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Arithmetic Flashcards

(watch video)

Real number: any number that can be shown on the number line

$a + b = b + a$

$a \times b = b \times a$

$(a + b) + c = a + (b + c)$

$(a \times b) \times c = a \times (b \times c)$

$a(b + c) = ab + ac$

$1 \times a = a$

$a \div 1 = a$

$a \times 0 = 0$

$a + 0 = a$

$a \div a = 1 \quad (\text{as long } a \neq 0)$

Arithmetic Flashcards

(watch video)

Adding a positive number

- Move right along the number line

Subtracting a positive number

- Move left along the number line

Adding a negative number

- Same as subtracting a positive number $a+(-b)=a-b$

Subtracting a negative number

- Same as adding a positive number $a-(-b)=a+b$

(watch video)

$$(\text{positive}) \times \text{or} \div (\text{positive}) = \text{positive}$$
$$(\text{negative}) \times \text{or} \div (\text{positive}) = \text{negative}$$
$$(\text{positive}) \times \text{or} \div (\text{negative}) = \text{negative}$$
$$(\text{negative}) \times \text{or} \div (\text{negative}) = \text{positive}$$

2 like signs produce a positive number
2 different signs produce a negative number

Arithmetic Flashcards

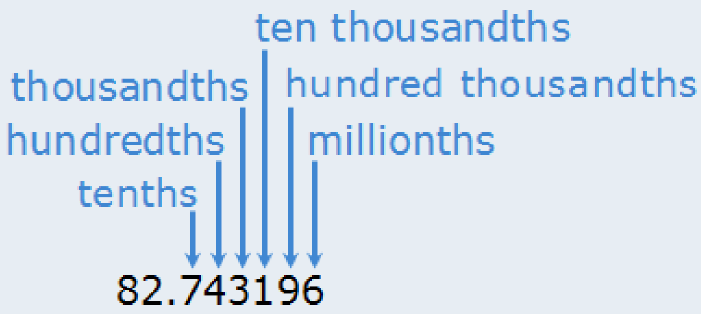
(watch video)

Parentheses	Brackets
Exponents	Exponents
Multiplication	Division
Division	Multiplication
Addition	Addition
Subtraction	Subtraction

absolute value: a number's distance from zero on the number line

Arithmetic Flashcards

(watch video)



Next digit is 0, 1, 2, 3 or 4 ➡ round down
Next digit is 5, 6, 7, 8 or 9 ➡ round up

Adding and subtracting decimals

- Line up the decimals
- Add additional zeros (or assume there are zeros)

Multiplying decimals

- Find the total number of digits to the right of each decimal
- Ignore the decimals and find the product
- Take product and move the decimal place to the left

Dividing decimals

- Move both decimals until the divisor becomes an integer
- Divide, keeping the decimal in the same location

(watch video)

Multiplying by powers of 10

- Move the decimal 1 space right for each zero

Dividing by powers of 10

- Move the decimal 1 space left for each zero

(watch video)

Equivalent fractions

$$\frac{1}{2}=\frac{5}{10} \qquad \frac{7}{9}=\frac{14}{18} \qquad \frac{3}{5}=\frac{30}{50}$$

- Create equivalent fractions by multiplying/dividing the numerator and denominator by the same number

$$\frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15} \qquad \frac{24}{36}=\frac{24\div 4}{36\div 4}=\frac{6}{9}$$

$$\frac{10}{11}=\frac{10\times 7}{11\times 7}=\frac{70}{77} \qquad \frac{35}{45}=\frac{35\div 5}{45\div 5}=\frac{7}{9}$$

(watch video)

Converting entire fractions into mixed numbers

$$\frac{7}{2} = 3\frac{1}{2}$$

entire fraction mixed number

- Determine how many times the denominator divides into the numerator (this becomes the whole number portion)
- The remainder becomes the numerator of the new fraction
- The denominator remains the same

(watch video)

Converting mixed numbers into entire fractions

- Multiply the whole number by the denominator, and add the product to the numerator
- The result becomes the new numerator and the denominator remains the same

$$6\frac{1}{3} = \frac{19}{3}$$

$$11\frac{2}{7} = \frac{79}{7}$$

Arithmetic Flashcards

Converting fractions to decimals

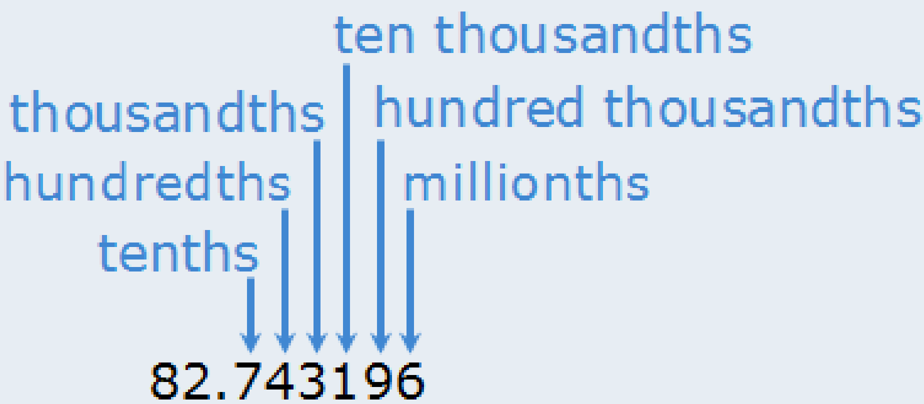
(watch video)

$\frac{1}{2}$	0.5
$\frac{1}{3}$	~ 0.333
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{6}$	~ 0.166
$\frac{1}{7}$	~ 0.14
$\frac{1}{8}$	0.125
$\frac{1}{9}$	~ 0.11

Converting decimals to fractions

(watch video)

- Find the place value of the last digit
- Write a fraction with that place value as the denominator



Arithmetic Flashcards

(watch video)

$$n = \frac{n}{1}$$

$\frac{n}{0}$ is undefined

$\frac{n}{n} = 1$ (as long as $n \neq 0$)

$\frac{1}{\frac{a}{b}} = \frac{b}{a}$ (as long as $a \neq 0$ and $b \neq 0$)

$\frac{a}{b} \times \frac{b}{a} = 1$ (as long as $a \neq 0$ and $b \neq 0$)

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

Arithmetic Flashcards

(watch video)

- Bigger numerator → bigger value
- Smaller numerator → smaller value
- Bigger denominator → smaller value
- Smaller denominator → bigger value

Increase numerator and denominator by same amount → fraction approaches 1

Arithmetic Flashcards

(watch video)

Adding and subtracting fractions

- Create equivalent fractions with the same denominator
- Add/subtract the numerators
- Keep the denominator the same

Multiplying fractions

- Multiply numerators, and multiply denominators
- Convert to entire fractions before multiplying
- When possible, simplify fractions before multiplying
- When possible, "cross simplify" before multiplying

Dividing fractions

- Multiply by the reciprocal of the divisor

Arithmetic Flashcards

(watch video)

$$\frac{abc}{def} = \frac{a}{d} \times \frac{b}{e} \times \frac{c}{f}$$

$$\frac{a+b+c}{d+e+f} \neq \frac{a}{d} + \frac{b}{e} + \frac{c}{f}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

$$\frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$$

$$\frac{a}{b} \times b = a \quad (\text{as long as } b \neq 0)$$

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$

Arithmetic Flashcards

(watch video)

$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	~ 0.333	~ 33.3%
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{6}$	~ 0.166	~ 16.6%
$\frac{1}{7}$	~ 0.14	~ 14%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{9}$	~ 0.11	~ 11.1%

Conversions (decimal to percent)

- Move decimal two places to the right

Conversions (percent to decimal)

- Move decimal two places to the left

(watch video)

The **part** is **some percent** of the **whole**

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$

p percent of x is $y \rightarrow \left(\frac{p}{100}\right)(x) = y$

(watch video)

10 percent of y

- move decimal 1 space to the left

1 percent of y

- move decimal 2 spaces to the left

What is 15 percent of 62?

$$\begin{array}{r} 10\% \text{ of } 62 = 6.2 \\ +\quad 5\% \text{ of } 62 = 3.1 \\ \hline 15\% \text{ of } 62 = 9.3 \end{array}$$

Arithmetic Flashcards

(watch video)

$$\%\ \text{change} = \frac{\text{change}}{\text{original value}} \rightarrow \text{(then rewrite as a percent)}$$

$$\%\ \text{change} = \frac{\text{change} \times 100}{\text{original value}}$$

$$\text{new} = \left(1 \pm \frac{\text{percent change}}{100}\right) \times \text{original}$$

Arithmetic Flashcards

(watch video)

Compound interest

$$\text{final} = P\left(1+\frac{r}{c}\right)^{nc}$$

P = principal
r = annual interest rate (as a decimal)
c = number of "compoundings" per year
n = number of years

simple interest

interest = (principal)(rate)(time)

- For short time periods, consider incremental calculations

e.g.,

	$40,000
after 1 month ➡	+ $400
	$40,400
after 2 months ➡	+ $404
	$40,804

(watch video)

"For every x there are y . . ." ➡ ratio question

Equivalent ratios

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc$$

Portioning into ratios

- Add the terms in the ratio and let the sum = T
- Divide the total quantity into T equal parts
- Divide the T equal parts into the target ratio

(watch video)

Combining Ratios

Strategy 1

- Find equivalent ratios until there are matching terms
- Combine

Strategy 2

- Solve one ratio
- Apply results to the other ratio

(watch video)

- Ratios can be used to solve simple rate questions

Powers & Roots Flashcards

(watch video)

base → 2^5 ← exponent

1 raised to any power is equal to 1

0 raised to any nonzero power is equal to 0

Any nonzero number raised to the power of 0 is equal to 1

Any number, x, raised to the power of 1 is equal to x

An odd exponent preserves the sign of the base

An even exponent always yields a positive result

* as long as the base $\neq 0$

(watch video)

Exponential Growth

Positive bases

If $x > 1$, then the value of x^n increases as n increases

If $0 < x < 1$, then the value of x^n approaches zero as n increases

Negative bases

If $x < -1$, then the magnitude of x^n increases as n increases, but the sign oscillates

If $-1 < x < 0$, then the magnitude of x^n decreases as n increases

(watch video)

Squaring Integers Ending in 5

$7 \times 8 = 56$ $\quad\quad$ $10 \times 11 = 110$

$75^2 = 5625$ $\quad\quad$ $105^2 = 11025$

<u>Technique</u>

- Let *n* be the number before the 5
- Write the product of *n* and *n*+1, followed by 25

Powers & Roots Flashcards

(watch video)

Quotient law

$$\frac{x^a}{x^b} = x^{a-b}$$

Product law

$$(x^a)(x^b) = x^{a+b}$$

Power of a power law

$$(x^a)^b = x^{ab}$$

Powers & Roots Flashcards

(watch video)

$$x^{-n} = \frac{1}{x^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

Powers & Roots Flashcards

(watch video)

Power of a product law

$$(x^a y^b)^n = x^{an} y^{bn}$$

Power of a quotient law

$$\left(\frac{x^a}{y^b}\right)^n = \frac{x^{an}}{y^{bn}}$$

Combining bases law

$$x^n y^n = (xy)^n$$

Combining bases law

$$\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

(watch video)

What is the units digit of 53^{35} ?

$53^1 = 53$
$53^2 = ---9$
$53^3 = ---7$
$53^4 = ---1$
$53^5 = ---3$
$53^6 = ---9$
$53^7 = ---7$
$53^8 = ---1$
.
.
.
$53^{35} =$

cycle = 4 ➡ When n is divisible by 4, the units digit of 53^n is 1

$53^{32} = ---1$
$53^{33} = ---3$
$53^{34} = ---9$
$53^{35} = ---7$

(watch video)

$\sqrt{n}$ = a number (greater than or equal to zero) that, when squared, equals n

Properties

- If $n < 0$, $\sqrt{n}$ has no real value
- If $n \geq 0$, then $\sqrt{n} \geq 0$

$\sqrt{x^2} = |x|$

If $0 < x < 1$ then $\sqrt{x} > x$

If $x > 1$ then $\sqrt{x} < x$

Powers & Roots Flashcards

(watch video)

$\sqrt[r]{n}$ = a number that, when raised to the power of r, equals n

A root will have, at most, 1 value

$\sqrt[n]{x} = x^{\frac{1}{n}}$

Powers & Roots Flashcards

(watch video)

$$\left(\sqrt[n]{x}\right)\left(\sqrt[n]{y}\right) = \sqrt[n]{xy}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$$

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Simplifying Square Roots

(watch video)

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, . . .

$\sqrt{700} = \sqrt{100 \times 7}$

- Rewrite the number inside the root as the product of a perfect square and some other number

$= \sqrt{100} \times \sqrt{7}$

- Rewrite the root as the product of 2 roots

$= 10\sqrt{7}$

- Simplify the root of the perfect square

(watch video)

Operations with Roots

Multiply the parts outside the root and multiply the parts inside the root

Divide the parts outside the root, and divide the parts inside the root

$\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ ✗

$n\sqrt{a+b} = \sqrt{na+nb}$ ✗

(watch video)

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$ → $$32^{\frac{3}{5}} = \sqrt[5]{32^3}$$

$$x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$$ → $$32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = (2)^3 = 8$$

$$x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$$ $$81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$$

(watch video)

Equations with Exponents

1) Rewrite with equal bases
2) Apply following rule
3) Solve resulting equation

If $b^x = b^y$ then $x = y$
$(b \neq 0, 1, -1)$

(watch video)

"Fixing" the Denominator

$$\frac{6\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

Multiply top and bottom by the root in the denominator

$$\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-2\sqrt{5}} \times \frac{\sqrt{3}+2\sqrt{5}}{\sqrt{3}+2\sqrt{5}}$$

Multiply top and bottom by the conjugate of the denominator

Definitions (watch video)

expression: collection of one or more terms combined using addition and/or subtraction

examples: $w^3 - 3x^2 + 5y$

$x - 1$

$\frac{2x^4}{5} + \frac{1}{y^3} - 5x^2y + x - 3y + 9$

monomial: expression with 1 term

examples: $14, \ 5x, \ 8xy^3, \ \frac{jk}{5m^3}$

binomial: expression with 2 terms

examples: $x^2 + 3y$

$w - 8$

polynomial: expression with 1 or more terms

Simplifying Expressions

(watch video)

- Like terms can be combined (added/subtracted)

  e.g., $2x + 7x = 9x$

- To add expressions in parentheses, remove the parentheses

  e.g., $(3x - 2y) + (x - 7y) = 3x - 2y + x - 7y$

- To subtract expressions in parentheses, add the "opposites"

  e.g., $(3x - 2y) - (x - 7y) = (3x - 2y) + (-x + 7y)$

(watch video)

Multiply members of the same "family"

e.g., $(5y^3)(4y^4) = 20y^7$

Multiply each term in the parentheses by the term in front

e.g., $3(2x+5) = 6x+15$

Multiplying two binomials

(watch video)

First

Outer

Inner

Last

$$(x+2)(x+7) = x^2 + 7x + 2x + 14$$
$$= x^2 + 9x + 14$$

$$(3y-4)(2y-5) = 6y^2 - 15y - 8y + 20$$
$$= 6y^2 - 23y + 20$$

$$(2x+y)(x-7y) = 2x^2 - 14xy + xy - 7y^2$$
$$= 2x^2 - 13xy - 7y^2$$

Multiplying two binomials

(watch video)

First
Outer
Inner
Last

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

(watch video)

Greatest Common Factor Factoring

- Find the greatest common factor (divisor) of all terms
- Place the greatest common factor in front of the parentheses
- Determine which terms must be inside the parentheses to get the desired product

(watch video)

Difference of Squares Factoring

- Watch out for **differences** of squares

$$a^2 - b^2 = (a+b)(a-b)$$

(watch video)

Quadratic Polynomial Factoring

$$x^2 + nx + p = (x + a)(x + b)$$

$n = a + b$

$p = ab$

Factoring – Putting it all Together

(watch video)

1. Factor out the greatest common factor
2. Factor further (if possible)

Example: $2x^6 - 2x^2 = 2x^2\left(x^4 - 1\right)$

$$= 2x^2\left(x^2 + 1\right)\left(x^2 - 1\right)$$

$$= 2x^2\left(x^2 + 1\right)(x + 1)(x - 1)$$

(watch video)

Simplifying Rational Expressions

$$\frac{x^3+4x^2+3x}{x^3+2x^2-3x}=\frac{x\left(x^2+4x+3\right)}{x\left(x^2+2x-3\right)}$$

$$=\frac{x(x+3)(x+1)}{x(x+3)(x-1)}$$

$$=\frac{x+1}{x-1}$$

$$\frac{x^3+4x^2+3x}{x^3+2x^2-3x}=\frac{x+1}{x-1}$$ *for all values of x for which both expressions are defined*

(watch video)

Golden Rule of Equation Solving

What you do to one side of the equation, you must do to the other side

- Isolate the variable by performing the same operations to both sides
- "solution" = "root"

(watch video)

$$\frac{x}{10}+\frac{4}{5}=\frac{x}{12}+1$$

$$\frac{60}{1}\left(\frac{x}{10}+\frac{4}{5}\right)=\frac{60}{1}\left(\frac{x}{12}+1\right)$$

Multiply both sides by the least common multiple of the denominators

$$\frac{60x}{10}+\frac{240}{5}=\frac{60x}{12}+\frac{60}{1}$$

$$6x+48=5x+60$$

$$x+48=60$$

$$x=12$$

(watch video)

$$\frac{7}{6x-6}=\frac{3}{2x+2}$$

If $\dfrac{a}{b}=\dfrac{c}{d}$ then $ad=bc$

$$7(2x+2)=3(6x-6)$$

$$14x+14=18x-18$$

$$14=4x-18$$

$$32=4x$$

$$8=x$$

(watch video)

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

- Most (all) quadratic equations can be solved by factoring
- Solvable quadratic equations will have 1 or 2 unique solutions (roots)

Quadratic formula

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(b^2 - 4ac\right) < 0 \Rightarrow \text{no solution exists}$$

$$\left(b^2 - 4ac\right) \geq 0 \Rightarrow \text{solution exists}$$

(watch video)

2 Equations with 2 Unknowns

Substitution Method

1) Solve one equation for one variable
2) Take the other equation and replace the chosen variable with its equivalent expression from the first equation
3) Solve for the variable
4) Plug the solution into any equation to solve for the other variable

(watch video)

2 Equations with 2 Unknowns

Elimination Method

1) Manipulate equations until you have matching coefficients for one variable
2) Add or subtract the 2 equations to eliminate one variable
3) Solve for the remaining variable
4) Plug the solution into any equation to solve for the other variable

(watch video)

Number of Solutions

- Solve as usual
  - find solution ➡ 1 solution
  - identical equations ➡ infinite solutions
  - $0x + 0y =$ nonzero value ➡ zero solutions

(watch video)

Solving 3 Equations with 3 Unknowns

1) Solve one equation for one variable
2) Take the other two equations and replace the chosen variable with its equivalent expression from the first equation
3) Solve for the two remaining variables
4) Plug the solution into any equation to solve for the third variable

or

1) Solve using the elimination method

Equations with Square Roots

(watch video)

Square root

1) Eliminate square root by squaring both sides
2) Solve for variable
3) Check for extraneous roots

n^{th} root

1) Raise both sides by power of n
2) Solve for variable
3) If n is even, check for extraneous roots

(watch video)

Equations with Exponents

1) Rewrite with equal bases
2) Apply following rule
3) Solve resulting equation

If $b^x = b^y$ then $x = y$

$(b \neq 0, 1, -1)$

(watch video)

Equations with Absolute Value

1) Apply rule $|x| = a \Rightarrow \begin{cases} x = a \\ x = -a \end{cases}$
2) Solve resulting equations
3) Check for extraneous roots

(watch video)

Strange Operators

- Use the "recipe" to evaluate

(watch video)

Solving Inequalities

- Adding and subtracting to/from both sides does not affect the inequality
- Multiplying and dividing both sides by a positive number does not affect the inequality
- Multiplying and dividing both sides by a negative number reverses the inequality

(watch video)

combining inequalities

$$w < x$$
$$x < y$$

$$w < x < y \Rightarrow w < y$$

Rewrite inequalities facing the same direction before trying to combine

(watch video)

adding inequalities

$$\begin{array}{r} A < B \\ + \quad C < D \\ \hline A + C < B + D \end{array}$$

$$\begin{array}{r} \text{Al} < \$15 \\ + \quad \text{Bob} < \$10 \\ \hline \text{Al} + \text{Bob} < \$25 \end{array}$$

The inequality signs must face the same direction before adding

(watch video)

subtracting inequalities

$$\begin{array}{r} 10 < 17 \\ -\quad 9 < 10 \\ \hline 1 < 7 \end{array}$$

$$\begin{array}{r} 10 < 11 \\ -\quad 4 < 8 \\ \hline 6 < 3 \end{array}$$

Do not subtract inequalities

Do not multiply inequalities

Do not divide inequalities

(watch video)

$|x| < a \Rightarrow -a < x < a$ (where a is positive)

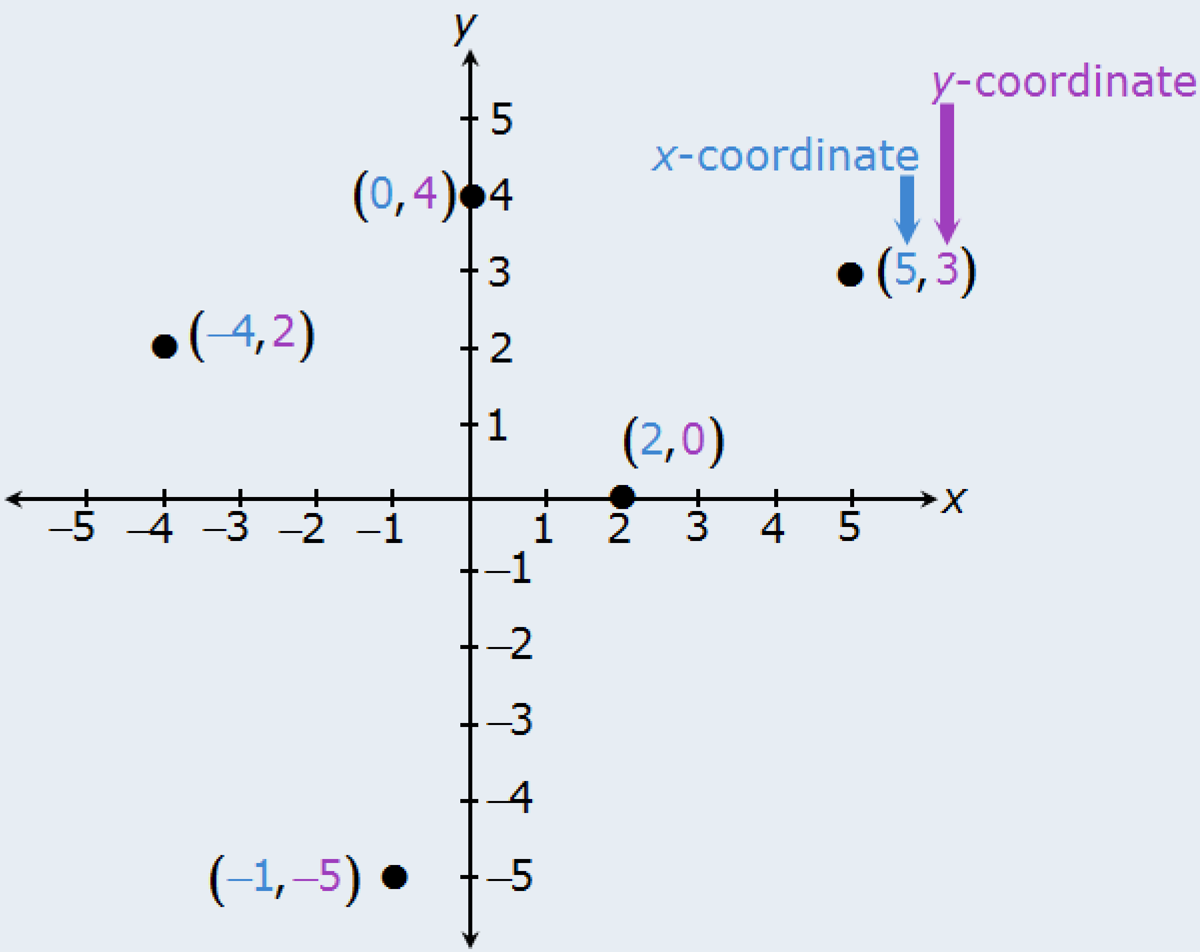
$|x| > a \Rightarrow x > a$ or $x < -a$ (where a is positive)

(watch video)

Quadratic Inequalities

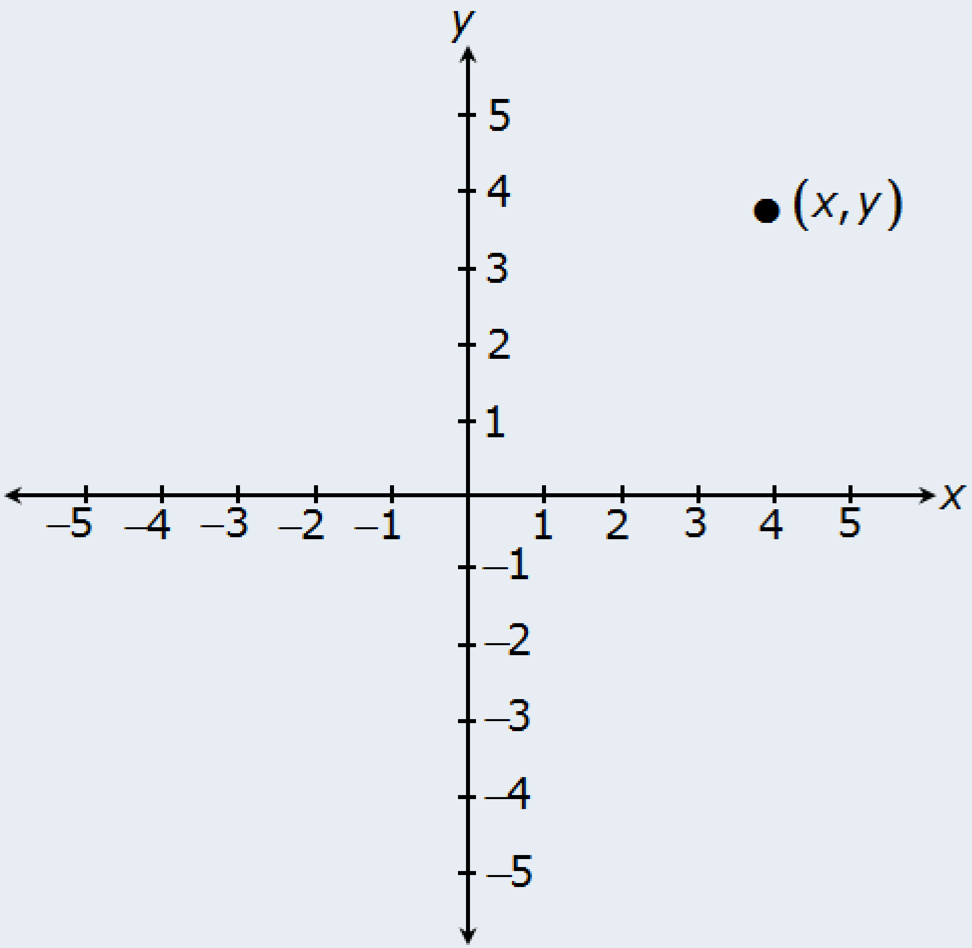
- Set the expression to equal zero
- Find solutions and record on number line
- Test number from each region
- Solve the inequality

(watch video)



- Every point in the coordinate plane is defined by a unique ordered pair of numbers (x, y)

(watch video)



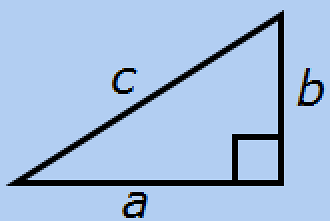
(watch video)

Distance Between Two Points

- Apply formula

Distance between points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

- Sketch and use Pythagorean Theorem



$$a^2 + b^2 = c^2$$

Graphing Lines

(watch video)

- In the coordinate plane, a line is a set of points such the coordinates of each point satisfy the given equation
- If the coordinates of a point satisfy the equation, then that point will lie on the line
- If the coordinates of a point do not satisfy the equation, then that point will not lie on the line

(watch video)

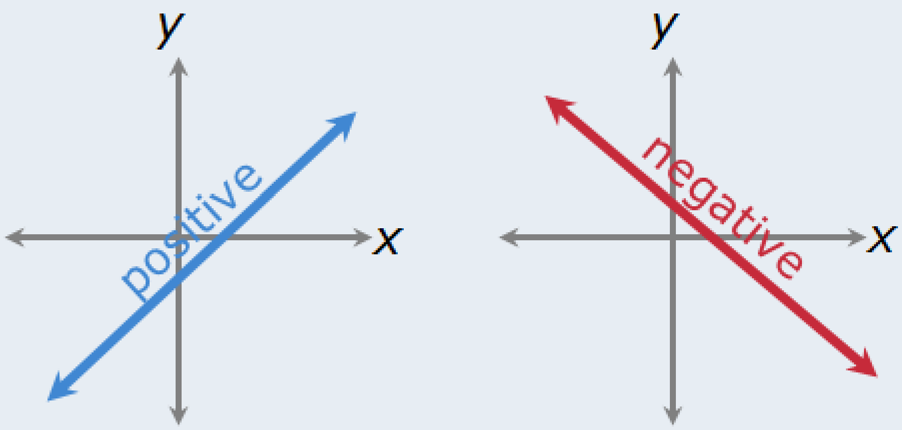
- The graph of the line $x=k$ will be a **vertical** line where all of the points have x-coordinates equal to k
- The graph of the line $y=k$ will be a **horizontal** line where all of the points have y-coordinates equal to k

(watch video)

Given (x_1, y_1) and (x_2, y_2)

$$\text{slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



As the magnitude of the slope increases, the line gets steeper

(watch video)

x-intercept: x-coordinate of point where the line intersects the x-axis

- To find the x-intercept, plug $y = 0$ into the equation

y-intercept: y-coordinate of point where the line intersects the y-axis

- To find the y-intercept, plug $x = 0$ into the equation

(watch video)

Slope *y*-intercept form

$$y = mx + b$$

slope (m) — *y*-intercept (b)

(watch video)

Writing equations from two given points

$$y = mx + b$$

1) Find the slope (m) of the line
2) Plug the value of m into the slope y-intercept equation
3) Plug the coordinates of one point into the equation
4) Solve for b
5) Write the equation in slope y-intercept form

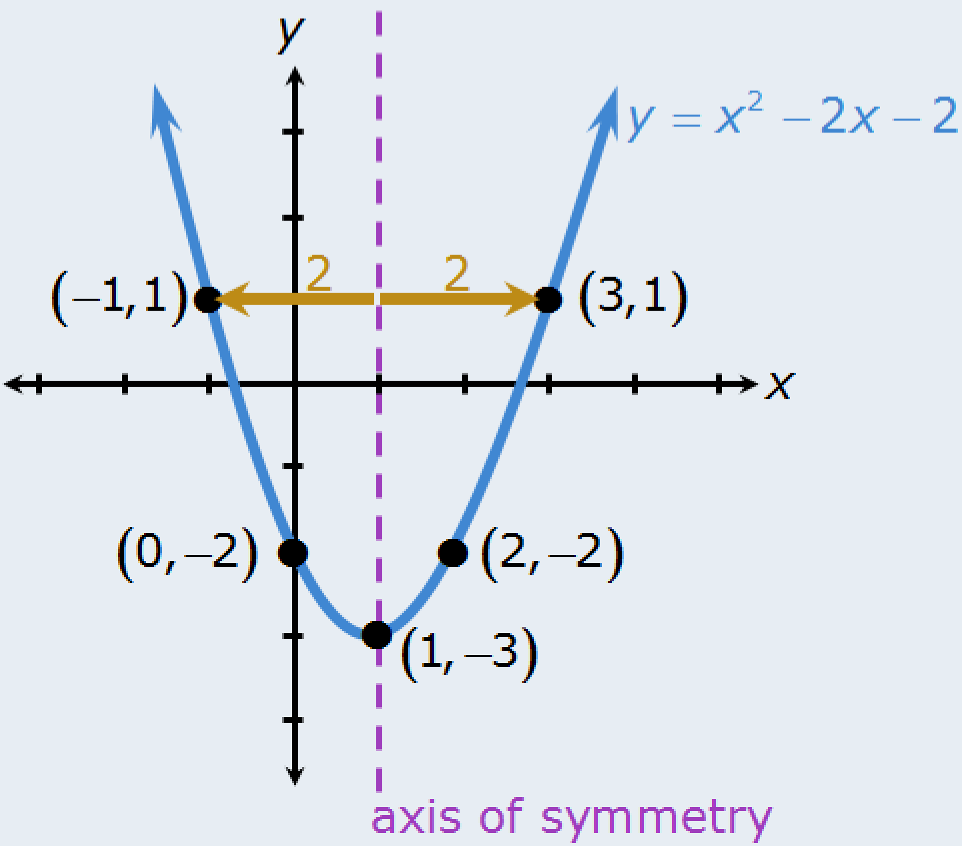
Practice

$(1, -2)$ and $(4, 7) \Rightarrow y = 3x - 5$

$(4, -9.5)$ and $(-2,\ 5.5) \Rightarrow y = -\frac{5}{2}x + \frac{1}{2}$

$y = ax^2 + bx + c$

(watch video)



(watch video)

Introduction to Word Problems

1. Understand the question and any restrictions
2. Consider testing the answer choices
3. Assign variables
4. Create an equation
5. Solve the equation (if necessary)
6. Reread question and confirm required value

(watch video)

Strategy for Testing Answer Choices

- Test answer choices, beginning with C
- Eliminate impossible answer choices

(watch video)

Assigning Variables

- Consider assigning the variable to the target value
- Consider writing a "word equation"
- It is often best to assign the variable to the smallest value
- Assign descriptive variables
- Look for relationships

(watch video)

Writing Equations

- Write a "word equation"
- Replace with algebraic expressions
- Solve for variable
- Reread the question and confirm required value

$$\text{profit} = \text{revenue} - \text{cost}$$

$$\text{total cost} = \text{price per item} \times \text{quantity purchased}$$

$$\text{total earnings} = \text{pay rate} \times \text{time worked}$$

(watch video)

Using More than 1 Variable

- Begin with one variable, but change to more variables if there are complex relationships between the unknown values
- n variables requires n equations

(watch video)

Past & Future Age Questions

- Create table with given times
- Create equation(s)
- Ensure that you have obtained the required information

(watch video)

$$\text{distance} = \text{rate} \times \text{time}$$

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\frac{(\text{Distance})}{(\text{Rate})(\text{Time})}$$

$$\frac{D}{RT}$$

$$\text{distance} = \text{rate} \times \text{time}$$

The time units must match before multiplying

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

The distance units must match before dividing

(watch video)

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time}}$$

- Assign variables if necessary

(watch video)

Multiple Trips and/or Multiple Travelers

- Consider possible word equations
- Use the equation with favorable variables

(watch video)

Shrinking/Expanding Gaps

- Observe the outcome after 1 unit of time
- Determine shrink/expansion rate
- Apply the rate to the question

(watch video)

Work Questions

$$\text{output} = \text{rate} \times \text{time}$$

$$\text{rate} = \frac{\text{output}}{\text{time}}$$

$$\text{time} = \frac{\text{output}}{\text{rate}}$$

$$\frac{(\text{Output})}{(\text{Rate})(\text{Time})}$$

$$\frac{O}{RT}$$

Find the output rates

- Add rates when there are two or more contributors (machines, workers, etc.)

(watch video)

Double Matrix Method - Example

In a shipment of 40 toys, each toy is either blue or green, and each toy is either large or small. In total, there are 30 small toys, and there are 14 blue toys. If the shipment contains 22 toys that are both small and green, how many toys are both large and blue?

A) 4

B) 6

C) 10

D) 12

E) 14

40	blue	green	
small	8	22	sum = 30
large	6	4	sum = 10
	sum = 14	26	

(watch video)

3-Criteria Venn Diagrams

- Draw 3 overlapping sets
- Fill in regions from the middle outwards

(watch video)

- **Recursive definitions** of sequences typically require us to start at the beginning of the sequence

(watch video)

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

The number of integers from x to y inclusive $= y - x + 1$

(watch video)

Growth Tables

- For incremental growth or decline draw a table
- Note changes for every time period

(watch video)

Mixture Questions

- Sketch the solution(s) with the parts separated
- Combine like parts

(watch video)

Variables in the Answer Choices

Algebraic approach

- Translate the information into an expression

Input–output approach

- Choose value(s) for the given variable(s)
- Use those values to calculate the required output
- Use the same values to evaluate each answer choice, and look for a matching output

Both strategies usually work

The algebraic approach is typically faster

(watch video)

Tips for the Algebraic Approach

- Use real numbers to determine the required operations to reach a certain goal
- Apply those operations to the given variables
- Try writing the expression in different ways

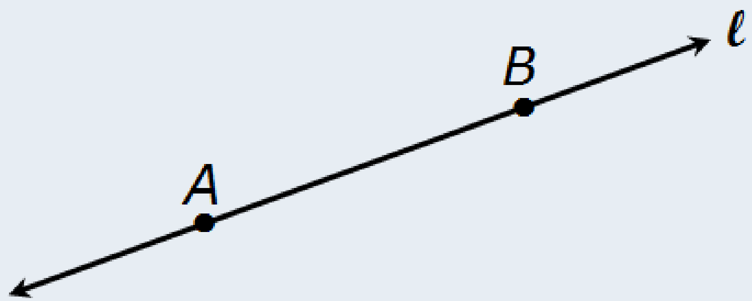
(watch video)

Tips for the Input-Output Approach

- Use small numbers and prime numbers
- Use different numbers
- In most cases, avoid using 0 and 1
- Avoid numbers that appear in the question
- Some numbers allow us to quickly eliminate answer choices

(watch video)

line: a straight path that extends without end in both directions



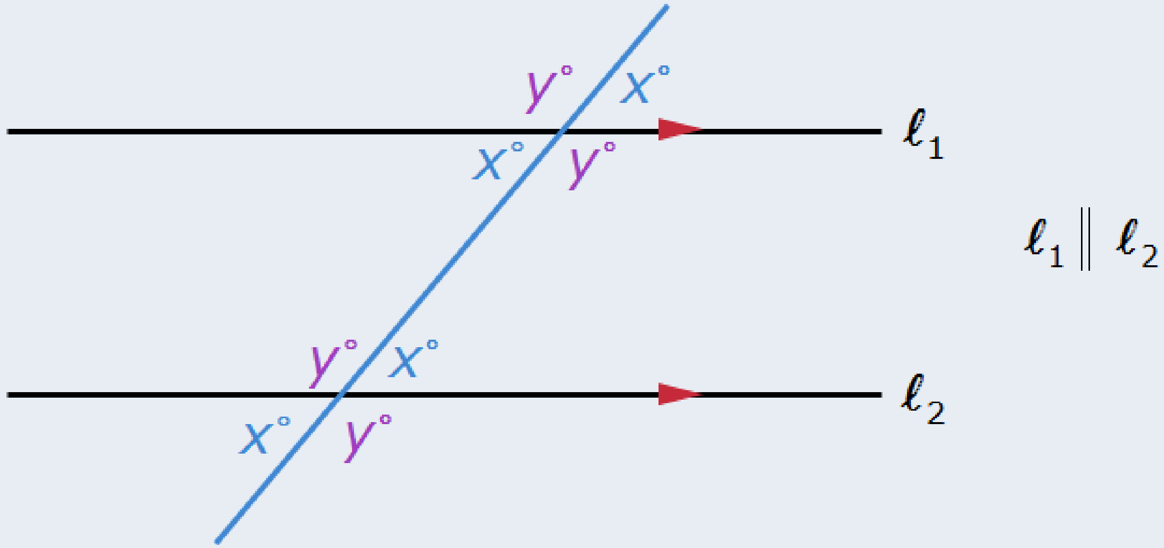
$\overline{AB}$: line segment

AB: length of line segment AB (e.g., DE=7)

(watch video)

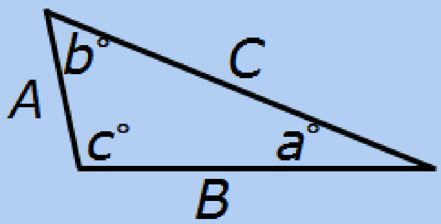
Angles on a line add to 180°

Opposite angles are equal



(watch video)

Angles in a triangle add to 180°



If $a < b < c$ then $A < B < C$

The sum of the lengths of any two sides of a triangle must be greater than the third side.

Given lengths of sides A and B

$|A - B| < 3^{rd}\text{ side} < A + B$

(watch video)

Assumptions about Geometric Figures

- Lines that appear straight can be assumed to be straight
- Angles are greater than zero degrees
- Do not make assumptions about angle measurements
- Do not make assumptions about parallelism
- Use visual estimation sparingly

(watch video)

- An isosceles triangle has 2 equal sides and 2 equal angles
- An equilateral triangle has 3 equal sides and 3 equal angles (60° each)

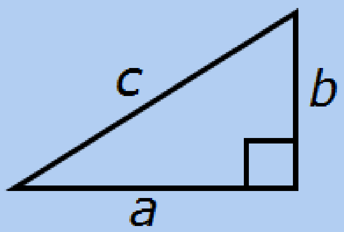
$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

$$\text{Area} = \frac{\sqrt{3} \times (\text{side})^2}{4}$$

- The altitudes of isosceles triangles and equilateral triangles bisect the base

Geometry Flashcards

(watch video)

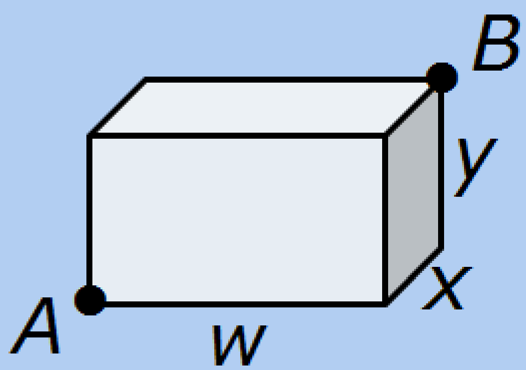


$$a^2 + b^2 = c^2$$

- Watch out for Pythagorean triples (and their multiples)

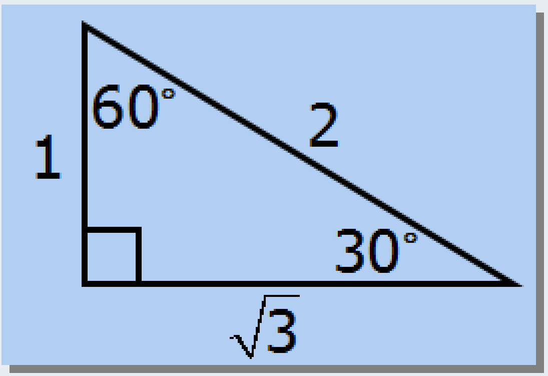
  3-4-5
  5-12-13
  8-15-17
  7-24-25

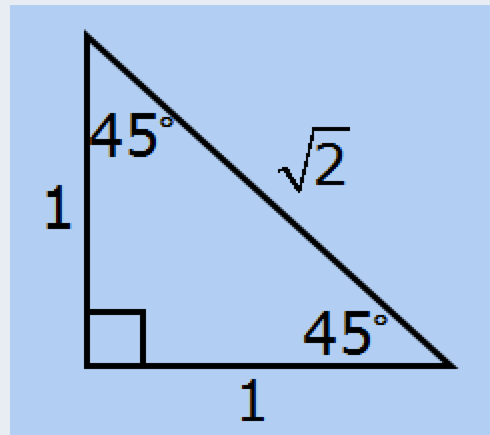
(watch video)



$$AB = \sqrt{w^2 + x^2 + y^2}$$

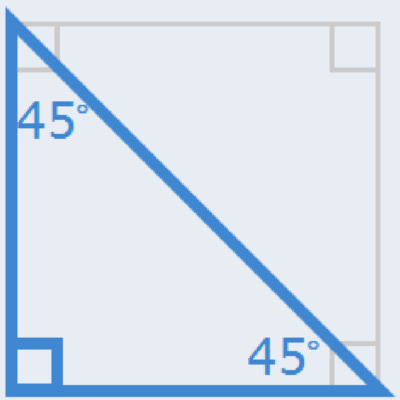
(watch video)



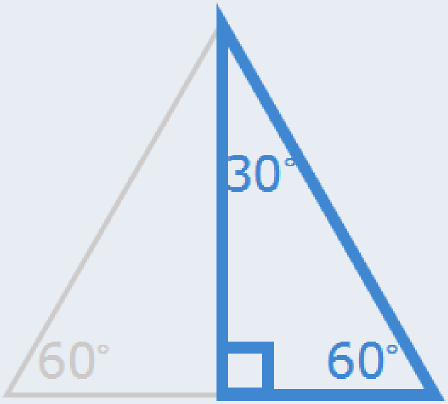


(watch video)

Square



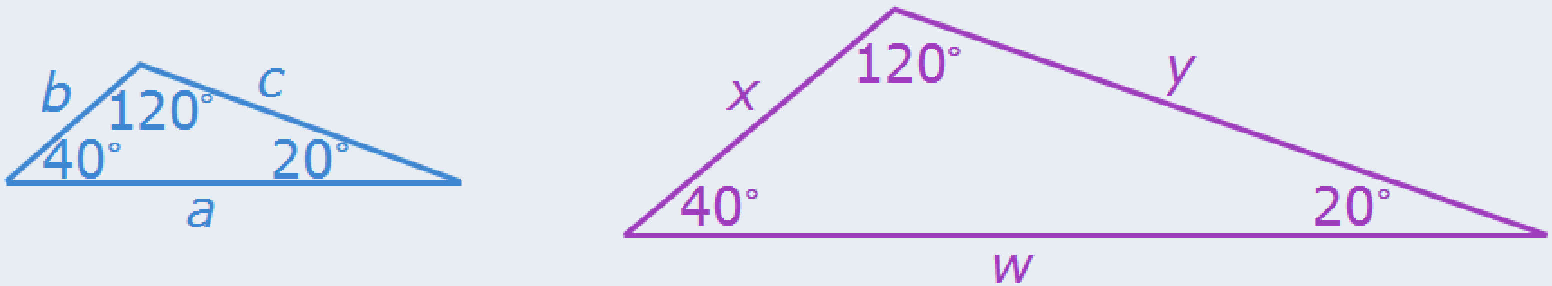
Equilateral Triangle



Watch out for special right triangles "hiding" in squares and equilateral triangles

(watch video)

Similar triangles have the same 3 angles in common



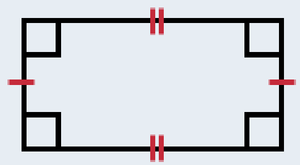
$$\frac{a}{w} = \frac{b}{x} = \frac{c}{y}$$

With similar triangles, the ratio of any pair of corresponding sides is the same

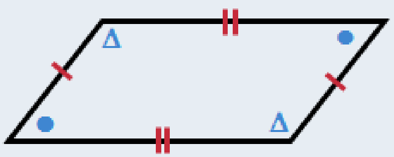
Geometry Flashcards

(watch video)

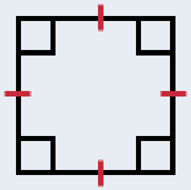
rectangle
- opposite sides parallel
- all angles are 90°



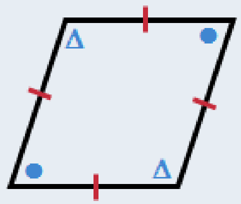
parallelogram
- opposite sides parallel



square
- opposite sides parallel



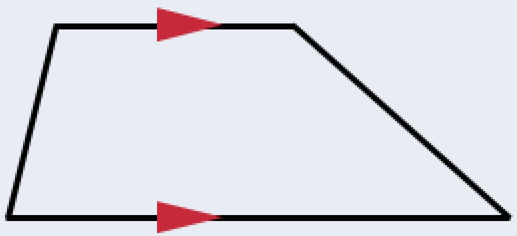
rhombus
- opposite sides parallel
- all sides are equal



(watch video)

trapezoid

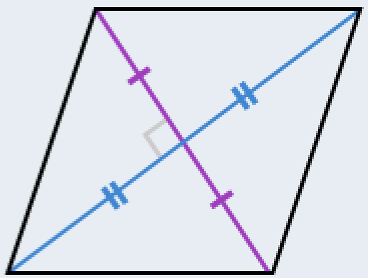
- 2 sides parallel



(watch video)

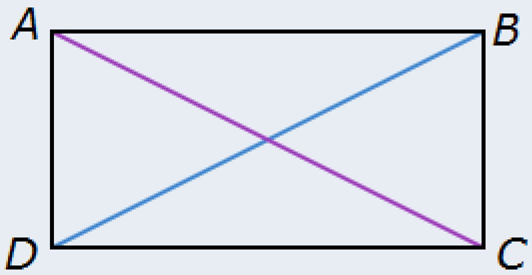
Rhombus (and square)

- diagonals are perpendicular bisectors



Rectangle (and square)

- diagonals are equal length

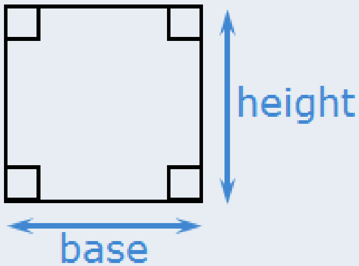


$AC = BD$

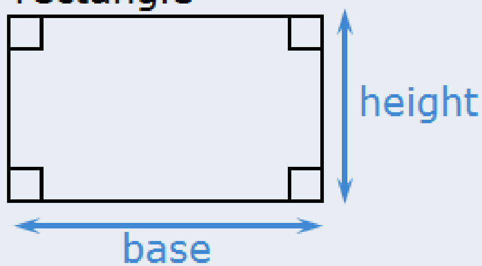
area = base × height

(watch video)

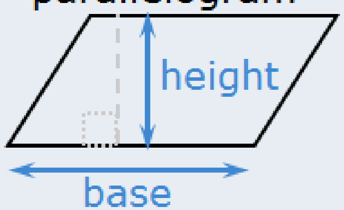
square



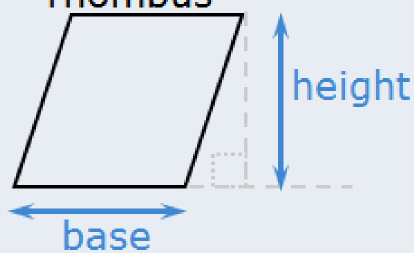
rectangle



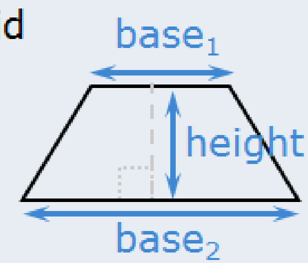
parallelogram



rhombus



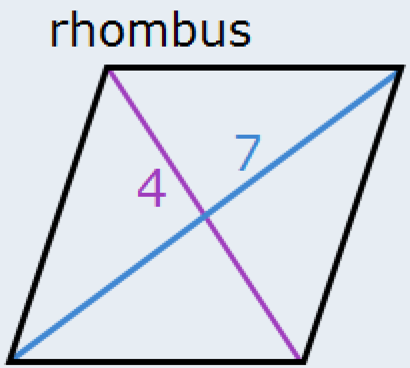
trapezoid



$$\text{area} = \left(\frac{\text{base}_1 + \text{base}_2}{2}\right) \times \text{height}$$

$$= \text{average of bases} \times \text{height}$$

(watch video)



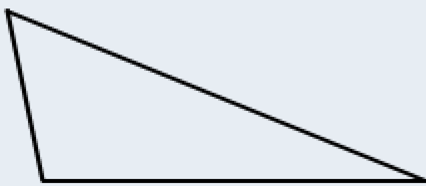
$$\text{area} = \frac{\text{diagonal}_1 \times \text{diagonal}_2}{2}$$

$$\text{area} = \frac{4 \times 7}{2} = \frac{28}{2} = 14$$

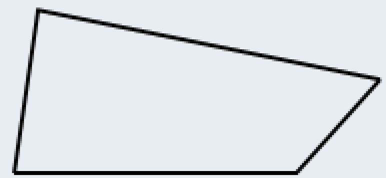
(watch video)

- Polygon: Closed figure formed by 3 or more line segments
- "polygon" ➡ "convex polygon" (all interior angles less than 180°)
- Regular polygon: equal sides and equal angles

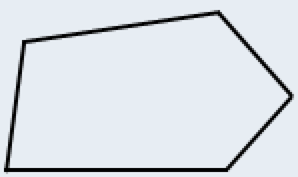
Triangle



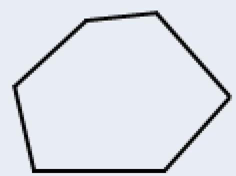
Quadrilateral



Pentagon



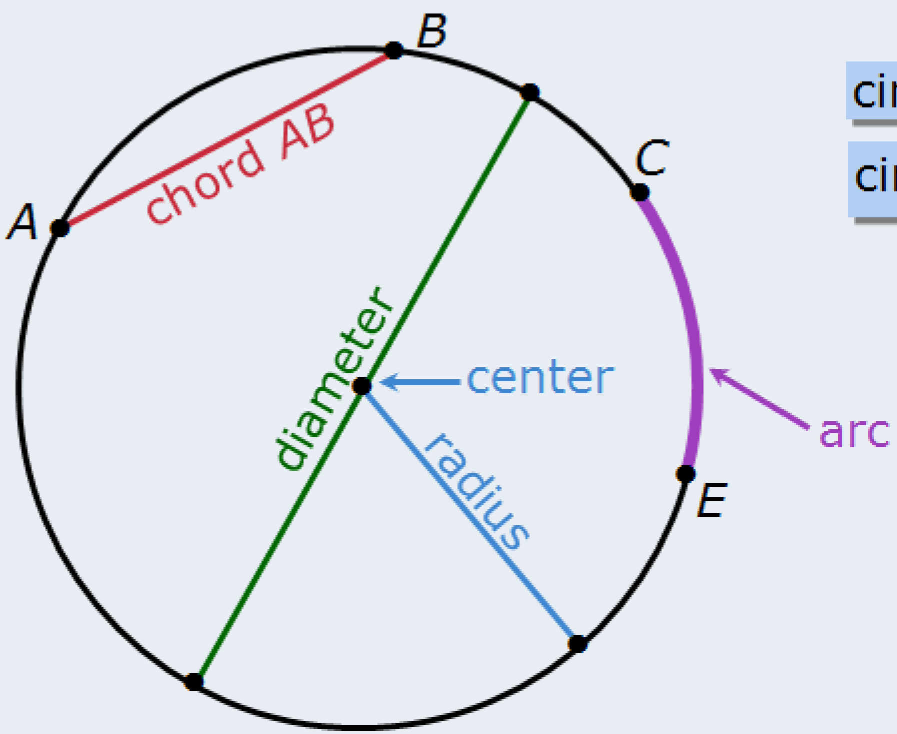
Hexagon



The sum of the interior angles in an N-sided polygon is equal to $180^\circ(N-2)$

Geometry Flashcards

(watch video)



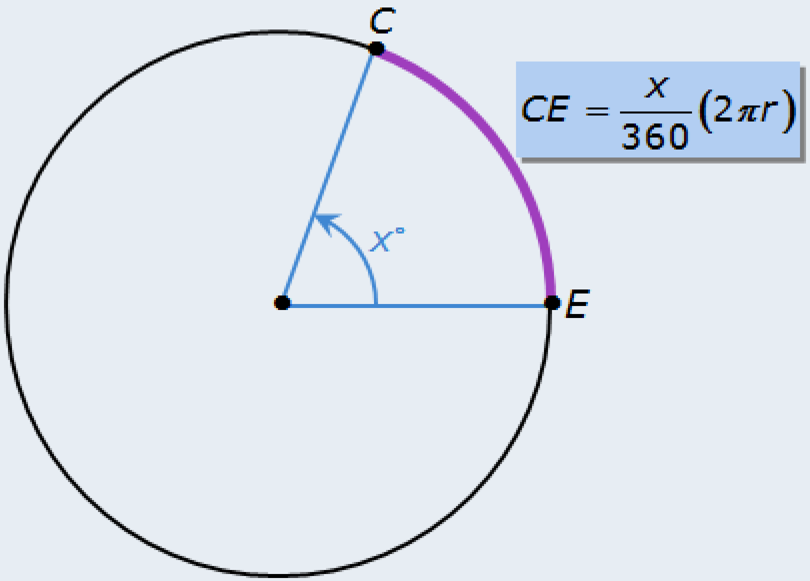
$$\text{circumference} = \pi \times \text{diameter}$$

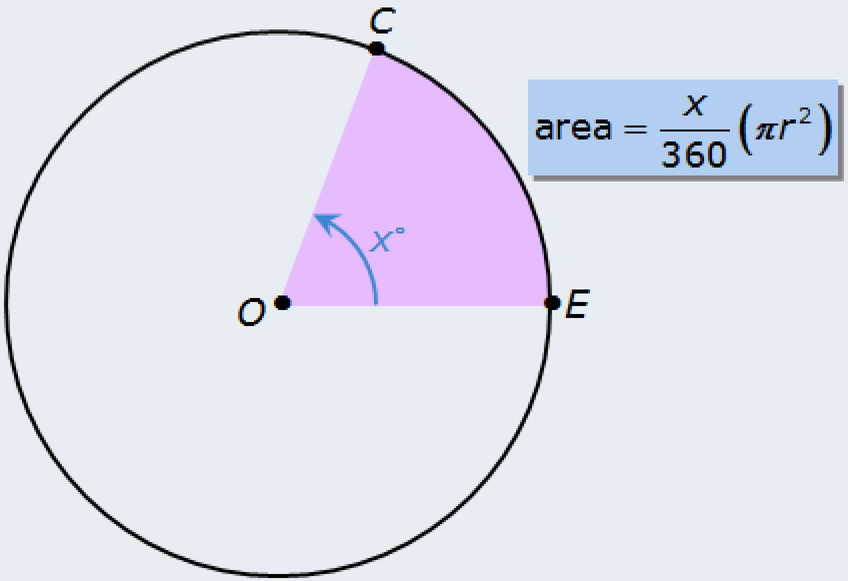
$$\text{circumference} = \pi \times (2 \times \text{radius})$$

$$\text{area} = \pi r^2$$

$$\pi \approx 3.14$$
$$\approx 3$$

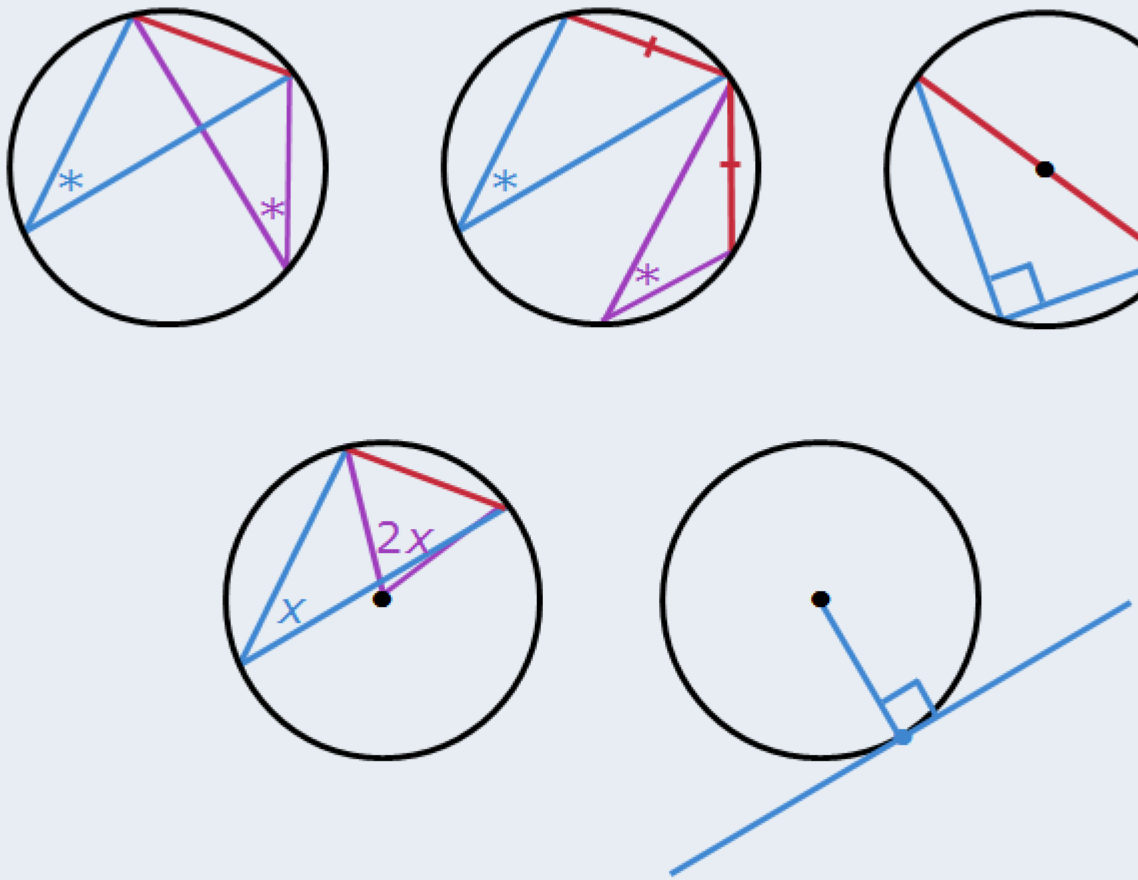
(watch video)



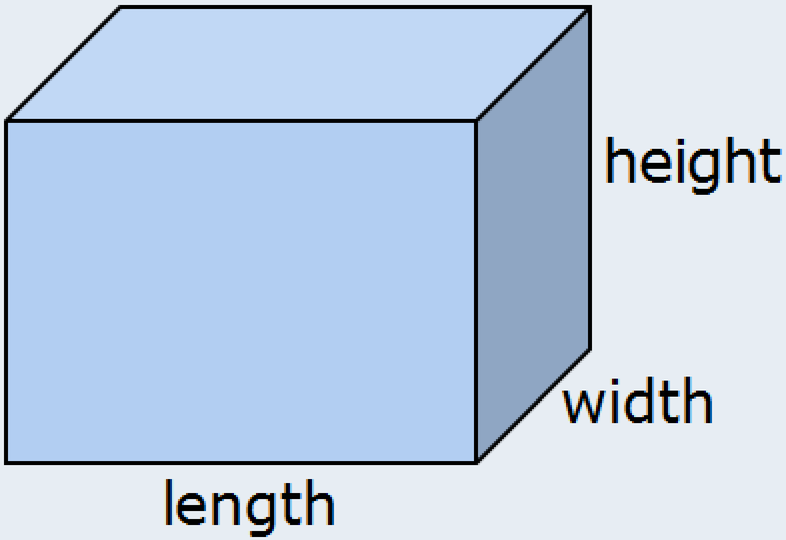


Circle Properties

(watch video)



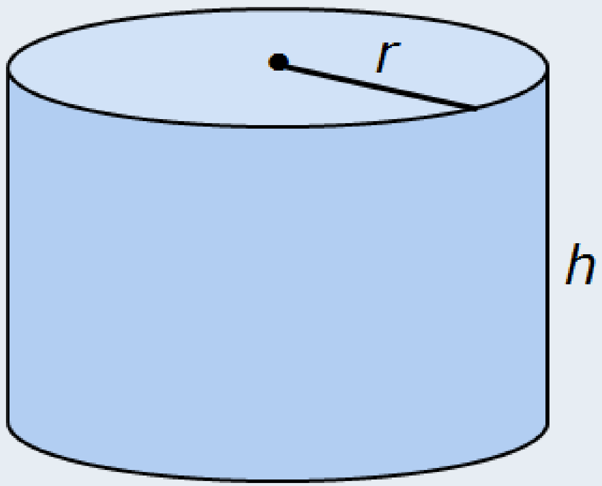
(watch video)



volume = length × width × height

surface area = sum of areas of all 6 sides

(watch video)



volume $= \pi r^2 h$

$$\begin{aligned}\text{surface area} &= \pi r^2 + \pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r\left(r + h\right)\end{aligned}$$

(watch video)

Conversions

- If conversion is required, relationship will be given
  - e.g., (1 kilometer = 1000 meters)
  - e.g., (1 mile = 5280 feet)
- Note: Relationships **not** given for units of time
  - e.g., (1 hour = 60 minutes)
  - e.g., (1 day = 24 hours)

(watch video)

Geometry Data Sufficiency Questions

- Do not estimate lengths and angles
- To find one length, requires at least one other length
- Sketch diagram and add information
- Mentally grab and move points and lines

Geometry Strategies

(watch video)

- Redraw figures
- Add all given information
- Add any information that can be deduced
- Add/extend lines
- Assign variables and use algebra
- Problem solving questions drawn to scale:
  - estimate to confirm calculations and guide guesses
- Two or more triangles and length required
  - look for similar triangles
- Right triangle:
  - use Pythagorean Theorem to relate sides
  - watch for Pythagorean Triples and special triangles
- Circle:
  - beware of circle properties (inscribed/central angles, tangent lines)
  - look for isosceles triangles
- Break areas/volumes into manageable pieces

(watch video)

If x and y are integers then:

$$
\begin{aligned}
\text{"}x \text{ is divisible by } y\text{"} &= \text{"when } x \text{ is divided by } y \text{ the remainder is 0"} \\
&= \text{"}y \text{ is a divisor of } x\text{"} \\
&= \text{"}y \text{ is a factor of } x\text{"} \\
&= \text{"}x \text{ equals } ky \text{ for some integer } k\text{"} \\
&= \text{"}x \text{ is a multiple of } y\text{"}
\end{aligned}
$$

GMAT questions typically focus on **positive** divisors/factors

(watch video)

Divisibility Rules

Divisible by	Characteristic
2	Units digit is 0, 2, 4, 6, or 8
3	Sum of digits is divisible by 3
4	2-digit # at the end is divisible by 4
5	Units digit is 0 or 5
6	The number is divisible by 2 AND by 3
9	Sum of digits is divisible by 9
10	Units digit is 0

(watch video)

Prime Number

A positive integer with exactly 2 positive divisors.

Primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, . . .

Note:

- 1 is not prime (only 1 positive divisor)
- 2 is the **only** even prime number

Integer Properties Flashcards

(watch video)

"x is divisible by y" = "when x is divided by y the remainder is 0"
= "y is a divisor of x"
= "y is a factor of x"
= "x equals ky for some integer k"
= "x is a multiple of y"
= "y is 'hiding' in the prime factorization of x"

6 is a divisor of W ➡ $W = 6 \times ? \times ? \times ? \times \cdots$
$= 2 \times 3 \times ? \times ? \times ? \times \cdots$

R is divisible by 88 ➡ $R = 88 \times ? \times ? \times ? \times \cdots$
$= 2 \times 2 \times 2 \times 11 \times ? \times ? \times ? \times \cdots$

(watch video)

- The **prime factorization** of each number is unique
- Prime factorization can help determine whether numbers are divisors
- If some number, *k*, is "hiding" in the prime factorization of a number, then *k* is a divisor of that number

(watch video)

Counting Divisors of Large Numbers

If $N = p^a \times q^b \times r^c \times \cdots$, where p, q, r (etc) are prime numbers, then the total number of positive divisors of N is $(a+1)(b+1)(c+1)\cdots$

(watch video)

Squares of Integers

- The prime factorization of a perfect square will have an even number of each prime.
- A perfect square will have an odd number of positive divisors

(watch video)

Divisor Rules

Given: *j*, *k*, *M* and *N* are integers:

- If *k* is a divisor of *N*, then *k* is a divisor of *NM*
- If *jk* is a divisor of *N*, then *j* is a divisor of N, and *k* is a divisor of *N*
- If *k* is not a divisor of *N*, then *jk* is not a divisor of *N*
- If *k* is a divisor of both *N* and *M*, then *k* is a divisor of *N*+*M* (and *N*–*M* and *M*–*N*)
- If *k* is a divisor *N*, but *k* is not a divisor of *M*, then *k* is not a divisor of *N*+*M* (or *N*–*M* or *M*–*N*)

Greatest Common Divisor

(watch video)

Find the greatest common divisor of 56 and 70:

$$56 = 2 \times 2 \times 2 \times 7$$
$$70 = 2 \times 5 \times 7$$
$$GCD = 2 \times 7$$
$$= 14$$

Find the greatest common divisor of 132, 198 and 330:

$$132 = 2 \times 2 \times 3 \times 11$$
$$198 = 2 \times 3 \times 3 \times 11$$
$$330 = 2 \times 3 \times 5 \times 11$$
$$GCD = 2 \times 3 \times 11$$
$$= 66$$

(watch video)

Least Common Multiple

Find the least common multiple of 12 and 56:

$$12 = 2 \times 2 \times 3$$
$$56 = 2 \times 2 \times 2 \times 7$$
$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 7$$
$$= 168$$

Find the least common multiple of 18 and 42:

$$18 = 2 \times 3 \times 3$$
$$42 = 2 \times 3 \times 7$$
$$\text{LCM} = 2 \times 3 \times 3 \times 7$$
$$= 126$$

(watch video)

$$(\text{GCD of } x \text{ and } y)(\text{LCM of } x \text{ and } y) = xy$$

Integer Properties Flashcards

(watch video)

$$\text{odd} \pm \text{odd} = \text{even}$$
$$\text{odd} \pm \text{even} = \text{odd}$$
$$\text{even} \pm \text{even} = \text{even}$$

$$\text{odd} \times \text{odd} = \text{odd}$$
$$\text{odd} \times \text{even} = \text{even}$$
$$\text{even} \times \text{even} = \text{even}$$

$\dfrac{\text{even}}{\text{even}}$ can be a non-integer, even or odd

If $\dfrac{\text{even}}{\text{odd}}$ is an integer, it will be even

$\dfrac{\text{odd}}{\text{even}}$ cannot be an integer

If $\dfrac{\text{odd}}{\text{odd}}$ is an integer, it will be odd

(watch video)

- Create a table to **test cases**
  - Use "E" and "O" and even/odd rules
  - Plug in values and evaluate
- Draw conclusions based on outcomes

(watch video)

. . .-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7,. . .

Every n^{th} integer is divisible by n

n consecutive integers ➡ 1 number must be divisible by n

Integer Properties Flashcards

(watch video)

divisor remainder

$$11 \div 4 = 2(3)$$

dividend quotient

$0 \leq \text{remainder} < \text{divisor}$

remainder

If $N \div D = Q(R)$, then the possible values of N are: $R, \quad R + D, \quad R + 2D, \quad R + 3D, \ldots$

If $N \div D = Q(R) \Rightarrow Q \times D + R = N$

remainder

(watch video)

$$\text{average} = \text{mean} = \frac{\text{sum of } n \text{ numbers}}{n}$$

$$\text{sum of } n \text{ numbers} = (\text{mean})(n)$$

median: the middlemost value when the numbers are arranged in ascending order

n is odd: median = middle number

n is even: median = average of the 2 middle numbers

mode: the number that occurs most frequently

(watch video)

If the numbers in a set are evenly spaced, then the mean and median of that set are equal

If the mean and median of a set are equal, then the numbers in that set **may or may not** be evenly spaced

Statistics Flashcards

(watch video)

$$\text{Weighted average} = (\text{proportion})\begin{pmatrix}\text{group}\\ \text{A}\\ \text{average}\end{pmatrix} + (\text{proportion})\begin{pmatrix}\text{group}\\ \text{B}\\ \text{average}\end{pmatrix} + \cdots$$

Group A average $= a$

Group B average $= b$

case 1) population A = population B

➡ average of combined group $= \dfrac{a+b}{2}$

case 2) population A is greater than population B

➡ average of combined group is closer to a

case 3) population B is greater than population A

➡ average of combined group is closer to b

gmat PREPNOW

(watch video)

range = greatest value – least value

Standard Deviation of $x_1, x_2, x_3, x_4, \ldots, x_n$

m = mean

n = number of values

$$SD = \sqrt{\frac{(x_1 - m)^2 + (x_2 - m)^2 + (x_3 - m)^2 + \cdot\cdot\cdot + (x_n - m)^2}{n}}$$

Informal definition

Standard deviation is the average distance the data values are away from the mean.

(watch video)

$\text{variance} = (\text{standard deviation})^2$

<u>units of standard deviation</u>

If the standard deviation of a set of numbers is k, then k = 1 *unit of standard deviation*

(watch video)

When tackling counting questions, consider listing and counting.

(watch video)

Fundamental Counting Principle

If a task is comprised of stages, where

one stage can be accomplished in ***A*** ways,
another stage can be accomplished in ***B*** ways,
another stage can be accomplished in ***C*** ways,
and so on,

then the total number of ways to accomplish the task is

$A \times B \times C \times \ldots$

Can I take the task of "building" possible outcomes and break it into individual stages?

(watch video)

Arranging *n* Unique Objects

n unique objects can be arranged in *n*! ways

Factorial Notation

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$n! = n \times (n-1) \times (n-2) \times \cdot \;\cdot\; \cdot \times 3 \times 2 \times 1$$

$$0! = 1$$

(watch video)

- Counting question with **restrictions**
  - Adhere to the restriction
  - Apply the Restrictions Rule

Restrictions Rule

# of ways to follow a restriction = # of ways to ignore restriction − # of ways to break restriction

- Restrictions Rule is useful for questions involving "at least" and "at most"

(watch video)

MISSISSIPPI Rule

- When arranging objects, determine whether the objects are unique

Arranging Objects When Some are Alike

Given n objects where ***A*** are alike, another ***B*** are alike, another ***C*** are alike and so on, the number of ways to arrange the n objects is

$$\frac{n!}{(A!)(B!)(C!)\cdots}$$

(watch video)

Combination: A selection from a set of unique objects where the order of the selected objects does not matter.

- Choosing committee members is a popular theme

Combination Formula

r objects can be selected from a set of n unique objects in ${}_nC_r$ ways, where:

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(watch video)

When to use Combinations

General Strategy

- If possible, break the required task into stages
- "Does the outcome of each stage differ from the outcomes of other stages?"

No ➡ Combination

Yes ➡ Fundamental Counting Principle or other strategy

(watch video)

Calculating Combinations Shortcut

r objects can be selected from a set of n unique objects in $_{n}C_{r}$ ways

$$_{n}C_{r} = \frac{\text{first } r \text{ values of } n!}{r!}$$

(watch video)

Counting Strategies

1. List outcomes
2. If restrictions exist, consider applying the Restrictions Rule
3. If possible, break the required task into stages
4. Ask, "Do the outcomes of each stage differ from the outcomes of other stages?"

    No ➡ combination
    Yes ➡ continue below

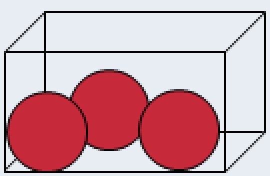
5. Determine the number of ways to accomplish each stage, beginning with the most restrictive stage(s)
6. Apply the Fundamental Counting Principle
7. Arranging objects that are not unique may require the MISSISSIPPI Rule

(watch video)

- The probability of an event = the likelihood that the event will occur
- $0 \leq$ probability of an event ≤ 1

P(Event A) = 0 ➡ Event A will not occur

P(Event A) = 1 ➡ Event A will definitely occur



P(selected ball is green) = 0

P(selected ball is red) = 1

(watch video)

Probability of an Event

In an experiment where each outcome is equally likely, the probability that event A will occur is:

$$P(A) = \frac{\text{number of outcomes where A occurs}}{\text{total number of possible outcomes}}$$

Calculating the denominator first will often help you gain insight into a question

(watch video)

Complement

$$P(\text{event happens}) = 1 - P(\text{event DOES NOT happen})$$

Possible Uses

- When calculating P(event DOES NOT happen) is easier
- Questions with "at least" and "at most"

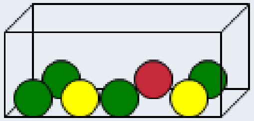
(watch video)

Mutually Exclusive Events

Two events are mutually exclusive if both events cannot occur together.

Example

A ball is randomly selected from the box



Event A: The ball is red

Event B: The ball is yellow

Can both events occur together?

No ➡ The events are mutually exclusive

➡ $P(\text{A and B}) = 0$

(watch video)

"or" Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If events A and B are mutually exclusive, then P(A and B) = 0

Events A and B are mutually exclusive

$$\Rightarrow P(A \text{ or } B) = P(A) + P(B)$$

(watch video)

$$P(A\ \text{AND}\ B) = P(A) \times P(B|A)$$

$P(B|A)$ = probability of event B given that event A has occurred

(watch video)

Events A and B are **dependent** if the occurrence of one event affects the probability of the other.
If Events A and B are dependent, then:

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Events A and B are **independent** if the occurrence of one event does not affect the probability of the other. If Events A and B are independent, then:

$$P(A \text{ and } B) = P(A) \times P(B)$$

(watch video)

Rewriting Questions

"or" Probabilities
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Dependent Events
$P(A \text{ and } B) = P(A) \times P(B|A)$

Complement
$P(A) = 1 - P(\text{NOT } A)$

Independent Events
$P(A \text{ and } B) = P(A) \times P(B)$

Questions

- What must occur to get this outcome?
- Will it be faster to use the complement?
- Are the events mutually exclusive?
- Are the events independent?

(watch video)

General Probability Strategies

- Consider using the complement
- Determine the general approach
  - Basic probability formula
  - Probability rules
- Basic probability formula
  - Equally likely outcomes?
  - List or use counting techniques
- Probability rules
  - Rewrite question by asking, "What must occur?"
  - "or" probability ➡ test for mutually exclusivity
  - "and" probability ➡ test for independence

(watch video)

Guessing Strategies

- Use your instincts
- For questions that may involve the complement, eliminate any answer choice that does not combine with another answer choice to add to 1