MUST DO

QUANT TOPIC – WISE EXCERSIE
(700-800 LEVEL QUESTIONS)

TOPIC 1 – GENERAL ARITHMETIC

Venn Diagram / Overlapping SETS

1.

For an overlapping sets problem we can use a double-set matrix to organize our information and solve. Because the values are in percents, we can assign a value of 100 for the total number of interns at the hospital. Then, carefully fill in the matrix based on the information provided in the problem. The matrix below details this information. Notice that the variable x is used to detail the number of interns who receive 6 or more hours of sleep, 70% of whom reported no feelings of tiredness.

	Tired	Not Tired	TOTAL
6 or more hours	$.3x$	$.7x$	x
Fewer than 6 hours	75		80
TOTAL			100

In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. Thus, the boldfaced entries below were derived using the above matrix.

	Tired	Not Tired	TOTAL
6 or more hours	**6**	**14**	**20**
Fewer than 6 hours	75	**5**	80
TOTAL	**81**	**19**	100

We were asked to find the percentage of interns who reported no feelings of tiredness, or 19% of the interns.

The correct answer is C.

2.

This is an overlapping sets problem concerning two groups (students in either band or orchestra) and the overlap between them (students in both band and orchestra).

If the problem gave information about the students only in terms of percents, then a smart number to use for the total number of students would be 100. However, this problem gives an actual number of students ("there are 119 students in the band") in addition to the percentages given. Therefore, we cannot assume that the total number of students is 100.

Instead, first do the problem in terms of percents. There are three types of students: those in band, those in orchestra, and those in both. 80% of the students are in only one group. Thus, 20% of the students are in both groups. 50% of the students are in the band only. We can use those two figures to determine the percentage of students left over: 100% - 20% - 50% = 30% of the students are in the orchestra only.

Great - so 30% of the students are in the orchestra only. But although 30 is an answer choice, watch out! The question doesn't ask for the percentage of students in the orchestra only, it asks for the number of students in the orchestra only. We must figure out how many students are in Music High School altogether.

The question tells us that 119 students are in the band. We know that 70% of the students are in the band: 50% in band only, plus 20% in both band and orchestra. If we let x be the total number of students, then 119 students are 70% of x, or 119 = .7x. Therefore, x = 119 / .7 = 170 students total.

The number of students in the orchestra only is 30% of 170, or .3 × 170 = 51.

The correct answer is B.

3.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Because the values here are percents, we can assign a value of 100 to the total number of lights at Hotel California. The information given to us in the question is shown in the matrix in boldface. An x was assigned to the lights that were "Supposed To Be Off" since the values given in the problem reference that amount. The other values were filled in using the fact that in a double-set matrix the sum of the first two rows equals the third and the sum of the first two columns equals the third.

	Supposed To Be On	Supposed To Be Off	TOTAL
Actually on		**$0.4x$**	**80**
Actually off	**$0.1(100 - x)$**	$0.6x$	20
TOTAL	$100 - x$	**x**	**100**

Using the relationships inherent in the matrix, we see that:
$0.1(100 - x) + 0.6x = 20$
$10 - 0.1x + 0.6x = 20$
$0.5x = 10$ so $x = 20$

We can now fill in the matrix with values:

	Supposed To Be On	Supposed To Be Off	TOTAL
Actually on	72	8	80
Actually off	8	12	20
TOTAL	80	20	100

Of the 80 lights that are actually on, 8, or 10% percent, are supposed to be off.

The correct answer is D.

4.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Because the given values are all percentages, we can assign a value of 100 to the total number of people in country Z. The matrix is filled out below based on the information provided in the question.

The first sentence tells us that 10% *of all of the people* do have their job of choice but do not have a diploma, so we can enter a 10 into the relevant box, below. The second sentence tells us that 25% *of those who do not have their job of choice* have a diploma. We don't know how many people do not have their job of choice, so we enter a variable (in this case, x) into that box. Now we can enter 25% of those people, or 0.25x, into the relevant box, below. Finally, we're told that 40% of all of the people have their job of choice.

	University Diploma	NO University Diploma	TOTAL
Job of Choice		10	40
NOT Job of Choice	$0.25x$		x
TOTAL			100

In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. Thus, the boldfaced entries below were derived using relationships (for example: $40 + x = 100$, therefore $x = 60$. $0.25 \times 60 = 15$. And so on.).

	University Diploma	NO University Diploma	TOTAL
Job of Choice	**30**	10	40
NOT Job of Choice	**15**	**45**	**60**
TOTAL	***45***	**55**	100

We were asked to find the percent of the people who have a university diploma, or 45%.

The correct answer is B.

5.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. The boldfaced values were given in the question. The non-boldfaced values were derived using the fact that in a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. The variable p was used for the total number of pink roses, so that the total number of pink and red roses could be solved using the additional information given in the question.

	Red	Pink	White	TOTAL
Long-stemmed			**0**	80

Short-stemmed	5	**15**	**20**	**40**
TOTAL	100 - *p*	*p*	20	**120**

The question states that the percentage of red roses that are short-stemmed is equal to the percentage of pink roses that are short stemmed, so we can set up the following proportion:

$$\frac{5}{100 - p} = \frac{15}{p}$$

$5p = 1500 – 15p$

$p = 75$

This means that there are a total of 75 pink roses and 25 red roses. Now we can fill out the rest of the double-set matrix:

	Red	Pink	White	TOTAL
Long-stemmed	20	60	**0**	80
Short-stemmed	5	**15**	**20**	**40**
TOTAL	25	75	20	**120**

Now we can answer the question. 20 of the 80 long-stemmed roses are red, or 20/80 = 25%.

The correct answer is B.

6.

You can solve this problem with a matrix. Since the total number of diners is unknown and not important in solving the problem, work with a hypothetical total of 100 couples. Since you are dealing with percentages, 100 will make the math easier.

Set up the matrix as shown below:

	Dessert	**NO dessert**	**TOTAL**
Coffee			
NO coffee			
TOTAL			**100**

Since you know that 60% of the couples order BOTH dessert and coffee, you can enter that number into the matrix in the upper left cell.

	Dessert	**NO dessert**	**TOTAL**
Coffee	**60**		
NO coffee			
TOTAL			100

The next useful piece of information is that 20% of the couples who order dessert don't order coffee. **But be careful!** The problem does not say that 20% of the *total* diners order dessert and don't order coffee, so you CANNOT fill in 40 under "dessert, no coffee" (first column, middle row). Instead, you are told that 20% **of the couples who order dessert** don't order coffee.

Let x = total number of couples who order dessert. Therefore you can fill in $.2x$ for the number of couples who order dessert but no coffee.

	Dessert	**NO dessert**	**TOTAL**
Coffee	60		
NO coffee	$.2x$		
TOTAL	x		100

Set up an equation to represent the couples that order dessert and solve:

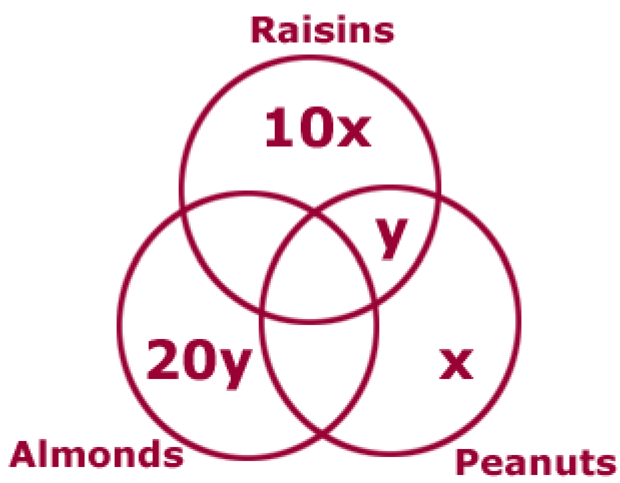
$$60 + .2x = x$$
$$60 = .8x$$
$$x = 75$$

75% of all couples order dessert. Therefore, there is only a 25% chance that the next couple the maitre 'd seats will *not* order dessert. The correct answer is B.

7.

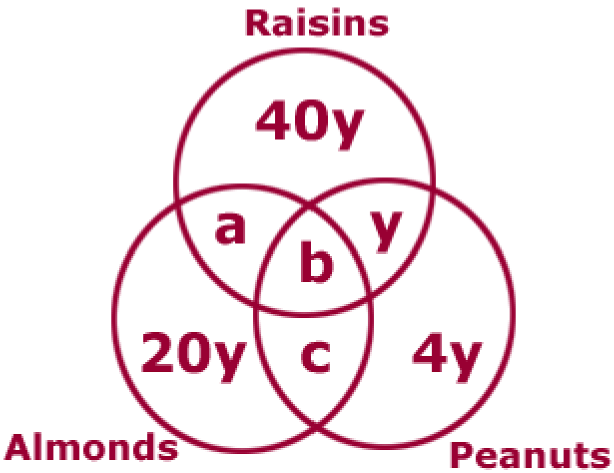
This problem involves 3 overlapping sets. To visualize a 3 set problem, it is best to draw a Venn Diagram.

We can begin filling in our Venn Diagram utilizing the following 2 facts: (1) The number of bags that contain only raisins is 10 times the number of bags that contain only peanuts. (2) The number of bags that contain only almonds is 20 times the number of bags that contain only raisins and peanuts.



Next, we are told that the number of bags that contain only peanuts (which we have represented as x) is one-fifth the number of bags that contain only almonds (which we have represented as 20y).

This yields the following equation: $x = (1/5)20y$ which simplifies to $x = 4y$. We can use this information to revise our Venn Diagram by substituting any x in our original diagram with 4y as follows:



Notice that, in addition to performing this substitution, we have also filled in the remaining open spaces in the diagram with the variable a, b, and c.

Now we can use the numbers given in the problem to write 2 equations. First, the sum of all the expressions in the diagram equals 435 since we are told that there are 435 bags in total. Second, the sum of all the expressions in the almonds circle equals 210 since we are told that 210 bags contain almonds.

$435 = 20y + a + b + c + 40y + y + 4y$
$210 = 20y + a + b + c$
Subtracting the second equation from the first equation, yields the following:

$225 = 40y + y + 4y$
$225 = 45y$
$5 = y$

Given that $y = 5$, we can determine the number of bags that contain only one kind of item as follows:

The number of bags that contain only raisins = $40y = 200$
The number of bags that contain only almonds = $20y = 100$
The number of bags that contain only peanuts = $4y = 20$

Thus there are 320 bags that contain only one kind of item. The correct answer is D.

8.

For this overlapping set problem, we want to set up a two-set table to test our possibilities. Our first set is vegetarians vs. non-vegetarians; our second set is students vs. non-students.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT			
NON-STUDENT		15	
TOTAL	*x*	*x*	?

We are told that each non-vegetarian non-student ate exactly one of the 15 hamburgers, and that nobody else ate any of the 15 hamburgers. This means that there were exactly 15 people in the non-vegetarian non-student category. We are also told that the total number of vegetarians was equal to the total number of non-vegetarians; we represent this by putting the same variable in both boxes of the chart.

The question is asking us how many people attended the party; in other words, we are being asked for the number that belongs in the bottom-right box, where we have placed a question mark.

The second statement is easier than the first statement, so we'll start with statement (2).

(2) INSUFFICIENT: This statement gives us information only about the cell labeled "vegetarian non-student"; further it only tells us the number of these guests as a *percentage* of the total guests. The 30% figure does not allow us to calculate the actual number of any of the categories.

SUFFICIENT: This statement provides two pieces of information. First, the vegetarians attended at the rate, or in the ratio, of 2:3 students to non-students. We're also told that this 2:3 rate is half the rate for non-vegetarians. In order to double a rate, we double the first number; the rate for non-vegetarians is 4:3 We can represent the actual numbers of non-vegetarians as $4a$ and $3a$ and add this to the chart below. Since we know that there were 15 non-vegetarian non-students, we know the missing common multiple, a, is $15/3 = 5$. Therefore, there were $(4)(5) = 20$ non-vegetarian students and $20 + 15 = 35$ total non-vegetarians (see the chart below). Since the same number of vegetarians and non-vegetarians attended the party, there were also 35 vegetarians, for a total of 70 guests.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT		$4a$ or 20	
NON-STUDENT		$3a$ or 15	
TOTAL	*x* or 35	*x* or 35	? or 70

The correct answer is A.

9.

In an overlapping set problem, we can use a double set matrix to organize the information and solve.

From information given in the question, we can fill in the matrix as follows:

	GREY	WHITE	TOTALS
BLUE		> 3	
BROWN			
TOTALS			55

The question is asking us if the total number of blue-eyed wolves (fourth column, second row) is greater than the total number of brown-eyed wolves (fourth column, third row).

(1) INSUFFICIENT. This statement allows us to fill in the matrix as below. We have no information about the total number of brown-eyed wolves.

	GREY	WHITE	TOTALS
BLUE	$4x$	$3x$	**7x**
BROWN			
TOTALS			55

(2) INSUFFICIENT. This statement allows us to fill in the matrix as below. We have no information about the total number of blue-eyed wolves.

	GREY	WHITE	TOTALS
BLUE			
BROWN	$1y$	$2y$	$\mathbf{3y}$
TOTALS			55

TOGETHER, statements (1) + (2) are SUFFICIENT. Combining both statements, we can fill in the matrix as follows:

	GREY	WHITE	TOTALS
BLUE	$4x$	$3x$	**7x**
BROWN	$1y$	$2y$	$\mathbf{3y}$
TOTALS	$\mathbf{4x + y}$	$\mathbf{3x + 2y}$	55

Using the additive relationships in the matrix, we can derive the equation $7x + 3y = 55$ (notice that adding the grey and white totals yields the same equation as adding the blue and brown totals).

The original question can be rephrased as "Is $7x > 3y$?"

On the surface, there *seems* to NOT be enough information to solve this question. However, we must consider some of the restrictions that are placed on the values of x and y:

(1) ***x* and *y* must be integers** (we are talking about numbers of wolves here and looking at the table, *y*, 3*x* and 4*x* must be integers so *x* and *y* must be integers)

(2) **x must be greater than 1** (the problem says there are more than 3 blue-eyed wolves with white coats so 3x must be greater than 3 or x > 1)

Since *x* and *y* must be integers, there are only a few *x*,*y* values that satisfy the equation $7x + 3y = 55$. By trying all integer values for x from 1 to 7, we can see that the only possible x,y pairs are:

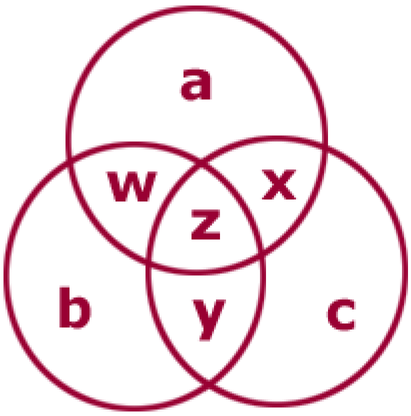
X	y	7x	3y
1	16	7	48
4	**9**	**28**	**27**
7	**2**	**49**	**6**

Since *x* cannot be 1, the only two pairs yield 7*x* values that are greater than the corresponding 3*y* values (28 > 27 and 49 > 6).

The correct answer is C.

10.

This is a problem that involves three overlapping sets. A helpful way to visualize this is to draw a Venn diagram as follows:



Each section of the diagram represents a different group of people. Section *a* represents those residents who are members of only club *a*. Section *b* represents those residents who are members of only club *b*. Section *c* represents those residents who are members of only club *c*. Section *w* represents those residents who are members of only clubs *a* and *b*. Section *x* represents those residents who are members of only clubs *a* and *c*. Section *y* represents those residents who are members of only clubs *b* and *c*. Section *z* represents those residents who are members of all three clubs.

The information given tells us that $a + b + c = 40$. One way of rephrasing the question is as follows: Is $x > 0$? (Recall that *x* represents those residents who are member of fitness clubs A and C but not B).

Statement (1) tells us that $z = 2$. Alone, this does not tell us anything about *x*, which could, for example, be 0 or 10, among many other possibilities. This is clearly not sufficient to answer the question.

Statement (2) tells us that $w + y = 8$. This alone does not give us any information about x, which, again could be 0 or a number of other values.
In combining both statements, it is *tempting* to assert the following.

We know from the question stem that $a + b + c = 40$. We also know from statement one that $z = 2$. Finally, we know from statement two that $w + y = 8$. We can use these three pieces of information to write an equation for all 55 residents as follows:

$a + b + c + w + x + y + z = 55.$
$(a + b + c) + x + (w + y) + (z) = 55.$
$40 + x + 8 + 2 = 55$
$x = 5$

This would suggest that there are 5 residents who are members of both fitness clubs A and C but not B.

However, this assumes that all 55 residents belong to at least one fitness club. Yet, this fact is not stated in the problem. It is possible then, that 5 of the residents are not members of *any* fitness club. This would mean that 0 residents are members of fitness clubs A and C but not B.

Without knowing how many residents are not members of *any* fitness club, we do not have sufficient information to answer this question.

The correct answer is E: Statements (1) and (2) TOGETHER are NOT sufficient.

Percentages

11.

Bob put 20 gallons of gasohol into his car's gas tank, consisting of 5% ethanol and 95% gasoline. Chemical Mixture questions can be solved by using a mixture chart.

SUBSTANCES	AMOUNT	PERCENTAGE
ETHANOL	1	5%
GASOLINE	19	95%
TOTALS	20	100%

This chart indicates that there is 1 gallon of ethanol out of the full 20 gallons, since 5% of 20 gallons is 1 gallon.

Now we want to add x gallons of ethanol to raise it to a 10% ethanol mixture. We can use another mixture chart to represent the altered solution.

SUBSTANCES	AMOUNT	PERCENTAGE
ETHANOL	$1 + x$	10%

GASOLINE	19	90%
TOTALS	$20 + x$	100%

Therefore, the following equation can be used to solve for x:

$$\frac{1 + x}{20 + x} = 10\%$$

$1 + x = 2 + 0.1x$
$0.9x = 1$
$x = 10/9$

The correct answer is C.

12.

We can solve this question as a VIC (Variable in answer choices) by plugging in values for x, y and z:

x	percent mark-up (1st)	10
y	percent discount (2nd)	20
z	original price	100

If a $100 item is marked up 10% the price becomes $110. If that same item is then reduced by 20% the new price is $88.

If we plug $x = 10$, $y = 20$, $z = 100$ into the answer choices, only answer choice (A) gives us 88:

$$\frac{10{,}000(100) + 100(100)(10 - 20) - (10)(20)(100)}{10{,}000} = 88$$

Alternatively we could have solved this algebraically.

A price markup of x percent is the same as multiplying the price z by $\left(1 + \frac{x}{100}\right)$.

A price discount of y percent is the same as by multiplying by $\left(1 - \frac{y}{100}\right)$.

We can combine these as: $z(1 + x/100)(1 - y/100)$.

This can be simplified to: $10{,}000z + 100z(x - y) - xyz$

$\overline{\text{10,000}}$

The correct answer is A.

13.

Let M be the number of the cameras produced in 1995.

[M/(1+y%)]/(1+x%)=1000

M = 1000+10x+10y+xy/10

Knowing that x+y+ xy/100 = 9.2, M can be solve out.

Answer is B

14.

The profit is sale price-cost

For one stock, (sale price-cost)/cost=20%, its cost is 96/1.2=80

For the other stock, (sale price-cost)/cost=-20%, cost is 96/0.8=120

Then, the total profit for the two stocks is:

96-80+96-120 = –8

Answer is C

Work / Rate

15.

Tom's individual rate is 1 job / 6 hours or 1/6.

During the hour that Tom works alone, he completes 1/6 of the job (using *rt* = *w*).

Peter's individual rate is 1 job / 3 hours.
Peter joins Tom and they work together for another hour; Peter and Tom's respective individual rates can be added together to calculate their combined rate: 1/6 + 1/3 = 1/2. Working together then they will complete 1/2 of the job in the 1 hour they work together.

At this point, 2/3 of the job has been completed (1/6 by Peter alone + 1/2 by Peter and Tom), and 1/3 remains.

When John joins Tom and Peter, the new combined rate for all three is: 1/6 + 1/3 + 1/2 = 1. The time that it will take them to finish the remaining 1/3 of the job can be solved:
$rt = w \longrightarrow (1)(t) = 1/3 \longrightarrow t = 1/3.$

The question asks us for the fraction of the job that Peter completed. In the hour that Peter worked with Tom he alone completed: $rt = w \longrightarrow w = (1/3)(1) = 1/3$ of the job. In the last 1/3 of an hour that all three worked together, Peter alone completed: (1/3)(1/3) = 1/9 of the job.

Adding these two values together, we get 1/3 + 1/9 of the job = 4/9 of the job.

The correct answer is E.

16.

We can solve this problem as a VIC (Variable In answer Choice) and plug in values for the variable *x*. Let's say *x* = 6. (Note that there is a logical restriction here in terms of the value of *x*. Lindsay has to have a rate of less than less than 1 room per hour if she needs Joseph's help to finish in an hour).

If Lindsay can paint 1/6 of the room in 20 minutes (1/3 of an hour), her rate is 1/2.
$rt = w$
$r(1/3) = 1/6$
$r = 1/2$

Let *J* be the number of hours it takes Joseph to paint the entire room. Joseph's rate then is 1/*J*. Joseph and Lindsay's combined rate is 1/2 + 1/*J*, which can be simplified:

$1/2 + 1/J \longrightarrow J / 2J + 2 / 2J \longrightarrow (J + 2) / 2J$

If the two of them finish the room in one hour, using the formula of $rt = w$, we can solve for *J*.

$rt = w$ and $t = 1$ (hour), $w = 1$ (job)

$((J + 2) / 2J)(1) = 1 \longrightarrow J + 2 = 2J \longrightarrow J = 2$

That means that Joseph's rate is 1/2, the same as Lindsay's. The question though asks us what fraction of the room Joseph would complete in 20 minutes, or 1/3 of an hour.
$rt = w$
$(1/2)(1/3) = w$
$w = 1/6$

Now we must look at the answer choices to see which one is equal to 1/6 when we plug in $x = 6$. Only C works: (6 – 3) / 18 = 1/6.

The correct answer is C.

17.

The combined rate of individuals working together is equal to the sum of all the individual working rates.

Let s = rate of a smurf, e = rate of an elf, and f = rate of a fairy. A rate is expressed in terms of treehouses/hour. So for instance, the first equation below says that a smurf and an elf working together can build 1 treehouse per 2 hours, for a rate of 1/2 treehouse per hour.
$s + e = 1/2$

2) $s + 2f = 1/2$
$e + f = 1/4$

The three equations can be combined by solving the first one for s in terms of e, and the third equation for f in terms of e, and then by substituting both new equations into the middle equation.

1) $s = 1/2 – e$

2) $(1/2 – e) + 2 (1/4 – e) = 1/2$

3) $f = 1/4 – e$

Now, we simply solve equation 2 for e:

$(1/2 – e) + 2 (1/4 – e) = 1/2$
$2/4 – e + 2/4 – 2e = 2/4$
$4/4 – 3e = 2/4$
$-3e = -2/4$
$e = 2/12$
$e = 1/6$

Once we know e, we can solve for s and f:

$s = 1/2 – e$
$s = 1/2 – 1/6$
$s = 3/6 – 1/6$
$s = 2/6$
$s = 1/3$
$f = 1/4 – e$
$f = 1/4 – 1/6$
$f = 3/12 – 2/12$
$f = 1/12$

We add up their individual rates to get a combined rate:

$e + s + f =$
$1/6 + 1/3 + 1/12 =$
$2/12 + 4/12 + 1/12 = 7/12$

Remembering that a rate is expressed in terms of treehouses/hour, this indicates that a smurf, an elf, and a fairy, working together, can produce 7 treehouses per 12 hours. Since we want to know the number of hours per treehouse, we must take the reciprocal of the rate. Therefore we conclude that it takes them 12 hours per 7 treehouses, which is equivalent to 12/7 of an hour per treehouse.

The correct answer is D.

18.

Since this is a work rate problem, we'll use the formula *rate* × *time* = *work*. Since we'll be calculating times, we'll use it in the form *time* = *work* / *rate*.

First let T_0 equal the time it takes to paint the houses under the speedup scenario. T_0 will equal the sum of the following two values:
1. the time it takes to paint y houses at a rate of x
2. the time it takes to paint $(80 - y)$ houses at a rate of $1.25x$.

$$T_0 = \frac{y}{x} + \frac{(80 - y)}{1.25x}$$

$$T_0 = \frac{1.25y}{1.25x} + \frac{(80 - y)}{1.25x}$$

$$T_0 = \frac{(80 + 0.25y)}{1.25x}$$

Then let T_1 equal the time it takes to paint all 80 houses at the steady rate of x.

$$T_1 = \frac{80}{x}$$

The desired ratio is $\frac{T_0}{T_1}$. This equals T_0 times the reciprocal of T_1.

$$\frac{T_0}{T_1} = \frac{80 + 0.25y}{1.25x} \times \frac{x}{80}$$

$$\frac{T_0}{T_1} = \frac{80+0.25y}{100}$$

$$\frac{T_0}{T_1} = 0.8 + 0.0025y$$

As a quick check, note that if $y = 80$, meaning they paint ALL the houses at rate x before bringing in the extra help, then $T_0/T_1 = 1$ as expected.

The correct answer is B.

SPEED DISTANCE

19.

To determine Bill's average rate of movement, first recall that Rate × Time = Distance. We are given that the moving walkway is 300 feet long, so we need only determine the time elapsed during Bill's journey to determine his average rate.

There are two ways to find the time of Bill's journey. First, we can break down Bill's journey into two legs: walking and standing. While walking, Bill moves at 6 feet per second. Because the walkway moves at 3 feet per second, Bill's foot speed along the walkway is 6 – 3 = 3 feet per second. Therefore, he covers the 120 feet between himself and the bottleneck in (120 feet)/(3 feet per second) = 40 seconds.

Now, how far along is Bill when he stops walking? While that 40 seconds elapsed, the crowd would have moved (40 seconds)(3 feet per second) = 120 feet. Because the crowd already had a 120 foot head start, Bill catches up to them at 120 + 120 = 240 feet. The final 60 feet are covered at the rate of the moving walkway, 3 feet per second, and therefore require (60 feet)/(3 feet per second) = 20 seconds. The total journey requires 40 + 20 = 60 seconds, and Bill's rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

This problem may also be solved with a shortcut. Consider that Bill's journey will end when the crowd reaches the end of the walkway (as long as he catches up with the crowd before the walkway ends). When he steps on the walkway, the crowd is 180 feet from the end. The walkway travels this distance in (180 feet)/(3 feet per second) = 60 seconds, and Bill's average rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

The correct answer is E.

20.

Use S, R and B to represent the individual race times of Stephanie, Regine, and Brian respectively. The problem tells us that Stephanie and Regine's combined times exceed Brian's time by 2 hours. Therefore:

$$S + R = B + 2$$

In order to win the race, an individual's time must be less than one-third of the the combined times of all the runners. Thus, in order for Brian to win the race (meaning that Brain would have the lowest time), his time would need to be less than one-third of the combined times for all the runners. This can be expressed as follows:

$$B < \frac{1}{3}(S + R + B)$$

This inequality can be simplified as follows:

$$B < \frac{1}{3}(S + R + B)$$
$$3B < S + R + B$$
$$2B < S + R$$

Using the fact that $S + R = B + 2$, the inequality can be simplified even further:

$$2B < S + R$$
$$2B < B + 2$$
$$B < 2$$

This tells us that in order for Brian to win the race, his time must be less than 2 hours. However, this is impossible! We know that the fastest Brian runs is 8 miles per hour, which means that the shortest amount of time in which he could complete the 20 mile race is 2.5 hours.

This leaves us with Stephanie and Regine as possible winners. Since the problem gives us identical information about Stephanie and Regine, we cannot eliminate either one as a possible winner. Thus, the correct answer is D: Stephanie or Regine could have won the race

21.

If we want Brenda's distance to be twice as great as Alex's distance, we can set up the following equation:

$2(4T) = R(T - 1)$, where $4T$ is Alex's distance (rate × time) and $R(T - 1)$ is Brenda's distance (since Brenda has been traveling for one hour less).

If we simplify this equation to isolate the T (which represents Alex's total time), we get:

$2(4T) = R(T - 1)$

$8T = RT - R$
$R = RT - 8T$
$R = T(R - 8)$

$$\frac{R}{R-8} = T$$

This is choice C.

22.

The key to solving this question lies in understanding the mathematical relationship that exists between the speed (s), the circumference of the tires (c) and the number of revolutions the tires make per second (r). It makes sense that if you keep the speed of the car the same but increase the size of the tires, the number of revolutions that the new tires make per second should go down. What, however, is the exact relationship?

Sometimes the best way to come up with a formula expressing the relationship between different variables is to analyze the labels (or units) that are associated with those variables. Let's use the following units for the variables in this question (even though they are slightly different in the question):

c: inches/revolution
s: inches/sec
r: revolutions/sec

The labels suggest: (rev/sec) x (inches/rev) = (inches/sec), which means that $rc = s$.

When the speed is held constant, as it is in this question, the relationship $rc = s$ becomes $rc = k$. r and c are inversely proportional to one another. When two variables are inversely proportional, it means that whatever factor you multiply one of the variables by, the other one changes by the inverse of that factor. For example if you keep the speed constant and you double the circumference of the tires, the rev/sec will be halved. In this way the product of c and r is kept constant.

In this question the circumferences of the tires are given in inches, the speed in miles per hour, and the rotational speed in revolutions per second. However, the discrepancies here don't affect the fundamental mathematical relationship of inverse proportionality: if the speed is kept constant, the rev/sec of the tires will change in an inverse manner to the circumference of the tires.

Let's assign c_1 = initial circumference; c_2 = new circumference
r_1 = initial rev/sec ; r_2 = new rev/sec

Since the speeds are held constant:

$c_1 r_1 = c_2 r_2$
$r_2 = (c_1/c_2) r_1$
$r_2 = (28/32) r_1$
$r_2 = (7/8) r_1$

If the new rev/sec is 7/8 of the previous rev/sec, this represents a 1/8 or 12.5% decrease and the correct answer is (B). Relationships of inverse proportionality are important in any word problem on the GMAT involving a formula in the form of $xy = z$ and in which z is held constant.

23

The crux of this problem is recalling the average speed formula:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

In this particular case, since Martha drove at one speed for some time and at another speed for the remainder of the trip, the total time will be the sum of the times spent at the two speeds. Let T_1 be the time spent traveling at the first speed and let T_2 be the time spent traveling at the second speed. Martha's average speed can then be expressed as:

$$\text{average speed} = \frac{\text{total distance}}{T_1 + T_2}$$

Since we do not know the total distance, we can call it *d*. We do not know either T_1 or T_2, but we can express them in terms of *d* by recalling that $T = \frac{D}{R}$, where *D* is the distance and *R* is the rate.

Let's find T_1 first. Since Martha traveled the first *x* percent of the journey at 60 miles per hour, *D* for that portion of the trip will be equal to $\frac{dx}{100}$ and T_1 will therefore be equal to $\frac{\frac{dx}{100}}{60}$.

Now let's find T_2. The remaining distance in Martha's trip can be expressed as $d - \frac{dx}{100}$. Therefore, T_2 will be equal to $\frac{d - \frac{dx}{100}}{50}$. We can plug these into our average rate formula and simplify:

$$\frac{d}{\frac{\frac{dx}{100}}{60} + \frac{\left(d - \frac{dx}{100}\right)}{50}} \rightarrow$$

$$\frac{\cancel{d}}{\frac{\cancel{d}x}{6000} + \frac{\cancel{d}}{50} - \frac{\cancel{d}x}{5000}} \rightarrow$$

$$\frac{1}{\frac{x}{6000} + \frac{1}{50} - \frac{x}{5000}} \rightarrow$$

$$\frac{1}{\frac{5x}{30000} + \frac{600}{30000} - \frac{6x}{30000}} \rightarrow$$

$$\frac{1}{\frac{600 - x}{30000}} \rightarrow$$

$$\frac{30000}{600 - x}$$

We cannot reduce this fraction any further. Therefore, the numerator of Martha's average speed is 30,000.

The correct answer is E.

24.

We are asked to find the time that it takes Train B to travel the entire distance between the two towns.

> SUFFICIENT: This tells us that B started traveling 1 hour after Train A started traveling. From the question we know that Train A had been traveling for 2 hours when the trains passed each other. Thus, train B, which started 1 hour later, must have been traveling for 2 - 1 = 1 hour when the trains passed each other.

Let's call the point at which the two trains pass each other Point P. Train A travels from Town H to Point P in 2 hours, while Train B travels from Town G to Point P in 1 hour. Adding up these distances and times, we have it that the two trains covered the entire distance between the towns in 3 (i.e. 2 + 1) hours of combined travel time. Since both trains travel at the same rate, it will take 3 hours for either train to cover the entire distance alone. Thus, from Statement (1) we know that it will take Train B 3 hours to travel between Town G and Town H.

> INSUFFICIENT: This provides the rate for Train B. Since both trains travel at the same rate, this is also the rate for Train A. However, we have no information about when Train B started traveling (relative to when Train A started traveling) and we have no information about the distance between Town G and Town H. Thus, we cannot calculate any information about time.

The correct answer is A.

SI / CI / Population Growth

25.

We need to consider the formula for compound interest for this problem: $F = P(1 + r)^x$, where F is the final value of the investment, P is the principal, r is the interest rate per compounding period as a decimal, and x is the number of compounding periods (NOTE: sometimes the formula is written in terms of the annual interest rate, the number of compounding periods per year and the number of years). Let's start by manipulating the given expression for r:

$$r = 100\left(\sqrt{\frac{v+q}{p}} - 1\right) \rightarrow \frac{r}{100} = \sqrt{\frac{v+q}{p}} - 1 \rightarrow 1+\frac{r}{100} = \sqrt{\frac{v+q}{p}} \rightarrow$$

$$\left(1+\frac{r}{100}\right)^2 = \left(\sqrt{\frac{v+q}{p}}\right)^2 \rightarrow \left(1+\frac{r}{100}\right)^2 = \frac{v+q}{p} \rightarrow p\left(1+\frac{r}{100}\right)^2 = v+q \rightarrow$$

$$v = p\left(1+\frac{r}{100}\right)^2 - q$$

Let's compare this simplified equation to the compound interest formula. Notice that r in this simplified equation (and in the question) is not the same as the r in the compound interest formula. In the formula, the r is already expressed as a decimal equivalent of a percent, in the question the interest is r percent. The simplified equation, however, deals with this discrepancy by dividing r by 100.

In our simplified equation, the cost of the share of stock (p), corresponds to the principal (P) in the formula, and the final share price (v) corresponds to the final value (F) in the formula. Notice also that the exponent 2 corresponds to the x in the formula, which is the number of compounding periods. By comparing the simplified equation to the compound interest formula, we see that the equation tells us that the share rose at the daily interest rate of p percent for TWO days. Then the share lost a value of q dollars on the third day, i.e. the "$– q$" portion of the expression. If the investor bought the share on Monday, she sold it three days later on Thursday.

The correct answer is B.

5.

Compound interest is computed using the following formula:

$F = P\ (1 + r/n)^{nt}$, where
F = Final value
P = Principal
r = annual interest rate
n = number of compounding periods per year
t = number of years

From the question, we can deduce the following information about the growth during this period:

At the end of the x years, the final value, F, will be equal to 16 times the principal (the money is growing by a factor of 16).

Therefore, $F = 16P$.
$r = .08$ (8% annual interest rate)
$n = 4$ (compounded quarterly)
$t = x$ (the question is asking us to express the time in terms of x number of years)

We can write the equation
$16P = P\,(1 + .08/4)^{4x}$
$16 = (1.02)^{4x}$

Now we can take the fourth root of both sides of the equation. (i.e.the equivalent of taking the square root twice) We will only consider the positive root because a negative 2 doesn't make sense here.
$16^{1/4} = [(1.02)^{4x}]^{1/4}$
$2 = (1.02)^{x}$

The correct answer is B.

RATIOS

27.

We are told that bag B contains red and white marbles in the ration 1:4. This implies that W_B, the number of white marbles in bag B, must be a multiple of 4.

What can we say about W_A, the number of white marbles in bag A? We are given *two* ratios involving the white marbles in bag A. The fact that the ratio of red to white marbles in bag A is 1:3 implies that W_A must be a multiple of 3. The fact that the ratio of white to blue marbles in bag A is 2:3 implies that W_A must be a multiple of 2. Since W_A is both a multiple of 2 *and* a multiple of 3, it must be a multiple of 6.

We are told that $W_A + W_B = 30$. We have already figured out that W_A must be a multiple of 6 and that W_B must be a multiple of 4. So all we need to do now is to test each candidate value of W_A (i.e. 6, 12, 18, and 24) to see whether, when plugged into $W_A + W_B = 30$, it yields a value for W_B that is a multiple of 4. It turns out that $W_A = 6$ and $W_A = 18$ are the only values that meet this criterion.

Recall that the ratio of red to white marbles in bag A is 1:3. If there are 6 white marbles in bag A, there are 2 red marbles. If there are 18 white marbles in bag A, there are 6 red marbles. Thus, the number of red marbles in bag A is either 2 or 6. Only one answer choice matches either of these numbers.

The correct answer is D.

TOPIC 2 – STATISTICS

MEAN

2.

This question deals with weighted averages. A weighted average is used to combine the averages of two or more subgroups and to compute the overall average of a group. The two subgroups in this question are the men and women. Each subgroup has an average weight (the women's is given in the question; the men's is given in the first statement). To calculate the overall average weight of the group, we would need the averages of each subgroup along with the ratio of men to women. The ratio of men to women would determine the weight to give to each subgroup's average. However, this question is not asking for the weighted average, but is simply asking for the ratio of women to men (i.e. what percentage of the competitors were women).

(1) INSUFFICIENT: This statement merely provides us with the average of the other subgroup – the men. We don't know what weight to give to either subgroup; therefore we don't know the ratio of the women to men.

(2) SUFFICIENT: If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the competitors must have been women. Consider the following rule and its proof.

RULE: The ratio that determines how to weight the averages of two or more subgroups in a weighted average ALSO REFLECTS the ratio of the distances from the weighted average to each subgroup's average.

Let's use this question to understand what this rule means. If we start from the solution, we will see why this rule holds true. The average weight of the men here is 150 lbs, and the average weight of the women is 120 lbs. There are twice as many men as women in the group (from the solution) so to calculate the weighted average, we would use the formula [1(120) + 2(150)] / 3. If we do the math, the overall weighted average comes to 140.

Now let's look at the distance from the weighted average to the average of each subgroup.

Distance from the weighted avg. to the avg. weight of the men is 150 – 140 = 10.

Distance from the weighted avg. to the avg. weight of the women is 140 – 120 = 20.

Notice that the weighted average is twice as close to the men's average as it is to the women's average, and notice that this reflects the fact that there were twice as many men as women. In general, the ratio of these distances will always reflect the relative ratio of the subgroups.

T he correct answer is (B), Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.

3.

First, let's use the average formula to find the current mean of set S: Current mean of set

S = (sum of the terms)/(number of terms): (sum of the terms)

= (7 + 8 + 10 + 12 + 13) = 50

(number of terms) = 5 50/5 = 10

Mean of set S after integer n is added = 10 × 1.2 = 12

Next, we can use the new average to find the sum of the elements in the new set and compute the value of integer n. Just make sure that you remember that after integer n is added to the set, it will contain 6 rather than 5 elements.

Sum of all elements in the new set = (average) × (number of terms) = 12 × 6 = 72

Value of integer n = sum of all elements in the new set – sum of all elements in the original set = 72 – 50 = 22

The correct answer is D.

4.

Let x = the number of 20 oz. bottles

48 – x = the number of 40 oz. bottles

The average volume of the 48 bottles in stock can be calculated as a weighted average:

$$\frac{x(20) + (48 - x)(40)}{48} = 35$$

$x = 12$

Therefore there are 12 twenty oz. bottles and 48 – 12 = 36 forty oz. bottles in stock. If no twenty oz. bottles are to be sold, we can calculate the number of forty oz. bottles it would take to yield an average volume of 25 oz:
Let n = number of 40 oz. bottles

$$\frac{(12)(20) + (n)(40)}{n + 12} = 25$$

$(12)(20) + 40n = 25n + (12)(25)$

$15n = (12)(25) - (12)(20)$

$15n = (12)(25 - 20)$

$15n = (12)(5)$

$15n = 60$

$n = 4$

Since it would take 4 forty oz. bottles along with 12 twenty oz. bottles to yield an average volume of 25 oz, 36 – 4 = 32 forty oz. bottles must be sold.

The correct answer is D.

5.

The average number of vacation days taken this year can be calculated by dividing the total number of vacation days by the number of employees. Since we know the total number of employees, we can rephrase the question as: How many total vacation days did the employees of Company X take this year?

(1) INSUFFICIENT: Since we don't know the specific details of how many vacation days each employee took the year before, we cannot determine the actual numbers that a 50% increase or a 50% decrease represent. For example, a 50% increase for someone who took 40 vacation days last year is going to affect the overall average more than the same percentage increase for someone who took only 4 days of vacation last year.

(2) SUFFICIENT: If three employees took 10 more vacation days each, and two employees took 5 fewer vacation days each, then we can calculate how the number of vacation days taken this year differs from the number taken last year:

(10 more days/employee)(3 employees) – (5 fewer days/employee)(2 employees) = 30 days – 10 days = 20 days

20 additional vacation days were taken this year.

In order to determine the total number of vacation days taken this year (i.e., in order to answer the rephrased question), we need to determine the number of vacation days taken last year. The 5 employees took an average of 16 vacation days each last year, so the total number of vacation days taken last year can be determine by taking the product of the two:

(5 employees)(16 days/employee) = 80 days

80 vacation days were taken last year. Hence, **the total number of vacation days taken this year was 100 days.**

Note: It is not necessary to make the above calculations -- it is simply enough to know that you have enough information in order to do so (i.e., the information given is sufficient)!

The correct answer is B.

6.

The question is asking us for the *weighted* average of the set of men and the set of women. To find the weighted average of two or more sets, you need to know the average of each set and the ratio of the number of members in each set. Since we are told the average of each set, this question is really asking for the ratio of the number of members in each set.

(1) SUFFICIENT: This tells us that there are twice as many men as women. If *m* represents the number of men and *w* represents the number of women, this statement tells us that $m = 2f$. To find the weighted average, we can sum the total weight of all the men and the total weight of all the women, and divide by the total number of people. We have an equation as follows:

M * 150 + F * 120 / M + F

Since this statement tells us that $m = 2f$, we can substitute for m in the average equation and average now = 140. Notice that we don't need the actual number of men and women in each set but just the ratio of the quantities of men to women.

(2) INSUFFICIENT: This tells us that there are a total of 120 people in the room but we have no idea how many men and women. This gives us no indication of how to weight the averages.

The correct answer is A.

7.

We're asked to determine whether the average number of runs, per player, is greater than 22. We are given one piece of information in the question stem: the ratio of the number of players on the three teams.

The simple average formula is just $A = S/N$ where A is the average, S is the total number of runs and N is the total number of players. We have some information about N: the ratio of the number of players. We have no information about S.

SUFFICIENT. Because we are given the individual averages for the team, we do not need to know the actual number of members on each team. Instead, we can use the ratio as a proxy for the actual number of players. (In other words, we don't need the actual number; the ratio is sufficient because it is in the same proportion as the actual numbers.)

If we know both the average number of runs scored and the ratio of the number of players, we can use the data to calculate:

# RUNS	RELATIVE # PLAYERS	R*P
30	2	60
17	5	85
25	3	75

The S, or total number of runs, is 60 + 85 + 75 = 220. The N, or number of players, is 2 + 5 + 3 = 10. A = 220/10 = 22. The collective, or weighted, average is 22, so we can definitively answer the question: No. (Remember that "no" is a sufficient answer. Only "maybe" is insufficient.)

INSUFFICIENT. This statement provides us with partial information about S, the sum, but we need to determine whether it is sufficient to answer the question definitively. "Is at least" means S is greater than or equal to 220. We know that the minimum number of players, or N, is 10 (since we can't have half a player). If N is 10 and S is 220, then A is 220/10 = 22 and we can answer the question No: 22 is not greater than 22. If N is 10 and S is 221, then A is 221/10 = 22.1 and we can answer the question Yes: 22.1 is greater than 22. We cannot answer the question definitively with this information.

The correct answer is A.

MEDIAN

8.

One approach to this problem is to try to create a Set T that consists of up to 6 integers and has a median equal to a particular answer choice. The set {–1, 0, 4) yields a median of 0. Answer choice A can be eliminated. The set {1, 2, 3} has an average of 2. Thus, $x = 2$. The median of this set is also 2. So the median = x. Answer choice B can be eliminated. The set {–4, –2, 12} has an average of 2. Thus, $x = 2$. The median of this set is –2. So the median = $-x$. Answer choice C can be eliminated. The set {0, 1, 2} has 3 integers. Thus, $y = 3$. The median of this set is 1. So the median of the set is (1/3)y. Answer choice D can be eliminated. As for answer choice E, there is no possible way to create Set T with a median of (2/7)y. Why? We know that y is either 1, 2, 3, 4, 5, or 6. Thus, (2/7)y will yield a value that is some fraction with denominator of 7.

The possible values of (2/7)y are as follows: $\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, 1\frac{1}{7}, 1\frac{3}{7}, 1\frac{5}{7}$

However, the median of a set of integers must always be either an integer or a fraction with a denominator of 2 (e.g. 2.5, or 5/2). So (2/7)y cannot be the median of Set T.

The correct answer is E.

9.

Since S contains only consecutive integers, its median is the average of the extreme values a and b.

We also know that the median of S is $\frac{3}{4}b$. We can set up and simplify the following equation:

$$\frac{a+b}{2} = \frac{3b}{4} \rightarrow$$
$$4a + 4b = 6b \rightarrow$$
$$4a = 2b \rightarrow$$
$$2a = b$$

Since set Q contains only consecutive integers, its median is also the average of the extreme values, in this case b and c. We also know that the median of Q is $\frac{7}{8}c$. We can set up and simplify the following equation:

$$\frac{b+c}{2} = \frac{7c}{8} \rightarrow$$
$$8b + 8c = 14c \rightarrow$$
$$8b = 6c \rightarrow$$
$$4b = 3c$$

We can find the ratio of a to c as follows: Taking the first equation, $2a = b \rightarrow 8a = 4b$

and the second equation, $4b = 3c$ and setting them equal to each other, yields the following:

$$8a = 3c \rightarrow \frac{a}{c} = \frac{3}{8}.$$

Since set R contains only consecutive integers, its median is the average of the extreme values a and c: $\frac{a+c}{2}$. We can use the ratio $\frac{a}{c} = \frac{3}{8}$ to substitute $\frac{3c}{8}$ for a:

$$\frac{\frac{3c}{8} + c}{2} \rightarrow$$
$$\frac{\frac{11c}{8}}{2} \rightarrow$$
$$\frac{11}{16}c$$

Thus the median of set R is $\frac{11}{16}c$.

The correct answer is C.

10.

The set $R_n = R_{n-1} + 3$ describes an evenly spaced set: each value is three more than the previous. For example the set could be 3, 6, 9, 12 . . .

For any evenly spaced set, the mean of the set is always equal to the median. A set of consecutive integers is an example of an evenly spaced set. If we find the mean of this set, we will be able to find the median because they are the same.

(1) INSUFFICIENT: This does not give us any information about the value of the mean. The only other way to find the median of a set is to know every term of the set.

(2) SUFFICIENT: The mean must be the median of the set since this is an evenly spaced set. This statement tells us that mean is 36. Therefore, the median must be 36.

The correct answer is B.

11.

To get the maximum length of the shortest piece, we must let other values as little as possible.

That is, the values after the median should equal the median, and the value before the median should be equal to each other.

Let the shortest one be x:

x+x+140*3=124*5

x=100

MODE

12.

Statement 1 tells us that the difference between any two integers in the set is less than 3. This information alone yields a variety of possible sets. For example, one possible set (in which the difference between any two integers is less than 3) might be: (x, x, x, $x + 1$, $x + 1$, $x + 2$, $x + 2$). Mode = x (as stated in question stem). Median = $x + 1$. Difference between median and mode = 1.

Alternately, another set (in which the difference between any two integers is less than 3) might look like this: ($x – 1$, x, x, $x + 1$). Mode = x (as stated in the question stem). Median = x. Difference between median and mode = 0.

We can see that statement (1) is not sufficient to determine the difference between the median and the mode.

Statement (2) tells us that the average of the set of integers is x. This information alone also yields a variety of possible sets. For example, one possible set (with an average of x) might be: ($x – 10$, x, x, $x + 1$, $x + 2$, $x + 3$, $x + 4$). Mode = x (as stated in the question stem). Median = $x + 1$. Difference between median and mode = 1. Alternately, another set (with an average of x) might look like this: ($x – 90$, x, x, $x + 15$, $x + 20$, $x + 25$, $x + 30$). Mode = x (as stated in the question stem). Median = $x + 15$. Difference between median and mode = 15. We can see that statement (2) is not sufficient to determine the difference between the median and the mode. Both statements taken together imply that the only possible members of the set are $x – 1$, x, and $x + 1$ (from the fact that the difference between any two integers in the set is less than 3) and that every $x – 1$ will be balanced by an $x + 1$ (from the fact that the average of the set is x). Thus, x will lie in the middle of any such set and therefore x will be the median of any such set. If x is the mode and x is also the median, the difference between these two measures will always be 0.

The correct answer is C:

BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

RANGE

16.

(1) INSUFFICIENT: Statement (1) tells us that the range of S is less than 9. The range of a set is the positive difference between the smallest term and the largest term of the set. In this case, knowing that the range of set S is less than 9, we can answer only MAYBE to the question "Is $(x + y) < 18$". Consider the following two examples: Let $x = 7$ and $y = 7$. The range of S is less than 9 and $x + y < 18$, so we conclude YES. Let $x = 10$ and $y = 10$. The range of S is less than 9 and $x + y > 18$, so we conclude NO. Because this statement does not allow us to answer definitively Yes or No, it is insufficient.

(2) SUFFICIENT: Statement (2) tells us that the average of x and y is less than the average of the set S. Writing this as an inequality: $(x + y)/2 < (7 + 8 + 9 + 12 + x + y)/6$; $(x + y)/2 < (36 + x + y)/6$;

$3(x + y) < 36 + (x + y)$; $2(x + y) < 36$; $x + y < 18$.

Therefore, statement (2) is SUFFICIENT to determine whether $x + y < 18$.

TOPIC 3 – INEQUALITIES+ABSOLUTEVALUE (MODULUS)

1.

There are two characteristics of x that dictate its exponential behavior. First of all, it is a decimal with an absolute value of less than 1. Secondly, it is a negative number.

I. True. x^3 will always be negative (negative × negative × negative = negative), and x^2 will always be positive (negative × negative = positive), so x^3 will always be less than x^2.

II. True. x^5 will always be negative, and since x is negative, $1 - x$ will always be positive because the double negative will essentially turn $1 - x$ into $1 + |x|$. Therefore, x^5 will always be less than $1 - x$.

III. True. One useful method for evaluating this inequality is to plug in a number for x. If $x = - 0.5$,

$x^4 = (-0.5)^4 = 0.0625$

$x^2 = (-0.5)^2 = 0.25$

To understand why this works, it helps to think of the negative aspect of x and the decimal aspect of x separately.

Because x is being taken to an even exponent in both instances, we can essentially ignore the negative aspect because we know the both results will be positive.

The rule with decimals between 0 and 1 is that the number gets smaller and smaller in absolute value as the exponent gets bigger and bigger. Therefore, x^4 must be smaller in absolute value than x^2.

The correct answer is E.

2.

(1) INSUFFICIENT: We can solve this absolute value inequality by considering both the positive and negative scenarios for the absolute value expression $|x + 3|$.

If $x > -3$, making $(x + 3)$ positive, we can rewrite $|x + 3|$ as $x + 3$:

$x + 3 < 4$
$x < 1$

If $x < -3$, making $(x + 3)$ negative, we can rewrite $|x + 3|$ as $-(x + 3)$:
$-(x + 3) < 4$
$x + 3 > -4$
$x > -7$
If we combine these two solutions we get $-7 < x < 1$, which means we can't tell whether x is positive.

(2) INSUFFICIENT: We can solve this absolute value inequality by considering both the positive and negative scenarios for the absolute value expression $|x – 3|$.

If $x > 3$, making $(x – 3)$ positive, we can rewrite $|x – 3|$ as $x – 3$:
$x – 3 < 4$
$x < 7$

If $x < 3$, making $(x – 3)$ negative, we can rewrite $|x – 3|$ as $-(x – 3)$ OR $3 – x$

$3 – x < 4$
$x > -1$

If we combine these two solutions we get $-1 < x < 7$, which means we can't tell whether x is positive.

(1) AND (2) INSUFFICIENT: If we combine the solutions from statements (1) and (2) we get an overlapping range of $-1 < x < 1$. We still can't tell whether x is positive.

The correct answer is E.

3.

The question asks whether x is greater than y. The question is already in its most basic form, so there is no need to rephrase it; we can go straight to the statements.

(1) INSUFFICIENT: The fact that x2 is greater than y does not tell us whether x is greater than y. For example, if x = 3 and y = 4, then x2 = 9, which is greater than y although x itself is less than y. But if x = 5 and y = 4, then x2 = 25, which is greater than y and x itself is also greater than y.

(2) INSUFFICIENT: We can square both sides to obtain x < y2. As we saw in the examples above, it is possible for this statement to be true whether y is less than or greater than x (just substitute x for y and vice-versa in the examples above).

(1) AND (2) INSUFFICIENT: Taking the statements together, we know that x < y2 and y < x2, but we do not know whether x > y. For example, if x = 3 and y = 4, both of these inequalities hold (3 < 16 and 4 < 9) and x < y. But if x = 4 and y = 3, both of these inequalities still hold (4 < 9 and 3 < 16) but now x > y.

The correct answer is E.

4.

The equation in question can be rephrased as follows:
$x^2y – 6xy + 9y = 0$
$y(x^2 – 6x + 9) = 0$
$y(x – 3)^2 = 0$

Therefore, one or both of the following must be true:
$y = 0$ or
$x = 3$

It follows that the product xy must equal either 0 or 3y. This question can therefore be rephrased "What is y?"

(1) INSUFFICIENT: This equation cannot be manipulated or combined with the original equation to solve directly for x or y. Instead, plug the two possible scenarios from the original equation into the equation from this statement:

If $x = 3$, then $y = 3 + x = 3 + 3 = 6$, so $xy = (3)(6) = 18$.
If $y = 0$, then $x = y - 3 = 0 - 3 = -3$, so $xy = (-3)(0) = 0$.

Since there are two possible answers, this statement is not sufficient.

(2) SUFFICIENT: If $x^3 < 0$, then $x < 0$. Therefore, x cannot equal 3, and it follows that $y = 0$. Therefore, $xy = 0$.

The correct answer is B.

5.

For their quotient to be less than zero, *a* and *b* must have opposite signs. In other words, if the answer to the question is "yes," EITHER *a* is positive and *b* is negative OR *a* is negative and *b* is positive.

The question can be rephrased as the following: "Do *a* and *b* have opposite signs?"

(1) INSUFFICIENT: a^2 is always positive so for the quotient of a^2 and b^3 to be positive, b^3 must be positive. That means that *b* is positive. This does not however tell us anything about the sign of *a*.

(2) INSUFFICIENT: b^4 is always positive so for the product of *a* and b^4 to be negative, a must be negative. This does not however tell us anything about the sign of *b*.

(1) AND (2) SUFFICIENT: Statement 1 tells us that *b* is positive and statement 2 tells us that *a* is negative. The yes/no question can be definitively answered with a "yes."

The correct answer is C.

6.

The question asks about the sign of *d*.

(1) INSUFFICIENT: When two numbers sum to a negative value, we have two possibilities:
Possibility A: Both values are negative (e.g., $e = -4$ and $d = -8$)

Possibility B: One value is negative and the other is positive.(e.g., $e = -15$ and $d = 3$).

(2) INSUFFICIENT: When the difference of two numbers produces a negative value, we have three possibilities:

Possibility A: Both values are negative (e.g., $e = -20$ and $d = -3$)

Possibility B: One value is negative and the other is positive (e.g., $e = -20$ and $d = 3$).

Possibility C: Both values are positive (e.g., $e = 20$ and $d = 30$)

(1) AND (2) SUFFICIENT: When d is ADDED to e, the result (-12) is greater than when d is SUBTRACTED from e. This is only possible if d is a positive value. If d were a negative value than adding d to a number would produce a smaller value than subtracting d from that number (since a double negative produces a positive). You can test numbers to see that d must be positive and so we can definitively answer the question using both statements.

7.

Because we know that $|a| = |b|$, we know that a and b are equidistant from zero on the number line. But we do not know anything about the signs of a and b (that is, whether they are positive or negative). Because the question asks us which statement(s) MUST be true, we can eliminate any statement that is not always true. To prove that a statement is not always true, we need to find values for a and b for which the statement is false.

I. NOT ALWAYS TRUE: a does not necessarily have to equal b. For example, if $a = -3$ and $b = 3$, then $|-3| = |3|$ but $-3 \neq 3$.

II. NOT ALWAYS TRUE: $|a|$ does not necessarily have to equal $-b$. For example, if $a = 3$ and $b = 3$, then $|3| = |3|$ but $|3| \neq -3$.

III. NOT ALWAYS TRUE: $-a$ does not necessarily have to equal $-b$. For example, if $a = -3$ and $b = 3$, then $|-3| = |3|$ but $-(-3) \neq -3$.

The correct answer is E.

8.

Let us start be examining the conditions necessary for $|a|b > 0$. Since $|a|$ cannot be negative, both $|a|$ and b must be positive. However, since $|a|$ is positive whether a is negative or positive, the only condition for a is that it must be non-zero.

Hence, the question can be restated in terms of the necessary conditions for it to be answered "yes":

"Do both of the following conditions exist: *a* is non-zero AND *b* is positive?"

(1) INSUFFICIENT: In order for $a = 0$, $|a^b|$ would have to equal 0 since 0 raised to any power is always 0. Therefore (1) implies that a is non-zero. However, given that a is non-zero, b can be *anything* for $|a^b| > 0$ so we cannot determine the sign of b.

(2) INSUFFICIENT: If $a = 0$, $|a| = 0$, and $|a|^b = 0$ for any b. Hence, a must be non-zero and the first condition (a is not equal to 0) of the restated question is met. We now need to test whether the second condition is met. (Note: If a had been zero, we would have been able to conclude right away that (2) is sufficient because we would answer "no" to the question is $|a|b > 0$?) Given that a is non-zero, $|a|$ must be positive integer. At first glance, it seems that b must be positive because a positive integer raised to a negative integer is typically fractional (e.g., $a^{-2} = 1/a^2$). Hence, it appears that b cannot be negative. However, there is a special case where this is not so:

If $|a| = 1$, then b could be anything (positive, negative, or zero) since $|1|^b$ is always equal to 1, which is a non-zero integer . In addition, there is also the possibility that $b = 0$. If $|b| = 0$, then $|a|^0$ is always 1, which is a non-zero integer.

Hence, based on (2) alone, we cannot determine whether b is positive and we cannot answer the question.

An alternative way to analyze this (or to confirm the above) is to create a chart using simple numbers as follows:

a	***b***	**Is $|a|^b$ a non-zero integer?**	**Is $|a|b > 0$?**
1	2	Yes	Yes
1	-2	Yes	No
2	1	Yes	Yes
2	0	Yes	No

We can quickly confirm that (2) alone does not provide enough information to answer the question.

(1) AND (2) INSUFFICIENT: The analysis for (2) shows that (2) is insufficient because, while we can conclude that a is non-zero, we cannot determine whether b is positive. (1) also implies that a is non-zero, but does not provide any information about b other than that it could be anything. Consequently, (1) does not add any information to (2) regarding b to help answer the question and (1) and (2) together are still insufficient. (Note: As a quick check, the above chart can also be used to analyze (1) and (2) together since all of the values in column 1 are also consistent with (1)).

20.

$X-2Y < -6 \Rightarrow -X + 2Y > 6$

Combined $X - Y > -2$, we know $Y > 4$

$X - Y > -2 \Rightarrow -2X + 2Y < 4$

Combined $X - 2Y < -6$, we know $-X < -2 \Rightarrow X > 2$

Therefore, $XY > 0$

Answer is C

10.

The $|x| + |y|$ on the left side of the equation will always add the positive value of x to the positive value of y, yielding a positive value. Therefore, the $-x$ and the $-y$ on the right side of the equation must also each yield a positive value. The only way for $-x$ and $-y$ to each yield positive values is if both x and y are negative.

(A) FALSE: For $x + y$ to be greater than zero, either x or y has to be positive.

(B) TRUE: Since x has to be negative and y has to be negative, the sum of x and y will always be negative.

(C) UNCERTAIN: All that is certain is that x and y have to be negative. Since x can have a larger magnitude than y and vice-versa, $x - y$ could be greater than zero.

(D) UNCERTAIN: All that is certain is that x and y have to be negative. Since x can have a larger magnitude than y and vice versa, $x - y$ could be less than zero.

(E) UNCERTAIN: As with choices (C) and (D), we have no idea about the magnitude of x and y. Therefore, $x^2 - y^2$ could be either positive or negative.

Another option to solve this problem is to systematically test numbers. With values for x and y that satisfy the original equation, observe that both x and y have to be negative.

If
$x = -4$ and $y = -2$, we can eliminate choices (A) and (C). Then, we might choose numbers such that y has a greater magnitude than x, such as $x = -2$ and $y = -4$. With these values, we can eliminate choices (D) and (E).

The correct answer is B.

11.

This question cannot be rephrased since it is already in a simple form.

(1) INSUFFICIENT: Since x^2 is positive whether x is negative or positive, we can only determine that x is not equal to zero; x could be either positive or negative.

(2) INSUFFICIENT: By telling us that the expression $x \cdot |y|$ is not a positive number, we know that it must either be negative or zero. If the expression is negative, x must be negative ($|y|$ is never negative). However if the expression is zero, x or y could be zero.

(1) AND (2) INSUFFICIENT: We know from statement 1 that x cannot be zero, however, there are still two possibilities for x: x could be positive (y is zero), or x could be negative (y is anything).

The correct answer is E.

32.

(1) INSUFFICIENT: This expression provides only a range of possible values for x.

(2) SUFFICIENT: Absolute value problems often -- ***but not always*** -- have multiple solutions because the expression *within* the absolute value bars can be either positive or negative even though the absolute value of the expression is always positive. For example, if we consider the equation $|2 + x| = 3$, we have to consider the possibility that $2 + x$ is already positive and the possibility that $2 + x$ is negative. If $2 + x$ is positive, then the equation is the same as $2 + x = 3$ and $x = 1$. But if $2 + x$ is negative, then it must equal -3 (since $|-3| = 3$) and so $2 + x = -3$ and $x = -5$.

So in the present case, in order to determine the possible solutions for *x*, it is necessary to solve for *x* under both possible conditions.

For the case where $x > 0$:

$x = 3x - 2$
$-2x = -2$
$x = 1$

For the case when $x < 0$:

$x = -1(3x - 2)$ We multiply by -1 to make *x* equal a negative quantity.
$x = 2 - 3x$
$4x = 2$
$x = 1/2$

Note however, that the second solution $x = 1/2$ contradicts the stipulation that $x < 0$, hence there is no solution for *x* where $x < 0$. Therefore, $x = 1$ is the only valid solution for (2).

The correct answer is B.

13.

(1) INSUFFICIENT: If we test values here we find two sets of possible x and y values that yield conflicting answers to the question.

x	$\sqrt{x}$	y	Is $x > y$?
4	2	1	YES
1/4	1/2	1/3	NO

(2) INSUFFICIENT: If we test values here we find two sets of possible x and y values that yield conflicting answers to the question.

x	x^3	y	Is $x > y$?
2	8	1	YES
-1/2	-1/8	-1/4	NO

(1) AND (2) SUFFICIENT: Let's start with statement 1 and add the constraints of statement 2. From statement 1, we see that *x* has to be positive since we are taking the square root of *x*. There is no point in testing negative values for *y* since a positive value for *x* against a negative *y* will always yield a yes to the question. Lastly, we should consider *x* values between 0 and 1 and greater than 1 because proper fractions behave different than integers with regard to exponents. When we try to come up with *x* and *y* values that fit both conditions, we must adjust the two variables so that *x* is always greater than *y*.

x	$\sqrt{x}$	x^3	y	Is $x > y$?
2	1.4	8	1	YES
1/4	1/2	1/64	1/128	YES

Logically it also makes sense that if the cube and the square root of a number are both greater than another number than the number itself must be greater than that other number.

The correct answer is C.

14.

The question "Is $|x|$ less than 1?" can be rephrased in the following way.

Case 1: If $x > 0$, then $|x| = x$. For instance, $|5| = 5$. So, if $x > 0$, then the question becomes "Is x less than 1?"

Case 2: If $x < 0$, then $|x| = -x$. For instance, $|-5| = -(-5) = 5$. So, if $x < 0$, then the question becomes "Is $-x$ less than 1?" This can be written as follows:

$-x < 1$?
or, by multiplying both sides by -1, we get
$x > -1$?

Putting these two cases together, we get the fully rephrased question:
"Is $-1 < x < 1$ (and x not equal to 0)"?

Another way to achieve this rephrasing is to interpret absolute value as distance from zero on the number line. Asking "Is $|x|$ less than 1?" can then be reinterpreted as "Is x less than 1 unit away from zero on the number line?" or "Is $-1 < x < 1$?" (The fact that x does not equal zero is given in the question stem.)

(1) INSUFFICIENT: If $x > 0$, this statement tells us that $x > x/x$ or $x > 1$. If $x < 0$, this statement tells us that $x > x/-x$ or $x > -1$. This is not enough to tell us if $-1 < x < 1$.

(2) INSUFFICIENT: When $x > 0$, $x > x$ which is not true (so $x < 0$). When $x < 0$, $-x > x$ or $x < 0$. Statement (2) simply tells us that x is negative. This is not enough to tell us if $-1 < x < 1$.

(1) AND (2) SUFFICIENT: If we know $x < 0$ (statement 2), we know that $x > -1$ (statement 1). This means that $-1 < x < 0$. This means that x is definitely between -1 and 1.

The correct answer is C.

15.

The question stem gives us three constraints:

1) a is an integer.

2) b is an integer.

3) a is farther away from zero than b is (from the constraint that $|a| > |b|$).

When you see a problem using absolute values, it is generally necessary to try positive and negative values for each of the variables. Thus, we should take the information from the question, and see what it tells us about the signs of the variables.

For b, we should try negative, zero, and positive values. Nothing in the question stem eliminates any of those possibilities. For a, we only have to try negative and positive values. Why not $a = 0$? We know that b must be closer to zero than a, so a cannot equal zero because there is no potential value for b that is closer to zero than zero itself. So to summarize, the possible scenarios are:

a	*b*
neg	neg
neg	0
neg	Pos
pos	Neg
pos	0
pos	Pos

(1) INSUFFICIENT: This statement tells us that a is negative, ruling out the positive a scenarios above. Remember that a is farther away from zero than b is.

a	*b*	$a \cdot \lvert b \rvert$	$a - b$	Is $a \cdot \lvert b \rvert < a - b$?
neg	neg	neg · |neg| = neg · pos = more negative	neg (far from 0) – neg (close to 0) = neg (far from 0) + pos (close to 0) = less neg	Is more neg < less neg? **Yes.**
neg	0	neg · |0| = neg · 0 = 0	neg – 0 = neg	Is 0 < neg? **No.**
neg	pos	neg · |pos| = neg · pos = at least as negative as *a*, since *b* could be 1 or greater	neg – pos = more negative than *a*	Is at least as negative as *a* < more negative than *a* ? **It depends.**

For some cases the answer is "yes," but for others the answer is "no." Therefore, statement (1) is insufficient to solve the problem.

(2) INSUFFICIENT: This statement tells us that a and b must either have the same sign (for $ab > 0$), or one or both of the variables must be zero (for $ab = 0$). Thus we can rule out any scenario in the original list that doesn't meet the constraints from this statement.

a	*b*	$a \cdot \lvert b \rvert$	$a - b$	Is $a \cdot \lvert b \rvert < a - b$?
neg	neg	neg · |neg|	neg (far from 0) - neg (close to 0)	Is more negative < less

		= neg · pos = more negative	= neg (far from 0) + pos (close to 0) = less negative	negative? **Yes.**
neg	0	neg · |0| = 0	neg - 0 = neg	Is 0 < neg? **No.**
pos	0	pos · |0| = 0	pos - 0 = pos	Is 0 < pos? **Yes.**
pos	pos	pos · |pos| = more positive	pos (far from 0) - pos (close to 0) = less positive	Is more positive < less positive? **No.**

For some cases the answer is "yes," but for others the answer is "no." Therefore, statement (2) is insufficient to solve the problem.

(1) & (2) INSUFFICIENT: For the two statements combined, we must consider only the scenarios with negative *a* and either negative or zero *b*. These are the scenarios that are on the list for both statement (1) and statement (2).

a	b	$a \cdot \mid b \mid$	$a - b$	Is $a \cdot \mid b \mid < a - b$?
neg	neg	neg · |neg| = neg · pos = more negative	neg (far from 0) - neg (close to 0) = neg (far from 0) + pos (close to 0) = less negative	Is more negative < less negative? **Yes**
neg	0	neg · |0| = 0	neg – 0 = neg	Is 0 < neg? **No**

For the first case the answer is "yes," but for the second case the answer is "no." Thus the two statements combined are not sufficient to solve the problem.

The correct answer is E.

16.

Note that one need not determine the values of both x and y to solve this problem; the value of product xy will suffice.

(1) SUFFICIENT: Statement (1) can be rephrased as follows:

$-4x - 12y = 0$
$-4x = 12y$
$x = -3y$

If x and y are non-zero integers, we can deduce that they must have opposite signs: one positive, and the other negative. Therefore, this last equation could be rephrased as

$|x| = 3|y|$

We don't know whether x or y is negative, but we do know that they have the opposite signs. Converting both variables to absolute value cancels the negative sign in the expression $x = -3y$.

We are left with two equations and two unknowns, where the unknowns are $|x|$ and $|y|$:

$|x| + |y| = 32$

$|x| - 3|y| = 0$

Subtracting the second equation from the first yields

$4|y| = 32$
$|y| = 8$

Substituting 8 for $|y|$ in the original equation, we can easily determine that $|x| = 24$. Because we know that one of either x or y is negative and the other positive, xy must be the negative product of $|x|$ and $|y|$, or -8(24) = -192.

(2) INSUFFICIENT: Statement (2) also provides two equations with two unknowns:

$|x| + |y| = 32$
$|x| - |y| = 16$

Solving these equations allows us to determine the values of $|x|$ and $|y|$: $|x| = 24$ and $|y| = 8$. However, this gives no information about the sign of x or y. The product xy could either be -192 or 192.

The correct answer is A.

17.

(1) INSUFFICIENT: Since this equation contains two variables, we cannot determine the value of y. We can, however, note that the absolute value expression $|x^2 - 4|$ must be greater than or equal to 0. Therefore, $3|x^2 - 4|$ must be greater than or equal to 0, which in turn means that $y - 2$ must be greater than or equal to 0. If $y - 2 \geq 0$, then $y \geq 2$.

(2) INSUFFICIENT: To solve this equation for y, we must consider both the positive and negative values of the absolute value expression:

If $3 - y > 0$, then $3 - y = 11$
$y = -8$

If $3 - y < 0$, then $3 - y = -11$
$y = 14$

Since there are two possible values for y, this statement is insufficient.

(1) AND (2) SUFFICIENT: Statement (1) tells us that y is greater than or equal to 2, and statement (2) tells us that $y = -8$ or 14. Of the two possible values, only 14 is greater than or equal to 2. Therefore, the two statements together tell us that y must equal 14.

The correct answer is C.

18.

The question asks whether x is positive. The question is already as basic as it can be made to be, so there is no need to rephrase it; we can go straight to the statements.

(1) SUFFICIENT: Here, we are told that $|x + 3| = 4x – 3$. When dealing with equations containing variables and absolute values, we generally need to consider the possibility that there may be more than one value for the unknown that could make the equation work. In order to solve this particular equation, we need to consider what happens when $x + 3$ is positive and when it is negative (remember, the absolute value is the same in either case). First, consider what happens if $x + 3$ is positive. If $x + 3$ is positive, it is as if there were no absolute value bars, since the absolute value of a positive is still positive:
$x + 3 = 4x – 3$
$6 = 3x$
$2 = x$

So when x + 3 is positive, $x = 2$, a positive value. If we plug 2 into the original equation, we see that it is a valid solution:
$|2 + 3| = 4(2) – 3$
$|5| = 8 – 3$
$5 = 5$

Now let's consider what happens when $x + 3$ is negative. To do so, we multiply $x + 3$ by -1:
$-1(x + 3) = 4x – 3$
$-x – 3 = 4x – 3$
$0 = 5x$
$0 = x$

But if we plug 0 into the original equation, it is *not* a valid solution:
$|0 + 3| = 4(0) – 3$
$|3| = 0 – 3$
$3 = -3$

Therefore, there is no solution when $x + 3$ is negative and we know that 2 is the only solution possible and we can say that x is definitely positive.

Alternatively, we could have noticed that the right-hand side of the original equation must be positive (because it equals the absolute value of $x + 3$). If $4x – 3$ is positive, x must be positive. If x were negative, $4x$ would be negative and a negative minus a positive is negative.

(2) INSUFFICIENT: Here, again, we must consider the various combinations of positive and negative for both sides. Let's first assume that both sides are positive (which is equivalent to assuming both sides are negative):
$|x – 3| = |2x – 3|$
$x – 3 = 2x – 3$
$0 = x$

So when both sides are positive, $x = 0$. We can verify that this solution is valid by plugging 0 into the original equation:
$|0 – 3| = |2(0) – 3|$
$|-3| = |-3|$
$3 = 3$

Now let's consider what happens when only one side is negative; in this case, we choose the right-hand side:
$|x – 3| = |2x – 3|$
$x – 3 = -(2x – 3)$

$x - 3 = -2x + 3$
$3x = 6$
$x = 2$

We can verify that this is a valid solution by plugging 2 into the original equation:
$|2 - 3| = |2(2) - 3|$
$|-1| = |1|$
$1 = 1$

Therefore, both 2 and 0 are valid solutions and we cannot determine whether *x* is positive, since one value of *x* is zero, which is not positive, and one is positive.

The correct answer is A.

19.

The expression is equal to *n* if $n \geq 0$, but $-n$ if $n \leq 0$. This means that EITHER
$n < 1$ if $n \geq 0$

OR

$-n < 1$ (that is, $n > -1$) if $n \leq 0$.
If we combine these two possibilities, we see that the question is really asking whether $-1 < n < 1$.

(1) INSUFFICIENT: If we add *n* to both sides of the inequality, we can rewrite it as the following:
$n^x < n$.

Since this is a Yes/No question, one way to handle it is to come up with sample values *that satisfy this condition* and then see whether these values give us a "yes" or a "no" to the question.

$n = ½$ and $x = 2$ are legal values since $(1/2)^2 < 1/2$

These values yield a YES to the question, since *n* is between -1 and 1.

$n = -3$ and $x = 3$ are also legal values since $3^{-3} = 1/27 < 3$

These values yield a NO to the question since *n* is greater than 1.

With legal values yielding a "yes" and a "no" to the original question, statement (1) is insufficient.

(2) INSUFFICIENT: $x^{-1} = -2$ can be rewritten as $x = -2^{-1} = -½$. However, this statement contains no information about *n*.

(1) AND (2) SUFFICIENT: If we combine the two statements by plugging the value for x into the first statement, we get $n^{-½} < n$.
The only values for n that satisfy this inequality are greater than 1.

Negative values for *n* are not possible. Raising a number to the exponent of -½ is equivalent to taking the reciprocal of the square root of the number. However, it is not possible (within the real number system) to take the square root of a negative number.

A fraction less than 1, such as ½, becomes a LARGER number when you square root it ($½^{½} = \sim 0.7$). However, the new number is still less than 1. When you reciprocate that value, you get a number ($½^{-½} = \sim 1.4$) that is LARGER than 1 and therefore LARGER than the original value of n.

Finally, all values of n greater than 1 satisfy the inequality $n^{-½} < n$.
For instance, if $n = 4$, then $n^{-½} = ½$. Taking the square root of a number larger than 1 makes the number smaller, though still greater than 1 -- and then taking the reciprocal of that number makes the number smaller still.

Since the two statements together tell us that n must be greater than 1, we know the definitive answer to the question "Is n between -1 and 1?" Note that the answer to this question is "No," which is as good an answer as "Yes" to a Yes/No question on Data Sufficiency.

The correct answer is (C).

20.

(1) INSUFFICIENT: Statement (1) is insufficient because y is unbounded when both x and k can vary. Therefore y has no definite maximum.

To show that y is unbounded, let's calculate y for a special sequence of (x, k) pairs. The sequence starts at (-2, 1) and doubles both values to get the next (x, k) pair in the sequence.

$y_1 = | -2 - 1 | - | -2 + 1 | = 3 - 1 = 2$
$y_2 = | -4 - 2 | - | -4 + 2 | = 6 - 2 = 4$
$y_3 = | -8 - 4 | - | -8 + 4 | = -12 + 4 = 8$
etc.

In this sequence y doubles each time so it has no definite maximum, so statement (1) is insufficient.

(2) SUFFICIENT: Statement (2) says that $k = 3$, so $y = | x - 3 | - | x + 3 |$. Therefore to maximize y we must maximize $| x - 3 |$ while simultaneously trying to minimize $| x + 3 |$. This state holds for very large negative x. Let's try two different large negative values for x and see what happens:

If $x = -100$ then:
$y = |-100 - 3| - |-100 + 3|$
$y = 103 - 97 = 6$

If $x = -101$ then:
$y = |-101 - 3| - |-101 + 3|$
$y = 104 - 98 = 6$

We see that the two expressions increase at the same rate, so their difference remains the same. As x decreases from 0, y increases until it reaches 6 when $x = -3$. As x decreases further, y remains at 6 which is its maximum value.

The correct answer is B.

21.

Simplifying the original expression yields:

$$\frac{8xy^3 + 8x^3y}{1} = \frac{2x^2y^2}{2^{-3}}$$

$$2^{-3}(8xy^3 + 8x^3y) = 1(2x^2y^2)$$

$$\frac{1}{8}\left(8xy^3 + 8x^3y\right) = 2x^2y^2$$

$$\frac{8xy^3 + 8x^3y}{8} = 2x^2y^2$$

$$\frac{8\left(xy^3 + x^3y\right)}{8} = 2x^2y^2$$

$$xy^3 + x^3y = 2x^2y^2$$

$$xy^3 + x^3y - 2x^2y^2 = 0$$

$$xy\left(y^2 + x^2 - 2xy\right) = 0$$

$$xy(y - x)^2 = 0$$

Therefore: xy = 0 or y — x = 0. Our two solutions are: xy = 0 or y = x.

Statement (1) says y > x so y cannot be equal to x. Therefore, xy = 0. Statement (1) is sufficient.

Statement (2) says x < 0. We cannot say whether x = y or xy = 0. Statement (2) is not sufficient.

The correct answer is A.

22.

We can rephrase the question by opening up the absolute value sign. In other words, we must solve all possible scenarios for the inequality, remembering that the absolute value is always a positive value. The two scenarios for the inequality are as follows:

If x > 0, the question becomes "Is x < 1?"
If x < 0, the question becomes: "Is x > -1?"

We can also combine the questions: "Is -1 < x < 1?"

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: There are three possible equations here if we open up the absolute value signs:

1. If x < -1, the values inside the absolute value symbols on both sides of the equation are negative, so we must multiply each through by -1 (to find its opposite, or positive, value):

$|x + 1| = 2|x - 1| \longrightarrow -(x + 1) = 2(1 - x) \longrightarrow x = 3$

(However, this is invalid since in this scenario, $x < -1$.)

2. If $-1 < x < 1$, the value inside the absolute value symbols on the left side of the equation is positive, but the value on the right side of the equation is negative. Thus, only the value on the right side of the equation must be multiplied by -1:

$|x + 1| = 2|x - 1| \longrightarrow x + 1 = 2(1 - x) \longrightarrow x = 1/3$

3. If $x > 1$, the values inside the absolute value symbols on both sides of the equation are positive. Thus, we can simply remove the absolute value symbols:

$|x + 1| = 2|x - 1| \longrightarrow x + 1 = 2(x - 1) \longrightarrow x = 3$

Thus $x = 1/3$ or 3. While 1/3 is between -1 and 1, 3 is not. Thus, we cannot answer the question.

(2) INSUFFICIENT: There are two possible equations here if we open up the absolute value sign:

1. If $x > 3$, the value inside the absolute value symbols is greater than zero. Thus, we can simply remove the absolute value symbols:

$|x - 3| > 0 \longrightarrow x - 3 > 0 \longrightarrow x > 3$

2. If $x < 3$, the value inside the absolute value symbols is negative, so we must multiply through by -1 (to find its opposite, or positive, value).

$|x - 3| > 0 \longrightarrow 3 - x > 0 \longrightarrow x < 3$

If x is either greater than 3 or less than 3, then x is anything but 3. This does not answer the question as to whether x is between -1 and 1.

(1) AND (2) SUFFICIENT: According to statement (1), x can be 3 or 1/3. According to statement (2), x cannot be 3. Thus using both statements, we know that $x = 1/3$ which IS between -1 and 1.

23.

We can rephrase this question as: "Is a farther away from zero than b, on the number-line?" We can solve this question by picking numbers:

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.
(1) INSUFFICIENT: Picking values that meet the criteria $b < -a$ demonstrates that this is not enough information to answer the question.

a	b	Is $\|a\| > \|b\|$?
2	-5	NO
-5	2	YES

(2) INSUFFICIENT: We have no information about b.

(1) AND (2) INSUFFICIENT: Picking values that meet the criteria $b < -a$ and $a < 0$ demonstrates that this is not enough information to answer the question.

a	b	Is $|a| > |b|$?
-2	-5	NO
-5	2	YES

The correct answer is E.

24.

Since $|r|$ is always positive, we can multiply both sides of the inequality by $|r|$ and rephrase the question as: Is $r^2 < |r|$? The only way for this to be the case is if r is a nonzero fraction between -1 and 1.

(1) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If $r = -1/2$, $|r| = 1/2$ and $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if $r = 4$, $|r| = 4$ and $r^2 = 16$, and the answer to the question is NO.

(2) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If $r = 1/2$, $|r| = 1/2$ ans $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if $r = -4$, $|r| = 4$ and $r^2=16$, and the answer to the question is NO.

(1) AND (2) SUFFICIENT: Together, the statements tell us that r is between -1 and 1. The square of a proper fraction (positive or negative) will always be smaller than the absolute value of that proper fraction.

The correct answer is C.

25.

One way to solve equations with absolute values is to solve for x over a series of intervals. In each interval of x, the sign of the expressions within each pair of absolute value indicators does not change.

In the equation $|x-2|-|x-3|=|x-5|$, there are 4 intervals of interest:

$x < 2$: In this interval, the value inside each of the three absolute value expressions is negative.

$2 < x < 3$: In this interval, the value inside the first absolute value expression is positive, while the value inside the other two absolute value expressions is negative.

$3 < x < 5$: In this interval, the value inside the first two absolute value expressions is positive, while the value inside the last absolute value expression is negative.

$5 < x$: In this interval, the value inside each of the three absolute value expressions is positive.

Use each interval for x to rewrite the equation so that it can be evaluated without absolute value signs.

For the first interval, x < 2, we can solve the equation by rewriting each of the expressions inside the absolute value signs as negative (and thereby remove the absolute value signs):

$$-x+2-(-x+3)=-x+5$$
$$-x+2+x-3=-x+5$$
$$x=6$$

Notice that the solution x = 6 is NOT a valid solution since it lies outside the interval x < 2. (Remember, we are solving the equation for x SUCH THAT x is within the interval of interest).

For the second interval 2 < x < 3, we can solve the equation by rewriting the expression inside the first absolute value sign as positive and by rewriting the expressions inside the other absolute values signs as negative:

$$x-2-(-x+3)=-x+5$$
$$x-2+x-3=-x+5$$
$$3x=10$$
$$x=\frac{10}{3}$$

Notice, again, that the solution $x=\frac{10}{3}$ is NOT a valid solution since it lies outside the interval 2 < x < 3.

For the third interval 3 < x < 5, we can solve the equation by rewriting the expressions inside the first two absolute value signs as positive and by rewriting the expression inside the last absolute value sign as negative:

$$x-2-(x-3)=-x+5$$
$$x-2-x+3=-x+5$$
$$x=4$$

The solution x = 4 is a valid solution since it lies within the interval 3 < x < 5.

Finally, for the fourth interval 5 < x, we can solve the equation by rewriting each of the expressions inside the absolute value signs as positive:

$$x-2-(x-3)=x-5$$
$$x-2-x+3=x-5$$
$$x=6$$

The solution x = 6 is a valid solution since it lies within the interval 5 < x.

We conclude that the only two solutions of the original equation are x = 4 and x = 6.

Only answer choice C contains all of the solutions, both 4 and 6, as part of its set.

Therefore, C is the correct answer.

26.

Theoretically, any absolute value expression represents two scenarios. For example $|x| = x$ when $x > 0$, and $|x| = -x$, when $x < 0$. Thus, statement (1) can be rewritten in the following manner:

$|x - x^2| = 2$ when $x^2 > 0$

$|x - (-x^2)| = 2$ when $x^2 < 0$ (BUT x^2 is *never* less than 0; this scenario DOES NOT exist)

We can further simplify the expression into two scenarios:

I. $x - x^2 = 2$ when $x - x^2 > 0$ ($0 < x < 1$ **)

II. $-(x - x^2) = 2$ or $x^2 - x = 2$ when $x - x^2 < 0$ ($x < 0$ or $x > 1$ **)

Scenario I can be rewritten as $x^2 - x + 2 = 0$. On the GMAT, a quadratic in the form of $x^2 + bx + c = 0$ can be factored by finding which two factors of *c* (including negative factors) add up to *b*, paying special attention to the sign of *b* and *c*. There are no such factors in this equation (neither 1,2 nor -1,-2 add up to -1); therefore the quadratic cannot be factored and there are no integer solutions here. Alternatively, you can use the quadratic formula to see that this quadratic has no *real* solutions because if we compare this quadratic to the standard form of a quadratic $ax^2 + bx + c = 0$, we see that $a = 1, b = -1$, and $c = 2$. For any quadratic to have real roots, the expression $b^2 - 4ac$ must be positive, and in this case it is not: $(-1)^2 - 4(1)(2) = -7$.

Scenario II can be rewritten as $x^2 - x - 2 = 0$. This quadratic can be factored: $(x - 2)(x + 1) = 0$, with solutions x = -1 or 2. Notice that these two solutions are consistent with the conditions for this scenario, namely $x < 0$ or $x > 1$. It is important to always check potential solutions to an absolute value expression against the conditions that defined that scenario. Whenever a certain scenario for an absolute value expression yields an answer *that violates the very condition that defined that scenario*, that answer is null and void.

Scenario II therefore yields two solutions, x = -1 or 2, so statement (1) is insufficient.

Statement (2), $|x^2 - |x|| = 2$ can first be rewritten using the following two scenarios:

III. $|x^2 - x| = 2$ when $x > 0$

IV. $|x^2 - (-x)| = 2$ or $|x^2 + x| = 2$ when $x < 0$

Furthermore each of these scenarios has two scenarios:

IIIA. $x^2 - x = 2$ when $x^2 - x > 0$ ($x < 0$ or $x > 1$ **)

IIIB. $-(x^2 - x) = 2$ or $x - x^2 = 2$ when $x^2 - x < 0$ ($0 < x < 1$ **)

IVA. $x^2 + x = 2$ when $x^2 + x > 0$ ($x < -1$ or $x > 0$ **)

IVB. $-(x^2 + x) = 2$ or $-x^2 - x = 2$ when $x^2 + x < 0$ ($-1 < x < 0$ **)

Notice that these four scenarios are subject to their specific conditions, as well as to the general conditions for scenarios I and II above ($x > 0$ or $x < 0$, respectively)

Scenario IIIA can be rewritten as $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$, so it has two solutions, $x = -1, 2$. HOWEVER, one of these solutions, $x = -1$, *violates the condition* for all scenario I's which says that $x > 0$. Therefore, according to scenario IA there is only one solution, $x = 2$.

Scenario IIIB can be rewritten as $x^2 - x + 2 = 0$, which offers no integer solutions (see above).

Scenario IVA can be rewritten as $x^2 + x - 2 = 0$ or $(x+2)(x-1) = 0$, so it has two solutions, $x = -2, 1$. HOWEVER, one of these solutions, $x = 1$, *violates the condition* for all scenario II's which says that $x < 0$. Therefore, according to scenario IIA there is only one solution, $x = -2$.

Scenario IVB can be rewritten as $x^2 + x + 2 = 0$, which offers no integer solutions (see above).

Taking all four scenarios of statement (2) into account (IIIA, IIIB, IVA, IVB), $x = -2, 2$, so statement (2) is NOT sufficient.

When you take statements (1) and (2) together, x must be 2 so the answer is C.

An alternative, easier approach to this problem would be to set up the different scenarios WITHOUT concentrating on the conditions. Whatever solutions you come up with could then be verified by plugging them back into the appropriate equation. For example, in scenario IIIA of statement (2), the $x = -1$ could have been eliminated as a possible answer choice by simply trying it back in the equation $\left|x^2 - |x|\right| = 2$. We discovered the *reason* why it doesn't work above – it violates one of the conditions for the scenario. However, often times the *reason* is of little significance on the GMAT.

** The above conditions were simplified in the following manner:

For example $x - x^2 > 0$ implies that $x(1-x) > 0$. We can use the solutions to the "parallel" quadratic $x(1-x) = 0$, i.e. $x = 0$ or 1,
to help us think about the inequality. The solutions to the corresponding quadratic must be the endpoints of the ranges that are the solution to the inequality. Just try numbers in these various ranges: less than 0 (e.g. -1), in between 0 and 1 (e.g. $\frac{1}{2}$) and greater than 1 (e.g. 2). In this case only the $\frac{1}{2}$ satisfies the expression $x - x^2 > 0$, so the condition here is

$0 < x < 1$.

27.

For $|a| + |b| > |a + b|$ to be true, a and b must have opposite signs. If a and b have the same signs (i.e. both positive or both negative), the expressions on either side of the inequality will be the same. The question is really asking if a and b have opposite signs.
(1) INSUFFICIENT: This tells us that $|a| > |b|$. This implies nothing about the signs of a and b.

(2) INSUFFICIENT: Since the absolute value of a is always positive, this tells us that $b < 0$. Since we don't know the sign of a, we can't answer the question.

(1) AND (2) INSUFFICIENT: We know the sign of *b* from statement 2 but statement 1 does not tell us the sign of *a*. For example, if $b = -4$, a could be 5 or -5.

The correct answer is E

28.

First, let's simplify the question:

$$\left(x^{-1} + y^{-1}\right)^{-1} > \left(\left(x^{-1}\right)\left(y^{-1}\right)\right)^{-1}$$

$$\left(\frac{1}{x} + \frac{1}{y}\right)^{-1} > \left(\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)\right)^{-1}$$

$$\left(\frac{x+y}{xy}\right)^{-1} > \left(\frac{1}{xy}\right)^{-1}$$

$$\text{Is } \frac{xy}{x+y} > xy \text{ ?}$$

(1) SUFFICIENT: If we plug $x = 2y$ into our simplified question we get the following:

Is $2y^2/3y > 2y^2$? Since $2y^2$ must be positive we can divide both sides of the inequality by $2y^2$ which leaves us with the following: Is $1/3y > 1$? If we investigate this carefully, we find that if y is an nonzero integer, $1/3y$ is never greater than 1. Try $y = 2$ and $y = -2$, In both cases $1/3y$ is less than 1.

(2) INSUFFICIENT: Let's plug in values to investigate this statement. According to this statement, the *x* and *y* values we choose must have a positive sum. Let's choose a set of values that will yield a positive *xy* and a set of values that will yield a negative *xy*.

x	y	
3	1	$xy/(x + y) < xy$
3	-1	$xy/(x + y) > xy$

This not does yield a definitive yes or no answer so statement (2) is not sufficient.

The correct answer is A.

TOPIC 4 – NUMBERS

Types of Numbers

1.

In order for $\frac{ab}{cd}$ to be positive, *ab* and *cd* must share the same sign; that is, both either positive or negative. There are two sets of possibilities for achieving this sufficiency. First, if all four integers share the same sign- positive or negative- both *ab* and *cd* would be positive. Second, if any two of the four integers are positive while the other two are negative, *ab* and *cd* must share the same sign. The following table verifies this claim:

Positive Pair	Negative Pair	*ab* Sign	*cd* Sign
a, *b*	*c*, *d*	+	+
a, *c*	*b*, *d*	-	-
a, *d*	*b*, *c*	-	-
b, *c*	*a*, *d*	-	-
b, *d*	*a*, *c*	-	-
c, *d*	*a*, *b*	+	+

For the first and last cases, $\frac{ab}{cd}$ will be positive. On the other hand, it can be shown that if only one of the four

integers is positive and the other three negative, or vice versa, $\frac{ab}{cd}$ must be negative. This question can most tidily be rephrased as “Among the integers *a*, *b*, *c* and *d*, are an even number (zero, two, or all four) of the integers positive?”

(1) SUFFICIENT: This statement can be rephrased as *ad* = -*bc*. For the signs of *ad* and *bc* to be opposite one another, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

(2) SUFFICIENT: For the product *abcd* to be negative, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

The correct answer is D.

2.

1) J = 2 * 3 * 5 => J has 3 different prime factors, insufficient.

2) K = 2^3 * 5^3 => K has 2 different prime factors, insufficient.

1 + 2 => J has more different prime factors

Answer is C

3.

For statement 1, for example, 0, 0, 0, 2 can fulfill the requirement. Insufficient. So, B

4.

For 1, 3*2>3, * can be multiply or add, while (6*2)*4=6*(2*4).

For 2, 3*1=3, * can be multiply or divide. The information cannot determine whether (6*2)*4=6*(2*4).

Answer is A

5.

x+4=1, y+4=-5=> x=-3, y=-9, z+4=m

x+e=7, y+e=n => e=10, n=1, z=0, z+e=10

z=0 and z+4=m, m=4 =>m+n=4+1=5

ODDS and EVENS

6.

(1) SUFFICIENT: If $z/2$ = even, then $z = 2 \times$ even. Thus, z must be even, because it is the product of 2 even numbers.

Alternatively, we could list numbers according to the criteria that $z/2$ is even.
$z/2$: 2, 4, 6, 8, 10, etc.
Multiply the entire list by the denominator 2 to isolate the possible values of z:
z: 4, 8, 12, 16, 20, etc. All of those values are even.

(2) INSUFFICIENT: If $3z$ = even, then z = even/3. There are no odd and even rules for division, mainly because there is no guarantee that the result will be an integer. For example, if $3z = 6$, then z is the even integer 2. However, if $3z = 2$, then $z = 2/3$, which is not an integer at all.

The danger in evaluating this statement is forgetting about the fractional possibilities. A way to avoid that mistake is to create a full list of numbers for z that meet the criteria that $3z$ is even.
$3z$: 2, 4, 6, 8, 10, 12, etc.

Divide the entire list by the coefficient 3 to isolate the possible values of z:
z: 2/3, 4/3, 2, 8/3, 10/3, 4, etc. Some of those values are even, but others are not.

The correct answer is A.

7.

We can first simplify the exponential expression in the question:
$b^{a+1} - ba^b$
$b(b^a) - b(a^b)$
$b(b^a - a^b)$
So we can rewrite this question then as is $b(b^a - a^b)$ odd? Notice that if either b or $b^a - a^b$ is even, the answer to this question will be no.

(1) SUFFICIENT: If we simplify this expression we get $5a - 8$, which we are told is odd. For the difference of two numbers to be odd, one must be odd and one must be even. Therefore $5a$ must be odd, which means that a itself must be odd. To determine whether or not this is enough to dictate the even/oddness of the expression $b(b^a - a^b)$, we must consider two scenarios, one with an odd b and one with an even b:

a	b	$b(b^a - a^b)$	odd/even
3	1	$1(1^3 - 3^1) = -2$	even
3	2	$2(2^3 - 3^2) = -2$	even

It turns out that for both scenarios, the expression $b(b^a - a^b)$ is even.

(2) SUFFICIENT: It is probably easiest to test numbers in this expression to determine whether it implies that b is odd or even.

b	$b^3 + 3b^2 + 5b + 7$	odd/even
2	$2^3 + 3(2^2) + 5(2)+ 7 = 37$	odd
1	$1^3 + 3(1^2) + 5(1) + 7 = 16$	even

We can see from the two values that we plugged that only even values for b will produce odd values for the expression $b^3 + 3b^2 + 5b + 7$, therefore b must be even. Knowing that b is even tells us that the product in the question, $b(b^a - a^b)$, is even so we have a definitive answer to the question.

The correct answer is D, EACH statement ALONE is sufficient to answer the question.

8.

Since the digits of G are halved to derive those of H, the digits of G must both be even. Therefore, there are only 16 possible values for G and H and we can quickly calculate the possible sums of G and H:

G	H	$G + H$
88	44	132
86	43	129
84	42	126
82	41	123
68	34	102
66	33	99
64	32	96
62	31	93
48	24	72
46	23	69

44	22	66
42	21	63
28	14	42
26	13	39
24	12	36
22	11	33

Alternately, we can approach this problem algebraically. Let's call x and y the tens digit and the units digit of G. Thus H can be expressed as $5x + .5y$. And the sum of G and H can be written as $15x + 1.5y$.

Since we know that x and y must be even, we can substitute $2a$ for x and $2b$ for y and can rewrite the expression for the sum of G and H as: $15(2a) + 1.5(2b) = 30a + 3b$. This means that the sum of G and H must be divisible by 3, so we can eliminate C and E.

Additionally, since we know that the maximum value of G is less than 100, then the maximum value of H must be less than 50. Therefore the maximum value of G + H must be less than 150. This eliminates answer choices A and B. This leaves answer choice D, 129. This can be written as 86 + 43.

The correct answer is D.

9.

Sequence problems are often best approached by charting out the first several terms of the given sequence. In this case, we need to keep track of n, t_n, and whether t_n is even or odd.

n	**t_n**	**Is t_n even or odd?**
0	$t_n = 3$	Odd
1	$t_1 = 3 + 1 = \mathbf{4}$	Even
2	$t_2 = 4 + 2 = \mathbf{6}$	Even
3	$t_3 = 6 + 3 = \mathbf{9}$	Odd
4	$t_4 = 9 + 4 = \mathbf{13}$	Odd
5	$t_5 = 13 + 5 = \mathbf{18}$	Even
6	$t_6 = 18 + 6 = \mathbf{24}$	Even
7	$t_7 = 24 + 7 = \mathbf{31}$	Odd
8	$t_8 = 31 + 8 = \mathbf{39}$	Odd

Notice that beginning with n = 1, a pattern of even-even-odd-odd emerges for t_n.

Thus t_n is even when n = 1, 2 . . . 5, 6 . . . 9, 10 . . . 13, 14 . . . etc. Another way of conceptualizing this pattern is that t_n is even when n is either
(a) 1 plus a multiple of 4 (n = 1, 5, 9, 13, etc.) or
(b) 2 plus a multiple of 4 (n = 2, 6, 10, 14, etc).

From this we see that only Statement (2) is sufficient information to answer the question. If n – 1 is a multiple of 4, then n is 1 plus a multiple of 4. This means that t_n is always even.

Statement (1) does not allow us to relate n to a multiple of 4, since it simply tells us that n + 1 is a multiple of 3. This means that n could be 2, 5, 8, 11, etc. Notice that for n = 2 and n = 5, t_n is in fact even. However, for n = 8 and n = 11, t_n is odd.

Thus, Statement (2) alone is sufficient to answer the question but Statement (1) alone is not.

The correct answer is B.

Units digits, factorial powers

10.

When a number is divided by 10, the remainder is simply the units digit of that number. For example, 256 divided by 10 has a remainder of 6. This question asks for the remainder when an integer power of 2 is divided by 10. If we examine the powers of 2 (2, 4, 8, 16, 32, 64, 128, and 256…), we see that the units digit alternates in a consecutive pattern of 2, 4, 8, 6. To answer this question, we need to know which of the four possible units digits we have with 2^p.

(1) INSUFFICIENT: If s is even, we know that the product rst is even and so is p. Knowing that p is even tells us that 2^p will have a units digit of either 4 or 6 ($2^2 = 4$, $2^4 = 16$, and the pattern continues).

(2) SUFFICIENT: If $p = 4t$ and t is an integer, p must be a multiple of 4. Since every fourth integer power of 2 ends in a 6 ($2^4 = 16$, $2^8 = 256$, etc.), we know that the remainder when 2^p is divided by 10 is 6.

The correct answer is B.

11.

For problems that ask for the units digit of an expression, yet seem to require too much computation, remember the Last Digit Shortcut. Solve the problem step-by-step, but recognize that you only need to pay attention to the last digit of every intermediate product. Drop any other digits.

So, we can drop any other digits in the original expression, leaving us to find the units digit of:

$(4)^{(2x+1)}(3)^{(x+1)}(7)^{(x+2)}(9)^{(2x)}$

This problem is still complicated by the fact that we don't know the value of x. In such situations, it is often a good idea to look for patterns. Let's see what happens when we raise the bases 4, 3, 7, and 9 to various powers. For example: $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, and so on. The units digit of the powers of three follow a pattern that repeats every fourth power: 3, 9, 7, 1, 3, 9, 7, 1, and so on. The patterns for the other bases are shown in the table below:

	Exponent						
Base	**1**	**2**	**3**	**4**	**5**	**6**	**Pattern**
4	4	6	4	6	4	6	4, 6, repeat
3	3	9	7	1	3	9	3, 9, 7, 1, repeat
7	7	9	3	1	7	9	7, 9, 3, 1, repeat
9	9	1	9	1	9	1	9, 1, repeat

The patterns repeat at least every fourth term, so let's find the units digit of $(4)^{(2x+1)}(3)^{(x+1)}(7)^{(x+2)}(9)^{(2x)}$ for at least four consecutive values of x:

$x = 1$: units digit of $(4^3)(3^2)(7^3)(9^2)$ = units digit of (4)(9)(3)(1) = units digit of 108 = 8

$x = 2$: units digit of $(4^5)(3^3)(7^4)(9^4)$ = units digit of (4)(7)(1)(1) = units digit of 28 = 8

$x = 3$: units digit of $(4^7)(3^4)(7^5)(9^6)$ = units digit of (4)(1)(7)(1) = units digit of 28 = 8

$x = 4$: units digit of $(4^9)(3^5)(7^6)(9^8)$ = units digit of (4)(3)(9)(1) = units digit of 108 = 8

The units digit of the expression in the question must be 8.

Alternatively, note that x is a positive integer, so $2x$ is always even, while $2x + 1$ is always odd. Thus,
$(4)^{(2x+1)} = (4)^{(\text{odd})}$, which always has a units digit of 4
$(9)^{(2x)} = (9)^{(\text{even})}$, which always has a units digit of 1

That leaves us to find the units digit of $(3)^{(x+1)}(7)^{(x+2)}$. Rewriting, and dropping all but the units digit at each intermediate step,

$(3)^{(x+1)}(7)^{(x+2)}$
$= (3)^{(x+1)}(7)^{(x+1)}(7)$
$= (3 \times 7)^{(x+1)}(7)$
$= (21)^{(x+1)}(7)$
$= (1)^{(x+1)}(7) = 7$, for any value of x.

So, the units digit of $(4)^{(2x+1)}(3)^{(x+1)}(7)^{(x+2)}(9)^{(2x)}$ is (4)(7)(1) = 28, then once again drop all but the units digit to get 8.

$\boxed{\text{The correct answer is D.}}$

$\boxed{12.}$

In order to answer this, we need to recognize a common GMAT pattern: the difference of two squares. In its simplest form, the difference of two squares can be factored as follows: $x^2 - y^2 = (x + y)(x - y)$. Where, though, is the difference of two squares in the question above? It pays to recall that all even exponents are squares. For example,

$$x^4 = (x^2)(x^2)$$
$$x^{56} = (x^{28})(x^{28}).$$

Because the numerator in the expression in the question is the difference of two even exponents, we can factor it as the difference of two squares and simplify:

$$\frac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} = a$$

$$\frac{\cancel{\left[(13!)^8 + (13!)^4\right]}\left[(13!)^8 - (13!)^4\right]}{\cancel{(13!)^8 + (13!)^4}} = a$$

$$(13!)^8 - (13!)^4 = a$$

$$(13!)^4\left[(13!)^4 - 1\right] = a$$

$$(13!)^4 - 1 = \frac{a}{(13!)^4}$$

The units digit of the left side of the equation is equal to the units digit of the right side of the equation (which is what the question asks about). Thus, if we can determine the units digit of the expression on the left side of the equation, we can answer the question.

Since $(13!) = 13 \times 12 \times 11 \times 10 \ldots \times 1$, we know that 13! contains a factor of 10, so its units digit must be 0.

Similarly, the units digit of $(13!)^4$ will also have a units digit of 0. If we subtract 1 from this, we will be left with a number ending in 9.

Therefore, the units digit of $\frac{a}{(13!)^4}$ is 9.

The correct answer is E.

13.

We know from the question that *x* and *y* are integers and also that they are both greater than 0. Because we are only concerned with the units digit of *n* and because both bases end in 3 (243 and 463), we simply need to know $x + y$ to figure out the units digit for *n*. Why? Because, to get the units digit, we are simply going to complete the operation $3^x \times 3^y$ which, using our exponent rules, simplifies to $3^{(x+y)}$.

So we can rephrase the question as "What is $x + y$?"

(1) SUFFICIENT: This tells us that $x + y = 7$. Therefore, the units digit of the expression in the question will be the same as the units digit of 3^7.

(2) INSUFFICIENT: This gives us no information about *y*.

The correct answer is A.

DECIMALS

14.

To determine the value of $10 - x$, we must determine the exact value of *x*. To determine the value of *x*, we must find out what digits *a* and *b* represent. Thus, the question can be rephrased: What is *a* and what is *b*?

(1) INSUFFICIENT: This tells us that *x* rounded to the nearest hundredth must be 1.44. This means that *a*, the hundredths digit, might be either 3 (if the hundredths digit was rounded up to 4) or 4 (if the hundredths digit was rounded down to 4). This statement alone is NOT sufficient since it does not give us a definitive value for *a* and tells us nothing about *b*.

(2) SUFFICIENT: This tells us that *x* rounded to the nearest thousandth must be 1.436. This means, that *a*, the hundredths digit, is equal to 3. As for *b*, the thousandths digit, we know that it is followed by a 5 (the ten-thousandths digit); therefore, if *x* is rounded to the nearest thousandth, *b* must rounded UP. Since *b* is rounded UP to 6, then we know that *b* must be equal to 5. Statement (2) alone is sufficient because it provides us with definitive values for both *a* and *b*.

The correct answer is B.

15.

For fraction p/q to be a terminating decimal, the numerator must be an integer and the denominator must be an integer that can be expressed in the form of 2^x5^y where *x* and *y* are nonnegative integers. (Any integer divided by a power of 2 or 5 will result in a terminating decimal.)

The numerator p, 2^a3^b, is definitely an integer since a and b are defined as integers in the question.

The denominator q, $2^c3^d5^e$, could be rewritten in the form of 2^x5^y if we could somehow eliminate the expression 3^d. This could happen if the power of 3 in the numerator (b) is greater than the power of 3 in the denominator (d), thereby canceling out the expression 3^d. Thus, we could rephrase this question as, is $b > d$?

(1) INSUFFICIENT. This does not answer the rephrased question "is $b > d$"? The denominator q is not in the form of 2^x5^y so we cannot determine whether or not p/q will be a terminating decimal.

(2) SUFFICIENT. This answers the question "is $b > d$?"

The correct answer is B.

Sequences and Series

16.

To find each successive term in S, we add 1 to the previous term and add this to the reciprocal of the previous term plus 1.

$S_1 = 201$

$$S_2 = (201+1) + \frac{1}{(201+1)} = 202 + \frac{1}{202}$$

$$S_3 = \left(202 + \frac{1}{202} + 1\right) + \frac{1}{(202 + \frac{1}{202} + 1)} = 203 + \frac{1}{202} + \frac{1}{203 + \frac{1}{202}}$$

The question asks to estimate (Q), the sum of the first 50 terms of S. If we look at the endpoints of the intervals in the answer choices, we see have quite a bit of leeway as far as our estimation is concerned. In fact, we can simply ignore the fractional portion of each term. Let's use $S_2 \approx 202$, $S_3 \approx 203$. In this way, the sum of the first 50 terms of S will be approximately equal to the sum of the fifty consecutive integers 201, 202 … 250.

To find the sum of the 50 consecutive integers, we can multiply the mean of the integers by the number of integers since average = sum / (number of terms).

The mean of these 50 integers = (201 + 250) / 2 = 225.5

Therefore, the sum of these 50 integers = 50 x 225.5 = 11,275, which falls between 11,000 and 12,000. The correct answer is C.

17.

Since the membership of the new group increases by 700% every 10 months, after the first 10-month period the new group will have (8)(4) members (remember that to increase by 700% is to increase eightfold). After the second 10-month period, it will have (8)(8)(4) members. After the third 10-month period, it will have (8)(8)(8)(4) members. We can now see a pattern: the number of members in the new group can be expressed as $(4)(8^x)$, where x is the number of 10-month periods that have elapsed.

Since the membership of the established group doubles every 5 months (remember, to increase by 100% is to double), it will have (2)(4096) after the first 5-month period. After another 5 months, it will have (2)((2)(4096)) members. After another 5 months, it will have (2)((2)(2)(4096)). We can now see a pattern: the number of members in the established group will be equal to $(4096)(2^y)$, where *y* represents the number of 5-month periods that have elapsed.

The question asks us after how many months the two groups will have the same number of members.

Essentially, then, we need to know when $(4096)(2^y) = (4)(8^x)$. Since *y* represents the number of 5-month periods and *x* represents the number of 10-month periods, we know that *y* = 2*x*. We can rewrite the equation as $(4096)(2^{2x}) = (4)(8^x)$. We now need to solve for *x*, which represents the number of 10-month periods that elapse before the two groups have the same number of members.

The next step we need to take is to break all numbers down into their prime factors:

$4 = 2^2$
$8 = 2^3$
$4096 = 2^{12}$

We can now rewrite the equation:

$(2^{12})(2^{2x}) = (2^2)(2^3)^x \rightarrow$
$2^{2x+12} = (2^2)(2^{3x}) \rightarrow$
$2^{2x+12} = 2^{3x+2}$

Since the bases are equal on both sides of the equation, the exponents must be equal as well. Therefore, it must be true that $2x + 12 = 3x + 2$. We can solve for *x*:

$2x + 12 = 3x + 2 \rightarrow$
$10 = x$

If *x* = 10, then 10 ten-month periods will elapse before the two groups have equal membership rolls, for a total of 100 months.

The correct answer is E.

Remainders, Divisibility

18.

First consider an easier expression such as $10^5 - 560$. Doing the computation yields 99,440, which has 2 9's followed by 440.

From this, we can extrapolate that $10^{25} - 560$ will have a string of 22 9's followed by 440.

Now simply apply your divisibility rules:

You might want to skip 11 first because there is no straightforward rule for divisibility by 11. You can always return to this if necessary. [One complex way to test divisibility by 11 is to assign opposite signs to adjacent digits and then to add them to see if they add up to 0. For example, we know that 121 is divisible by 11 because -1 +2 -1 equals zero. In our case, the twenty-two 9s, when assigned opposite signs, will add up to zero, and so will the digits of 440, since +4 -4 +0 equals zero.]

If the last three digits of the number are divisible by 8, the number is divisible by 8. Since 440 is divisible by 8, the entire expression is divisible by 8.

If the last two digits of the number are divisible by 4, the number is divisible by 4. Since 40 is divisible by 4, the enter expression is divisible by 4.

If a number ends in 0 or 5, it is divisible by 5. Since the expression ends in 0, it is divisible by 5.
For a number to be divisible by three, the sum of the digits must be divisible by three. The sum of the 22 9's will be divisible by three but when you add the sum of the last three digits, 8 (4 + 4 + 0), the result will not be divisible by 3. Thus, the expression will NOT be divisible by 3.

The correct answer is E.

19.

The remainder is what is left over after 4 has gone wholly into *x* as many times as possible. For example, suppose that *x* is 10. 4 goes into 10 two whole times ($2 \times 4 = 8 < 10$), but not quite three times ($3 \times 4 = 12 > 10$). The remainder is what is left over: $10 – 8 = 2$.

(1) INSUFFICIENT: This statement tells us that $x/3$ must be an odd integer, because that is the only way we would have a remainder of 1 after dividing by 2. Thus, *x* is (3 × an odd integer), and (odd × odd = odd), so *x* must be an odd multiple of 3. The question stem tells us that *x* is positive. So, *x* could be any positive, odd integer that is a multiple of 3: 3, 9, 15, 21, 27, 33, 39, 45, etc. Now we need to answer the question “when *x* is divided by 4, is the remainder equal to 3?” for every possible value of *x* on the list. For $x = 15$, the answer is “yes,” since 15/4 results in a remainder of 3. For $x = 9$, the answer is “no,” since 9/4 results in a remainder of 1. The answer to the question might be “yes” or “no,” depending on the value of *x*, so we are not able to give a definite answer based on the information given.

(2) INSUFFICIENT: This statement tells us that *x* is a multiple of 5. The question stem tells us that *x* is a positive integer. So, *x* could be any positive integer that is a multiple of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, etc. Now we need to answer the question “when *x* is divided by 4, is the remainder equal to 3?” for every possible value of *x* on the list. For $x = 15$, the answer is “yes,” since 15/4 results in a remainder of 3. For $x = 5$, the answer is “no,” since 5/4 results in a remainder of 1. The answer to the question might be “yes” or “no,” depending on the value of *x*, so we are not able to give a definite answer based on the information given.

(1) AND (2) INSUFFICIENT: From the two statements, we know that *x* is an odd multiple of 3 and that *x* is a multiple of 5. In order for *x* to be both a multiple of 3 and 5, it must be a multiple of 15 ($15 = 3 \times 5$). The question stem tells us that *x* is a positive integer. So, *x* could be any odd, positive integer that is a multiple of 15: 15, 45, 75, 105, etc. Now we need to answer the question “when *x* is divided by 4, is the remainder equal to 3?” for every possible value of *x* on the list. For $x = 15$, the answer is “yes,” since 15/4 results in a remainder of 3. For $x = 45$, the answer is “no,” since 45/4 results in a remainder of 1. The answer to the question might be “yes” or “no,” depending on the value of *x*, so we are not able to give a definite answer based on the information given.

The correct answer is E.

20.

A remainder, by definition, is always smaller than the divisor and always an integer. In this problem, the divisor is 7, so the remainders all must be integers smaller than 7. The possibilities, then, are 0, 1, 2, 3, 4, 5, and 6. In order to calculate the sum, we need to know which remainders are created.

(1) INSUFFICIENT: The range is defined as the difference between the largest number and the smallest number in a given set of integers. In this particular question, a range of 6 indicates that the difference between the largest remainder and the smallest remainder is 6. However, this does not tell us any information about the rest of the remainders; though we know the smallest term is 0, and the largest is 6, the other remainders could be any values between 0 and 6, which would result in varying sums.

(2) SUFFICIENT: By definition, when we divide a consecutive set of seven integers by seven, we will get one each of the seven possibilities for remainder. For example, let's pick 11, 12, 13, 14, 15, 16, and 17 for our set of seven integers (x1 through x7). The remainders are as follows:
x1 = 11. 11/7 = 1 remainder 4
x2 = 12. 12/7 = 1 remainder 5
x3 = 13. 13/7 = 1 remainder 6
x4 = 14. 14/7 = 2 remainder 0
x5 = 15. 15/7 = 2 remainder 1
x6 = 16. 16/7 = 2 remainder 2
x7 = 17. 17/7 = 2 remainder 3

Alternatively, you can solve the problem algebraically. When you pick 7 consecutive integers on a number line, one and only one of the integers will be a multiple of 7. This number can be expressed as $7n$, where n is an integer. Each of the other six consecutive integers will cover one of the other possible remainders: 1, 2, 3, 4, 5, and 6. It makes no difference whether the multiple of 7 is the first integer in the set, the middle one or the last. To prove this consider the set in which the multiple of 7 is the first integer in the set. The seven consecutive integers will be: $7n$, $7n + 1$, $7n + 2$, $7n + 3$, $7n + 4$, $7n + 5$, $7n + 6$. The sum of the remainders here would be $0 + 1 + 2 + 3 + 4 + 5 + 6 = 21$.

The correct answer is B.

21.

(1) INSUFFICIENT: At first glance, this may seem sufficient since if 5 is the remainder when k is divided by j, then there will *always* exist a positive integer m such that $k = jm + 5$. In this case, m is equal to the integer quotient and 5 is the remainder. For example, if $k = 13$ and $j = 8$, and 13 divided by 8 has remainder 5, it *must* follow that there exists an m such that $k = jm + 5$: $m = 1$ and $13 = (8)(1) + 5$.

However, the logic does not go the other way: 5 is not necessarily the remainder when k is divided by j. For example, if $k = 13$ and $j = 2$, there exists an m ($m = 4$) such that $k = jm + 5$: $13 = (2)(4) + 5$, consistent with statement (1), yet 13 divided by 2 has remainder 1 rather than 5.

When $j < 5$ (e.g., $2 < 5$); this means that j can go into 5 (e.g., 2 can go into 5) at least one more time, and consequently m is not the true quotient of k divided by j and 5 is not the true remainder. Similarly, if we let $k = 14$ and $j = 3$, there exists an m (e.g., $m = 3$) such that statement (1) is also satisfied [i.e., $14 = (3)(3) + 5$], yet the remainder when 14 is divided by 3 is 2, a different result than the first example.

Statement (1) tells us that $k = jm + 5$, where m is a positive integer. That means that $k/j = m + 5/j$ = integer + $5/j$. Thus, the remainder when k is divided by j is either 5 (when $j > 5$), or equal to the remainder of $5/j$ (when j is 5 or less). Since we do not know whether j is greater than or less than 5, we cannot determine the remainder when k is divided by j.

(2) INSUFFICIENT: This only gives the range of possible values of j and by itself does not give any insight

as to the value of the remainder when k is divided by j.

(1) AND (2) SUFFICIENT: Statement (1) was not sufficient because we were not given whether $5 > j$, so we could not be sure whether j could go into 5 (or k) any additional times. However, (2) tells us that $j > 5$, so we now know that j cannot go into 5 any more times. This means that m is the exact number of times that k can be divided by j and that 5 is the true remainder.

Another way of putting this is: From statement (1) we know that $k/j = m + 5/j$ = integer + $5/j$. From statement (2) we know that $j > 5$. Therefore, the remainder when k is divided by j must always be 5.

The correct answer is C.

22.

(1) INSUFFICIENT: We are told that $5n/18$ is an integer. This does not allow us to determine whether $n/18$ is an integer. We can come up with one example where $5n/18$ is an integer and where $n/18$ is **NOT** an integer. We can come up with another example where $5n/18$ is an integer and where $n/18$ **IS** an integer.

Let's first look at an example where $5n/18$ is equal to the integer 1.

If $\frac{5n}{18} = 1$, then $\frac{n}{18} = \frac{1}{5}$. In this case $n/18$ is NOT an integer.

Let's next look at an example where $5n/18$ is equal to the integer 15.

If $\frac{5n}{18} = 15$, then $\frac{n}{18} = 3$ - In this case $n/18$ IS an integer.

Thus, Statement (1) is NOT sufficient.

(2) INSUFFICIENT: We can use the same reasoning for Statement (2) that we did for statement (1). If $3n/18$ is equal to the integer 1, then $n/18$ is NOT an integer. If $3n/18$ is equal to the integer 9, then $n/18$ IS an integer.
(1) AND (2) SUFFICIENT: If $5n/18$ and $3n/18$ are both integers, $n/18$ must itself be an integer. Let's test some examples to see why this is the case.

The first possible value of n is 18, since this is the first value of n that ensures that **both** $5n/18$ and $3n/18$ are integers. If $n = 18$, then $n/18$ is an integer. Another possible value of n is 36. (This value also ensures that both $5n/18$ and $3n/18$ are integers). If $n = 36$, then $n/18$ is an integer.

A pattern begins to emerge: the fact that $5n/18$ AND $3n/18$ are both integers limits the possible values of n to multiples of 18. Since n must be a multiple of 18, we know that $n/18$ must be an integer. The correct answer is C.

Another way to understand this solution is to note that according to (1), $n = (18/5)$*integer, and according to (2), $n = 6$*integer. In other words, n is a multiple of both 18/5 and 6. The least common multiple of these two numbers is 18. In order to see this, write 6 = 30/5. The LCM of the numerators 18 and 30 is 90. Therefore, the LCM of the fractions is 90/5 = 18.

Again, the correct answer is C.

23.

This question can be rephrased: Is $r - s$ divisible by 3? Or, are r and s each divisible by 3?

Statement (1) tells us that r is divisible by 735. If r is divisible by 735, it is also divisible by all the factors of 735. 3 is a factor of 735. (To test whether 3 is a factor of a number, sum its digits; if the sum is divisible by 3, then 3 is a factor of the number.) However, statement (1) does not tell us anything about whether or not s is divisible by 3. Therefore it is insufficient.

Statement (2) tells us that $r + s$ is divisible by 3. This information alone is insufficient. Consider each of the following two cases:

CASE ONE: If $r = 9$, and $s = 6$, $r + s = 15$ which is divisible by 3, and $r - s = 3$, which is also divisible by 3.
CASE TWO: If $r = 7$ and $s = 5$, $r + s = 12$, which is divisible by 3, but $r - s = 2$, which is NOT divisible by 3.

Let's try both statements together. There is a mathematical rule that states that if two integers are each divisible by the integer x, then the sum or difference of those two integers is also divisible by x.

We know from statement (1) that r is divisible by 3. We know from statement (2) that $r + s$ is divisible by 3. Using the converse of the aforementioned rule, we can deduce that s is divisible by 3. Then, using the rule itself, we know that the difference $r - s$ is also divisible by 3.

The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

24.

Statement (1) tells us that n is a prime number.

If $n = 2$, the lowest prime number, then $n^2 - 1 = 4 - 1 = 3$, which is *not* divisible by 24.

If $n = 3$, the next prime number, then $n^2 - 1 = 9 - 1 = 8$ which is *not* divisible by 24.

However, if $n = 5$, then $n^2 - 1 = 25 - 1 = 24$, which *is* divisible by 24.

Thus, statement (1) alone is not sufficient.

Now let us examine statement (2). By design, this number is large enough so that it would not be easy to check numbers directly. Thus, we need to go straight to number properties.

For an expression to be divisible by 24, it must be divisible by 2, 2, 2, and 3 (since this is the prime factorization of 24). In other words, the expression must be divisible by 2 at least three times and by 3 at least once.

The expression $n^2 - 1 = (n - 1)(n + 1)$.

If we think about 3 consecutive integers, with n as the middle number, the expression $n^2 - 1$ is the product of the smallest number $(n - 1)$ and the largest number $(n + 1)$ in the consecutive set.

Given 3 consecutive positive integers, the product of the smallest number and the largest number will be divisible by 2 three times if the middle number is odd. Thus, if n is odd, the product $(n - 1)(n + 1)$ must be divisible by 2 three times.

(Consider why: If the middle number of 3 consecutive integers is odd, then the smallest and largest numbers of the set will be consecutive even integers - their product must therefore be divisible by 2 at least twice. Further, since the smallest and the largest number are consecutive even integers, one of them must be divisible by 4. Thus the product of the smallest and largest number must actually be divisible by 2 at least three times!)

Additionally, given 3 consecutive positive integers, exactly ONE of those three numbers must be divisible by 3. To ensure that the product of the smallest number and the largest number will be divisible by 3, the middle number must NOT be divisible by 3. Thus, for the expression $n^2 - 1$ to be divisible by 24, n must be odd and must NOT be divisible by 3.

Statement (2) alone tells us that $n > 191$. Since, this does not tell us whether n is even, odd, or divisible by 3, it is not sufficient to answer the question.

Taking statements (1) and (2) together, we know that n is a prime number greater than 191. Every prime number greater than 3 must, by definition, be ODD (since the only even prime number is 2), and must, by definition, NOT be divisible by 3 (otherwise it would not be prime!)

Thus, so long as n is a prime number greater than 3, the expression $n^2 - 1$ will always be divisible by 24. The correct answer is C: Statements (1) and (2) TAKEN TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient.

25.

We are first told that the sum of all the digits of the number q is equal to the three-digit number $x13$. Then we are told that the number q itself is equal to $10^n - 49$. Finally, we are asked for the value of n.

The first step is to recognize that $10^n - 49$ will have to equal a series of 9's ending with a 5 and a 1 (99951, for example, is $10^5 - 49$). So q is a series of 9's ending with a 5 and a 1. Since the sum of all the digits of q is equal to $x13$, we know that $x13$ is the sum of all those 9's plus 5 plus 1. So if we subtract 5 and 1 from $x13$, we are left with $x07$.

This three-digit number x07 is the sum of all the 9's alone. So $x07$ must be a multiple of 9. For any multiple of 9, the sum of all the digits of that multiple must itself be a multiple of 9 (for example, 585 = (9)(65) and 5 + 8 + 5 = 18, which is a multiple of 9). So it must be true that $x + 0 + 7$ is a multiple of 9. The only single-digit value for x that will yield a multiple of 9 when added to 0 and 7 is 2. Therefore, $x = 2$ and the sum of all the 9's in q is 207.

Since 207 is a multiple of 9, we can set up the equation $9y = 207$, where y is a positive integer. Solving for y, we get $y = 23$. So we know q consists of a series of twenty-three 9's followed by a 5 and a 1: 9999999999999999999999951. If we add 49 to this number, we get 10,000,000,000,000,000,000,000,000.

Since the exponent in every power of 10 represents the number of zeroes (e.g., $10^2 = 100$, which has two zeroes; $10^3 = 1000$, which has three zeroes, etc.), we must be dealing with 10^{25}. Thus $n = 25$.

$\boxed{\text{The correct answer is B.}}$

26.

1). n could be 22,24, 26, ...insufficient

2). n=28K+3, then (28K+3)/7, the remainder is 3

$\boxed{\text{Answer is B}}$

27.

1). The general term is x=6k+3. So, the remainder is 3. [look for the "general term" in this page, you can find the explanation about it.]

2). The general term is x=12k+3. So, remainder is 3 as well.

Answer is D

28.

If x divided by 11 has a quotient of y and a remainder of 3, x can be expressed as $x = 11y + 3$, where y is an integer (by definition, a quotient is an integer). If x divided by 19 also has a remainder of 3, we can also express x as $x = 19z + 3$, where z is an integer.

We can set the two equations equal to each other:
$11y + 3 = 19z + 3$
$11y = 19z$

The question asks us what the remainder is when y is divided by 19. From the equation we see that $11y$ is a multiple of 19 because z is an integer. y itself must be a multiple of 19 since 11, the coefficient of y, is not a multiple of 19.

If y is a multiple of 19, the remainder must be zero.

The correct answer is A

Factors, Divisors, Multiples, LCM, HCF

29.

The factors of 104 are {1, 2, 4, 8, 13, 26, 52, 104}. If $x – 1$ is definitely one of these factors, OR if $x – 1$ is definitely NOT one of these factors, then the statement is sufficient.

(1) INSUFFICIENT: x could be any one of the infinite multiples of 3. If $x = 3$, $x – 1$ would equal 2, which is a factor of 104. If $x = 6$, $x – 1$ would equal 5, which is not a factor of 104.

(2) INSUFFICIENT: The factors of 27 are {1, 3, 9, 27}. Subtracting 1 from each of these values yields the set {0, 2, 8, 26}. The non-zero values are all factors of 104, but 0 is not.

(1) AND (2) SUFFICIENT: Given that x is a factor of 27 and also divisible by 3, x must equal one of 3, 9 or 27. $x – 1$ must therefore equal one of 2, 8, or 26 - all factors of 104.

The correct answer is C.

30.

88,000 is the product of an unknown number of 1's, 5's, 11's and x's. To figure out how many x’s are multiplied to achieve this product, we have to figure out what the value of x is. Remember that a number's prime box shows all of the prime factors that when multiplied together produce that number: 88,000's prime box contains one 11, three 5’s, and six 2’s, since $88{,}000 = 11 \times 5^3 \times 2^6$.

The 11 in the prime box must come from a red chip, since we are told that $5 < x < 11$ and therefore x could not have 11 as a factor. In other words, the factor of 11 definitely did not come from the selection of a purple chip, so we can ignore that factor for the rest of our solution.

So, turning to the remaining prime factors of 88,000: the three 5's and six 2's. The 2's must come from the purple chips, since the other colored chips have odd values and thus no factor of two. Thus, we now know something new about x: it must be even. We already knew that $5 < x < 11$, so now we know that x is 6, 8, or 10.

However, x cannot be 6: $6 = 2 \times 3$, and our prime box has no 3's.

x seemingly might be 10, because $10 = 2 \times 5$, and our prime box does have 2's and 5's. However, our prime box for 88,000 only has three 5's, so a maximum of three chips worth 10 points are possible. But that leaves three of the six factors of 2 unaccounted for, and we know those factors of two must have come from the purple chips.

So x must be 8, because $8 = 2^3$ and we have six 2's, or two full sets of three 2's, in the prime box. Since x is 8, the chips selected must have been 1 red (one factor of 11), 3 green (three factors of 5), 2 purple (two factors of 8, equivalent to six factors of 2), and an indeterminate number of blue chips.

The correct answer is B.

31.

The problem asks us to find the greatest possible value of (length of x + length of y), such that x and y are integers and $x + 3y < 1{,}000$ (note that x and y are the numbers themselves, not the lengths of the numbers - lengths are always indicated as "length of x" or "length of y," respectively).

Consider the extreme scenarios to determine our possible values for integers x and y based upon our constraint $x + 3y < 1{,}000$ and the fact that both x and y have to be greater than 1. If $y = 2$, then $x \leq 993$. If $x = 2$, then $y \leq 332$. Of course, x and y could also be somewhere between these extremes.

Since we want the maximum possible sum of the lengths, we want to maximize the length of our x value, since this variable can have the largest possible value (up to 993). The greatest number of factors is calculated by using the smallest prime number, 2, as a factor as many times as possible. $2^9 = 512$ and $2^{10} = 1{,}024$, so our largest possible length for x is 9.

If x itself is equal to 512, that leaves 487 as the highest possible value for $3y$ (since $x + 3y < 1{,}000$). The largest possible value for integer y, therefore, is 162 (since 487 / 3 = 162 remainder 1). If $y < 162$, then we again use the smallest prime number, 2, as a factor as many times as possible for a number less than 162. Since $2^7 = 128$ and $2^8 = 256$, our largest possible length for y is 7.

If our largest possible length for x is 9 and our largest possible length for y is 7, our largest sum of the two lengths is $9 + 7 = 16$.

What if we try to maximize the length of the y value rather than that of the x value? Our maximum y value is 332, and the greatest number of prime factors of a number smaller than 332 is $2^8 = 256$, giving us a length of 8 for y. That leaves us a maximum possible value of 231 for x (since $x + 3y < 1{,}000$). The greatest number of prime factors of a number smaller than 231 is $2^7 = 128$, giving us a length of 7 for x. The sum of these lengths is $7 + 8 = 15$, which is *smaller* than the sum of 16 that we obtained when we maximized the x value. Thus 16, not 15, is the maximum value of (length of x + length of y).

The correct answer is D.

32.

The greatest common divisor is the largest integer that evenly divides both 35*x* and 20*y*.

(A) CAN be the greatest common divisor. First, $35x / 5 = 7x$, which is an integer for every possible value of *x*. Second, $20y / 5 = 4y$, which is an integer for every possible value of *y*. Therefore, 5 is a common divisor. It will be the greatest common divisor when 7*x* and 4*y* share no other factors. To illustrate, if $x = 1$ and $y = 1$, then $35x = 35$ and $20y = 20$, and their greatest common divisor is 5.

(B) CAN be the greatest common divisor. First, $\frac{35x}{5(x - y)} = \frac{7x}{x - y}$, which can be an integer

in certain cases: when *y* is a multiple of $(x - y)$. So, this answer choice will be a divisor if both *x* and *y* are multiples of $(x - y)$. Since *x* and *y* are integers, the easiest way to meet the requirement is to select $(x - y) = 1$, for example $x = 3$ and $y = 2$. To illustrate, if $x = 3$ and $y = 2$, then $35x = 105$ and $20y = 40$, and their greatest common divisor is $5 = 5(x - y)$.

in certain cases: when x is a multiple of (x – y). Second, $\frac{20y}{5(x - y)} = \frac{4y}{x - y}$, which can be an integer

(C) CANNOT be the greatest common divisor. Regardless of the values of *x* and *y*, $35x / 20x = 35/20 = 7/4$, which is not an integer. Therefore, 20*x* does not evenly divide one of the numbers in question. It is not a divisor, and certainly not the greatest common divisor.

(D) CAN be the greatest common divisor. First, $35x / 20y = 7x / 4y$, which can be an integer in certain cases: one such case is when $x = 4$ and $y = 1$. Second, $20y / 20y = 1$, which is an integer. To illustrate, if $x = 4$ and $y = 1$, then $35x = 140$ and $20y = 20$, and their greatest common divisor is $20 = 20y$.

(E) CAN be the greatest common divisor. First, $35x / 35x = 1$, which is an integer. Second, $20y / 35x = 4y / 7x$, which can be an integer in certain cases: one such case is when $x = 1$ and $y = 7$. To illustrate, if $x = 1$ and $y = 7$, then $35x = 35$ and $20y = 140$, and their greatest common divisor is $35 = 35x$.

$\boxed{\text{The correct answer is C.}}$

33.

Let's start by breaking 80 down into its prime factorization: $80 = 2 \times 2 \times 2 \times 2 \times 5$. If p^3 is divisible by 80, p^3 must have 2, 2, 2, 2, and 5 in its prime factorization. Since p^3 is actually $p \times p \times p$, we can conclude that the prime factorization of $p \times p \times p$ must include 2, 2, 2, 2, and 5.

Let's assign the prime factors to our *p*'s. Since we have a 5 on our list of prime factors, we can give the 5 to one of our *p*'s:

p: 5
p:
p:

Since we have four 2's on our list, we can give each *p* a 2:

p: 5 × 2

p: 2
p: 2

But notice that we still have one 2 leftover. This 2 must be assigned to one of the p's:

p: 5 × 2 × 2
p: 2
p: 2

We must keep in mind that each p is equal in value to any other p. Therefore, all the p's must have exactly the same prime factorization (i.e. if one p has 5 as a prime factor, all p's must have 5 as a prime factor). We must add a 5 and a 2 to the 2nd and 3rd p's:

p: 5 × 2 × 2 = 20
p: 5 × 2 × 2 = 20
p: 5 × 2 × 2 = 20

We conclude that p must be at least 20 for p^3 to be divisible by 80. So, let's count how many factors 20, or p, has:

1 × 20
2 × 10
4 × 5

20 has 6 factors. If p must be at least 20, p has at least 6 distinct factors.

The correct answer is C.

34.

(1) INSUFFICIENT: For a number to be divisible by 16, it must be divisible by 2 four times. The expression $m(m + 1)(m + 2) \ldots (m + n)$ represents the product of $n + 1$ consecutive integers. For example if $n = 5$, the expression represents the product of 6 consecutive integers.

To find values of n for which the product will be divisible by 16 let's first consider $n = 3$. This implies a product of four consecutive integers. In any set of four consecutive integers, two of the integers will be even and two will be odd. In addition one of the two even integers will be a multiple of 4, since every other *even* integer on the number line is a multiple of 4 (4 yes, 6 no, 8 yes, etc...). This accounts for a total of *three* 2's that can definitely divide into the product of 4 consecutive integers. This however is not enough.

With five integers (i.e. $n = 4$), there *may or may not* be an additional 2: there could be two or three even integers. Six integers (i.e. $n = 5$) will guarantee three even integers and a product that is divisible by four 2's. Anything above six integers will naturally be divisible by 16 as well, so we can conclude that n is greatern than or equal to 5.

(2) INSUFFICIENT: This expression factors to $(n - 4)(n - 5) = 0$. There are two solutions for n, 4 or 5.

(1) AND (2) TOGETHER SUFFICIENT: If n must be greater than or equal to 5 and either 4 or 5, then n must be equal to 5.

The correct answer is C.

35.

(1) INSUFFICIENT: a and b could be 12 and 8, with a greatest common factor of 4; or they could be 11 and 7, with a greatest common factor of 1.

(2) INSUFFICIENT: This statement tells us that b is a multiple of 4 but we have no information about a.

(1) AND (2) SUFFICIENT: Together, we know that b is a multiple of 4 and that a is the next consecutive multiple of 4. For any two positive consecutive multiples of an integer n, n is the greatest common factor of those multiples, so the greatest common multiple of a and b is 4.

The correct answer is C.

36.

First, let's rephrase the complex wording of this question into something easier to handle. The question asks whether x is a non-integer which is the reverse of an easier question: Is x an integer?

Surely, if we can answer this question, we can answer the original question.

We can isolate x by rewriting the given equation as follows: $x = \frac{ab}{30}$.

In order for x to be an integer, ab must be divisible by 30. In order for a number to be divisible by 30, it must have 2, 3, and 5 as prime factors (since 30 = 2 x 3 x 5).

Thus, the question becomes: Does ab have 2, 3, and 5 as prime factors?

We know from the question that a and b are consecutive positive integers. Thus, either a or b is an even number, which means that the product ab must be divisible by 2.

Since we know that 2 is a prime factor of ab, the question can be further simplified: Does ab have 3 and 5 as prime factors?

Statement (1) tells us that 21 is a factor of a^2 which means that 3 and 7 are prime factors of a^2.

We can deduce from this that 3 and 7 must also be factors of a itself. (How? We know that a is a positive integer, which means that a^2 is a perfect square. All prime factors of perfect squares come in pairs. Thus, if a^2 is divisible by 3, then a^2 must be divisible by a pair of 3's, which means that a itself must be divisible by at least *one* 3. You can test this using a real value for a^2.)

Knowing that 3 is a prime factor of a tells us that 3 is a factor of ab but is not sufficient to answer our rephrased question, since we know nothing about whether 5 is a factor of ab.

Statement (2) tells us that 35 is a factor of b^2 which means that 5 and 7 are prime factors of b^2.

Using the same logic as for the previous statement, we can deduce that 5 and 7 must be factors of b itself. Knowing that 5 is a prime factor of b tells us that 5 is a factor of ab but it is not sufficient to answer our rephrased question, since we know nothing about whether 3 is a factor of ab.

If we combine both statements, we know that ab must be divisible by both 3 and 5, which is sufficient information to answer the original question. The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

37.

We have 5,10,15,20,25,30 and some 2s to see how many zeros there are in the tail of the result 5x2, 15x2 contributes 1 zero each

10, 20, 30 => 3 zeros

25x4 => be careful, it contributes 2 zeros

So totally there are 1x2+3+2=7 zeros in the tails of 30!

For 1, d can be 1,2,3,4,5,6,7

For 2, d can be any integer >6

Together, d can only be 7 to fulfill both requirements.

Answer is C

38.

1). n=k1*5;t=k2*5(k1,k2 are natural numbers), n*t=k1*k2*5^2. k1 and k2 are unknown, thus, we cannot obtain the greatest prime factor of nt.

2). 105=1*3*5*7.

The least common multiple of n and t is 105=>the greatest value of n and t is 105, and the other one must be less than 105 and be composed with numbers of 1, 3, 5, 7.

So, the greatest prime factor is 7

Answer is B

Digits

39.

There are 3!, or 6, different three-digit numbers that can be constructed using the digits *a*, *b*, and *c*:

abc	*bac*	*cab*
acb	*bca*	*cba*

The value of any one of these numbers can be represented using place values. For example, the value of *abc* is 100*a* + 10*b* + *c*.
Therefore, you can represent the sum of the 6 numbers as:

$$\begin{array}{r}100a + 10b + c\\ 100a + b + 10c\\ 10a + 100b + c\\ a + 100b + 10c\\ 10a + b + 100c\\ \underline{a + 10b + 100c}\\ 222a + 222b + 222c = 222(a + b + c)\end{array}$$

x is equal to $222(a + b + c)$. Therefore, x must be divisible by 222.

$\boxed{\text{The correct answer is E.}}$

40.

The question asks for the value of the three-digit number *SSS* and tells us that *SSS* is the sum of the three-digit numbers *ABC* and *XYZ*. We can represent this relationship as:

$$\begin{array}{r}ABC\\ \underline{+XYZ}\\ SSS\end{array}$$

Statement (1) tells us that $S = 1.75X$. Since X is a digit between 0 and 9, inclusive, S must equal 7. This is because X must be a multiple of 4 in order for $1.75X$ to yield an integer (remember that 1.75 is the decimal equivalent of 7/4). Therefore, X must be either 4 or 8. However, X cannot be 8 because $1.75(8) = 14$, which is not a digit and thus cannot be the value of S. So X must be 4 and $1.75(4) = 7$. Therefore, the value of *SSS* is 777. Statement (1) is sufficient.

Statement (2) tells us that $S^2 = \frac{49}{8}ZX$. If we take the square root of both sides and simplify, we get:

$$\sqrt{S^2} = \sqrt{\frac{49}{8}ZX} \rightarrow$$
$$S = 7\sqrt{\frac{ZX}{8}}$$

Since S is an integer, $7\sqrt{\frac{ZX}{8}}$ must be an integer as well. And since S must be less than 10, $7\sqrt{\frac{ZX}{8}}$ must also be less than 10. The only way in the present circumstances for this to happen is if $\sqrt{\frac{ZX}{8}} = 1$. Therefore, $S = (7)(1) = 7$ and the value of *SSS* is 777. Statement (2) is sufficient.

$\boxed{\text{The correct answer is D: Either statement alone is sufficient.}}$

TOPIC 5 – GEOMETRY

Part 1: Lines and Angles

1.

(1) INSUFFICIENT: We don't know any of the angle measurements.

(2) INSUFFICIENT: We don't know the relationship of x to y.

(1) AND (2) INSUFFICIENT: Because $l1$ is parallel to $l2$, we know the relationship of the four angles at the intersection of $l2$ and $l3$ ($l3$ is a transversal cutting two parallel lines) and the same four angles at the intersection of $l1$ and $l3$. We do not, however, know the relationship of y to those angles because we do not know if $l3$ is parallel to $l4$.

The correct answer is E.

2.

We are given two triangles and asked to determine the degree measure of z, an angle in one of them.

The first step in this problem is to analyze the information provided in the question stem. We are told that $x - q = s - y$. We can rearrange this equation to yield $x + y = s + q$. Since $x + y + z = 180$ and since $q + s + r = 180$, it must be true that $z = r$. We can now look at the statements.

Statement (1) tells us that $xq + sy + sx + yq = zr$. In order to analyze this equation, we need to rearrange it to facilitate factorization by grouping like terms: $xq + yq + sx + sy = zr$. Now we can factor:

$$xq + yq + sx + sy = zr \rightarrow$$
$$q(x + y) + s(x + y) = zr \rightarrow$$
$$(x + y)(q + s) = zr$$

Since $x + y = q + s$ and $z = r$, we can substitute and simplify:

$$(x + y)(q + s) = zr \rightarrow$$
$$(x + y)(x + y) = (z)(z) \rightarrow$$
$$\sqrt{(x + y)^2} = \sqrt{z^2} \rightarrow$$
$$x + y = z$$

Is this sufficient to tell us the value of z? Yes. Why? Consider what happens when we substitute z for $x + y$:

$$x + y + z = 180 \rightarrow$$
$$z + z = 180 \rightarrow$$
$$2z = 180 \rightarrow$$
$$z = 90$$

It is useful to remember that when the sum of two angles of a triangle is equal to the third angle, the triangle must be a right triangle. Statement (1) is sufficient.

Statement (2) tells us that $zq - ry = rx - zs$. In order to analyze this equation, we need to rearrange it:

$$zq - ry = rx - zs \rightarrow$$
$$zq + zs = rx + ry \rightarrow$$
$$z(q + s) = r(x + y) \rightarrow$$
$$z = \frac{r(x + y)}{(q + s)} \rightarrow$$
$$\frac{z}{r} = \frac{x + y}{q + s}$$

Is this sufficient to tell us the value of z? No. Why not? Even though we know the following:

$z = r$
$x + y = q + s$
$x + y + z = 180$
$q + r + s = 180$

we can find different values that will satisfy the equation we derived from statement (2):

$$\frac{90}{90} = \frac{30 + 60}{40 + 50}$$
or
$$\frac{100}{100} = \frac{40 + 40}{10 + 70}$$

These are just two examples. We could find many more. Since we cannot determine the value of z, statement (2) is insufficient.

The correct answer is A: Statement (1) alone is sufficient, but statement (2) is not.

3.

The question asks us to find the degree measure of angle a. Note that a and e are equal since they are vertical angles, so it's also sufficient to find e.

Likewise, you should notice that $e + f + g = 180$ degrees. Thus, to find e, it is sufficient to find $f + g$. The question can be rephrased to the following: "What is the value of $f + g$?"

(1) SUFFICIENT: Statement (1) tells us that $b + c = 287$ degrees. This information allows us to calculate $f + g$. More specifically:

$b + c = 287$
$(b + f) + (c + g) = 180 + 180$ Two pairs of supplementary angles.
$b + c + f + g = 360$
$287 + f + g = 360$
$f + g = 73$

(2) INSUFFICIENT: Statement (2) tells us that $d + e = 269$ degrees. Since $e = a$, this is equivalent to $d + a = 269$. There are many combinations of d and a that satisfy this constraint, so we cannot determine a unique value for a.

The correct answer is A.

Part 2: Triangles

4.

Because angles *BAD* and *ACD* are right angles, the figure above is composed of three *similar* right triangles: *BAD*, *ACD* and *BCA.* [Any time a height is dropped from the right angle vertex of a right triangle to the opposite side of that right triangle, the three triangles that result have the same 3 angle measures. This means that they are similar triangles.] To solve for the length of side *CD*, we can set up a proportion, based on the relationship between the similar triangles *ACD* and *BCA*: BC/AC = CA/CD or ¾ = 4/CD or CD = 16/3.

The correct answer is D.

5.

(1) INSUFFICIENT: This tells us that *AC* is the height of triangle *BAD* to base *BD*. This does not help us find the length of *BD*.

(2) INSUFFICIENT: This tells us that *C* is the midpoint of segment *BD*. This does not help us find the length of *BD*.

(1) AND (2) SUFFICIENT: Using statements 1 and 2, we know that *AC* is the perpendicular bisector of *BD*. This means that triangle *BAD* is an isosceles triangle so side AB must have a length of 5 (the same length as side *AD*). We also know that angle *BAD* is a right angle, so side *BD* is the hypotenuse of right isosceles triangle *BAD*. If each leg of the triangle is 5, the hypotenuse (using the Pythagorean theorem) must be $5\sqrt{2}$.

The correct answer is C.

6.

(1) SUFFICIENT: If we know that ABC is a right angle, then triangle ABC is a right triangle and we can find the length of BC using the Pythagorean theorem. In this case, we can recognize the common triple 5, 12, 13 - so BC must have a length of 12.

(2) INSUFFICIENT: If the area of triangle ABC is 30, the height from point C to line AB must be 12 (We know that the base is 5 and area of a triangle = 0.5 × base × height). There are only two possibilities for such such a triangle. Either angle CBA is a right triangle, and CB is 12, or angle BAC is an obtuse angle and the height from point C to length AB would lie outside of the triangle. In this latter possibility, the length of segment BC would be greater than 12.

The correct answer is A.

7

The perimeter of a triangle is equal to the sum of the three sides.

(1) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

(2) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three

sides.

Together, the two statements are SUFFICIENT. Triangle ABC is an isosceles triangle which means that there are theoretically 2 possible scenarios for the lengths of the three sides of the triangle: (1) AB = 9, BC = 4 and the third side, AC = 9 **OR** (1) AB = 9, BC = 4 and the third side AC = 4.

These two scenarios lead to two different perimeters for triangle ABC, HOWEVER, upon careful observation we see that the second scenario is an IMPOSSIBILITY. A triangle with three sides of 4, 4, and 9 is not a triangle. Recall that any two sides of a triangle must sum up to be greater than the third side. 4 + 4 < 9 so these are not valid lengths for the side of a triangle.

Therefore the actual sides of the triangle must be AB = 9, BC = 4, and AC = 9. The perimeter is 22.

The correct answer is C.

8.

The question stem tells us that ABCD is a rectangle, which means that triangle ABE is a right triangle.

The formula for the area of any triangle is: 1/2 (Base X Height).

In right triangle ABE, let's call the base AB and the height BE. Thus, we can rephrase the questions as follows: **Is 1/2 (AB X BE) greater than 25?**

Let's begin by analyzing the first statement, taken by itself. Statement (1) tells us that the length of AB = 6. While this is helpful, it provides no information about the length of BE. Therefore there is no way to determine whether the area of the triangle is greater than 25 or not.

Now let's analyze the second statement, taken by itself. Statement (2) tells us that length of diagonal AE = 10. We may be tempted to conclude that, like the first statement, this does not give us the two pieces of information we need to know (that is, the lengths of AB and BE respectively). However, knowing the length of the diagonal of the right triangle actually does provide us with some very relevant information about the lengths of the base (AB) and the height (BE).

Consider this fact: Given the length of the diagonal of a right triangle, it **IS** possible to determine the maximum area of that triangle.

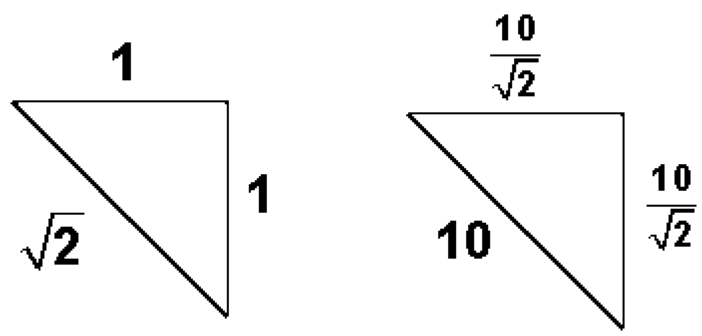
How? The right triangle with the largest area will be an isosceles right triangle (where both the base and height are of equal length).

If you don't quite believe this, see the end of this answer for a visual proof of this fact. (See "visual proof" below).

Therefore, given the length of diagonal AE = 10, we can determine the largest possible area of triangle ABE by making it an isosceles right triangle.

If you plan on scoring 700+ on the GMAT, you should know the side ratio for all isosceles right triangles (also known as 45-45-90 triangles because of their degree measurements).

That important side ratio is $1:1:\sqrt{2}$ where the two **1**'s represent the two legs (the base and the height) and $\sqrt{2}$ represents the diagonal. Thus if we are to construct an isosceles right triangle with a diagonal of 10, then, using the side ratios, we can determine that each leg will have a length of $\frac{10}{\sqrt{2}}$.



Now, we can calculate the area of this isosceles right triangle:

$$\frac{1}{2}(AB \times BE) = \frac{1}{2}\left(\frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{100}{2}\right) = \frac{1}{2}(50) = 25$$

Since an isosceles right triangle will yield the maximum possible area, we know that 25 is the maximum possible area of a right triangle with a diagonal of length 10.

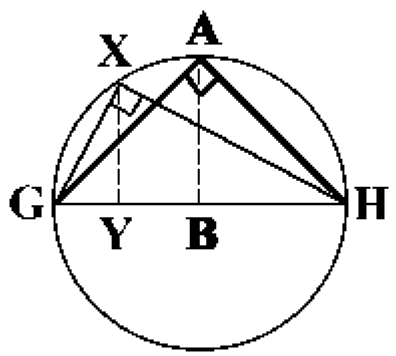
Of course, we don't really know if 25 is, in fact. the area of triangle ABE, but we do know that 25 is the maximum possible area of triangle ABE. Therefore we are able to answer our original question: Is the area of triangle ABE greater than 25? *NO it is not greater than 25, because the maximum area is 25.*

Since we can answer the question using Statement (2) alone, the correct answer is B.

Visual Proof:

Given a right triangle with a fixed diagonal, why will an ISOSCELES triangle yield the triangle with the greatest area?

Study the diagram below to understand why this is always true:



In the circle above, GH is the diameter and AG = AH. Triangles GAH and GXH are clearly both right triangles (any triangle inscribed in a semicircle is, by definition, a right triangle).

Let's begin by comparing triangles GAH and GXH, and thinking about the area of each triangle. To determine this area, we must know the base and height of each triangle.

Notice that both these right triangles share the same diagonal (GH). In determining the area of both triangles, let's use this diagonal (GH) as the base. Thus, the bases of both triangles are equal.

Now let's analyze the height of each triangle by looking at the lines that are perpendicular to our base GH. In triangle GAH, the height is line AB. In triangle GXH, the height is line XY.

Notice that the point A is HIGHER on the circle's perimeter than point X. This is because point A is directly above the center of the circle, it the highest point on the circle.

Thus, the perpendicular line extending from point A down to the base is LONGER than the perpendicular line extending from point X down to the base. Therefore, the height of triangle GAH (line AB) is greater than the height of triangle GXH (line XY).

Since both triangles share the same base, but triangle GAH has a greater height, then the area of triangle GAH must be greater than the area of triangle GXH.

We can see that no right triangle inscribed in the circle with diameter GH will have a greater area than the isosceles right triangle GAH.

(Note: Another way to think about this is by considering a right triangle as half of a rectangle. Given a rectangle with a fixed perimeter, which dimensions will yield the greatest area? The rectangle where all sides are equal, otherwise known as a square! Test it out for yourself. Given a rectangle with a perimeter of 40, which dimensions will yield the greatest area? The one where all four sides have a length of 10.)

9.

Since BC is parallel to DE, we know that Angle ABC = Angle BDE, and Angle ACB = Angle CED. Therefore, since Triangle ABC and Trianlge ADE have two pairs of equal angles, they must be ***similar triangles***. Similar triangles are those in which all corresponding angles are equal and the lengths of corresponding sides are in proportion.

For Triangle ABC, let the base = **b**, and let the height = **h**.

Since Triangle ADE is similar to triangle ABC, apply multiplier "**m**" to **b** and **h**. Thus, for Triangle ADE, the base = **mb** and the height = **mh**.

Since the Area of a Triangle is defined by the equation $\frac{\text{base} \times \text{height}}{2}$, and since the problem tells us that the area of trianlge ABC is $\frac{1}{12}$ the area of Triangle ADE, we can write an equation comparing the areas of the two triangles:

$$\frac{b \times h}{2} = \frac{1}{12}\left(\frac{mb \times mh}{2}\right)$$

Simplifying this equation yields:

$$bh = \frac{1}{12} mbmh$$
$$12bh = m^2 bh$$
$$12 = m^2$$
$$\sqrt{12} = m$$
$$2\sqrt{3} = m$$

Thus, we have determined that the multipier (**m**) is $2\sqrt{3}$. Therefore the length of $AE = 2\sqrt{3} \times AC$.

We are told in the problem that $AC = 3$, so $AE = 3\left(2\sqrt{3}\right) = 6\sqrt{3}$.

The problem asks us to solve for x, which is the difference between the length of **AE** and the length of **AC**.

Therefore, $x = AE - AC = 6\sqrt{3} - 3$.

10.

The formula for the area of a triangle is 1/2(bh). We know the height of ΔABC. In order to solve for area, we need to find the length of the base. We can rephrase the question:

What is BC?

(1) INSUFFICIENT: If angle ABD = 60°, ΔABD must be a 30-60-90 triangle. Since the proportions of a 30-60-90 triangle are x: x$\sqrt{3}$: 2x (shorter leg: longer leg: hypotenuse), and AD = 6$\sqrt{3}$, BD must be 6. We know nothing about DC.

(2) INSUFFICIENT: Knowing that AD = 6$\sqrt{3}$, and AC = 12, we can solve for CD by recognizing that ΔACD must be a 30-60-90 triangle (since it is a right triangle and two of its sides fit the 30-60-90 ratio), or by using the Pythagorean theorem. In either case, CD = 6, but we know nothing about BD.

(1) AND (2) SUFFICIENT: If BD = 6, and DC = 6, then BC = 12, and
the area of ΔABC = 1/2(bh) = 1/2(12)(6$\sqrt{3}$) = 36$\sqrt{3}$.

The correct answer is C

11.

Since *BE* $\parallel$ *CD*, triangle *ABE* is similar to triangle *ACD* (parallel lines imply two sets of equal angles). We can use this relationship to set up a ratio of the respective sides of the two triangles:

$$\frac{AB}{AC} = \frac{AE}{AD}$$

$$\frac{3}{6} = \frac{4}{AD}$$ So $AD = 8$.

We can find the area of the trapezoid by finding the area of triangle *CAD* and subtracting the area of triangle *ABE*.

Triangle *CAD* is a right triangle since it has side lengths of 6, 8 and 10, which means that triangle *BAE* is also a right triangle (they share the same right angle).
Area of trapezoid = area of triangle *CAD* – area of triangle *BAE*

= (1/2)*bh* – (1/2)*bh*
= 0.5(6)(8) – 0.5(3)(4)

$= 24 – 6$
$= 18$

The correct answer is B

12.

Triangle *DBC* is inscribed in a semicircle (that is, the hypotenuse CD is a diameter of the circle). Therefore, angle *DBC* must be a right angle and triangle *DBC* must be a right triangle.

(1) SUFFICIENT: If the length of *CD* is twice that of *BD*, then the ratio of the length of *BD* to the length of the hypotenuse *CD* is 1 : 2. Knowing that the side ratios of a 30-60-90 triangle are 1 : $\sqrt{3}$: 2, where 1 represents the short leg, $\sqrt{3}$ represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle *DBC* is a 30-60-90 triangle. Since side *BD* is the short leg, angle x, the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle *DBC* is inscribed in a semicircle, it must be a right triangle. So, angle *DBC* is 90 degrees. If $y = 60$, $x = 180 – 90 – 60 = 30$.

The correct answer is D.

13.

We are given a right triangle that is cut into four smaller right triangles. Each smaller triangle was formed by drawing a perpendicular from the right angle of a larger triangle to that larger triangle's hypotenuse. When a right triangle is divided in this way, two similar triangles are created. And each one of these smaller similar triangles is also similar to the larger triangle from which it was formed.

Thus, for example, triangle *ABD* is similar to triangle *BDC*, and both of these are similar to triangle *ABC*. Moreover, triangle *BDE* is similar to triangle *DEC*, and each of these is similar to triangle *BDC*, from which they were formed. If *BDE* is similar to *BDC* and *BDC* is similar to *ABD*, then *BDE* must be similar to *ABD* as well.

Remember that similar triangles have the same interior angles and the ratio of their side lengths are the same. So the ratio of the side lengths of *BDE* must be the same as the ratio of the side lengths of *ABD*. We are given the hypotenuse of *BDE*, which is also a leg of triangle *ABD*. If we had even one more side of *BDE*, we would be able to find the side lengths of *BDE* and thus know the ratios, which we could use to determine the sides of *ABD*.

(1) SUFFICIENT: If $BE = 3$, then *BDE* is a 3-4-5 right triangle. *BDE* and *ABD* are similar triangles, as discussed above, so their side measurements have the same proportion. Knowing the three side measurements of *BDE* and one of the side measurements of *ABD* is enough to allow us to calculate *AB*.

To illustrate:
$BD = 5$ is the hypotenuse of *BDE*, while *AB* is the hypotenuse of *ABD*.
The longer leg of right triangle *BDE* is $DE = 4$, and the corresponding leg in *ABD* is $BD = 5$.

Since they are similar triangles, the ratio of the longer leg to the hypotenuse should be the same in both *BDE* and *ABD*.
For *BDE*, the ratio of the longer leg to the hypotenuse = 4/5.
For *ABD*, the ratio of the longer leg to the hypotenuse = $5/AB$.

Thus, $4/5 = 5/AB$, or $AB = 25/4 = 6.25$

(2) SUFFICIENT: If *DE* = 4, then *BDE* is a 3-4-5 right triangle. This statement provides identical information to that given in statement (1) and is sufficient for the reasons given above.

The correct answer is D.

Part 3: Quadrilaterals

14.

(1) INSUFFICIENT: The diagonals of a parallelogram bisect one another. Knowing that the diagonals of quadrilateral *ABCD* (i.e. *AC* and *BD*) bisect one another establishes that *ABCD* is a parallelogram, but not necessarily a rectangle.
(2) INSUFFICIENT: Having one right right angle is not enough to establish a quadrilateral as a rectangle.

(1) AND (2) SUFFICIENT: According to statement (1), quadrilateral *ABCD* is a parallelogram. If a parallelogram has one right angle, all of its angles are right angles (in a parallelogram opposite angles are equal and adjacent angles add up to 180), therefore the parallelogram is a rectangle.

The correct answer is C.

15.

(1) SUFFICIENT: The diagonals of a rhombus are perpendicular bisectors of one another. This is in fact enough information to prove that a quadrilateral is a rhombus.

(2) SUFFICIENT: A quadrilateral with four equal sides is by definition a rhombus.

The correct answer is D.

16.

(1) INSUFFICIENT: Not all rectangles are squares.

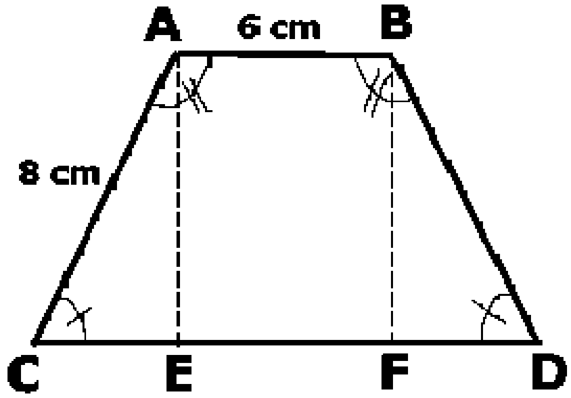
(2) INSUFFICIENT: Not every quadrilateral with two adjacent sides that are equal is a square. (For example, you can easily draw a quadrilateral with two adjacent sides of length 5, but with the third and fourth sides not being of length 5.)

(1) AND (2) SUFFICIENT: *ABCD* is a rectangle with two adjacent sides that are equal. This implies that all four sides of *ABCD* are equal, since opposite sides of a rectangle are always equal. Saying that *ABCD* is a rectangle with four equal sides is the same as saying that ABCD is a square.

The correct answer is C.

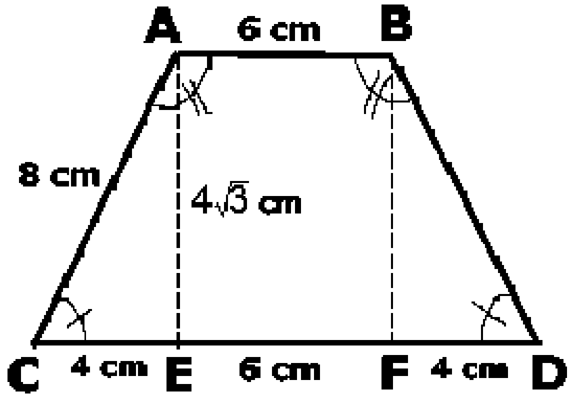
17.

The area of a trapezoid is equal to the average of the bases multiplied by the height. In this problem, you are given the top base (AB = 6), but not the bottom base (CD) or the height. (Note: 8 is NOT the height!) In order to find the area, you will need a way to figure out this missing data.
Drop 2 perpendicular lines from points A and B to the horizontal base CD, and label the points at which the lines meet the base E and F, as shown.



EF = AB = 6 cm. The congruent symbols in the drawing tell you that Angle A and Angle B are congruent, and that Angle C and Angle D are congruent. This tells you that AC = BD and CE = FD.

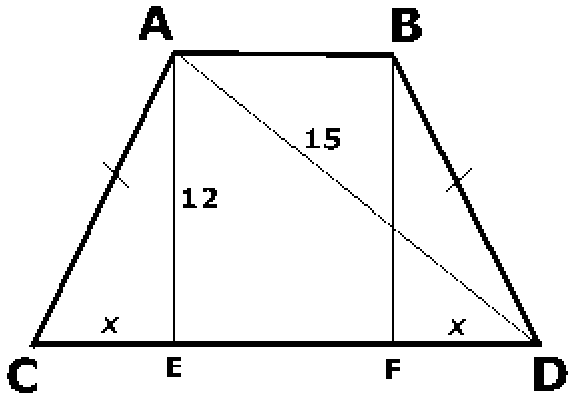
Statement (1) tells us that Angle A = 120. Therefore, since the sum of all 4 angles must yield 360 (which is the total number of degrees in any four-sided polygon), we know that Angle B = 120, Angle C = 60, and Angle D = 60. This means that triangle ACE and triangle BDF are both 30-60-90 triangles. The relationship among the sides of a 30-60-90 triangle is in the ratio of $x : x\sqrt{3} : 2x$, where x is the shortest side. For triangle ACE, since the longest side AC = 8 , CE = 4 and AE = $4\sqrt{3}$. The same measurements hold for triangle BFD. Thus we have the length of the bottom base (4 + 6 + 4) and the height and we can calculate the area of the trapezoid.



Statement (2) tells us that the perimeter of trapezoid ABCD is 36. We already know that the lengths of sides AB (6), AC (8), and BD (8) sum to 22. We can deduce that CD = 14. Further, since EF = 6, we can determine that CE = FD = 4. From this information, we can work with either Triangle ACE or Triangle BDF, and use the Pythagorean theorem to figure out the height of the trapezoid. Now, knowing the lengths of both bases, and the height, we can calculate the area of the trapezoid.

The correct answer is D: EACH statement ALONE is sufficient.

18.



By sketching a drawing of trapezoid ABDC with the height and diagonal drawn in, we can use the Pythagorean theorem to see the ED = 9. We also know that ABDC is an isosceles trapezoid, meaning that AC = BD; from this we can deduce that CE = FD, a value we will call x. The area of a trapezoid is equal to the average of the two bases multiplied by the height.
The bottom base, CD, is the same as CE + ED, or x + 9. The top base, AB, is the same as ED – FD, or 9 – x.

Thus the average of the two bases is $\frac{(x+9)+(9-x)}{2} = \frac{18}{2} = 9$.

Multiplying this average by the height yields the area of the trapezoid: $9 \times 12 = 108$.

The correct answer is D.

19.

Assume the larger red square has a side of length x + 4 units and the smaller red square has a side of length x - 4 units. This satisfies the condition that the side length of the larger square is 8 more than that of the smaller square.

Therefore, the area of the larger square is $(x + 4)^2$ or $x^2 + 8x + 16$. Likewise, the area of the smaller square is $(x - 4)^2$ or $x^2 - 8x + 16$. Set up the following equation to represent the combined area:

$(x^2 + 8x +16) + (x^2 - 8x +16) = 1000$
$2x^2 + 32 = 1000$
$2x^2 = 968$

It is possible, but not necessary, to solve for the variable x here.

The two white rectangles, which are congruent to each other, are each x + 4 units long and x - 4 units high. Therefore, the area of either rectangle is (x + 4)(x - 4), or $x^2 - 16$. Their combined area is $2(x^2 - 16)$, or $2x^2 - 32$.

Since we know that $2x^2 = 968$, the combined area of the two white rectangles is 968 - 32, or 936 square units.

The correct answer is B.

20.

This question is simply asking if the two areas--the area of the circle and the area of quadrilateral ABCD--are equal.

We know that the area of a circle is equal to πr^2, which in this case is equal to $\pi(XY)^2$. If ABCD is a square or a rectangle, then its area is equal to the length times the width. Thus in order to answer this question, we will need to be given (1) the exact shape of quadrilateral ABCD (just because it appears visually to be a square or a rectangle does not mean that it is) and (2) some relationship between the radius of the circle and the side(s) of the quadrilateral that allows us to relate their respective areas.

Statement 1 appears to give us exactly what we need. Using the information given, one might deduce that since all of its sides are equal, quadrilateral ABCD is a square. Therefore, its area is equal to one of its sides squared or $(AB)^2$. Substituting for the value of AB given in this statement, we can calculate that the area of ABCD equals $(\sqrt{\pi}XY)^2 = \pi XY^2$. This suggests that the area of quadrilateral ABCD *is* in fact equal to the area of the circle. However this reasoning is INCORRECT.

A common trap on difficult GMAT problems is to seduce the test-taker into making assumptions that are not verifiable; this is particularly true when unspecified figures are involved. Despite the appearance of the drawing and the fact that all sides of ABCD are equal, ABCD does not HAVE to be a square. It could, for example, also be a rhombus, which is a quadrilateral with equal sides, but one that is not necessarily composed of four right angles. The area of a rhombus is not determined by squaring a side, but rather by taking half the product of the diagonals, which do not have to be of equal length. Thus, the information in Statement 1 is NOT sufficient to determine the shape of ABCD. Therefore, it does not allow us to solve for its area and relate this area to the area of the circle.

Statement 2 tells us that the diagonals are equal--thus telling us that ABCD has right angle corners (The only way for a quadrilateral to have equal diagonals is if its corners are 90 degrees.) Statement 2 also gives us a numerical relationship between the diagonal of ABCD and the radius of the circle. If we assume that ABCD is a square, this relationship would allow us to determine that the area of the square and the area of the circle are equal. *However, once again, we cannot assume that ABCD is a square.*

Statement 2 tells us that ABCD has 90 degree angle corners but it does not tell us that all of its sides are equal; thus, ABCD could also be a rectangle. If ABCD is a rectangle then its length is not necessarily equal to its width which means we are unable to determine its exact area (and thereby relate its area to that of the circle). Statement 2 alone is insufficient.

Given BOTH statements 1 and 2, we are assured that ABCD is a square since only squares have *both* equal sides AND equal length diagonals. Knowing that ABCD *must* be a square, we can use either numerical relationship given in the statements to confirm that the area of the quadrilateral is equal to the area of the circle.

The correct answer is C: Both statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

21.

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. The opposite sides of a parallelogram also have equal length.

(1) SUFFICIENT: We know from the question stem that opposite sides *PS* and *QR* are parallel, while this statement tells us that they also have equal lengths. The opposite sides *PQ* and *RS* must also be parallel and equal in length. This is the definition of a parallelogram, so the answer to the question is “Yes.”

(2) INSUFFICIENT: We know from the question stem that opposite sides *PS* and *QR* are parallel, but have no information about their respective lengths. This statement tells us that the opposite sides *PQ* and *RS* are equal in length, but we don't know their respective angles; they might be parallel, or they might not be. According to the information given, *PQRS* could be a trapezoid with *PS* not equal to *QR*. On the other hand, *PQRS* could be a parallelogram with *PS* = *QR*. The answer to the question is uncertain.

The correct answer is A.

22.

The rectangular yard has a perimeter of 40 meters (since the fence surrounds the perimeter of the yard). Let's use l for the length of the fence and w for the width.

Perimeter = $2l + 2w$
$40 = 2l + 2w$
$20 = l + w$

The area of the yard = lw
$64 = lw$

If we solve the perimeter equation for w, we get $w = 20 - l$.
Plug this into the area equation:
$64 = l(20 - l)$
$64 = 20l - l^2$
$l^2 - 20l + 64 = 0$
$(l - 16)(l - 4) = 0$
$l = 4$ or 16

This means the width is either 16 or 4 ($w = 20 - l$).
By convention, the length is the longer of the two sides so the length is 16.

We could also solve this question by backsolving the answer choices.

Let's start with C, the middle value. If the length of the yard is 12 and the perimeter is 40, the width would be 8 (perimeter – $2l = 2w$). With a length of 12 and a width of 8, the area would be 96. This is too big of an area.

It may not be intuitive whether we need the length to be longer or shorter, based on the above outcome. Consider the following geometric principle: **for a fixed perimeter, the maximum area will be achieved when the values for the length and width are closest to one another.** A 10×10 rectangle has a much bigger area than an 18×2 rectangle. Put differently, when dealing with a fixed perimeter, the greater the disparity between the length and the width, the smaller the area.

Since we need the area to be smaller than 96, it makes sense to choose a longer length so that the disparity between the length and width will be greater.

When we get to answer choice E, we see that a length of 16 gives us a width of 4 (perimeter – $2l = 2w$). Now the area is in fact $16 \times 4 = 64$.

The correct answer is E

23.

A rhombus is a parallelogram with four sides of equal length. Thus, AB = BC = CD = DA.

The diagonals of a parallelogram bisect each other, meaning that AC and BD intersect at their midpoints, which we will call E. Thus, AE = EC = 4 and BE = ED = 3. Since ABCD is a rhombus, diagonals AC and BD are also perpendicular to each other.

Labeling the figure with the lengths above, we can see that the rhombus is divided by the diagonals into four right triangles, each of which has one side of length 3 and another side of length 4.

Remembering the common right triangle with side ratio = 3: 4: 5, we can infer that the unlabeled hypotenuse of each of the four triangles has length 5.

Thus, AB = BC = CD = DA = 5, and the perimeter of ABCD is 5 × 4 = 20.

The correct answer is C.

Topic 4: Circles

24.

Let us say that line segment RT has a length of 1. RT is the radius of circle R, so circle R has a radius of 1. Line segment QT is the diameter of circle R, so it has a length of 2 (twice the radius of circle R). Segment QT also happens to be the radius of circle Q, which therefore has a radius of 2. Line segment PT, being the diameter of circle Q, has a length of 4. Segment PT also happens to be the radius of circle P, which therefore has a radius of 4. The question is asking us what fraction of circle P is shaded. The answer will be (shaded area) ÷ (area of circle P). The area of circle P is $\pi(4)^2$, which equals 16π. The shaded area is just the area of circle Q (i.e. $\pi\,(2)^2$, which equals 4π) minus the area of circle R (i.e. $\pi\,(1)^2$, which equals π).

Therefore, the answer to our question is

(4π –π) / 16π = 3/16

25.

Consider the *RT* = *D* formula (rate × time = distance).

D, the total distance traveled by the car, will be equal to *C*, the circumference of each tire, times *N*, the number of 360° rotations made by each tire. Thus, we can rewrite our formula as *RT* = *CN*.

The question is asking us how many 360° rotations each tire makes in 10 minutes. In other words, it is asking us for the value of *N* when *T* = 10. We can rewrite our equation thus:

R(10) = *CN*

N = *R*(10)/*C*

Clearly, in order to answer the question “what is *N*” we need to know both *R* and *C*.

(1) INSUFFICIENT: This only gives us *R*, the speed of the car.

(2) INSUFFICIENT: This gives us the radius of the tire, which enables us to find *C*, the circumference of the tire. Knowing *C* is not, however, sufficient to figure out *N*.

(1) AND (2) SUFFICIENT: Now that we know both *R* and *C*, we can figure out what *N* is from the equation $N = R(10)/C$.

The correct answer is C.

26.

If we know the ratio of the radii of two circles, we know the ratio of their areas. Area is based on the square of the radius [A = π (radius)^2]. If the ratio of the radii of two circles is 2:1, the ratio of their areas is 2^2: 1^2 or 4:1.

In this question, the radius of the larger circular sign is twice that of the smaller circular sign; therefore the larger sign's area is four times that of the smaller sign and would require 4 times as much paint.

The correct answer is D.

27.

First, we need the area of the smallest quarter-circle. The smallest quarter-circle will have a radius of (x - 4) and thus an area of $\frac{(x-4)^2\pi}{4}$ (we must divide by 4 because it is a quarter-circle).

Now, we need the area of the middle band. We can find this by subtracting the area of the second quarter-circle from that of the third: $\frac{(x-2)^2\pi}{4} - \frac{(x-3)^2\pi}{4}$.

Now, we need the area of the outer band. We can find this by subtracting the area of the second-largest quarter-circle from that of the largest: $\frac{(x)^2\pi}{4} - \frac{(x-1)^2\pi}{4}$.

Finally, we need to add all these areas:

$$\left(\frac{(x)^2\pi}{4} - \frac{(x-1)^2\pi}{4}\right) + \left(\frac{(x-2)^2\pi}{4} - \frac{(x-3)^2\pi}{4}\right) + \frac{(x-4)^2\pi}{4} \rightarrow$$

$$\frac{\pi\left(\left(x^2 - \left(x^2 - 2x + 1\right)\right) + \left(\left(x^2 - 4x + 4\right) - \left(x^2 - 6x + 9\right)\right) + x^2 - 8x + 16\right)}{4} \rightarrow$$

$$\frac{\pi\left(x^2 - x^2 + 2x - 1 + x^2 - 4x + 4 - x^2 + 6x - 9 + x^2 - 8x + 16\right)}{4} \rightarrow$$

$$\frac{\left(x^2 - 4x + 10\right)\pi}{4}$$

Note that we factored pi out of every term and that you must remember to distribute the minus signs as if they were -1.

The correct answer is A.

28.

The probability of the dart landing outside the square can be expressed as follows:

$$P = \frac{\text{area of circle} - \text{area of square}}{\text{area of circle}}$$

To find the area of the circle, we need to first determine the radius. Since arc ADC has a length of $\pi\left(\sqrt{x}\right)$, we know that the circumference of the circle must be double this length or $2\pi\left(\sqrt{x}\right)$. Using the formula for circumference, we can solve for the radius:

$$2\pi r = 2\pi\left(\sqrt{x}\right)$$

$$r = \sqrt{x}$$

Therefore, the area of the circle $= \pi r^2 = \pi\left(\sqrt{x}\right)^2 = \pi(x)$.

To find the area of the square, we can use the fact that the diagonal of the square is equal to twice the radius of the circle. Thus, the diagonal of the square is $2\sqrt{x}$. Since the diagonal of the square represents the hypotenuse of a 45-45-90 triangle, we use the side ratios of this special triangle $(1:1:\sqrt{2})$ to determine that each side of the square measures $\frac{2\sqrt{x}}{\sqrt{2}} = \sqrt{2x}$. Thus, the area of the square $= (\sqrt{2x})^2 = 2x$.

Now we can calculate the total probability that the dart will land inside the circle but outside the square:

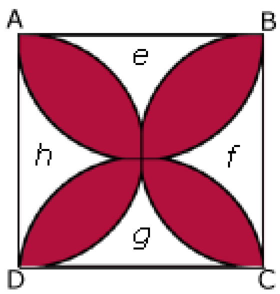
$$P = \frac{\text{area of circle} - \text{area of square}}{\text{area of circle}} = \frac{\pi(x) - 2x}{\pi(x)} = 1 - \frac{2}{\pi}$$

Interestingly, the probability does not depend on x at all, meaning that this probability holds true for any square inscribed in a circle (regardless of size).

29.

Instead of trying to find the area of the red shaded region directly by somehow measuring the area of the "cloverleaf," it is easier to find the area of the unshaded portions and subtract that from the area of the square as a whole.

Let's call the four uncolored portions *e*, *f*, *g*, and *h*, respectively.



To find the combined area of *h* and *f*, we need to take the area of the square and subtract the combined area of the semicircles formed by arcs AB and DC.

To make the problem less abstract, it helps to pick a number for the variable *x*. Let's say each side of the square has a length of 8. Therefore, the area of the square is 64.

Since the square has a side of length 8, the diameter of each semicircle is 8, which means that the radius is 4. So the area of each semicircle is $\frac{\pi r^2}{2} = \frac{16\pi}{2} = 8\pi$.

If we take the area of the square and subtract the combined area of the two semicircles formed by arcs AB and DC, we get $64 - 2(8\pi) = 64 - 16\pi$. This represents the combined area of *h* and *f*. The combined area of *e* and *g* is the same as that of *h* and *f*, so altogether the four uncolored portions have an area of $2(64 - 16\pi) = 128 - 32\pi$.

If we take the area of the square and subtract out the area of the uncolored regions, we are left with the area of the colored red region:

$$64 - (128 - 32\pi) \rightarrow 64 - 128 + 32\pi \rightarrow -64 + 32\pi \rightarrow 32\pi - 64$$

Now we need to plug in 8 for *x* in each answer choice. The one that produces the expression $32\pi - 64$ is the correct answer. The only one to do so is choice (B).

(Note: One other way to think about the problem is to recognize that the combined area of the 4 semicircles (AB, AD, BC, BD) minus the area of the square is equal to the area of the red shaded region. Then one can logically proceed from this by picking a value for *x* in a manner similar to the procedure explained above.)

30.

Triangle *DBC* is inscribed in a semicircle (that is, the hypotenuse CD is a diameter of the circle). Therefore, angle *DBC* must be a right angle and triangle *DBC* must be a right triangle.

(1) SUFFICIENT: If the length of *CD* is twice that of *BD*, then the ratio of the length of *BD* to the length of the hypotenuse *CD* is 1 : 2. Knowing that the side ratios of a 30-60-90 triangle are 1 : $\sqrt{3}$: 2, where 1 represents the short leg, $\sqrt{3}$ represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle *DBC* is a 30-60-90 triangle. Since side *BD* is the short leg, angle *x*, the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle *DBC* is inscribed in a semicircle, it must be a right triangle. So, angle *DBC* is 90 degrees. If *y* = 60, *x* = 180 – 90 – 60 = 30.

The correct answer is D

31.

Given that line *CD* is parallel to the diameter, we know that angle *DCB* and angle *CBA* are equal. Thus *x* = 30°.

First, let's calculate the length of arc *CAE*. Since arc *CAE* corresponds to an inscribed angle of 60° (2*x* = 2×30° = 60°), it must correspond to a central angle of 120° which is 1/3 of the 360° of the circle. Thus we can take 1/3 of the circumference to give us the arc length *CAE*. The circumference is given as $2\pi r$, where *r* is the radius. Thus the circumference equals 10π and arc length *CAE* equals $(10/3)\pi$.

Now we need to calculate the length of *CB* and *BE*. Since they have the same angle measure, these lengths are equal so we can just find one length and double it. Let us find the length of *CB*. If we draw a line from *A* to *C* we have a right traingle because any inscribed triangle that includes the diameter is a right triangle. Also, we know that *x* = 30° so we have a 30-60-90 triangle. The proportions of the length of the sides of a 30-60-90 triangle are 1-$\sqrt{3}$-2 for the side opposite each respective angle. We know the hypotenuse is the diameter which is 2*r = 10. So length AC must equal 5 and length CB must equal 5$\sqrt{3}$.

Putting this all together gives us (10/3)π + 2 × 5$\sqrt{3}$ = (10/3)π + 10$\sqrt{3}$

The correct answer is D.

Topic 5: Polygons

32.

We are told that the smallest angle measures 136 degrees--this is the first term in the consecutive set. If the polygon has S sides, then the largest angle--the last term in the consecutive set--will be (S - 1) more than 136 degrees.

The sum of consecutive integers = $(\text{Average Term}) \times (\#\text{ of Terms}) = \frac{\text{First} + \text{Last}}{2} \times (\#\text{ of Terms})$.

Given that there are S terms in the set, we can plug in for the first and last term as follows:

$\frac{(136) + (136 + (S-1))}{2} \times (S)$ = sum of the angles in the polygon.

We also know that the sum of the angles in a polygon = $180(S-2)$ where S represents the number of sides.

Therefore: $180(S-2) = \frac{(136 + 136 + (S-1))}{2} \times (S)$. We can solve for S by cross-multiplying and simplifying as follows:

$$2(180)(S-2) = (272 + (S-1)) \times S$$
$$360S - 720 = (271 + S) \times S$$
$$360S - 720 = 271S + S^2$$
$$S^2 - 89S + 720 = 0$$

A look at the answer choices tells you to try (S - 8), (S - 9), or (S - 10) in factoring.

As it turns out $(S-9)(S-80) = 0$, which means S can be 9 or 80. However S cannot be 80 because this creates a polygon with angles greater than 180.

Therefore S equals 9; there are 9 sides in the polygon.

33.

The relationship between the number of sides in a polygon and the sum of the interior angles is given by 180(n – 2) = (sum of interior angles), where n is the number of sides. Thus, if we know the sum of the interior angles, we can determine the number of sides. We can rephrase the question as follows: "What is the sum of the interior angles of Polygon X?"

(1) SUFFICIENT: Using the relationship $180(n-2)$ = (sum of interior angles), we could calculate the sum of the interior angles for all the polygons that have fewer than 9 sides. Just the first two are shown below; it would take too long to calculate all of the possibilities:

polygon with 3 sides: $180(3-2)$ or $180 \times 1 = 180$
polygon with 4 sides: $180(4-2)$ or $180 \times 2 = 360$

Notice that each interior angle sum is a multiple of 180. Statement 1 tells us that the sum of the interior angles is divisible by 16. We can see from the above that each possible sum will consist of 180 multiplied by some integer.

The prime factorization of 180 is ($\underline{2} \times \underline{2} \times 3 \times 3 \times 5$). The prime factorization of 16 is ($\underline{2} \times \underline{2} \times 2 \times 2$). Therefore, two of the $\underline{2}$'s that make up 16 can come from the 180, but the other two 2's will have to come from the integer that is multiplied by 180. Therefore, the difference $(n-2)$ must be a multiple of 2×2, or 4. Our possibilities for $(n-2)$ are:

3 sides: $180(3-2)$ or 180×1
4 sides: $180(4-2)$ or 180×2
5 sides: $180(5-2)$ or 180×3
6 sides: $180(6-2)$ or 180×4
7 sides: $180(7-2)$ or 180×5
8 sides: $180(8-2)$ or 180×6

Only the polygon with 6 sides has a difference $(n-2)$ that is a multiple of 4.

(2) INSUFFICIENT: Statement (2) tells us that the sum of the interior angles of Polygon X is divisible by 15. Therefore, the prime factorization of the sum of the interior angles will include 3×5. Following the same procedure as above, we realize that both 3 and 5 are included in the prime factorization of 180. As a result, every one of the possibilities can be divided by 15 regardless of the number of sides.

The correct answer is A.

34.

Shaded area = Area of the hexagon – (area of circle O) – (portion of circles A, B, C, D, E, F that is in the hexagon)

With a perimeter of 36, the hexagon has a side that measures 6. The regular hexagon is comprised of six identical equilateral triangles, each with a side measuring 6. We can find the area of the hexagon by finding the area of the equilateral triangles.

The height of an equilateral triangle splits the triangle into two 30-60-90 triangles (Each 30-60-90 triangle has sides in the ratio of 1: $\sqrt{3}$: 2). Because of this, the area for an equilateral triangle can be expressed in terms of one side. If we call the side of the equilateral triangle, s, the height must be $(s\sqrt{3}) / 2$ (using the 30-60-90 relationships).

The area of a triangle = 1/2 × base × height, so the area of an equilateral triangle can be expressed as: $1/2 \times s \times (s\sqrt{3}) / 2 = 1/2 \times 6 \times (3\sqrt{3}) = 9\sqrt{3}$.

Area of hexagon ABCDEF = $6 \times 9\sqrt{3} = 54\sqrt{3}$.

For circles A, B, C, D, E, and F to have centers on the vertices of the hexagon and to be tangent to one another, the circles must be the same size. Their radii must be equal to half of the side of the hexagon, 3. For circle O to be tangent to the other six circles, it too must have a radius of 3.

Area of circle O = $\pi r^2 = 9\pi$.

To find the portion of circles A, B, C, D, E, and F that is inside the hexagon, we must consider the angles of the regular hexagon. A regular hexagon has external angles of 360/6 = 60°, so it has internal angles of 180 – 60 = 120°. This means that each circle has 120/360 or 1/3 of its area inside the hexagon.

The area of circles A, B, C, D, E, and F inside the hexagon = 1/3(9π) × 6 circles = 18π.
Thus, the shaded area = $54\sqrt{3} - 9\pi - 18\pi = 54\sqrt{3} - 27\pi$.

The correct answer is E.

General Solids (Cube, Box, Sphere)

35.

The question stem tells us that the four spheres and three cubes are arranged in order of increasing volume, with no two solids of the same type adjacent in the lineup. This allows only one possible arrangement: sphere, cube, sphere, cube, sphere, cube, sphere.

Then we are told that the ratio of one solid to the next in line is constant. This means that to find the volume of any solid after the first, one must multiply the volume of the previous solid by a constant value. If, for example, the volume of the smallest sphere were 2, the volume of the first cube (the next solid in line) would be 2*x*, where *x* is the constant. The volume of the second sphere (the third solid in line) would be 2(*x*)(*x*) and the volume of the second cube (the fourth solid in line) would then be 2(*x*)(*x*)(*x*), and so on. By the time we got to the largest sphere, the volume would be $2x^6$.

We are not given the value of the constant, but are told that the radius of the smallest sphere is 1/4 that of the largest. We can use this information to determine the value of the constant. First, if the radius of the smallest sphere is *r*, then the radius of the largest sphere must be 4*r*. So if the volume of the smallest sphere is $\frac{4}{3}\pi r^3$, then the volume of the largest sphere must be $\frac{4}{3}\pi(4r)^3$ or $\frac{4}{3}\pi 64r^3$. So the volume of the largest sphere is 64 times larger than that of the smallest.

Using the information about the constant from the question stem, we can set up and simplify the following equation:

$$\frac{4}{3}\pi 64r^3 = \frac{4}{3}\pi r^3 x^6 \rightarrow$$
$$64 = x^6 \rightarrow$$
$$2 = x$$

Therefore, the value of the constant is 2. This means that the volume of each successive solid is twice that of the preceding solid. We are ready to look at the statements.

Statement (1) tells us that the volume of the smallest cube is 72π. This means that the volume of the smallest sphere (the immediately preceding solid) must be half, or 36π. If we have the volume of the smallest sphere, we can find the radius of the smallest sphere:

$$36\pi = \frac{4}{3}\pi r^3 \rightarrow$$
$$\left(\frac{3}{4}\right)36\pi = \left(\frac{3}{4}\right)\frac{4}{3}\pi r^3 \rightarrow$$
$$27\pi = \pi r^3 \rightarrow$$
$$27 = r^3 \rightarrow$$
$$3 = r$$

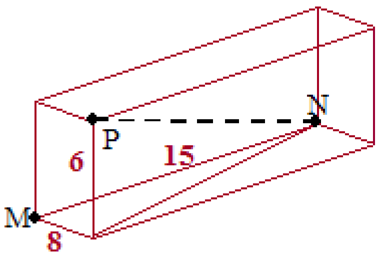
Therefore, the radius of the smallest sphere is 3. Statement (1) is sufficient.

Statement (2) tells us that the volume of the second largest sphere is 576π. Applying the same logic that we used to evaluate statement (1), we can find the radius of the smallest sphere by dividing 576π by 2 four times (because there are four solids smaller than the second largest sphere) until we get to the volume of the smallest sphere: 36π. At this point we can solve for the radius of the smallest sphere as we did in our analysis of statement (1) above. Statement (2) is sufficient.

The correct answer is D: either statement alone is sufficient to answer the question.

36.

The diagram represents the situation described in the problem.



The plane (P) is at an altitude of 6 miles and is flying due north towards The Airport (M). Notice that if the plane flies in a straight line toward The Airport it would be flying along the diagonal of a right triangle with sides of length 6 and 8. Thus, taking a direct approach, the plane would fly exactly 10 miles towards The Airport (a 6-8-10 right triangle).

However, the control tower instructs the plane to fly toward a new airport (N), which lies 15 miles due east of The Airport. If the plane flies in a straight line towards the new airport, it will be flying along the diagonal (the dotted black line) of the rectangular solid. To determine this length, we first need to determine the diagonal of the bottom face of the solid. Using the Pythagorean theorem, we can determine that the bottom face right triangle with sides of lengths 8 and 15 must have a diagonal of 17 miles (an 8-15-17 right triangle).

Now the dotted line diagonal of the solid can be calculated as follows:

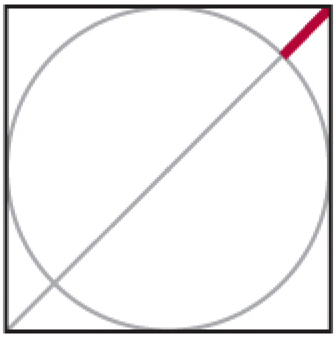
$$x^2 = 6^2 + 17^2$$
$$x^2 = 325$$
$$x = 5\sqrt{13}$$

In order to arrive at the new airport at 8:00 am, the pilot's flying time must remain 3 minutes, or 1/20 of an hour. The flight distance, however, has increased from 10 miles to $5\sqrt{13}$ miles. The plane's rate must increase accordingly. Using the formula that rate = distance ÷ time, we can calculate the rate increase as follows:

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} = \frac{(5\sqrt{13} - 10)\text{ miles}}{1/20\text{ hr}} = 100\sqrt{13} - 200\text{ miles/hr}$$

37.

The shortest distance from a vertex of the cube to the sphere would be ½ the length of the diagonal of the cube minus the radius of the sphere. To understand why, think of the parallel situation in two dimensions. In the diagram of the circle inscribed in the square to the right, the shortest possible distance from one of the vertices of the square to the circle would be ½ the diagonal of the square minus the radius of the circle.



The diagonal of a cube of side x is $x\sqrt{3}$. This can be found by applying the Pythagorean Theorem twice (first to find the diagonal of a face of the cube, $x\sqrt{2}$, and then to find the diagonal through the center, $x\sqrt{3}$). Like the sides of the circle in the diagram above, the sides of a sphere inscribed in a cube will touch the sides of the cube. Therefore, a sphere inscribed in a cube will have a radius equal to half the length of the side of that cube.

Diagonal of the cube = $x\sqrt{3}$ = $10\sqrt{3}$
Radius of the sphere = 5
½ diagonal of the cube – radius of the sphere = $5\sqrt{3} - 5 = 5(\sqrt{3} - 1)$

The correct answer is D.

38.

Let x be the length of an edge of the cube. We can find the length of *BC* by first finding the length of *CD*. *CD* must be $x\sqrt{2}$ since it is the hypotenuse of a 45-45-90 triangle with legs of length x.

Using the Pythagorean theorem, BC can be calculated: $BC = \sqrt{x^2 + (x\sqrt{2})^2} = x\sqrt{3}$

$AB = CD = x\sqrt{2}$, so $BC - AB = x\sqrt{3} - x\sqrt{2}$.
If we factor this expression and simplify, $x(\sqrt{3} - \sqrt{2}) \approx x(1.7 - 1.4) \approx 0.3x$.

Since $BC - AB \approx 0.3x$ and $AC = x$, the difference between BC and AB is equal to approximately 30% of AC.

The correct answer is C.

Cylinder

39.

The old volume is $\pi R^2 H$. Let's look at each answer choice to see which one is farthest away from twice this volume:

(A) a 100% increase to R and a 50% decrease to H:
The new volume = $\pi (2R)^2(.5H) = 2\pi R^2 H$ = exactly twice the original volume.

(B) a 30% decrease to R and a 300% increase to H:
The new volume = $\pi (.7R)^2(4H) = (.49)(4)\ \pi R^2 H \approx 2\pi R^2 H$ = approximately twice the original volume.

(C) a 10% decrease to R and a 150% increase to H:
The new volume = $\pi(.9R)^2(2.5H) = (.81)(2.5)\pi R^2 H \approx 2\pi R^2 H$ = approximately twice the original volume.

(D) a 40% increase to R and no change to H:
The new volume = $\pi(1.4R)^2(H) = (1.96)\pi R^2 H \approx 2\pi R^2 H$ = approximately twice the original volume.

(E) a 50% increase to R and a 20% decrease to H:
The new volume = $\pi(1.5R)^2(.8H) = (2.25)(.8)\pi R^2 H = 1.8\pi R^2 H$. This is the farthest away from twice the original volume.

The correct answer is E.

40.

To solve this problem we need to (a) find the volume of the tire and then (b) solve a rate problem to determine how long it will take to inflate the tire.

To find the volume of just the tire, we can find the volume of the entire object and then subtract out the volume of the hub. In order to do this, we will first need to determine the radius of the hub.

If the radius of the hub is r, then its area equals πr^2.

The area of the entire object is then $\pi(r+6)^2$.

This means that the area of just the tire equals $\pi(r+6)^2 - \pi r^2$.

The problem also tells us that the ratio of the area of the tire to the area of the entire object is 1/3. We can use this information to set up the following equation:

$$\frac{\text{area of hub}}{\text{area of tire}} = \frac{1}{3} = \frac{\pi r^2}{\pi (r+6)^2 - \pi r^2}$$

Now, we can solve for *r* as follows:

$$\frac{1}{3} = \frac{\pi r^2}{\pi (r+6)^2 - \pi r^2}$$
$$\frac{1}{3} = \frac{r^2}{(r+6)^2 - r^2}$$
$$3r^2 = (r^2 + 12r + 36) - r^2$$
$$3r^2 = 12r + 36$$
$$r^2 = 4r + 36$$
$$r^2 - 4r - 12 = 0$$
$$(r-6)(r+2) = 0$$

Thus, *r* is either 6 or –2. Since the radius must be positive, we know that *r* = 6.

To calculate volume, we simply multiply the area by a third dimension. This third dimension is the thickness of the tire (we'll call this *h)*, which is defined in the problem as 3 inches when the tire is fully inflated. Now, we can determine the volume of the tire using the following:

Volume of Entire Object – Volume of Hub = Volume of Tire

$$h\pi (r+6)^2 - h\pi r^2 = \text{Volume of Tire}$$

This can be simplified as followed:

$$= h\pi (6+6)^2 - h\pi 6^2$$
$$= 3\pi 12^2 - 3\pi 36$$
$$= 432\pi - 108\pi$$
$$= 324\pi$$

Thus, the volume of the tire is 324π cubic inches. Using the formula *work = rate × time,* we can set the volume of the tire as the total work to be done and use the rate given in the problem to determine the total time required to inflate the tire.

$$\text{Work} = \text{Rate} \times \text{Time}$$
$$324\pi in^3 = \frac{4\pi in^3}{\text{second}} \times t$$
$$t = 81 \text{ seconds}$$

The correct answer is D.

41.

The volume for a cylinder can be calculated by multiplying the area of the base times the height. The base is a circle with an area of r2, where r is the radius of the circle. Thus the volume of a cylinder is r2 × h, where h represents the height of the figure.

(1) SUFFICIENT: If we call the radius of the smaller cylinder r, and the height of the smaller cylinder h, the volume of the smaller cylinder would be r2h. If the radius of the larger cylinder is twice that of the smaller one, as is the height, the volume of the larger cylinder would be (2r)2(2h) = 8 r2h. The volume of the larger cylinder is eight times larger than that of the smaller one. If the contents of the smaller silo, which is full, are poured into the larger one, the larger one will be 1/8 full.

(2) INSUFFICENT: This question is about proportions, and this statement tells us nothing about volume of the smaller silo relative to the larger one.

The correct answer is A

42.

One of the cylinders has a height of 6 and a base circumference of 10; the other has a height of 10 and a base circumference of 6.

The cylinder with a height of 6 and a base circumference of 10 has a radius of $(5/\pi)$. Its volume is equal to $\pi r^2 h$, or $\pi(5/\pi)^2(6)$ or $150/\pi$.

The cylinder with a height of 10 and a base circumference of 6, however, has a radius of $(3/\pi)$. Its volume is equal to $\pi r^2 h$, or $\pi(3/\pi)^2(10)$ or $90/\pi$.

We can see that the volume of the cylinder with a height of 6 is $60/\pi$ inches greater than that of the cylinder with a height of 10. It makes sense in this case that the cylinder with the greater radius will have the greater volume since the radius is squared in the volume formula.

The correct answer is B.

TOPIC 6 – CO-ORDINATE GEOMETRY

1.

The two intersections: (0,4) and (y, 0)   So, 4 * y/ 2 = 12 => y= 6

Slope is positive => y is below the x-axis => y = -6

2.

Each side of the square must have a length of 10. If each side were to be 6, 7, 8, or most other numbers, there could only be four possible squares drawn, because each side, in order to have integer coordinates, would have to be drawn on the x- or y-axis. What makes a length of 10 different is that it could be the hyptoneuse of a pythagorean triple, meaning the vertices could have integer coordinates without lying on the x- or y-axis.

For example, a square could be drawn with the coordinates (0,0), (6,8), (-2, 14) and (-8, 6). (It is tedious and unnecessary to figure out all four coordinates for each square).

If we lable the square *abcd*, with *a* at the origin and the letters representing points in a clockwise direction, we can get the number of possible squares by figuring out the number of unique ways *ab* can be drawn.

a has coordinates (0,0) and *b* could have coordinates:

(-10,0)
(-8,6)
(6,8)
(0,10)
(6,8)
(8,6)
(10,0)
(8, -6)
(6, -8)
(0, 10)
(-6, -8)
(-8, -6)

There are 12 different ways to draw *ab*, and so there are 12 ways to draw *abcd*.

The correct answer is E.

3.

At the point where a curve intercepts the x-axis (i.e. the *x* intercept), the y value is equal to 0. If we plug $y = 0$ in the equation of the curve, we get $0 = (x - p)(x - q)$. This product would only be zero when x is equal to p or q. The question is asking us if (2, 0) is an x-intercept, so it is really asking us if either p or q is equal to 2.

(1) INSUFFICIENT: We can't find the value of *p* or *q* from this equation.

(2) INSUFFICIENT: We can't find the value of *p* or *q* from this equation.

(1) AND (2) SUFFICIENT: Together we have enough information to see if either *p* or *q* is equal to 2. To solve the two simultaneous equations, we can plug the *p*-value from the first equation, $p = -8/q$, into the second equation, to come up with $-2 + 8/q = q$.
This simplifies to $q^2 + 2q - 8 = 0$, which can be factored $(q + 4)(q - 2) = 0$, so $q = 2, -4$.
If $q = 2$, $p = -4$ and if $q = -4$, $p = 2$. Either way either *p* or *q* is equal to 2.

The correct answer is C.

4.

First, we determine the slope of line *L* as follows:

$$L = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{p-2} = \frac{-1}{p-2} = \frac{1}{2-p}$$

If line *m* is perpendicular to line *L*, then its slope is the negative reciprocal of line *L*'s slope. (This is true for all perpendicular lines.) Thus:

If the slope of $L = \frac{a}{b}$, then the slope of $m = -\frac{b}{a}$.

Therefore, the slope of line *m* can be calculated using the slope of line *L* as follows:

$$m = -\left(\frac{1}{\frac{1}{2-p}}\right) = p - 2$$

This slope can be plugged into the slope-intercept equation of a line to form the equation of line *m* as follows:

$y = (p - 2)x + b$

(where $(p - 2)$ is the slope and *b* is the y-intercept)

This can be rewritten as $y = px - 2x + b$ or $2x + y = px + b$ as in answer choice A.

An alternative method: Plug in a value for *p*. For example, let's say that $p = 4$.

Thus, the slope of line $L = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{4-2} = \frac{-1}{2} = -\frac{1}{2}$

The slope of line *m* is the negative inverse of the slope of line *L*. Thus, the slope of line *m* is 2.

Therefore, the correct equation for line *m* is the answer choice that yields a slope of 2 when the value 4 is plugged in for the variable *p*.

(A) $2x + y = px + 7$ **yields** $y = 2x + 7$

(B) $2x + y = -px$ **yields** $y = -6x$

(C) $x + 2y = px + 7$ **yields** $y = (3/2)x + 7/2$

(D) $y - 7 = x \div (p - 2)$ **yields** $y = (1/2)x + 7$

(E) $2x + y = 7 - px$ **yields** $y = -6x + 7$

Only answer choice A yields a slope of 2.

Choice A is therefore the correct answer.

5.

The distance between any two points (x,y) and (x_2,y_2) in the coordinate plane is defined by the distance formula.

$$
\begin{aligned}
D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(2A + 4 - A)^2 + \left(\sqrt{2A+9} - 0\right)^2} \\
&= \sqrt{(A + 4)^2 + \left(\sqrt{2A+9} - 0\right)^2} \\
&= \sqrt{A^2 + 8A + 16 + 2A + 9} \\
&= \sqrt{A^2 + 10A + 25} \\
&= \sqrt{(A+5)^2} \\
&= A + 5
\end{aligned}
$$

Thus, the distance between point K and point G is A + 5.

Statement (1) tells us that:

$$
\begin{aligned}
&A - 5A - 6 = 0 \\
&(A - 6)(A + 1) = 0
\end{aligned}
$$

Thus A = 6 or A = –1.

Using this information, the distance between point K and point G is either 11 or 4. This is not sufficient to answer the question.

Statement (2) alone tells us that A > 2, which is not sufficient to answer the question.

When we combine both statements, we see that A must be 6, which means the distance between point K and point G is 11. This is a prime number and we are able to answer the question.

The correct answer is C.

6.

If we put the equation $3x + 4y = 8$ in the slope-intercept form ($y = mx + b$), we get:

$y = (-3/4)x + 2$

This means that m (the slope) = -3/4 and b (the y-intercept) = 2.

We can graph this line by going to the point (0, 2) and going to the right 4 and down 3 to the point (0 + 4, 2 - 3) or (4, -1).

If we connect these two points, (0, 2) and (4, -1), we see that the line passes through quadrants I, II and IV.

The correct answer is C.

7.

To determine in which quadrant the point (p, $p – q$) lies, we need to know the sign of p and the sign of $p – q$.

(1) SUFFICIENT: If (p, q) lies in quadrant IV, p is positive and q is negative. $p – q$ must be positive because a positive number minus a negative number is always positive [e.g. 2 – (-3) = 5].

(2) SUFFICIENT: If (q, -p) lies in quadrant III, q is negative and p is positive. (This is the same information that was provided in statement 1).

The correct answer is D.

8.

Point B is on line AC, two-thirds of the way between Point A and Point C. To find the coordinates of point B, it is helpful to imagine that you are a point traveling along line AC.

When you travel all the way from point A to point C, your x-coordinate changes 3 units (from $x = 0$ to $x = 3$). Two-thirds of the way there, at point B, your x-coordinate will have changed 2/3 of this amount, i.e. 2 units. The x-coordinate of B is therefore $x = 0 + 2 = 2$.

When you travel all the way from point A to point C, your y-coordinate changes 6 units (from $y = -3$ to y = 3). Two-thirds of the way there, at point B, your y-coordinate will have changed 2/3 of this amount, i.e. 4 units. The y-coordinate of B is therefore $y = -3 + 4 = 1$.

Thus, the coordinates of point B are (2,1).

The correct answer is C.

9.

First, rewrite the line $y = 4 - 2x$ as $y = -2x + 4$. The equation is now in the form $y = mx + b$ where m represents the slope and b represents the y-intercept. Thus, the slope of this line is -2.

By definition, if line F is the perpendicular bisector of line G, the slope of line F is the negative inverse of the slope of line G. Since we are told that the line $y = -2x + 4$ is the perpendicular bisector of line segment RP, line segment RP must have a slope of $1/2$ (which is the negative inverse of).

Now we know that the slope of the line containing segment RP is $1/2$ but we do not know its y-intercept. We can write the equation of this line as $y = \frac{1}{2}x + b$, where b represents the unknown y-intercept.

To solve for b, we can use the given information that the coordinates of point R are (4, 1). Since point R is on the line $y = \frac{1}{2}x + b$, we can plug 4 in for x and 1 in for y as follows:

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(4) + b$$

$$-1 = b$$

Now we have a complete equation for the line containing segment RP: $y = \frac{1}{2}x - 1$

We also have the equation of the perpendicular bisector of this line: $y = -2x + 4$. To determine the point M at which these two lines intersect, we can set these two equations to equal each other as follows:

$$\frac{1}{2}x - 1 = -2x + 4$$
$$\frac{5}{2}x = 5$$

$$x = 2$$

Thus, the intersection point M has x-coordinate 2. Using this value, we can find the y coordinate of point M:

$$y = -2x + 4$$
$$y = -2(2) + 4$$
$$y = 0$$

Thus the perpendicular bisector intersects line segment RP at point M, which has the coordinates (2, 0). Since point M is on the bisector of RP, point M represents the midpoint on line segment RP; this means that it is equidistant from point R and point P.

We know point R has an x-coordinate of 4. This is two units away from the x-coordinate of midpoint M, 2. Therefore the x-coordinate of point P must also be two units away from 2, which is 0.

We know point R has a y-coordinate of 1. This is one unit away from the y-coordinate of midpoint M, 0. Therefore, the y-coordinate of point P must also be one unit away from 0, which is –1.

The coordinates of point P are $(0, -1)$.

The correct answer is D.

10.

Because we are given two points, we can determine the equation of the line. First, we'll calculate the slope by using the formula $(y_2 - y_1) / (x_2 - x_1)$:

$$\frac{[0 - (-5)]}{(7 - 0)} = \frac{-5}{7}$$

Because we know the line passes through (0,5) we have our *y*-intercept which is 5.

Putting these two pieces of information into the slope-intercept equation gives us $y = (-5/7)x + 5$. Now all we have to do is plug in the *x*-coordinate of each of the answer choices and see which one gives us the *y*-coordinate.

(A) (-14, 10)
$y = -5/7(-14) + 5 = 15$; this does not match the given *y*-coordinate.

(B) (-7, 5)
$y = -5/7(-7) + 5 = 10$; this does not match the given *y*-coordinate.

(C) (12, -4)
$y = -5/7(12) + 5 = -60/7 + 5$, which will not equal an integer; this does not match the given *y*-coordinate.

(D) (-14, -5)
$y = -5/7(-14) + 5 = -5$; this matches the given *y*-coordinate so we have found our answer.

(E) (21, -9)
$y = -5/7(21) + 5 = -15/7 + 5$, which will not equal an integer; this does not match the given *y*-coordinate. (Note that you do not have to test this answer choice if you've already discovered that D works.)

The correct answer is D

TOPIC 7 – PERMUTATION & COMBINATION

8.

In order to answer this question, we need to be able to determine the value of x. Thus, this question can be rephrased: What is x?

(1) SUFFICIENT: In analyzing statement (1), consider how many individuals would have to be available to create 126 different 5 person teams. We don't actually have to figure this out as long as we know that we *could* figure this out. Certainly by testing some values, we *could* figure this out. It turns out that if there are 9 available individuals, then we could create exactly 126 different 5-person teams (since 9! ÷ [(5!)(4!)] = 126). This value (9) represents $x + 2$. Thus x would equal 7.

(2) SUFFICIENT: The same logic applies to statement (2). Consider how many individuals would have to be available to create 56 different 3-person teams. Again, we don't actually have to figure this out as long as we know that we *could* figure this out. It turns out that if there are 8 available individuals, then we could create exactly 56 different 3-person teams (since 8! ÷ [(5!)(3!)] = 56). This value (8) represents $x + 1$. Thus x would equal 7. Statement (2) alone IS sufficient.

The correct answer is D.

9.

In order to determine how many 10-flavor combinations Sammy can create, we simply need to know how many different flavors Sammy now has. If Sammy had x flavors to start with and then threw out y flavors, he now has $x - y$ flavors. Therefore, we can rephrase this questions as: What is $x - y$? According to statement (1), if Sammy had $x - y - 2$ flavors, he could have made exactly 3,003 different 10-flavor bags. We could use the combination formula below to determine the value of $x - y - 2$, which is equal to n in the equation below:

$$\frac{n!}{10!(n-10)!} = 3{,}003$$

Solving this equation would require some time and more familiarity with factorials than is really necessary for the GMAT. However, keep in mind that you do not need to solve this equation; you merely need to be certain that the equation is solvable. (Note, if you begin testing values for n, you will soon find that $n = 15$.) Once we know the value of n, we can easily determine the value of $x - y$, which is simply 2 more than n. Thus, we know how many different flavors Sammy has, and could determine how many different 10-flavor combinations he could make. Statement (2) is tells us that $x = y + 17$. Subtracting y from both sides of the equation yields the equation $x - y = 17$. Thus, Sammy has 17 different flavors. This information is sufficient to determine the number of different 10-flavor combinations he could make.

The correct answer is D: Each statement ALONE is sufficient.

10.

The three-dice combinations fall into 3 categories of outcomes:

1) All three dice have the same number 2) Two of the dice have the same number with the third having a different number than the pair 3) All three dice have different numbers By calculating the number of combinations in each category, we can determine the total number of different possible outcomes by summing the number of possible outcomes in each category.

First, let's calculate how many combinations can be made if all 3 dice have the same number. Since there are only 6 numbers, there are only 6 ways for all three dice to have the same number (i.e., all 1's, or all 2's, etc.). Second, we can determine how many combinations can occur if only 2 of the dice have the same number. There are 6 different ways that 2 dice can be paired (i.e., two 1's, or two 2's or two 3's, etc.). For each given pair of 2 dice, the third die can be one of the five other numbers. (For example if two of the dice are 1's, then the third die must have one of the 5 other numbers: 2, 3, 4, 5, or 6.) Therefore there are 6 x 5 = 30 combinations of outcomes that involve 2 dice with the same number. Third, determine how many combinations can occur if all three dice have different numbers. Think of choosing three of the 6 items (each of the numbers) to fill three "slots." For the first slot, we can choose from 6 possible items. For the second slot, we can choose from the 5 remaining items. For the third slot, we can choose from the 4 remaining items. Hence, there are 6 x 5 x 4 = 120 ways to fill the three slots. However, we do not care about the order of the items, so permutations like {1,2,5}, {5, 2, 1}, {2, 5, 1}, {2, 1, 5}, {5, 1, 2}, and {1, 5, 2} are all considered to be the same result. There are 3! = 6 ways that each group of three numbers can be ordered, so we must divide 120 by 6 in order to obtain the number of combinations where order does not matter (every 1 combination has 6 equivalent permutations). Thus there are $120 \div 6 = 20$ combinations where all three dice have different numbers. The total number of combinations is the sum of those in each category or 6 + 30 + 20 = 56.

The correct answer is C.

11.

The first thing to recognize here is that this is a *permutation with restrictions* question. In such questions it is always easiest to tackle the restricted scenario(s) first. The restricted case here is when all of the men actually sit together in three adjacent seats. Restrictions can often be dealt with by considering the limited individuals as one unit. In this case we have four women (w_1, w_2, w_3, and w_4) and three men (m_1, m_2, and m_3). We can consider the men as one unit, since we can think of the 3 adjacent seats as simply 1 seat. If the men are one unit (m), we are really looking at seating 5 individuals (w_1, w_2, w_3, w_4, and m) in 5 seats. There are 5! ways of arranging 5 individuals in a row. This means that our group of three men is sitting in any of the "five" seats. Now, imagine that the one seat that holds the three men magically splits into three seats. How many different ways can the men arrange themselves in those three seats? 3!. To calculate the total number of ways that the men and women can be arranged in 7 seats such that the men **ARE** sitting together, we must multiply these two values: 5!3!. However this problem asks for the number of ways the theatre-goers can be seated such that the men are **NOT** seated three in a row. Logically, this must be equivalent to the following: (Total number of all seat arrangements) – (Number of arrangements with 3 men in a row). The total number of all seat arrangements is simply 7! so the final calculation is 7! – 5!3!.

The correct answer is C.

12.

It is important to first note that our point of reference in this question is all the possible subcommittees that include Michael. We are asked to find what percent of these subcommittees also include Anthony. Let's first find out how many possible subcommittees there are that must include Michael. If Michael must be on each of the three-person committees that we are considering, we are essentially choosing people to fill the two remaining spots of the committee. Therefore, the number of possible committees can be found by considering the number of different two-people groups that can be formed from a pool of 5 candidates (not 6 since Michael was already chosen). Using the anagram method to solve this combinations question, we assign 5 letters to the various board members in the first row. In the second row, two of the board members get assigned a Y to signify that they were chosen and the remaining 3 get an N, to signify that they were not chosen:

A	B	C	D	E
Y	Y	N	N	N

The number of different combinations of two-person committees from a group of 5 board members would be the number of possible anagrams that could be formed from the word YYNNN = 5! / (3!2!) = 10. Therefore there are 10 possible committees that include Michael. Out of these 10 possible committees, of how many will Anthony also be a member? If we assume that Anthony and Michael must be a member of the three-person committee, there is only one remaining place to fill. Since there are four other board members, there are four possible three-person committees with both Anthony and Michael. Of the 10 committees that include Michael, 4/10 or 40% also include Anthony.

The correct answer is C.

As an alternate method, imagine splitting the original six-person board into two equal groups of three. Michael is automatically in one of those groups of three. Now, Anthony could occupy any one of the other 5 positions -- the 2 on Michael's committee and the 3 on the other committee. Since Anthony has an equal chance of winding up in any of those positions, his chance of landing on Michael's committee is 2 out of 5, or 2/5 = 40%. Since that probability must correspond to the ratio of committees asked for in the problem, the answer is achieved.

Answer choice C is correct.

13.

The easiest way to solve this question is to consider the restrictions separately. Let’s start by considering the restriction that one of the parents must drive, temporarily ignoring the restriction that the two sisters won't sit next to each other. This means that…

2 people (mother or father) could sit in the driver’s seat
4 people (remaining parent or one of the children) could sit in the front passenger seat
3 people 0could sit in the first back seat
2 people could sit in the second back seat

1 person could sit in the remaining back seat

The total number of possible seating arrangements would be the product of these various possibilities: 2 × 4 × 3 × 2 × 1 = 48. We must subtract from these 48 possible seating arrangements the number of seating arrangements in which the daughters are sitting together. The only way for the daughters to sit next to each other is if they are both sitting in the back. This means that…

2 people (mother or father) could sit in the driver's seat
2 people (remaining parent or son) could sit in the front passenger seat

Now for the back three seats we will do something a little different. The back three seats must contain the two daughters and the remaining person (son or parent). To find out the number of arrangements in which the daughters are sitting adjacent, let's consider the two daughters as one unit. The remaining person (son or parent) is the other unit. Now, instead of three seats to fill, we only have two "seats," or units, to fill.

There are 2 × 1 = 2 ways to seat these two units.

However, the daughter-daughter unit could be d_1d_2 or d_2d_1

We must consider both of these possibilities so we multiply the 2 by 2! for a total of 4 seating possibilities in the back. We could also have manually counted these possibilities: d_1d_2X, d_2d_1X, Xd_1d_2, Xd_2d_1

Now we must multiply these 4 back seat scenarios by the front seat scenarios we calculated earlier:
(2 × 2) × 4 = 16
front   back

If we subtract these 16 "daughters-sitting-adjacent" scenarios from the total number of "parent-driving" scenarios, we get: 48 – 16 = 32

The correct answer is B.

14.

Ignoring Frankie's requirement for a moment, observe that the six mobsters can be arranged 6! or 6 x 5 x 4 x 3 x 2 x 1 = 720 different ways in the concession stand line. In each of those 720 arrangements, Frankie must be either ahead of or behind Joey. Logically, since the combinations favor neither Frankie nor Joey, each would be behind the other in precisely half of the arrangements. Therefore, in order to satisfy Frankie's requirement, the six mobsters could be arranged in 720/2 = 360 different ways.

The correct answer is D.

15.

We can imagine that the admissions committee will choose a "team" of students to receive scholarships. However, because there are three different levels of scholarships, it will not suffice to simply count the number of possible "scholarship teams." In other words, the committee must also place the “scholarship team” members according to scholarship level. So, order matters. If we knew the number of scholarships to be granted at each of the three scholarship levels, we could use the anagram method to count the number of ways the scholarships could be doled out among the 10 applicants. To show that this is true, let’s invent a hypothetical case for which there are 3 scholarships to be granted at each of the three levels. If we assign letters to the ten applicants from A to J, and if T represents a $10,000 scholarship, F represents a $5,000 scholarship, O represents a $1,000 scholarship, and N represents no scholarship, then the anagram grid would look like this:

A	B	C	D	E	F	G	H	I	J
T	T	T	F	F	F	O	O	O	N

Our “word” is TTTFFFOOON. To calculate the number of different “spellings” of this “word,” we use the following shortcut:

$$\frac{10!}{(3!)(3!)(3!)(1!)}$$

The 10 represents the 10 total applicants, the 3’s represent the 3 T’s, 3 F’s, and 3 O’s, and the 1 represents the 1 N. Simplifying this expression would yield the number of ways to distribute the scholarships among the 10 applicants. So, knowing the number of scholarships to be granted at each of the three scholarship levels allows us to calculate the answer to the question. Therefore, the rephrased question is: "How many scholarships are to be granted at each of the three scholarship levels?"

(1) INSUFFICIENT: While this tells us the total number of scholarships to be granted, we still don’t know how many from each level will be granted.

(2) INSUFFICIENT: While this tells us that the number of scholarships from each level will be equal, we still don’t know how many from each level will be granted.

(1) AND (2) SUFFICIENT: If there are 6 total scholarships to be granted and the same number from each level will be granted, there will be two $10,000 scholarships, two $5,000 scholarships, and two $1,000 scholarships granted.

The correct answer is C.

TOPIC 8 – PROBABILITY

1.

For probability, we always want to find the number of ways the requested event could happen and divide it by the total number of ways that any event could happen.

For this complicated problem, it is easiest to use combinatorics to find our two values. First, we find the total number of outcomes for the triathlon. There are 9 competitors; three will win medals and six will not. We can use the Combinatorics Grid, a counting method that allows us to determine the number of combinations without writing out every possible combination.

A	B	C	D	E	F	G	H	I
Y	Y	Y	N	N	N	N	N	N

Out of our 9 total places, the first three, A, B, and C, win medals, so we label these with a "Y." The final six places (D, E, F, G, H, and I) do not win medals, so we label these with an "N." We translate this into math: $9! / 3!6! = 84$. So our total possible number of combinations is 84. (Remember that ! means factorial; for example, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$.)

Note that although the problem seemed to make a point of differentiating the first, second, and third places, our question asks only whether the brothers will medal, not which place they will win. This is why we don't need to worry about labeling first, second, and third place distinctly.

Now, we need to determine the number of instances when at least two brothers win a medal. Practically speaking, this means we want to add the number of instances two brothers win to the number of instances three brothers win.

Let's start with all three brothers winning medals, where B represents a brother.

A	B	C	D	E	F	G	H	I
B	B	B	N	N	N	N	N	N

Since all the brothers win medals, we can ignore the part of the counting grid that includes those who don't win medals. We have $3! / 3! = 1$. That is, there is only one instance when all three brothers win medals.

Next, let's calculate the instances when exactly two brothers win medals.

A	B	C	D	E	F	G	H	I
B	B	Y	B'	N	N	N	N	N

Since brothers both win and don't win medals in this scenario, we need to consider both sides of the grid (i.e. the ABC side and the DEFGHI side). First, for the three who win medals, we have $3! / 2! = 3$. For the six who don't win medals, we have $6! / 5! = 6$. We multiply these two numbers to get our total number: $3 \times 6 = 18$. Another way to consider the instances of at least two brothers medaling would be to think of simple combinations with restrictions.

If you are choosing 3 people out of 9 to be winners, how many different ways are there to chose a specific set of 3 from the 9 (i.e. all the brothers)? Just one. Therefore, there is only one scenario of all three brothers medaling.
If you are choosing 3 people out of 9 to be winners, if 2 specific people of the 9 have to be a member of the winning group, how many possible groups are there? It is best to think of this as a problem of choosing 1 out of 7 (2 must be chosen). Choosing 1 out of 7 can be represented as 7! / 1!6! = 7. However, if 1 of the remaining 7 can not be a member of this group (in this case the 3rd brother) there are actually only 6 such scenarios. Since there are 3 different sets of exactly two brothers (B_1B_2, B_1B_3, B_2B_3), we would have to multiply this 6 by 3 to get 18 scenarios of only two brothers medaling.

The brothers win at least two medals in 18 + 1 = 19 circumstances. Our total number of circumstances is 84, so our probability is 19 / 84.

The correct answer is B.

2.

If set *S* is the set of all prime integers between 0 and 20 then:

S = {2, 3, 5, 7, 11, 13, 17, 19}

Let’s start by finding the probability that the product of the three numbers chosenis a number less than 31. To keep the product less than 31, the three numbers must be 2, 3 and 5. So, what is the probability that the three numbers chosen will be some combination of 2, 3, and 5?

Here’s the list all possible combinations of 2, 3, and 5:

case A: 2, 3, 5

case B: 2, 5, 3

case C: 3, 2, 5

case D: 3, 5, 2

case E: 5, 2, 3

case F: 5, 3, 2

This makes it easy to see that when 2 is chosen first, there are two possible combinations. The same is true when 3 and 5 are chosen first. The probability of drawing a 2, AND a 3, AND a 5 in case A is calculated as follows (remember, when calculating probabilities, AND means multiply):

case A: (1/8) x (1/7) x (1/6) = 1/336

The same holds for the rest of the cases.

case B: (1/8) x (1/7) x (1/6) = 1/336

case C: (1/8) x (1/7) x (1/6) = 1/336

case D: (1/8) x (1/7) x (1/6) = 1/336

case E: (1/8) x (1/7) x (1/6) = 1/336

case F: (1/8) x (1/7) x (1/6) = 1/336

So, a 2, 3, and 5 could be chosen according to case A, OR case B, OR, case C, etc. The total probability of getting a 2, 3, and 5, in any order, can be calculated as follows (remember, when calculating probabilities, OR means add):

(1/336) + (1/336) + (1/336) + (1/336) + (1/336) + (1/336) = 6/336

Now, let's calculate the probability that the sum of the three numbers is odd. In order to get an odd sum in this case, 2 must NOT be one of the numbers chosen. Using the rules of odds and evens, we can see that having a 2 would give the following scenario:

even + odd + odd = even

So, what is the probability that the three numbers chosen are all odd? We would need an odd AND another odd, AND another odd:

(7/8) x (6/7) x (5/6) = 210/336

The positive difference between the two probabilities is:

(210/336) – (6/336) = (204/336) = 17/28

The **correct answer is C.**

3.

Let's consider the different scenarios:

If Kate wins all five flips, she ends up with $15.
If Kate wins four flips, and Danny wins one flip, Kate is left with $13.
If Kate wins three flips, and Danny wins two flips, Kate is left with $11.
If Kate wins two flips, and Danny wins three flips, Kate is left with $9.
If Kate wins one flip, and Danny wins four flips, Kate is left with $7.
If Kate loses all five flips, she ends up with $5.

The question asks for the probability that Kate will end up with more than $10 but less than $15. In other words, we need to determine the probability that Kate is left with $11 or $13 (since there is no way Kate can end up with $12 or $14).

The probability that Kate ends up with $11 after the five flips:

Since there are 2 possible outcomes on each flip, and there are 5 flips, the total number of possible outcomes is $2 \times 2 \times 2 \times 2 \times 2 = 32$. Thus, the five flips of the coin yield 32 different outcomes.

To determine the probability that Kate will end up with $11, we need to determine how many of these 32 outcomes include a combination of exactly three winning flips for Kate.

We can create a systematic list of combinations that include three wins for Kate and two wins for Danny: DKKKD, DKKDK, DKDKK, DDKKK, KDKKD, KDKDK, KDDKK, KKDKD, KKDDK, KKKDD = 10 ways.

Alternatively, we can consider each of the five flips as five spots. There are 5 potential spots for Kate's first win. There are 4 potential spots for Kate's second win (because one spot has already been taken by Kate's first win). There are 3 potential spots for Kate's third win. Thus, there are $5 \times 4 \times 3 = 60$ ways for Kate's three victories to be ordered.

However, since we are interested only in unique winning combinations, this number must be reduced due to overcounting. Consider the winning combination KKKDD: This one winning combination has actually been counted 6 times (this is 3! or three factorial) because there are 6 different orderings of this one combination:

$K_1K_2K_3DD$, $K_1K_3K_2DD$, $K_2K_3K_1DD$, $K_2K_1K_3DD$, $K_3K_2K_1DD$, $K_3K_1K_2DD$

This overcounting by 6 is true for all of Kate's three-victory combinations. Therefore, there are only $60 \div 6 = 10$ ways for Kate to have three wins and end up with $11 (as we had discovered earlier from our systematic list).

The probability that Kate ends up with $13 after the five flips:

To determine the probability that Kate will end up with $13, we need to determine how many of the 32 total possible outcomes include a combination of exactly four winning flips for Kate.

Again, we can create a systematic list of combinations that include four wins for Kate and one win for Danny: KKKKD, KKKDK, KKDKK, KDKKK, DKKKK = 5 ways.

Alternatively, using the same reasoning as above, we can determine that there are $5 \times 4 \times 3 \times 2 = 120$ ways for Kate's four victories to be ordered. Then, reduce this by 4! (four factorial) or 24 due to overcounting. Thus, there are $120 \div 24 = 5$ ways for Kate to have four wins and end up with $13 (the same answer we found using the systematic list).

The total probability that Kate ends up with either $11 or $13 after the five flips:

There are 10 ways that Kate is left with $11. There are 5 ways that Kate is left with $13.

Therefore, there are 15 ways that Kate is left with more than $10 but less than $15.

Since there are 32 possible outcomes, the correct answer is $\frac{15}{32}$, answer choice **D**.

4.

The easiest way to attack this problem is to pick some real, easy numbers as values for *y* and *n*. Let's assume there are 3 travelers (A, B, C) and 2 different destinations (1, 2). We can chart out the possibilities as follows:

Destination 1	Destination 2
ABC	
AB	C
AC	B
BC	A
	ABC
C	AB
B	AC
A	BC

Thus there are 8 possibilities and in 2 of them all travelers end up at the same destination. Thus the probability is 2/8 or 1/4. By plugging in $y = 3$ and $n = 2$ into each answer choice, we see that only answer choice D yields a probability of 1/4.

Alternatively, consider that each traveler can end up at any one of *n* destinations. Thus, for each traveler there are *n* possibilities. Therefore, for *y* travelers, there are n^y possible outcomes.

Additionally, the "winning" outcomes are those where all travelers end up at the same destination. Since there are *n* destinations there are *n* "winning" outcomes.

Thus, the probability = $\frac{\text{winning outcomes}}{\text{total outcomes}} = \frac{n}{n^y} = \frac{1}{n^{y-1}}$.

The answer is D.

5.

The question require us to determine whether Mike's odds of winning are better if he attempts 3 shots instead of 1. For that to be true, his odds of making 2 out of 3 must be better than his odds of making 1 out of 1.

There are two ways for Mike to at least 2 shots: Either he hits 2 and misses 1, or he hits all 3:

Odds of hitting 2 and missing 1 $p \times p \times (1-p)$	**# of ways to hit 2 and miss 1** 3 (HHM, HMH, MHH)	**Total Probability** $3p^2(1-p)$
Odds of hitting all 3 $p \times p \times p$	**# of ways to hit all 3** 1 (HHH)	p^3
Mike's probability of hitting *at least* 2 out of 3 free throws =		$3p^2(1-p)+p^3$

Now, we can rephrase the question as the following inequality:

Is $3p^2(1-p)+p^3 > p$? (Are Mike's odds of hitting at least 2 of 3 greater than his odds of hitting 1 of 1?)

This can be simplified as follows:

$$3p^2(1-p)+p^3 > p$$
$$3p^2-3p^3+p^3 > p$$
$$3p^2-2p^3 > p \qquad \text{(we can divide by } p \text{ since } p>0\text{)}$$
$$3p-2p^2 > 1$$
$$-2p^2+3p-1>0 \qquad \text{(divide by } -2\text{, flipping the inequality)}$$
$$p^2-1.5p+.5<0$$
$$(p-.5)(p-1)<0$$

In order for this inequality to be true, p must be greater than .5 but less than 1 (since this is the only way to ensure that the left side of the equation is negative). But we already know that p is less than 1 (since Mike occasionally misses some shots). Therefore, we need to know whether p is greater than .5. If it is, then the inequality will be true, which means that Mike will have a better chance of winning if he takes 3 shots.

Statement 1 tells us that p < .7. This does not help us to determine whether p > .5, so statement 1 is not sufficient.

Statement 2 tells us that p > .6. This means that p must be greater than .5. This is sufficient to answer the question.

The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

6.

Although this may be counter-intuitive at first, the probability that *any* card in the deck will be a heart before any cards are seen is 13/52 or 1/4.

One way to understand this is to solve the problem analytically for any card by building a probability "tree" and summing the probability of all of its "branches."

For example, let's find the probability that the 2nd card dealt from the deck is a heart. There are two mutually exclusive ways this can happen: (1) both the first and second cards are hearts or (2) only the second card is a heart.

CASE 1: Using the multiplication rule, the probability that the first card is a heart AND the second card is a heart is equal to the probability of picking a heart on the first card (or 13/52, which is the number of hearts in a full deck divided by the number of cards) times the probability of picking a heart on the second card (or 12/51, which is the number of hearts remaining in the deck divided by the number of cards remaining in the deck).

13/52 x 12/51 = 12/204

CASE 2: Similarly, the probability that the first card is a non-heart AND the second card is a heart is equal to the probability that the first card is NOT a heart (or 39/52) times the probability of subsequently picking a heart on the 2nd card (or 13/51).

39/52 x 13/51 = 39/204

Since these two cases are mutually exclusive, we can add them together to get the total probability of getting a heart as the second card: 12/204 + 39/204 = 51/204 = 1/4.

We can do a similar analysis for any card in the deck, and, although the probability tree gets more complicated as the card number gets higher, the total probability that the *n*th card dealt will be a heart will always end up simplifying to 1/4.

The correct answer is A.

7.

The chance of getting AT LEAST one pair of cards with the same value out of 4 dealt cards should be computed using the 1-*x* technique. That is, you should figure out the probability of getting NO PAIRS in those 4 cards (an easier probability to compute), and then subtract that probability from 1.

First card: The probability of getting NO pairs so far is 1 (since only one card has been dealt).

Second card: There is 1 card left in the deck with the same value as the first card. Thus, there are 10 cards that will NOT form a pair with the first card. We have 11 cards left in the deck.

Probability of NO pairs so far = 10/11.

Third card: Since we have gotten no pairs so far, we have two cards dealt with different values. There are 2 cards in the deck with the same values as those two cards. Thus, there are 8 cards that will not form a pair with either of those two cards. We have 10 cards left in the deck.

Probability of turning over a third card that does NOT form a pair in any way, GIVEN that we have NO pairs so far = 8/10.

Cumulative probability of avoiding a pair BOTH on the second card AND on the third card = product of the two probabilities above = (10/11) (8/10) = 8/11.

Fourth card: Now we have three cards dealt with different values. There are 3 cards in the deck with the same values; thus, there are 6 cards in the deck that will not form a pair with any of the three dealt cards. We have 9 cards left in the deck.

Probability of turning over a fourth card that does NOT form a pair in any way, GIVEN that we have NO pairs so far = 6/9.

Cumulative probability of avoiding a pair on the second card AND on the third card AND on the fourth card = cumulative product = (10/11) (8/10) (6/9) = 16/33.

Thus, the probability of getting AT LEAST ONE pair in the four cards is 1 - 16/33 = 17/33.

The correct answer is C.

TOPIC 9 – MISCELLANEOUS

Calculation, Exponents, Basic Algebra

1.

(1) INSUFFICIENT: You cannot simply divide both sides of the equation by p to obtain $pq = 1$. The reason is that you don't know whether or not p is zero -- and remember, you are not allowed to divide by zero! Instead, you must factor this equation.

First, subtract p from both sides to get: $pqp – p = 0$.
Then, factor out a common p to get: $p(pq – 1) = 0$. This means that either $pq = 1$ or $p = 0$.
(2) INSUFFICIENT: The same process applies here as with statement (1). Remember, you should never divide both sides of an equation by a variable that could be zero.

First, subtract q from both sides to get: $qpq – q = 0$.
Then, factor out a common q to get: $q(pq – 1) = 0$. This means that either $pq = 1$ or $q = 0$.

(1) AND (2) INSUFFICIENT: Together we still don't have enough information to solve. Either $pq = 1$ or both p and q are 0.

The correct answer is E

2.

From the problem statement, we know that $\frac{5^a 2^b 3^c}{5^d 2^e 3^f} = 3$.
We also know that the digits b, c, e, and f are integers from 0 to 9 and that the digits a and d are integers from 1 to 9 (they cannot be 0 since they are in the hundreds place).

For the statement above to be true, $5^a 2^b$ must equal $5^d 2^e$, and $\frac{3^c}{3^f} = 3^1$.
Therefore, $a = d, b = e$, and $c - f = 1$.

Since the only difference between *abc* and *def* is in the units digits, the difference between these three-digit numbers is equal to $c - f$, or 1.

The correct answer is A.