \times 100 = 16.67$

- The commonly used terms – cost price and marked price (list price) – are changed into wholesale purchase price and original retail price. One needs to understand that these terms denote the same entity as the commonly used terms.

Case 2: The Successive Percentage Change

While solving questions on percentages, many-a-times students face situations where they need to calculate successive percentage changes on a single value. While it is possible to calculate every individual change to obtain the final value, the process of doing so also becomes lengthy, time consuming, and increases the chance of making error. To avoid this, one can simplify the process by doing the calculation of successive percentage changes in an easier way.

Let us take an example to demonstrate the case of successive percentage change, in the context of the following problem on Sales and Purchase.

Example 2

e-GMAT example with explanation

When the new online movie channel Amajon Prime was launched, an initial discount of 20% was given to the customers on the 6 months' subscription fee. Due to poor response, the company decided to apply further discount of 30% for the next 6 months' fee. In the next year the company moved to a flat 50% discount criteria for whole 1-year subscription. What will be the profit or loss for a consumer in the next year, if the listed subscription fee is $20 per month in both years?

Solution:

Understanding the Question:

As per the context, this question provides us 2 yearly scenarios and the different discount schemes on those 2 years when a consumer goes for the yearly subscription of the movie channel.
The provided information:

- In the first 6 months of the 1st year, 20% discount was given on the subscription fee.
- In the remaining 6 months of the 1st year, an additional discount of 30% was given on the already discounted fee.

- In the 2nd year, the company moved to a flat discount scheme of 50% for one whole year of subscription.
- The listed subscription fee in both the years is $20 per month in both years.

We need to find out the amount of profit or loss for a consumer in the 2nd year's subscription, compared to the 1st year's subscription.

Important Observation:

There are three specific cases of discount percentages one need to consider separately.

- First 6 months in year 1: discount percentage is 20%
- Last 6 months in year 1: 30% **more** discount over the previous discounted price
- Second year: flat 50% discount throughout the whole year

In the second phase of year 1, the 30% discount is applicable over the previous discount of 20%.

Note that this does not mean a cumulative discount of 50%.

Let's look at the approach of solving the question:

The Correct Approach and Solution:

For the first 6 months of year 1,

The listed subscription fee is $20 per month, and the discount amount is 20%

- Hence, amount to be paid per month = 20 – 20% of 20 = 20 – 4 = **16**

For the last 6 months of year 1,

A further discount of 30% was applicable

- Hence, amount to be paid per month = 16 – 30% of 16 = 16 – 4.8 = **11.2**

Therefore, considering year 1 only,

- The total savings for a consumer = 6 (20 – 16) + 6 (20 – 11.2) = 6 (4 + 8.8) = 6 × 12.2 = **73.2**

For all the months in year 2,

The listed subscription fee is $20 per month, and the discount amount is 50%

- Hence, amount to be paid per month = 20 – 50% of 20 = 20 – 10 = **10**

Therefore, considering the whole 2nd year,

- The total savings for the customer = 12 (20 – 10) = 12 × 10 = **120**

As the savings in year 2 is more than the cumulative savings in year 1,

- The profit for consumer in year 2 = 120 – 73.2 = **46.8**

Inputs on Calculating Successive Percentage:

For the last 6 months of year 1, we can see there are two discount percentages applicable – first 20% and then again 30% on the previous discounted value.

In such cases where we have successive percentage change over the value of a single element, we can apply the following formula to find out the overall percentage change:

- If the value of an item is increased by a% first, followed by another b% increase, then the overall percentage increase can be calculated by **"**$a + b + \frac{ab}{100}$**"**
- For example, if the value of an item first increased by 10% and then again by 20%, the overall percentage increase will be $10 + 20 + \frac{10 \times 20}{100} = 32\%$

  We can also verify the result for the same.

  Assuming the initial value of the item as 100.

  After 10% increase, it will become = $100 \times 1.1 = 110$

  After further 20% increase, it will become = $110 \times 1.2 = 132$ (indicating an overall increase of 32%)
- The same formula will be equally applicable for the cases related to percentage decrease also. For example, if there is a% decrease in the place of a% increase, we should take the value of the percentage change as (-a) in place of a. Similarly, if both are indicating decrease (as in the case of two successive discounts), both a and b should be considered as negative, keeping the formula same. If the final result becomes negative, it indicates a percentage decrease.

Considering the question above, there are two successive decreases in the subscription fee – first 20% and then 30%.

- Therefore, the overall reduction percentage = $(-20) + (-30) + \frac{(-20) \times (-30)}{100} = -50 + 6 = -44$

  [negative sign indicates percentage decrease]
- As the overall decrease is 44%, the final value = 20 – 44% of 20

  = 20 – 8.8

  = 11.2

A Word of Caution:

For the last 6 months in year 1, there were two discount percentages which were simultaneously active – 20% and 30%. There is a tendency among students to replace these two percentage changes with a single cumulative percentage change of (20 + 30) = 50%. However, this approach is absolutely incorrect.

- As we have seen, two successive discounts of 20% and 30% are equivalent to a single discount of 44%, hence, those two discount values can never be replaced by a single cumulative discount of 50%

Key Takeaways

- If the value of an item has been changed by a% followed by b%, the overall percentage change is *not* equal to (a + b)%
- In case of a successive percentage change of a% and b% on a certain value, the overall cumulative percentage change can be calculated by

$$(a + b + \frac{ab}{100})\ \%$$

Also, in case of 3 successive percentage change, one can use this formula twice to get the final percentage change value.

Case 3: The Interest Calculation for a Specific Year

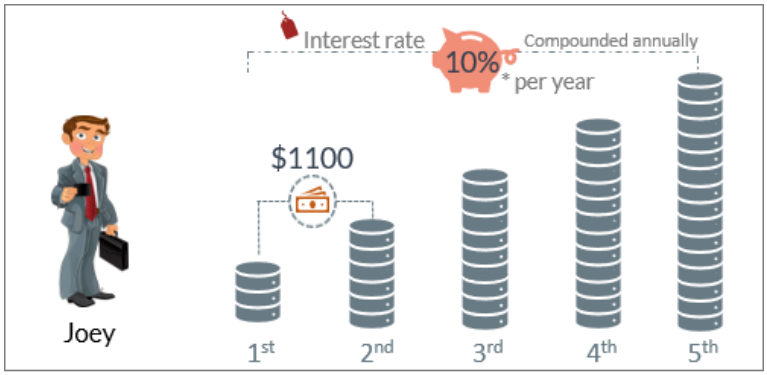
Example 3

e-GMAT example with explanation

Joey invested some amount in a scheme for 5 years, in which he was promised to get an interest of 10% per year, compounded annually. If the value of the investment at the beginning of year 2 is $1100, then find the difference between the interest earned by Joey for the 3rd year and the 4th year.

Solution:

Understanding the Question:



As per the context, this question is providing us a scenario where a certain investment is giving return at a fixed compounding rate.

The provided information:

- Joey invested the amount for 5 years
- The interest rate is 10% per year, compounded annually
- The value of the investment at the beginning of year 2 is $1100

We need to find out the difference between the interest earned in the 3rd year and the interest earned in the 4th year.

Important Observation:

Unlike most other questions of savings and interest, in this question we are asked about the interest earned in a specific year, and not the total interest earned till that year.

In a generalised way, we can define this as:

- Interest earned in the n^{th} year = total amount at the end of n years – total amount at the end of (n – 1) years

Correct Approach and Solution:

It is given that at the beginning of the 2nd year, the investment amounted to $1100

This is also equal to the amount at the end of the 1st year

As rate of interest (r) is 10%, if we assume the investment is P, then we can write

- $P (1 + \frac{r}{100})1 = 1100$

  Or, $P \times \frac{11}{10} = 1100$

  Or, $P = 1000$

Therefore, the amount that is invested by Joey is equal to $1000

Now, when we are trying to find out the interest for the 3rd year, we can write it as:

- 3rd year's interest = amount at the end of 3 years – amount at the end of 2 years

  Or, 3rd year's interest = $P (1 + \frac{10}{100})3 – P (1 + \frac{10}{100})2$

  Or, 3rd year's interest = $P (1 + \frac{10}{100})2 (1 + \frac{10}{100} – 1)$

  Or, 3rd year's interest = $P \times \frac{10}{100} \times (1 + \frac{10}{100})2 = \frac{P}{10} (1 + \frac{1}{10})2 = \frac{P}{10} (\frac{11}{10})2$

Similarly, when we are trying to find out the interest for the 4th year, we can write it as:

- 4th year's interest = amount at the end of 4 years – amount at the end of 3 years

  Or, 4th year's interest = $P (1 + \frac{10}{100})4 – P (1 + \frac{10}{100})3$

  Or, 4th year's interest = $P (1 + \frac{10}{100})3 (1 + \frac{10}{100} – 1)$

  Or, 4th year's interest = $P \times \frac{10}{100} \times (1 + \frac{10}{100})3 = \frac{P}{10} (1 + \frac{1}{10})3 = \frac{P}{10} (\frac{11}{10})3$

Therefore, the difference between the interest earned by Joey in the 3rd year and the 4th year:

- $\frac{P}{10}(\frac{11}{10})3 - \frac{P}{10}(\frac{11}{10})2$

$= \frac{P}{10}(\frac{11}{10})2\ [(\frac{11}{10}) - 1]$

$= \frac{P}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{1}{10}$

Replacing the value of P = 1000,

$= \frac{1000}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{1}{10}$

$= \frac{11 \times 11}{10}$

$= \frac{121}{10} = 12.10$

Hence, the difference in the interest earned by Joey in the 3rd and 4th year is $12.10

Inputs on Calculating the Interest on Specific Year:

If we look at the process carefully, we can see the interest earned in any specific year is calculated by:

- Amount at the end of that year – amount at the end of the previous year

We can actually generalise this into a single expression as follows:

Interest for the n^{th} year:

- Amount at the end of nth year – amount at the end of $(n-1)^{th}$ year

$= P(1 + \frac{r}{100})^n - P(1 + \frac{r}{100})^{n-1}$

$= P(1 + \frac{r}{100})^{n-1}(1 + \frac{r}{100} - 1)$

$\mathbf{= \frac{Pr}{100}(1 + \frac{r}{100})^{n-1}}$

Now, in this given expression, if we put n = 3 and r = 10, we get

- Interest earned in the 3^{rd} year $= P \times \frac{10}{100} \times (1 + \frac{10}{100})^2$

Similarly, if we put n = 4 and r = 10, we get

- Interest earned in the 4^{th} year $= P \times \frac{10}{100} \times (1 + \frac{10}{100})^3$

[Note that, in case of compound interest only, the interest for any two years will be different. In case of simple interest, the interest for any two years will always be same, and hence, can be calculated as:

- *Interest for 3^{rd} year = interest for 4^{th} year = interest for 1-year time = $\frac{Pr}{100}$]*

Key Takeaways

- For compound interest, where compounding is done annually, the interest earned in any specific year can be calculated using the expression *Amount at the end of nth year – amount at the end of (n-1)th year.* This can also be mathematically rewritten as follows:
    - $\frac{Pr}{100}(1+\frac{r}{100})^{n-1}$, where P is the amount invested, r is the rate of interest per annum, and n is the year of which we are trying to find the interest.

Key Takeaways from the article

- By default, profit/loss percentage is always calculated based on the cost price. However, if the question specifically mentions otherwise, one has to follow that defined base only to do the calculation.

- For successive percentage change of a% and b% on a single value, the overall percentage change can be calculated by using the formula $a + b + \frac{ab}{100}$
    - For any decrease in percentage, one needs to consider the negative percentage value in calculation [considering (-a) in place of a, if there is a decrease of a%]
    - if the final result is positive, it indicates percentage increase. However, if the final result is negative, it indicates percentage decrease.

- For any cases of compound interest, where compounding is done annually, the interest earned for any specific year can be calculated using the following:

  Interest for the nth year = $\frac{Pr}{100}(1+\frac{r}{100})^{n-1}$, where,
    - P = principal, r = rate of interest per annum

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