First of all, what does it mean precisely to be "more likely than not"? Well, since the probability OF something is 1 — the probability of NOT, these two quantities add up to 1. Something is "equally likely as not," then, if the probability of its occurring and the probability of its not occurring are both 50% — and therefore something is "more likely than not" as soon as its probability climbs the slightest bit above 50%, forcing the probability of "not" to dip just below 50%. Now, is there a quantity can we seek directly as the "not"? Well, the alternative outcome is that no two people share a birthday… in other words, that each new person we look at in the room has a birthday we haven't seen yet.

And the expression for this probability is delightfully straightforward:

There's a 100% chance the first person has a birthday we haven't seen yet, because we haven't seen anything yet. In other words, a 365/365 chance.

Then that first person has "used" one available calendar day, so there is a 364/365 chance that the second person we look at occupies an "unused" day.

If that goes well, two days will have been "used" and are no longer acceptable, so there's a 363/365 chance that the third person we look at was born on a still acceptable day.

And the fourth person? A 362/365 chance that he/she was born on a still acceptable day.

And so on.

And so extrapolating to n people, we calculate that the probability that all n people were born on DIFFERENT days is

(365/365)(364/365)(363/365)…((365-(n-1))/365).

[You might espy your permutations formula in the numerator here: (365)(364)(363)(365-(n-1)) = 365!/(365-n)! The denominator (365)(365)(365)…(365) is simply 365^n.]

So what's the probability that that's NOT the case… i.e. that not all n people were born on different days… i.e. that at least two share a birthday? Exactly 1 – the above. And we wanted 1 – the above to stop just above 50%.

Let's revisit our answer choices and do some thinking. It should be clear first of all that (A) is not right. In fact, if I have 366 people in a room, it is 100% certain that two of them will share a birthday, because there are only 365 different birthdays to go around. Next, it should be almost as clear that (B) is not right. With 365 people in the room, it is sooo close to certain that two will share a birthday, since each successive person we look at has a smaller and smaller chance of having an "unused" birthday as the collection of "used" birthdays grows, and since by the time we get to the last person, there is a mere 1/365 chance that he will preserve the different-ness of all birthdays, even in the unlikely event that we make it that far. (C) is a pretty tempting and popular answer, but it should trigger your there's-a-trap radar — would this question really help to separate high-end test-takers from low-end ones if all you had to do was take half of 365? No. So it's (D) or (E)… and (E) would seem shockingly low…

But it turns out, contrary to most people's intuition, that indeed, in a room of 23 people, it is more likely than not that two share a birthday! (In fact, if we went with (D) and put 60 people in the room, the probability that two would share a birthday is over 99%.) Recall our expression from above for the probability that at least two of n people share a birthday; let's check it for n = 23:

P(shared b'day among 23 people) = 1 – (365/365)(364/365)(363/365)… (343/365) = 50.7%.

We swear.