Today, as requested by Pratap, we are going to take removal/replacement in mixtures. For those of you who were looking forward to some more tricky probability questions, I will make up for your disappointment next week. Meanwhile, rest assured, replacement is a very interesting, not to mention useful, concept in GMAT. So brace yourself to learn some new things today.

First of all, many “replacement” questions are nothing but the plain old mixture questions, the type we discussed in this post, with an extra step. So don’t flip out the moment you read the word “replace.” Let me show you what I mean:

Example 1: If a portion of a 50% alcohol solution (in water) is replaced with 25% alcohol solution, resulting in a 30% alcohol solution, what percentage of the original alcohol was replaced?

(A) 3%
(B) 20%
(C) 66%
(D) 75%
(E) 80%

Solution: What this question says is that a solution of 25% alcohol (say solution 1) is mixed with another solution of 50% alcohol (say solution 2) to give us a 30% alcohol solution. We don’t know in what quantities they were mixed. Can we find out the ratio in which these two solutions were mixed? If you are not sure, check out this post.

w1/w2 = (A2 – Aavg)/(Aavg – A1) = (50 – 30)/(30 – 25) = 4/1

So the two solutions were mixed in the ratio 4:1. Mind you,

(volume of 25% alcohol solution) : (volume of 50% alcohol solution) = 4:1

Out of 5 parts of total solution obtained, 50% solution was 1 part while 4 parts was 25% solution. So what part of the 50% solution was removed and replaced by the 25% solution? Would you agree it is (4/5)th? We can say that 80% of the 50% solution was replaced by the 25% solution.

But the actual question is something else: What percentage of **original alcohol** was replaced?

We need to find the percentage of original alcohol that was replaced, not the percentage of original solution! Now, here is the interesting thing: Since the solution is homogenous, it you replace 80% of it, 80% of the original amount of alcohol in the solution will be replaced. So the answer will still be 80%. I will explain this point in detail since it is extremely important while dealing with the really treacherous replacement questions.

Let’s say we have 100 liters of 50% alcohol solution (so alcohol = 50 liters and water = 50 liters). When we remove 80% of the solution, we remove 80 liters of the solution. In the solution we remove, we will still have 50% alcohol i.e. we will have 40 liters alcohol and 40 liters water. In the 20 liters solution that is remaining, we will have 10 liters alcohol and 10 liters water. So amount of alcohol removed is 40/50 = 80%

Important Points to Remember:

1. When a fraction of a homogenous solution is removed, the percentage of either part does not change. If milk:water = 1:1 in initial solution, it remains 1:1 in the leftover solution.

2. When you add one component to a solution, the amount of the other component does not change. In milk and water solution, if you add water, amount of milk is still the same (not percentage but amount). If milk:water = 1:1 in 10 liters of solution, it means, milk = 5 liters and water = 5 liters. Now, if you add 2 liters of water, amount of water = 7 liters but

amount of milk is still 5 liters. The percentage of milk has changed but the amount of milk is still the same.

3. Amount of A = Concentration of A * Volume of the mixture

Amount = C*V

In a 10 liter mixture of milk and water, if milk is 50%, amount of milk = 50%*10 = 5 liter
When you add water to this solution, the amount of milk does not change (as discussed in point 2 above). The concentration of milk changes of course since the solution is diluted.

Amount of milk before addition = Amount of milk after addition

So Initial Concentration of milk * Initial Volume of solution = Final Concentration of milk * Final Volume of solution

Ci * Vi = Cf * Vf

Or

Cf = Ci * (Vi/Vf)

Remember, this is the relation between the initial and final concentration of milk since the amount of milk remains the same. The amount of water does not remain the same since more water is added. Hence, this relation does not hold for water.

Go through these points repeatedly till you are very comfortable with them!

Example 2: 10% of a 50% alcohol solution is replaced with water. From the resulting solution, again 10% is replaced with water. This step is repeated once more. What is the concentration of alcohol in the final solution obtained?

(A) 3%
(B) 20%
(C) 25%
(D) 36%
(E) 40%

Solution: In each step, we are replacing the solution with water. Every time we remove p% of the solution, the amount of alcohol goes down but the concentration of alcohol in the mixture remains the same (point 1 above). When we add water, the amount of alcohol remains the same.

Let’s try and perform the steps to see what happens:

Step 1: 10% of a 50% alcohol solution is removed – In the leftover solution, concentration of alcohol remains the same i.e. 50%. If initial volume of the solution was 10 liters, new volume is 9 liters.

Step 2: Water is added to the solution to replace the 10% shortfall – the concentration of alcohol changes now (but the amount of alcohol is still the same). Also, the volume of the solution is 10 liters again. In this new solution,

The concentration of alcohol after this step Cf1 = (50%)*(9/10) (using point 3)

Step 3: 10% of the solution with concentration of alcohol = Cf1 is removed – In the leftover solution, concentration of alcohol is still Cf1. The volume of the solution reduces to 9 liters again.

Step 4: Water is added to the solution to replace the 10% shortfall – The concentration of alcohol changes now. Also, the volume of the solution is 10 liters again. In this new solution,

The concentration of alcohol after this step Cf2 = Cf1*(9/10) = (50%)*(9/10) *(9/10)

Step 5: 10% of the solution with concentration of alcohol = Cf2 is removed – In the leftover solution, concentration of alcohol is still Cf2. The volume of the solution reduces to 9 liters again.

Step 6: Water is added to the solution to replace the 10% shortfall – The concentration of alcohol changes now. Also, the volume of the solution is 10 liters again. In this new solution,

The concentration of alcohol after this final step Cf3 = Cf2*(9/10) = .5*(9/10) *(9/10) *(9/10)

The concentration changes only when water is added. Each time water is added, the concentration becomes (9/10)th of the previous concentration.

Final concentration of alcohol = (50%) *(9/10) *(9/10) *(9/10) = 36.45%

Answer (D)

Now try the following question to see if the theory makes sense to you:

Example 2: 20% of a 40% alcohol solution is removed and replaced with water. From the resulting solution, again 20% is replaced with water. This step is repeated once more. What is the concentration of alcohol in the final solution obtained?

Solution:

Concentration of alcohol in the final solution = (40%) * (8/10) * (8/10) * (8/10) = 20.48%

I will leave you here with a complicated question. See if you can arrive at the answer on your own. If not, let me know!

Question 1: A container has 3 liters of pure lime juice. 1 liter from the container is taken out and 2 liter water is added. The process is repeated several times. After 19 such operations, quantity of lime juice in the mixture is

(A) 2/7 L
(B) 3/7 L
(C) 5/14 L
(D) 5/19 L
(E) 6/19L

This question can be solved in under a minute if you understand the concept of concentration and volume. Take your time and see if you can do it on your own!