Today, let's look at a question that involves inequalities and modulus and is best understood using the concept of sets. It is not a difficult question but it is still very tricky. You could easily get it right the first time around but if you get it wrong, it could take someone many trials before he/she is able to convince you of the right answer. Even after I write a whole post on it, I wouldn't be surprised if I see "but I still don't get it" in the comments below!

Anyway, enough of introduction! Let's get to the question now.

Question: If x/|x| < x, which of the following must be true about x?

(A) x > 1

(B) x > -1

(C) |x|< 1

(D) |x| = 1

(E) |x|^2 > 1

Solution:

First thing we do is tackle the mod. We know that |x| is just the absolute value of x.

So, x/|x| can take only 2 values: 1 or -1

If x is positive, x/|x| = 1 e.g. if x = 4, then 4/|4| = 1

If x is negative, x/|x| = -1 e.g. if x = -4, then -4/|-4| = -4/4 = -1

x cannot be 0 because we cannot have 0 in the denominator of an expression.

Now let's work on the inequality.

x > x/|x| implies x > 1 if x is positive or x > -1 if x is negative.

Hence, for this inequality to hold, either x > 1 (when x is positive) or -1 < x< 0 (when x is negative)

x can take many values e.g. -1/3, -4/5, 2, 5, 10 etc.

Now think – which of the following MUST BE TRUE about every value that x can take?

(A) x > 1

or

(B) x > -1

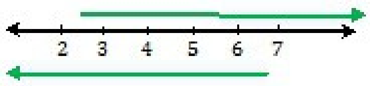
I hope that you agree that x > 1 doesn't hold for every possible value of x whereas x > -1 holds for every possible value of x. Mind you, every value greater than -1 need not be a possible value of x.

This concept might need some more work. Let me explain with another example.

Forget this question for a minute. Say instead you have this question:

Example 1: x > 2 and x < 7. What integral values can x take?

I guess most of you will come up with 3, 4, 5, 6. That's correct. I can represent this on the number line.

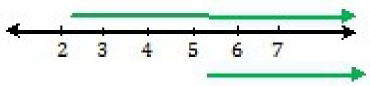


The top arrow shows x > 2 and the bottom arrow shows x < 7. You see that the overlapping area includes 3, 4, 5 and 6. That is the region that satisfies both the inequalities.

Now consider this:

Example 2: x > 2 or x > 5. What integral values can x take?

Let's draw that number line again.



So is the solution again the overlapping numbers i.e. all integers greater than 5? No. This question is different. x is greater than 2 OR greater than 5. This means that if x satisfies at least one of these conditions, it is included in your answer. Think of sets. AND means it should be in both the sets (i.e. the overlapping part) as was the case in example 1. OR means it should be in at least one of the sets. Hence, which values can x take? All integral values starting from 3 onwards i.e. 3, 4, 5, 6, 7, 8, 9 …

Now go back to this question. The solution is a one liner.

$x/|x|$ is either 1 or -1.

So $x > 1$ or $x > -1$

So which values can x take? All values included in at least one of the sets. Therefore, $x > -1$.

Note that the confusion lies only between the first two options. The other three options are rejected outright.

(C) $|x| < 1$ implies $-1 < x < 1$. Definitely doesn't hold.

(D) $|x| = 1$ implies $x = 1$ or -1. Definitely doesn't hold.

(E) $|x|^2 > 1$ implies either $x < -1$ or $x > 1$. Definitely doesn't hold.

So what do you say? Are you convinced that the answer is (B)?