Some of the trickiest questions in GMAT are based on positive/negative bases and powers. Today, let's look at some of their properties. First thing you must understand is that if the base is positive, it will stay positive no matter what the power . a^n is equal to a*a*a* … (n times). Since only positive numbers are multiplied with each other, the product will always be positive. (We cannot say the same thing about negative bases but let's ignore them in today's post.)

For example, 5^n must be positive no matter what the value of n.
The base 'a', which is positive, can belong to one of the two ranges – 'Greater than 1' or 'Between 0 and 1' (or it can be equal to 1). Let's see what happens to a^n is each case.

Case 1: Base 'a' greater than 1
If n is positive, a^n > 1 (for example, 3^2 = 9 which is greater than 1)
If n = 0, a^n = 1 (for example, 3^0 = 1)
If n is negative, 0 < a^n < 1 (for example, 3^(-2) = 1/9 which is less than 1)

We saw what happens if the base is greater than 1. Let's see what happens if the base is between 0 and 1.

Case 2: Base 'a' lies between 0 and 1
If n is positive, a^n < 1 (for example, (1/2)^2 = 1/4 which is less than 1)
If n = 0, a^n = 1 (for example, (1/2)^0 = 1)
If n is negative, a^n > 1(for example (1/2)^(-2) = 4 which is greater than 1)

If the base is equal to 1, it will stay 1 no matter what the power.

These relations hold the other way round too. If a is between 0 and 1, and a^n > 1, n must be negative etc. Plug in some values to understand them.

Let's look at a question dealing with these concepts now.

Question: In which one of the following choices must m be greater than n?

(A) 0.8^m = 0.8^n
(B) 0.8^m = 0.8^2n
(C) 0.8^m > 0.8^n
(D) 8^m < 8^n
(E) 8^m > 8^n

Solution:
The question asks you for the option where m MUST be greater than n. This means that there must be no values of m and n which satisfy the equation/inequality but where m is equal to or less than n. If we can find a single pair of values such that m = n or m < n which satisfy the equation/inequality, that equation/inequality cannot be the answer.

Options (A) and (B) are quite straight forward.

Option (A): Here, we can see that m must be equal to n. Hence this cannot be the answer.

Option (B): Values m = 0 and n = 0 hold for the equation but m is equal to n. Hence this cannot be the answer.

Let's go on to the inequalities now.

In the following three options, each term is positive (since a positive term to any power will stay positive). Therefore, we can take the term on the right to the left hand side so that we have to deal with a single base.

(C) (0.8)^m > (0.8)^n
(0.8)^(m-n) > 1

When the base is between 0 and 1 and the expression is greater than 1, what can we say about the power? We can say that the power must be negative. (Go and check back the third point of case 2.) Here, m-n must be negative i.e. m – n < 0 or m < n. Hence this cannot be the answer.

(D) 8^m < 8^n
8^(m – n) < 1

When the base is greater than 1 and the expression is less than 1, what can we say about the power? We can say that the power must be negative. (Go and check back the third point of case 1.) Here, m – n must be negative i.e. m – n < 0 or m < n. Hence this cannot be the answer.

(E) 8^m > 8^n
8^(m – n) > 1

When the base is greater than 1 and the expression is greater than 1, what can we say about the power? We can say that the power must be positive. (Go and check back the first point of case 1.) Here, m-n must be positive i.e. m – n > 0 or m > n. That is, m must be greater than n. That is what we were looking for! Option (E) must be the answer.

Answer (E)

This was a direct application of the rules we saw above. Hope you understand them well. They could help you extensively while dealing with equaltions/inequalities involving exponents.