You can't talk about probability without mentioning Las Vegas, and with the football gambling season looming it seems time for a probability-themed post with a gambling lead. As the football season approaches, you may well be placing your futures bets, noting that Alabama is a 9:2 favorite to win the collegiate national championship and that Indianapolis is a 13:2 favorite to win the Super Bowl.

The interesting thing about those futures bets is that when you select, say, Alabama, the sports book gets "everyone else". Will Alabama defend its title? Possibly, but the football season has so many hurdles (LSU, Auburn, the SEC championship game, the BCS championship game, injuries, Nick Saban leaving mid-season for another job…) that "everyone else" is a pretty good bet. Most seasons, a team that loses one game is eliminated from contention for the national championship, so a team would have to go undefeated in order to win. In a 12-game season, even if a team is 90% likely to win each game, that corresponds to:

9/10 * 9/10 * 9/10…. probability of going undefeated. For 12 games, the calculation would be (9/10)^12 = <30% chance of going undefeated.

And that's with a 90% chance of winning each game. Sure, Alabama will be favored against LSU, Auburn, and some of its other SEC opponents, but maybe it only has a 60% chance of winning those big games. That puts the undefeated odds down considerably (even with a returning Heisman winner).

How can this help you on the GMAT? As we've talked about, the opposite of "Alabama wins the national championship" is "anyone else wins the national championship", and that setup of complementary events (one and only one of those options will occur) allows you to tackle difficult problems with some insightful ease.

Here's an example of a GMAT problem that might test this subject:

Alabama is undefeated with four games remaining, with two at home and two away. If it has a 75% chance of winning its home games and a 50% chance of winning its away games, what is the probability that Alabama loses at least one of its last four games?

This question could get quite involved, as "at least one" loss has many permutations:

Loses the first and wins the next three (which would correspond to 1/4 * 3/4 * 1/2 * 1/2)

Loses the first and the last and wins the middle two (1/4 * 3/4 * 1/2 * 1/2)

Wins the first three and loses the last (3/4 * 3/4 * 1/2 * 1/2)

Loses all four (1/4 * 1/4 * 1/2 * 1/2)

Etc.

Each of the above corresponds to one sequence via which Alabama would lose at least one game, and you could drive yourself crazy (particularly if you bleed Crimson) thinking of all the possible ways to lose at least one game.

However, you can also look at it this way (through Crimson colored glasses):

If Alabama does NOT lose "at least one game" that means that it wins them all. And it's easier (and more fun for Bama fans) to calculate the odds of winning them all:

There is only one sequence that works – Win-Win-Win-Win:

3/4 * 3/4 * 1/2 * 1/2 = 9/64 chance that Alabama wins the rest.

Because there's a 100% chance that "something" happens, and we can divide that 100% into two comprehensive categories: "Alabama goes undefeated" or "Alabama loses at least one game", then we can take 100% and subtract "undefeated" to get "loses at least one":

1 – 9/64 = 55/64 probability that Alabama loses at least one of its last four games.

When you see questions that ask for the probability of "at least one" occurrence, the easiest way to calculate them is to calculate the probability of "no" occurrences and then subtract that from 100%. There are several sequences that could give you "at least one" (just the first, just the last, all of them, etc.) but only one that gives you "none" (all nones), so use that strategy to make quick, efficient work of probability questions on the GMAT.

What is your probability of missing at least one question on the math section of the GMAT? Well, it's 1 – getting-them-all-right, and the odds of your getting them all right just went up after reading this post!