Table of Contents

GMAT Study Guide - Notes

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Table of Contents

Borrowing a revised disclaimer from Slingfox's notes:

- See also my GMAT test debrief on Reddit (which has all the resources I used/recommend + general study strategies): https://www.reddit.com/r/MBA/comments/hfzvp9/from_710_to_770_50q45v8ir6awa_gmat_study_tips/
- If you've just started studying for the GMAT, do not start with this particular because it will likely be incomprehensible.
- These notes will likely be most useful to those who have made the effort to bring themselves up to speed on quant/verbal and completed at least 100-200 practice problems.
- A lot of the concepts here are based off of material contained in the various Manhattan GMAT guides + GMATClub's YouTube videos, so these notes will be most useful if you are familiar with or have access to those books.
- **Heads up:** My "Personal Notes" section includes problems and explanations taken from OG tests, so if you don't want to see any OG CAT problems, skip that section. This section was basically my own personal error log.
    - This section will probably be the least comprehensive since it is a personal error log (and by nature, error logs are deeply personalized), but a lot of people have been asking how I kept track of my questions and this was just how I kept track of the lessons learned/takeaways from problems I got wrong. I didn't use any complex software or anything, just a simple word/google doc for lessons learned and an excel sheet to keep track of which questions I was doing and what I got wrong.
- It is unlikely that you will fully understand everything here the first time. Even if you think you fully understand everything set forth herein, you will probably not fully appreciate how each concept is/can be applied to GMAT problems until deep into your GMAT practice/study.
- The "Quant Formulas" and "SC Idioms" sections are probably the most practical takeaways from this guide. The rest of the notes may or may not be helpful depending on your background.
- Most importantly: Keep practicing and don't give up! If you're stumped about something, use the internet and the various GMAT-centric message boards (such as GMAT Club) as a resource!
- Lastly, I do not guarantee the accuracy or validity of any information in this guide.

Table of Contents

Table of Contents

Quant

Quant Formulas / Concepts to Commit to Memory

FDP

- % Change = Change in Value / Original Value
- Precision in language
    - 200% increase from original = Original*(1+2) = Original * 3
    - 200% of original = Original * 2
    - x% decrease → Original * (1 - x/100)
- Memorize the following
    - First 6 factorials
        - 4! = 24
        - 5! = 120
        - 6! = 720
    - Fraction to decimal equivalents
        - 1/6 = .167
        - 1/7 = .142
        - 1/8 = .125
        - 1/9 = .11
        - 1/11 = .09
        - 1/12 = .083
        - 1/13 = .077
        - 22/7 = pi
    - Square root to decimal equivalents (rough approximations, for 5-8 just think about it in .2 increments starting from 2.25)
        - 2 - 1.41
        - 3 - 1.73
        - 5 - 2.25
        - 6 - 2.45
        - 7 - 2.65
        - 8 - 2.85
    - Perfect Squares (seeing these numbers should jump out)
        - $13^2 = 169$
        - $14^2 = 196$
        - $15^2 = 225$
        - $16^2 = 256$
        - $17^2 = 289$
        - $20^2 = 400$
        - $25^2 = 625$
        - $30^2 = 900$
    - Perfect cubes (up to 10) - seeing these numbers should jump out
        - $3^3 = 27$

        - $4^3 = 64$
        - $5^3 = 125$
        - $6^3 = 216$
        - $7^3 = 343$
        - $8^3 = 512$
        - $9^3 = 729$
        - $10^3 = 1000$

Geometry

Polygons

- Sum of interior angles = 180(n-2), where n is number of sides
    - Each interior angle = 180(n-2)/n
- Types
    - Triangle
        - Area = .5bh
        - Equilateral Triangles (IMPORTANT)
            - Area = s^2 * sqrt(3)/4
            - Height = s * sqrt(3)/2
            - Perimeter = 3a
        - Remember the following
            - 30-60-90 triangles have set ratio
                - leg, leg, hypotenuse
                - 30deg, 60deg, 90deg
                - x, x * sqrt(3), 2x
                - 1:sqrt(3):2x
            - 45-45-90 triangles have set ratio
                - leg, leg, hypotenuse
                - 45deg, 45deg, 90deg
                - x, x, x * sqrt(2)
                - 1:1:sqrt(2)
            - Angles of a Triangle
                - Sum of angles = 180
                - Angles correspond to their opposite sides
                    - Largest angle opposite largest side
                    - If two sides equal, opposite angles are equal
            - Triangle inequality law → Sum of any two sides of a triangle must be greater than the third side
                - Given two sides, third side must lie btwn diff. and sum of two other sides
            - Obtuse Triangle Rule
                - $c^2 > a^2 + b^2$
                - Good problem requiring knowledge of both Triangle Inequality Law AND Obtuse Triangle Rule

- Common Right Triangles to MEMORIZE
  - 3-4-5 (e.g. 6-8-10, 9-12-15, 12-16-20)
  - 5-12-13 (e.g. 10-24-26)
  - 8-15-17
  - 7-24-25
  - 9-40-41
- Parallelogram
  - Area = bh
- Rectangle
  - Area = lw
  - Diagonal = sqrt(w^2+l^2)
- Square
  - Area = s^2
  - Square has a larger area that any other quadrilateral with the same perimeter
  - Diagonal = s*sqrt(2)
  - For a square, can find area of a circumscribed circle if you know the side length
    - Side length will be diameter of the circle
- Rhombus
  - Area = base*height
  - Area = d1 * d2 / 2 (diagonal 1 * diagonal 2 / 2)
- Trapezoid
  - Area = (b1+b2)*h/2
  - Bases = parallel sides
  - Legs = non-parallel sides
- Regular Hexagon (regular meaning all sides are the same)
  - Diagonal = 2s (where s is a side length)
  - Can split a regular hexagon into 6 equilateral triangles

3D Geometry

- Cube
  - Volume = a^3
  - Surface Area = $6a^2$
  - Diagonal Length = sqrt(3) * a
- Cuboid
  - Volume = abc
  - Surface Area = 2(ab + bc + ca)
  - Diagonal = sqrt(a^2+b^2+c^2)
- Cylinder
  - Volume = pi * r^2 * h
  - Surface Area = 2*pi*r^2 + 2*pi*r*h
- Cone
  - Volume: ⅓ * pi * r^2 * h
- Sphere
  - Volume = 4/3 * pi * r^3

- Surface Area = 4 * pi * r^2

Algebra

- Quadratic equations ($ax^2 + bx + c$)
  - # of solutions
    - $b^2 < 4ac$ = no solution
    - $b^2 = 4ac$ = exactly 1 solution
    - $b^2 > 4ac$ = 2 solutions
  - Sum of roots = -b/a
  - Product of roots = c/a
- Quadratic Templates
  - Square of a sum
    - $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
      - Ex: Sum of 9999^2 and $10{,}001^2 = (10^4 - 1)^2 + (10^4+1)^2 = 2(10^8 + 1^2) = 200{,}000{,}002$
  - Square of a difference
    - $(a+b)^2 - (a-b)^2 = 4ab$
- Difference of Squares = most common identity
  - $a^2 - b^2 = (a+b)(a-b)$
    - Can take many forms
    - Ex: $25-x^2 = (5+x)(5-x)$
    - Ex: x-y = (sqrt(x)+sqrt(y))(sqrt(x)-sqrt(y))
- Useful Algebraic Identities
  - $(x+y)^2 = x^2+y^2+2xy$
    $(x-y)^2 = x^2+y^2-2xy$
    $x^2-y^2 = (x+y)(x-y)$
    $(x+y)^2-(x-y)^2 = 4xy$
    $(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$
    $x^3+y^3 = (x+y)(x^2+y^2-xy)$
    $x^3-y^3 = (x-y)(x^2+y^2+xy)$
    - The last 3 very rarely (if ever) come up, but first 4 are definitely common

Work/Distance

Distance

- For relative rate problems,
  - People walking towards each other, distance btwn them decreases at $r_1 + r_2$
  - People walking in same direction, distance btwn them decreases at $r_1 - r_2$
  - People walking opposite directions, distance btwn them increases at $r_1 + r_2$
- Xiggi's formula to find average speed, <u>only works when 2 distances are the same</u>
  - Average speed = (2 * $speed_1$ * $speed_2$) / ($speed_1$ + $speed_2$)
    - Shorthand: 2ab/ab

Table of Contents

Work

- If A completes certain amount of work in X hours, then A would complete 1/X of the work in one hour (reciprocal)
- If A completes 1/X of work in one hour and B completes 1/Y work in one hour, TOGETHER they can complete 1/X + 1/Y of work in one hour
- 1/X + 1/Y = 1/Z → A and B working together will complete 1/Z of work in one hour
    - Therefore, working together, they will complete the work in Z hours (take reciprocal)
    - 1/X - 1/Y = difference between the amount of work X and Y complete in one hour (or whatever the unit of time is)

Probability

- Arranging objects in a row
    - Combination of unordered collection of k objects taken from a set of n distinct objects
        - Order doesn't matter → AB = BA
        - C= n!/k!(n-k)!
    - Permutation of ordered collection of k objects taken from a set of n distinct objects
        - Order matters → AB != BA
        - P = n!/(n-k)!
- Probability of A OR B → P(A U B) = P(A) + P(B) - P(A ∩ B)
    - If they are mutually exclusive, would just be P(A) + P(B) (since P(A and B) = 0)
- Probability of A AND B → P(A ∩ B) = P(A) * P(B)

Number Properties

General

- Finding sum of factors of an integer
    - $n = a^p * b^q * c^r$ (a, b, c are prime factors; p, r, and r are powers)
        - Number of factors = (p+1)(q+1)(r+1) → this one is pretty common
        - Sum of factors of *n* = [$(a^{p+1}-1)*(b^{q+1}-1)*(c^r-1)$] / [(a-1)*(b-1)*(c-1)] → this one is rarer
- Last digit of product = last n digits of the product of last n digits of those integers
    - Last 2 digits of 845*9512*408*613 = last 2 digits of 45*12*8*13 = 540*104 = 40*4 = 160 = 60
    - Last digit of 1248 * 1247 = Last digit of 8 * 7 = Last digit of 56 = 6
- Last digit of exponent
    - xyz^n has same last digit as z^n (xyz are digits, not multiplied)
        - E.g. 127^{39} has same last digit as 7^{39}
    - Find cyclicity of z^n to determine answer
- 3 Patterns (unit digit of numbers raised to positive integer powers)
    - If unit digit is 0, 1, 5 or 6, all powers of such numbers end in 0, 1, 5 or 6, respectively
        - $(441)^{85}$ → will end in 1
    - If unit digit is 4 or 9, odd powers have same unit digit, even powers have same unit digit
        - $4^1 = 4$, $4^2 = 6$, $4^3 = 4$, $4^4 = 6$
        - $9^1 = 9$, $9^2 = 1$, $9^3 = 9$, $9^4 = 1$

    - If unit digit is 2, 3, 7 or 8, unit digit pattern repeats every 4th power
        - Example: 7
            - 7^1 → 7 → ends in 7
            - 7^2 → 49 → ends in 9
            - 7^3 → 343 → ends in 3
            - 7^4 → 2401 → ends in 1
            - 7^5 → 16807 → ends in 7 (starts repeating after 4^{th} power)
- Place Value
    - Sum of all numbers which can be formed by using the "n" digits
        - 1. Sum of all the numbers which can be formed by using the n digits without repetition is: (n-1)!*(sum of the digits)*(111…..n times)
        - 2. Sum of all the numbers which can be formed by using the n digits (repetition being allowed) is: n^{n-1}*(sum of the digits)*(111…..n times)
    - Example: Sum of all 3 digit numbers that can be constructed using 3, 4, 5 if each digit is used once?
        - (3)!(12)(111)

Factorials

- Finding number of powers of a prime number *p* in n!
    - $n/p + n/p^2 + \ldots$, while $p^x < n$
    - EX: What is the power of 2 in 25!
        - 25/2 + 25/4 + 25/8 + 25/16 = 12 + 6 + 3 + 1 = 22
- Number of trailing zeros in a factorial
    - $n / 5 + n / 5^2$, while $5^k \leq n$
    - EX: How many zeroes are in the end of 32!
        - 32/5 + 32/25 = 6 + 1 = 7

Absolute Value

- Properties
    - $|x| = \text{sqrt}(x^2)$
    - $|x| + |y| \geq |x+y|$

Remainder

- For y/x, y = divisor * quotient + remainder → y = xq + r or y/x = q+r/x
    - EX: If remainder is 7 when positive integer n is divided by 18, what is remainder when n is divided by 6?
        - n = 18q + 7
            - 18 is divisible by 6, so remainder will come from second term (7)
            - 7/6 = remainder of 1
    - If s/t = 64.12, what could be remainder of s/t?
        - s/t can be rewritten as → s = qt + r → s/t = q + r/t
            - r/t = 12/100 = 3/25 → r must be a multiple of 3
- Deriving general formula given two statements

- EX: When x is divided by 5, remainder is 3. When x is divided by 7, remainder is 4.
  - x = 5q+3
    - Possible #s: 3, 8, 13, 18, 23
  - x = 7q+4
    - Possible #s: 4, 11, 18, 25
  - For general formula, divisor will be the least common multiple between the divisors
    - LCM of 5 and 7 is 35
  - Remainder will be first common integer in two patterns
    - Remainder in this case will be 18 (bolded above)
  - General formula = x = 35q + 18
    - values = 18, 53, 88, 123, 158

GCF/LCM

- If you have two numbers, A and B
  - A*B = GCF*LCM
    - GCF * f_1 * GCF * f_2 = GCF * LCM
  - If given the GCF between 2 numbers, A and B, you know that
    - A = GCF * f_1
    - B = GCF * f_2
    - Where f_1 and f_2 are unique, co-primes (only integer that evenly divides them both is 1)

Sequences

- Average * # of terms = Sum (A = S / n)
- Consecutive integers
  - Counting consecutive integers: (last - fast + 1) → don't forget to include both extremes if inclusive!
  - Counting consecutive multiples (last multiple - first multiple) / increment + 1
  - For all evenly spaced sets, the average = (first + last) / 2
  - Sum of consecutive integers
    - Sum = average * number of terms
- Arithmetic, Geometric and Harmonic (take inverse of every term) Progressions
  - Arithmetic Mean = (a+b)/2 OR (a_1 + … + a_n) / n
    - a_n = a_1 + (n-1)*difference
    - Sum of n term AP w/ common difference d: = (n/2)(2a+(n-1)d))
  - Geometric Mean = sqrt(ab) OR (a_1 * … * a_n)$^{1/n}$
    - a_n = a_1 * r^{n-1}
    - Sum of n term GP with common ratio r: = a_1*(1-r^n)/(1-r)
    - Sum of infinite term GP with common ratio r = a_1/(1-r)
  - Harmonic Mean = 2ab / (a+b)
- Sum of first n consecutive integers

    - n(n+1)/2
    - Ex: Sum of first 100 consecutive positive integers?
        - 100(101)/2 = 50*101 = 5,050

Interest

- Simple Interest = principal * r * time
    - Time must be in same units used for time in the rate
        - EX: To find interest earned after 9 months with simple annual interest, time would be (9/12)
- Compound interest
    - Final Balance = principal * $(1+ (r/n)^{nt}$
        - r = rate
        - n = number of times compounded annually
        - t = number of years

Other

- Distance btwn 2 points = sqrt(dx^2+dy^2), where dx is diff. btwn x-coordinates and dy is diff. btwn y-coordinates
- 3 overlapping sets (Bunuel Explanation of the differences btwn the formulas + Example Problems here)
    - Total = A+B+C - (sum of 2 group overlaps) + all three + Neither
        - A+B+C+-(all2)+a3+n
        - different than EXACTLY 2 group overlaps since it will include the overlap for all 3 groups
    - Total = A+B+C - (sum of EXACTLY 2 group overlaps) - (2 * all 3) + Neither
        - A+B+C-(ex2)-(2*a3)+n
        - Only use when given or asked for information regarding EXACTLY 2 group overlaps
    - With two groups
        - Total = A + B - Both + Neither
- # of games played with n teams, if each team plays each other once
    - # of games played = n(n-1)/2
    - # of games each team plays = (n-1)
    - Same scenario as “n people, each shake their hand once, how many handshakes were there in total?”

Table of Contents

Manhattan Prep - Strategy Set - Quant

MGMAT Strategy Set - Guide 1 - Fractions / Decimals / Percents

- Digits / Decimals
    - Multiplication → to simplify multiplication, move decimals in opposite directions
        - Decimals
            - Ignore decimal point until end, and count digits to the right of the decimals
        - Large number * small number
            - Move decimals same number of places but in opposite directions
                - .0003 * 40,000 = 3 (4 right) * 4 (4 left) = 12
    - Division → to simplify division, move decimals in same direction
        - If decimal in dividend (12.42 / 3), can bring decimal point straight up and divide normally (e.g. 12 /3 and 42/ 3 = 4.14)
        - If decimal in divisor, shift decimal in divisor to make divisor a whole number, then do the above
- Test Cases for DS problems that allow for multiple possible starting points
    - Test to prove statement insufficient
        - Value
            - Sufficient: single numerical answer
            - Not sufficient: two or more possible answers
        - Yes / No
            - Sufficient: Always Yes or Always No
            - Not sufficient: Maybe or sometimes yes, sometimes no
- Fractions
    - Can often simplify double-decker fractions quickly by multiplying both top / bottom by common denominator
    - Cross multiply to compare fractions (e.g. is 8/13 < 7/11, check 88 < 91 = yes)
    - The product of a number and a reciprocal must be equal to 1
- Percents
    - Building percents: Can calculate percentages using a combo of percentages
        - e.g. 70% of a number is 50% + 10% + 10%
    - Formulas
        - Revenue = Price * Quantity Sold
        - % Change = Change in Value / Original Value
        - New Percent = New Value / Original Value (e.g. if price is decreased by 20%, new percent = 80%)
        - Compound Interest → Total Amount = P(1 + (r/n))^(nt)
            - P = principal, r = rate in decimals, n = number of times per year, t = number of years
- Choosing Smart Numbers
    - Only when all values are unknown
    - With percents, normally 100 is easiest to choose

    - With fractions, choose common denominator of fractions as smart number
    - Do not pick numbers that appear elsewhere in the problem
    - If choosing multiple numbers, choose different numbers, ideally with diff. properties (e.g. odd and even)
- Ratios
    - All ratios include an unknown multiplier (multiplier is always the same for all parts of the ratio)
    - Multiple ratios → create a common term corresponding to same quantity
    - x:y:z → x/y and y/z
- Estimation
    - If rounding two numbers, round one up and the other down to nearest fraction / percent to minimize rounding errors
- Extra FDPs
    - Raising decimal to higher power → rewrite decimal as product of an integer and a power of 10, then apply exponent (can apply to roots as well since root is number raised to fractional power)
    - If denominator is 9, 99, 999 or another number equal to power of 10 -1, numerator tells you the repeating digits
        - 3/11 = 27/99 → .272727
    - Terminating Decimals → If after being fully reduced, the denominator of a fraction has any prime factors besides 2 or 5, then its decimal will not terminal
    - Last digit shortcut → to find units digit of product / sum of integers, only pay attention to units digits of numbers you are working with… once you have all the last digits of each number, multiply or add them and find the units digit of the product / sum
        - e.g. units digit of 27 * 31 * 42 = 7 * 1 * 2 = 14 → 4 will be the units digit

MGMAT Strategy Set - Guide 2 - Algebra

- PEMDAS → when same level of priority (m/d and a/s), <u>always work from left to right</u>
- Abs. Value → equation of form |x| = a, where a > 0, solve equation for x = a and x = -a
- Exponents
    - Every number other than 0 raised to the power of 0 = 1
    - If x = x^2, x must be 0 or 1
    - Raising negative number to an odd power will always result in a negative number
    - If parentheses around negative number (e.g. (-2)^4), negative number to odd power = negative, to even power = positive
        - Otherwise, always negative → -2^4 = negative
    - When raising an exponential term to an exponent, multiply the exponents (e.g. (a^2)^2 = a^4)
    - Can factor expressions out of products
        - x = 4^20(21)
            - 21 = 3 * 7 (factors)
    - Any number raised to an even exponent becomes positive

    - Any equation involving only odd exponents or cube roots have only one solution
    - When multiplying numbers with same bases, add exponents (and subtract when dividing)
        - $5^2*5^4 = 5^6$
        - $5^5/5^2 = 5^3$
    - When bases are identical and no other bases exist, you can drop bases, rewrite exponents as an equation and solve
        - 2^6w = 2^(5w-5) → 6w = 5w - 5
- Roots
    - Fractional exponents
        - First, if exponent negative, take the reciprocal of base and change exponent to positive equivalent
        - Then, find all prime factors of base, then raise that to the base
            - e.g. 216^(⅓)
                - 216 = 3*3*3*2*2*2 = 6^3
                - 6^3(⅓) = 6
    - Can only separate / combine the product and quotient of two roots (not the sum or diff.)
        - e.g. sqrt(18) / sqrt(2) = sqrt(9)
        - sqrt(9) * sqrt (4) = sqrt(36)
- Quadratic equations
    - GMAT attempts to disguise quadratics by putting them in unconventional forms
        - e.g. 3w^2 = 6w is a quadratic
            - Don't just divide by sides by 3w, would miss one solution
            - Instead, 3w^2 - 6w = 0 and solve quadratic
        - e.g. 36/b = b - 5
    - When taking square root (e.g. of a perfect square), don't forget to take consider both positive AND negative square root
- Inequality
    - Common statements → implication
        - xy > 0 → x and y are both positive OR both negative
        - xy < 0 → x and y have different signs
        - x^2 - x < 0 → 0 < x < 1
    - Multiplying / dividing inequality by negative number flips sign of inequality
    - To convert multiple inequalities to a compound inequality, first line up variables then combine
    - You can add inequalities as long as the signs are first lined up to face the same direction
        - For DS inequality questions, test two statements individually, then if neither sufficient by itself, test statements together by
            - Adding inequalities together, adding the second inequality twice
            - Never subtract or divide two inequalities
    - Can square root inequalities if both sides are definitely not negative
    - If |x| < a,
        - If x positive, then x < a
        - If x negative, then x > a
    - If |x| > a,

        - If x positive, then $x > a$
        - If x negative, then $x < a$
- Test Cases for DS
    - When choosing numbers to test cases, only choose numbers that make statement true for each statement
- Extra Equation Strategies
    - For multiple absolute value equations, test two cases: one with same sign, and another in which one expression changes signs
        - Ex: $|x-2| = |2x-3|$
            - Test $(x-2) = (2x-3)$
            - Test $(x-2)= -(2x-3)$
    - Quadratic Formula (only important if shooting for 51, very few questions will make you use this since they can be factored): $(-b \pm \sqrt{b^2-4ac}) / 2a$
        - $b^2 - 4ac$ = discriminant
            - If discriminant $> 0 \rightarrow$ 2 solutions
            - If discriminant $= 0 \rightarrow$ 1 solution
            - If discriminant $< 0 \rightarrow$ no solutions
    - Combine statements w/ same variables
        - Ex: $xy = 2$, $xz = 8$, $yz = 5 \rightarrow x^2y^2z^2 = 80 \rightarrow xyz = \sqrt{80}$, or around 9
- Extra formulas strategies
    - Proportionality
        - $y = kx$, where k is a proportionality constant, x is input, and y is output
            - $k = y / x$
        - y_1/x_1 = before, y_2 / x_2 = after
            - Set equal to each other: $y_1/x_1 = y_2 / x_2$
    - Inverse proportionality
        - $y = k/x$, or $k = yx$
            - Set $y_1x_1 = y_2x_2$
    - For symmetry problems, choose a number and see what makes both equations equivalent
    - Range of a function is its possible y-values
- Extra inequalities strategies
    - Absolute value + inequality graphs
        - Ex: If $|x - 2| < 5$,
            - + test, remove abs. value and solve $\rightarrow x-2 < 5 \rightarrow x < 3$
            - - test, reverse sign inside abs. value, remove abs. value and solve $\rightarrow -(x-2) < 5 \rightarrow x > -3$
    - Reciprocals of inequalities
        - If you know the signs of the variables, flip the inequality unless x and y have different signs
            - Same signs = flip, if $x < y$, then $1/x > 1/y$
            - Different signs = don't flip, if $x < y$, then $1/x < 1/y$
    - Squaring inequalities
        - If signs are unclear of both sides, cannot square
        - If both sides have opposite signs, cannot square

        - Otherwise, can square
            - If both sides negative, flip inequality sign when you square
            - If both sides positive, don't flip inequality sign when you square
    - Remember to check both positive and negative cases with inequalities
        - Ex: If $4/x < ⅓$, what is the possible range of values for x?
            - Test value with x being positive, and test value with x being negative (and thus flipping inequality sign when multiplying both sides by x)
        - Ex: If $4 / x < -⅓ \rightarrow$ in this case, only check for negative case since x can never be positive

MGMAT Strategy Set - Guide 3 - Word Problems

- Translations
    - Common Relationships
        - TC = Unit Price * Quantity Sold
        - Profit = Rev - Cost
        - Total Earnings = Wage * Hours Worked
        - Miles = MPH * Hours
        - Miles = MPG * Gallons
    - Questions that ask for a single discrete variable are good for working backwards (e.g. test different answer choices)
        - If answers are in ascending order, start with B and D first, then can determine if answer has to be < B, > D, or in between the two
    - Often helpful to set up a RTD table for relevant problems
- Rates & Work
    - Rate * Time = Distance
    - Rate * Time = Work
    - For relative rate problems,
        - People walking towards each other, distance btwn them decreases at $r_1 + r_2$
        - People walking in same direction, distance btwn them decreases at $r_1 - r_2$
        - People walking opposite directions, distance btwn them increases at $r_1 + r_2$
    - Average Speed = Total Distance / Total Time
    - Xiggi's formula to find average speed, <u>only works when 2 distances are the same</u>
        - Average speed = $(2 * speed_1 * speed_2) / (speed_1 + speed_2)$
            - $2ab/(a+b)$
        - e.g. An elephant traveled 7 miles at an average rate of 4 miles per hour and then traveled the next 7 miles at an average rate of 1 mile per hour. What was the average speed, in miles per hour, of the elephant for the 14 miles?
            - $(2 * 4 * 1) / 5$
    - When 2+ workers completing same task, can add together the rates
- When choosing smart numbers, don't pick 0, 1 or a number already mentioned in the problem statement
    - When working with percentages, 100 is a great number to choose (unless problem already uses 100 or 100% somewhere, then should pick a different number)

    - When working with fractions, choose a common denominator for the total (e.g. if problem mentions ⅓ and ⅖, pick 15 as total)
- Overlapping sets
    - For overlapping sets, often a double-set matrix is helpful
- Statistics
    - Arithmetic mean, cost per employee shared equally, per capita income → all equal to average
        - Average * # of terms = Sum (A = S / n)
    - Std. Deviation
        - Will never be asked to calculate std. dev. → instead, look at average of spreads from the mean to get an estimate
        - SD of 0 means all numbers in the set are equal
- Weighted Averages
    - Teeter-totter method can be helpful
- Consecutive integers
    - Counting consecutive integers: (last - fast + 1) → don't forget to include both extremes if inclusive!
    - Counting consecutive multiples (last multiple - first multiple) / increment + 1
    - For all evenly spaced sets, the average = (first + last) / 2
    - Sum of consecutive integers
        - Sum = average * number of terms
            - e.g. sum of all integers from 20-100, inclusive
                - Average = 20 + 100 / 2 = 60
                - terms = 100 - 20 + 1 = 81
                - Sum = 60 * 81 = 4800 + 60 = 4860
- Check for hidden constraints!
    - E.g. When finding min / max, check whether you can have fractional units (people, cars, animals, etc… cannot be split into fractional parts)
        - E.g. if max number of students that can be accepted is 16.66, it is really 16 (since 17 would break the constraint)
- Can combine ratios
    - Ex: If ratio of hardcover to paper books is 22:3, total books is going to be a multiple of 25
- Combinations trick
    - If can only use chips of $5 and $7, which of the following cannot be paid out?
        - Subtract 5 from answer choices and see if multiple of 7 exists (or vice versa)
- Extra overlapping sets and consecutive integers
    - For problems with 3 overlapping sets, use a Venn diagram rather than double set matrix
        - Work from the inside out → e.g. start with overlap of all 3
- Product of consecutive integers
    - Product of k consecutive integers is always divisible by k factorial (k!)
        - Product of 4 consecutive integers is divisible by 4 * 3 * 2 * 1 = 24
    - Product of k consecutive multiples (x), product must have k x's as factors
        - EX: r, s, t are consecutive multiples of 3 → rst must have 3 3's as factors
            - e.g. 27 = 3 * 3 * 3

        - e.g. 54 = 3 * 3 * 3 * 2
- Summing consecutive integers tricks
    - With an even number of items, sum of all items never a multiple of the number of items
    - With an odd number of items, sum of all items is always a multiple of the number of items
- For scheduling problems, take into consideration the earliest / latest possible time slot for the events to be scheduled
    - Keep in mind that for years that are leap years, February has 29 days
    - Months alternate by 31 and 30 days, starting in January, with February being the one exception with 28 days, and reset count on august (starts with 31)
        - 31 28 31 30 31 30 31 31 30 31 30 31

MGMAT Strategy Set - Guide 4 - Geometry

- 3 principles
    - 1. If they don't tell you, don't assume (e.g. just cause some line looks like the diameter of a circle, it may not be unless they explicitly tell you)
    - 2. If they give you a piece of info, use it
    - 3. Know the rules / formulas
        - If a line passes through the center of a circle from one side to another, that line is a diameter
        - If one of the sides of a triangle inscribed in a circle is a diameter of the circle, the triangle must be a right triangle
            - For triangle ABC, if AC is a diameter, ABC is a right triangle and angle B is the right angle (side opposite center)
- Lines & Angles
    - Intersecting lines
        - Interior angles formed by intersecting lines sum up to 360 degrees (circle)
        - Inter angles combine to form a sum of 180 degrees (line)
        - Angles found opposite each other where two lines intersect are equal (vertical angles)
    - Parallel lines cut by a transversal → acute angles (< 90) and obtuse (> 90 but < 180)
- Polygons
    - Trapezoid (one pair of opposite sides is parallel, other is not)
    - Sum of Interior Angles of a Polygon = (n - 2) * 180, where n is the number of sides
        - Triangles = 180
        - Quadrilaterals = 360
        - Pentagon = 540
        - Hexagon = 720
    - Area Formulas
        - Area of Trapezoid = (($Base_1$ + $Base_2$) * Height) / 2
            - Height = line that is perpendicular to the two bases which are parallel
    - 3-dimensionals (rectangular solids + cubes)
        - Surface area = sum of all of the faces

- Both rectangular solids / cubes have six faces
  - Find area of all six sides
  - For rectangular solids, find area of top / bottom / sides
- Volume = Length * Width * Height (measured in cubic units)

- Remember: when fitting 3d objects into other 3d objects, knowing respective volumes is not enough… need to know the specific dimensions (length, width and height)

- Triangles & Diagonals
  - Angles of a Triangle
    - 2 key properties
      - Sum of angles = 180
      - Angles correspond to their opposite sides
        - Largest angle opposite largest side
        - If two sides equal, opposite angles are equal
  - Triangle inequality law → Sum of any two sides of a triangle must be greater than the third side
    - Given two sides, third side must lie btwn diff. and sum of two other sides
  - Common Right Triangles to MEMORIZE
    - 3-4-5 (e.g. 6-8-10, 9-12-15, 12-16-20)
    - 5-12-13 (e.g. 10-24-26)
    - 8-15-17
  - Isosceles Triangles & 45-45-90 Triangles
    - Isosceles right triangles = 45-45-90, have a set ratio, MEMORIZE
      - leg, leg, hypotenuse
      - 45deg, 45deg, 90deg
      - x, x, x * sqrt(2)
      - 1:1:sqrt(2)
    - Two 45-45-90's combine to form a square
  - Equilateral Triangles & 30-60-90 triangles
    - 30-60-90 triangles, have a set right ratio, MEMORIZE
      - leg, leg, hypotenuse
      - 30deg, 60deg, 90deg
      - x, x * sqrt(3), 2x
      - 1:sqrt(3):2x
    - Two 30-60-90's combine to form an equilateral triangle
  - Exterior Angles of Triangles
    - Exterior angle of a triangle equal to sum of two non-adjacent (opposite) interior angles of the triangle
  - Similar triangles
    - If two triangles have two angles in common, their corresponding sides must be in the same proportion
    - If two similar triangles have corresponding side lengths in ratio a:b, their areas will be in ratio a^2:b^2
- Circles & Cylinders
  - Circle

        - Circumference = distance around circle = 2 pi r = pi d
        - Area of circle = pi * r^2
    - Area of a Sector
        - Find area of the entire circle, then use central angle (angle whose vertex lies at center point of circle) to determine what fraction of entire circle is represented by the sector
            - Make sure to use central angle, not inscribed angle
                - Inscribed angle (from diameter) corresponds to half of central angle (from center, so from radius)
    - Inscribed triangles
        - If one of the sides of an inscribed triangle is a diameter of the circle, then triangle must be a right triangle
    - Cylinders / Volume
        - Volume of Cylinder = pi * r^2 * h
- Coordinate Plane
    - Horizontal Line (x-axis) = zero slope
    - Vertical Line (y-axis) = undefined slope
    - Distance btwn two points can be calculated by using the Pythagorean theorem
        - Draw a right triangle connecting the points / find the diagonal (distance)
    - Parallel lines have the same slope
        - If two lines in a plane do not intersect, they are parallel
    - Perpendicular lines (e.g. perpendicular bisectors) have negative reciprocal slopes
- Extra Geometry (good for advanced quant)
    - Rhombus: two pairs of parallel sides, all equal sides, a type of parallelogram
        - Area of Rhombus = ($Diagonal_1$ * $Diagonal_2$ / 2)
        - Diagonals of rhombus' are always perpendicular bisectors (cut each other in half at a 90deg angle)
            - Diagonals of a square are also perpendicular bisectors
            - Both rhombus' and squares have all sides equal, but a rhombus is not a square unless all angles are right angles
            - A rhombus = quadrilateral with all sides equal
            - Square = quadrilateral with all sides equal AND all interior angles equal
                - Rhombus not a square unless all sides equal and all sides have interior angles
                - Square is always a rhombus
    - Maximum Area
        - Quadrilaterals
            - Of all quadrilaterals with a given perimeter, the square has the largest area
            - Of all quadrilaterals with a given area, the square has the minimum perimeter
        - Parallelograms / Triangles
            - Area of triangle = .5 bh
            - Area of parallelogram = bh

            - If given two sides, max area by placing those sides perpendicular to each other
        - Area of Equilateral Triangle (with side *S*) = S^2 * sqrt(3) / 4
        - Diagonal of square (with side *S*) = S *sqrt(2)
        - Diagonal of square cube (with an edge of *S*) = S * sqrt (3)
        - Diagonal of rectangular box (deluxe Pythagorean theorem) → $d^2 = l^2 + w^2 + h^2$
        - Sectors & Arcs of Circles
            - Sector = slice of pizza, arc = crust
            - Perimeter of sector = length of arc + 2r
        - Cylinders & Surface Area
            - SA = 2 circles + rectangle = $2(pi * r^2) + 2 * pi * r * h$
        - Function Graphs & Quadratics
            - Quadratic functions = parabolas
                - To find y-intercepts, set x = 0 and find y = f(0)
                - To find x-intercepts, find values of x for which y = f(x) = 0
                    - To see how many times the parabola touches the x-axis, look at the discriminant
                        - If discriminant > 0 → 2 solutions
                        - If discriminant = 0 → 1 solution
                        - If discriminant < 0 → no solutions
- Look at Geometry Cheat Sheet at end of book

MGMAT Strategy Set - Guide 5 - Number Properties

- Divisibility & Primes
    - Divisibility rules
        - 3 - sum of digits divisible by 3
        - 4 - divisible by 2 twice or last two digits divisible by 4
        - 6 - divisible by both 2 and 3
        - 8 - divisible by 2 thrice or last three digits divisible by 8
        - 9 - sum of digits divisible by 9

2 - If the last digit is even, the number is divisible by 2.

3 - If the sum of the digits is divisible by 3, the number is also.

4 - If the last two digits form a number divisible by 4, the number is also.

5 - If the last digit is a 5 or a 0, the number is divisible by 5.

6 - If the number is divisible by both 3 and 2, it is also divisible by 6.

7 - Take the last digit, double it, and subtract it from the rest of the number, if the answer is divisible by 7 (including 0), then the number is divisible by 7.

8 - If the last three digits of a number are divisible by 8, then so is the whole number.

9 - If the sum of the digits is divisible by 9, so is the number.

10 - If the number ends in 0, it is divisible by 10.

11 - If you sum every second digit and then subtract all other digits and the answer is: 0, or is divisible by 11, then the number is divisible by 11.
Example: to see whether 9,488,699 is divisible by 11, sum every second digit: 4+8+9=21, then subtract the sum of

- Factors / Multiples
  - Factor = positive integer that divides evenly into an integer
  - Multiple = formed by multiplying an integer by any integer
  - Integers are both factors and multiples of itself, and 1 is a factor of every integer
- Divisibility and + / -
  - If you add or subtract multiples of N, the result is a multiple of N
  - If N is a divisor of x and y, then N is a divisor of x + y
- Primes
  - First prime number is 2 (also the only even prime)
  - Memorize 1st 10 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
  - Breaking down bases into prime factors
    - e.g. If $10^x = (4^y)(5^z)$, is x even?
      - $(2*5)^x = (2^2)^y * 5^z = 2^{2y} * 5^z \rightarrow$ x = 2y and x = z (if x = 2y and y is an integer, than yes, x is even)
- Factor Foundation Rule: if a is a factor of b, and b is a factor of c, then a is a factor of c
  - If you can make a number a from some combination of multiplying the prime factors of b, then a is a factor of b
- Divisor vs. Dividend
  - 8 / 5 → 8 is the dividend, 5 is the divisor

- Odds, Evens, Positives & Negatives
  - 0 is neither positive or negative, but it is even
  - Arithmetic Rules
    - Addition / Subtraction: if they are the same, the sum / difference will be even; if they are opposite, sum / diff. will be odd
      - Even ± Even = Even
      - Odd ± Odd = Even
      - Even ± Odd = Odd
    - Multiplication: if one even number is present, product will be even; if only odd numbers present, product will be odd
      - Even * Even = Even
      - Even * Odd = Even
      - Odd * Odd = odd
    - Divisibility: must test out with real numbers
  - Multiplying / Dividing Signed Numbers
    - If you have an even number of negative signs, answer is positive
    - If you have an odd number of negative signs, answer is negative
    - Check for possibility of 0 being included in the set; if 0 is included, the above doesn't hold
- Combinatorics
  - The words "AND" and "OR" in "how many different combinations... " problems
    - AND = multiply
    - OR = add
  - The number of ways of arranging n distinct objects, if there are no restrictions, is n! (n factorial)

        - Memorize first 6 factorials
            - 4! = 24
            - 5! = 120
            - 6! = 720
    - When arranging members of a group, create an anagram grid
        - If there are 5 diff. types of candies and each candy box can hold 2 kinds, how many different candy boxes can we make?
            - 5! / (2! * 3!)
            - numerator = n!
            - denominator = 2 in each box, 3 not in each box
    - How many different ways can the letters in a word be re-arranged?
        - e.g. LEVEL
            - 5! / (2! * 2!)
            - numerator is factorial of number of letters (n!)
            - denominator is factorial of the number of each repeated later (2 L's and 2E's)
                - If no repeated letters, answer would just be n!
- Probability
    - Probability of "x" happening → often easier to calculate the denominator first
    - Just as in in combinatorics,
        - AND = multiply
        - OR = add
    - The (1-x) probability trick
        - P ($x \geq 1$) = 1 - P ($x = 0$)
        - When question includes *at least* or *at most* language, 1 - P(Not A) usually faster than P(A)
- Extra Divisibility & Primes (good for advanced quant)
    - 3 facts about Primes
        - There are an infinite number of primes
        - There is no simple pattern for primes
        - Positive integers with exactly 2 factors must be prime, and positive integers with more than 2 factors are never prime
            - Any integer ≥ 2 has at least 2 factors, 1 and itself
    - Divisibility & + / -
        - If you add a multiple of N to a non-multiple of N, the result is a non-multiple of N (same for subtraction)
        - If you add two non-multiples of N, the result could be either a multiple or non-multiple of N (exception is 2)
    - Greatest Common Factor (GCF) and Least Common Multiple (LCM)
        - GCM vs LCM
            - GCM = largest divisor of 2+ integers; GCF is $\leq$ starting integers
            - LCM = smallest multiple of 2+ integers; LCM is $\geq$ starting integers
        - Find GCF and LCM of 2 numbers by placing prime factors into a Venn Diagram
            - Ex: 30 and 24

        - 30 - 5
        - Both: 2, 3
        - 24: 2, 2
    - GCF = product of shared region → 2 * 3
    - LCM = product of all primes in the diagram
    - Exception: if 2 numbers have no primes in common
        - GCF = 1
        - LCM = product of 2 numbers
- Finding GCF / LCM of 3+ numbers or large numbers using Prime Columns
    - Calculate prime factors for each integer
    - Create column for each prime factor found within any of integers / row for each integer
    - Place prime factor raised to a power
        - GCF = lowest count of each prime factor found across all integers
        - LCM = highest count of each prime factor found across all the integers
    - E.g. → GCF / LCM of 100, 140 and 250
        - Number | primes
        - $100 = 5 * 5 * 2 * 2 = 2^2 * 5^2 * 7^0$
        - $140 = 7 * 5 * 2 * 2 = 2^2 * 5^1 * 7^1$
        - $250 = 5 * 5 * 5 * 2 = 2^1 * 5^3 * 7^0$
            - $GCM = 2^1 * 5^1 * 7^0 = 10$
            - $LCM = 2^2 * 5^3 * 7^1 = 3500$
    - Working backwards from a GCF or LCM →
- Counting Factors / Primes
    - GMAT can ask the following
        - How many diff. prime factors?
        - How many total prime factors (length)? → add the exponents of the prime factors (if no exponent, count as 1)
        - How many total factors? → see below
    - Counting Total Factors
        - First factor into primes
        - If a number has prime factorization $a^x * b^y * c^z$, the number has $(x+1)(y+1)(z+1)$ different factors
            - e.g. $9450 = 945 * 10 = 189 * 5 * 5 * 2 = 21 * 9 * 5^2 * 2 = 3 * 7 * 3^2 * 5^2 * 2 = 2^1 * 3^3 * 5^2 * 7^1$ → (2)(4)(3)(2) factors → 48 factors
- Perfect Squares / Cubes
    - 4, 9, 25 are perfect squares (squares of other integers)
        - ALL perfect squares have an odd number of total factors
            - Any integer that has an odd number of total factors must be a perfect square
        - Prime factorization of perfect square contains only even powers of primes

        - If a number's prime factorization contains any odd powers of primes, then the number is not a perfect square
    - Same logic above extends into all other perfect powers (e.g. perfect cubes)
        - All powers of primes are multiples of 3 in the prime factorization of a perfect cube
  - Factorials & Divisibility
    - N! must be divisible by all integers from 1 to N
        - 10! + 7 must be a multiple of 7, since 10! and 7 are both multiples of 7
        - 10! + 15 must be a multiple of 15, b.c. 10! is divisible by 5 and 3 and 15 is divisible by 5 and 3
  - Creating Numbers w/ Certain Remainders
    - Dividend = Quotient * Divisor + Remainder
    - Remainder must ALWAYS be smaller than the divisor
  - Asking if p is divisible by 168 is the same thing as asking
    - First, find prime factors of 168 → $2^3 * 3 * 7$
    - Is there at least three 2's, one 3 and one 7 in the prime factorization of p?
  - For consecutive multiples of N, the GCF is N
    - E.g. for any pair of consecutive multiples of 4, GCF is 4
- Extra Combinatorics & Probability (TODO look back at this section)
  - Arrangements with constraints
    - E.g. how many ways can you arrange people sitting next to each other, but two specific people can't sit next to each other
        - 1) Find number of ways ignoring constraint
        - 2) Use glue method
            - Pretend those two people are one person and solve for number of ways again
            - Multiply by 2 (since person can be sitting together A-B and B-A) and subtract from step 1
  - Domino Effect → when outcome of the first event affects the probability of subsequent event (e.g. NOT replacing marbles in a jar)
  - Combinatorics and Domino Effect
    - When you have a symmetrical problem (a problem with multiple equivalent cases), calculate the probability of one case (often via domino-effect rule), then multiply by the number of cases
  - Probability Trees can be useful to keep track of branching possibilities and winning scenarios

Table of Contents

GMAT Club - Quant Guide

Number Theory

- 0 is even; 0 is neither negative nor positive
- Prime = number that has exactly two distinct divisors: 1 and itself
    - Infinite of primes; 2 is the smallest and only even prime
- Factors
    - *m* is a factor of *n* if there exists an integer *k* such that *n* = *k*m*
    - Rules
        - If *a* is a factor of *b* and *a* is a factor of *c*, *a* is a factor of (*b*+*c*)
        - If *a* is a factor of *b* and *b* is a factor of *c*, then *a* is a factor of c
        - If *a* is a factor of *b* and *b* is a factor of *a*, then a = b or a = -b
        - If *p* is a prime number and *p* is a factor of *ab*, then *p* is a factor of *a* or *p* is a factor of *b*
    - Finding sum of factors of an integer
        - $n = a^p * b^q * c^r$ (a, b, c are prime factors; p, r, and r are powers)
            - Sum of factors of *n* = $[(a^{p+1}-1)*(b^{q+1}-1)*(c^r-1)] / [(a-1)*(b-1)*(c-1)]$
            - Number of factors = (p+1)(q+1)(r+1)
    - GCF = prime factor, then multiply common factors, picking the lowest power of the common factors
    - LCM = prime factor, then multiply common factors, picking the highest power of the common factors
    - Perfect Squares (e.g. 16 = 4*4)
        - Number of distinct factors and sum of distinct factors is always odd
        - Always has even number of powers of prime factors
        - Always has odd number of odd factors and even number of even factors
    - Divisibility rules
        - 6 - if number is divisible by 2 and 3
        - 12 - if number is divisible by 3 and 4
    - Factorials
        - 0! = 1
        - Finding number of powers of a prime number *p* in n!
            - $n/p + n/p^2 + \ldots$, while $p^x < n$
            - EX: What is the power of 2 in 25!
                - 25/2 + 25/4 + 25/8 + 25/16 = 12 + 6 + 3 + 1 = 22
- Consecutive Integers
    - If n is odd, sum of consec. integers always divisible by n
    - If n is even, sum of consec. integers NEVER divisible by n
    - Product of n consec. integers always divisible by n!
- Evenly spaced sets
    - a_1 = first number, a_n = last number
        - Mean = median = $a_1 + a_n / 2$
        - Sum of elements in evenly spaced set = $((a_1 + a_n) / 2) * n$

        - This is the mean (and median) multiplied by number of terms
- Exponents
    - Even exponents = 2 solutions
    - Odd exponents = 1 solution
    - Divisibility
        - $a^n - b^n$ ALWAYS divisible by a-b
        - $a^n - b^n$ divisible by a+b if n is even
        - $a^n + b^n$ divisible by a+b if n is odd
- Last digit of product = last n digits of the product of last n digits of those integers
    - Last 2 digits of 845*9512*408*613 = last 2 digits of 45*12*8*13 = 540*104 = 40*4 = 160 = 60
- Last digit of exponent
    - xyz^n has same last digit as z^n (xyz are digits, not multiplied)
        - E.g. 127^{39} has same last digit as 7^{39}
    - Find cyclicity of z^n to determine answer
- Roots
    - When GMAT provides square root sign of an even root, ONLY accepted answer is the POSITIVE root
        - sqrt(25) = 5, NOT 5 and -5
        - However, $x^2 = 25$ has 2 solutions, 5 and -5
            - e.g. when variables are present, take both solutions
    - Odd roots will have same sign as the base
    - Memorize the following sqrts (approximations)
        - sqrt(2) = 1.41
        - sqrt(3) = 1.73
        - sqrt(5) = 2.24
        - sqrt(6) = 2.45
        - sqrt(7) = 2.65
        - sqrt(8) = 2.83
        - sqrt(10) = 3.16
- Percent Change = Change/Original * 100
- Simple Interest = principal * r * time
    - Time must be in same units used for time in the rate
        - EX: To find interest earned after 9 months with simple annual interest, time would be (9/12)
- Compound interest
    - Final Balance = principal * $(1+ (r/c)^{time*c}$
        - C = number of times compounded annually
        - T = number of years

Absolute Value

- Properties
    - $|x| = sqrt(x^2)$
    - $|x| + |y| \geq |x+y|$

Table of Contents

Factorials

- Number of trailing zeros in a factorial
  - $n / 5 + n / 5^2$, while $5^k \leq n$
  - EX: How many zeroes are in the end of 32!
    - $32/5 + 32/25 = 6 + 1 = 7$

Algebra

  - WATCH OUT: When you divide both sides by a variable (e.g. cancelling out *x* from both sides), you are implicitly assuming the variable cannot be zero (since you can't divide by zero)
  - Degree of an expression = highest power of variables present in the expression
  - Quadratic equations ($ax^2 + bx + c$)
    - of solutions
      - $b^2 < 4ac$ = no solution
      - $b^2 = 4ac$ = exactly 1 solution
      - $b^2 > 4ac$ = 2 solutions
    - Sum of roots = -b/a
    - Product of roots = c/a
    - Reducing the degree → often easy to see a simple variable substitution can reduce the degree
      - EX: $x^6-3x^3+2 = 0$
        - Let $y = x^3$
        - Equation becomes $y^2 - 3y + 2$ → (y-2)(y-1) → solutions are 2 and 1
  - Useful Algebraic Identities

$$(x+y)^2 = x^2+y^2+2xy$$
$$(x-y)^2 = x^2+y^2-2xy$$
$$x^2-y^2 = (x+y)(x-y)$$
$$(x+y)^2-(x-y)^2 = 4xy$$
$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$
$$x^3+y^3 = (x+y)(x^2+y^2-xy)$$
$$x^3-y^3 = (x-y)(x^2+y^2+xy)$$

Remainders

- Helpful properties
  - When a smaller integer is divided by a larger integer, the quotient is 0 and the remainder is the smaller integer for POSITIVE numbers
  - If a number is divided by 10, remainder is last digit of number
    - If a number is divided by 100, remainder is last two digits of number…
    - … and so on
- For y/x, y = divisor * quotient + remainder → $y = xq + r$ or $y/x = q+r/x$
  - EX: If remainder is 7 when positive integer n is divided by 18, what is remainder when n is divided by 6?

        - n = 18q + 7
            - 18 is divisible by 6, so remainder will come from second term (7)
            - 7/6 = remainder of 1
    - If s/t = 64.12, what could be remainder of s/t?
        - s/t can be rewritten as → s = qt + r → s/t = q + r/t
            - r/t = 12/100 = 3/25 → r must be a multiple of 3
- Any positive integer can yield only 3 reminders when divided by 3: 0, 1 or 2
- Deriving general formula given two statements
    - EX: When x is divided by 5, remainder is 3. When x is divided by 7, remainder is 4.
        - x = 5q+3
            - Possible s: 3, 8, 13, 18, 23
        - x = 7q+4
            - Possible s: 4, 11, 18, 25
        - For general formula, divisor will be the least common multiple between the divisors
            - LCM of 5 and 7 is 35
        - Remainder will be first common integer in two patterns
            - Remainder in this case will be 18 (bolded above)
        - General formula = x = 35q + 18

Distance / Speed / Time (DST) Word Problems

- Distance = Rate * Time (rate and speed are same thing)
- Create table for D, R and T as columns and break down rows into components (e.g. two different trains, or 1 component going forward and 1 component going back)

Work Word Problems

- If A completes certain amount of work in X hours, then A would complete 1/X of the work in one hour (reciprocal)
- If A completes 1/X of work in one hour and B completes 1/Y work in one hour, TOGETHER they can complete 1/X + 1/Y of work in one hour
- 1/X + 1/Y = 1/Z → A and B working together will complete 1/Z of work in one hour
    - Therefore, working together, they will complete the work in Z hours (take reciprocal)
    - 1/X - 1/Y = difference between the amount of work X and Y complete in one hour (or whatever the unit of time is)

Advanced Overlapping Set Problems

- For problems with 2 sets, use a double set matrix
- For problems with 3 overlapping sets, use these formulas
    - Total = A+B+C - (sum of 2 group overlaps) + all three + Neither
        - A+B+C+-(all2)+a3+n
        - different than EXACTLY 2 group overlaps since it will include the overlap for all 3 groups
    - Total = A+B+C - (sum of EXACTLY 2 group overlaps) - (2 * all 3) + Neither

        - A+B+C-(ex2)-(2*a3)+n
        - Only use when given or asked for information regarding EXACTLY 2 group overlaps
    - With two groups
        - Total = A + B - Both + Neither

Triangles

- Area
    - A = bh/2
    - A = sqrt(s(s-a)(s-b)(s-c)), where s = a+b+c/2 (semi-perimeter of triangle) and a, b and c are the sides
- Perimeter
    - For given perimeter, equilateral triangle has the largest area
    - For given area, equilateral triangle has the smallest perimeter
- Sides vs. Interior Angles
    - Shortest side always opposite shortest interior angle
    - Largest side always opposite largest interior angle
- Similar Triangles
    - Only necessary to determine two sets of angles are identical in order to conclude two triangles are similar
    - If two similar triangles have sides in ratio x/y, then areas are in ratio x^2/y^2
- Types of Triangles
    - Equilateral
        - Area = a^2 * sqrt(3)/4
        - Perimeter = 3a
        - Height = a * sqrt(3)/2
        - Radius of circumscribed circle = a * sqrt(3)/3
        - Radius of inscribed circle = a *sqrt(3)/6 (exactly half of radius of circumscribed circle)
    - Isosceles
        - To find base given leg and altitude: B = 2 * sqrt(L^2 - H^2)
        - To find leg given base and altitude: L = sqrt(A^2 + $(B/2)^2$)
        - To find altitude given base and leg: A = sqrt(L^2 - $(B/2)^2$)
    - Right Triangle
        - Right triangle w/ given hypotenuse has largest area when it's an isosceles triangle
        - Right triangle can never be equilateral
        - Remember the following Pythagorean triples (all are right triangles; any ratio or multiple of the following are Pythagorean triples)
            - 3,4,5 (e.g. also 6,10,15)
            - 5,12,13
            - 7,24,25
            - 8,15,17
            - 9,40,41
    - 30-60-90

        - Sides are in ratio: 1:sqrt(3):2 (1 is the side opposite smallest angle)
    - 45-45-90
        - Sides are in ratio: 1:1:sqrt(2)
        - Area = $S^2/2$
    - Triangles and circles
        - A right triangle inscribed in circle must have hypotenuse as diameter; reverse is also true → if diameter of circle is also triangle's hypotenuse, then triangle is a right triangle
        - For a circle inscribed inside a right triangle → r = (a+b-c) /2

Polygons

- Sum of interior angles = 180(n-2), where n is number of sides
- Each interior angle = 180(n-2)/n
- Types
    - Parallelogram
        - Area = bh
    - Rectangle
        - Diagonal = sqrt(w^2+h^2)
    - Square
        - Square has a larger area that any other quadrilateral with the same perimeter
        - Diagonal = s*sqrt(2)
            - Area = s^2 or $d^2/2$
        - Circumscribed Circle: A = pi/2
        - Inscribed Circle: A = pi/4
    - Rhombus
        - Area = base*height
        - Area = d1 * d2 / 2 (diagonal 1 * diagonal 2 / 2)
    - Trapezoid
        - Bases = parallel sides
        - Legs = non-parallel sides
        - Median = average length of bases: ((B1 + B2) / 2)

Circles

- Finding Area or Circumference given the other
    - Circumference = sqrt(4 * pi * Area)
    - Area = $Circumference^2$ / 4pi
- Semicircle
    - Perimeter = r(pi + 2)
- Chord
    - Length = 2 * sqrt(r^2 - d^2)
        - r = radius, d = perpendicular distance from chord to center of circle
- Measure of inscribed angle always half of measure of central angle

Table of Contents

Coordinate Geometry

- Distance btwn 2 points = sqrt(dx^2+dy^2), where dx is diff. btwn x-coordinates and dy is diff. btwn y-coordinates
- Midpoint = avg. of x/y coordinates of 2 endpoints (x1+x2/2, y1+y2/2)
- Lines
    - General Form: ax+by+c
        - -a/b = slope, -c/b = y-intercept
    - Point-int. form: y = mx+b
        - Equation of line passing through point (x1, y1) → $y-y_1 = m(x-x_1)$
    - Every line (except one that crosses origin OR is parallel to either of the axis; these cross two quadrants) crosses three quadrants
    - Perpendicular lines = slopes are negative reciprocals of one another
- Circle on a plane
    - $r^2 = (x-a)^2 + (y-b)^2$, where (a,b) is the center of the circle and (x,y) is any point on the circle
        - If circle on the origin, $r^2 = x^2 + y^2$
- Number Line
    - Distance btwn p and m = distance between p and n → |p-m| = |p-n|
- Parabolas → to see how many times the parabola touches the x-axis, look at the discriminant (b^2-4ac)
    - If discriminant > 0 → 2 solutions
    - If discriminant = 0 → 1 solution
    - If discriminant < 0 → no solutions

Standard Deviation

- Decrease/increase in all elements of a set by a CONSTANT PERCENTAGE will decrease/increase standard deviation by the same percentage
- Decrease/increase in all elements of a set by a CONSTANT VALUE will NOT decrease/increase standard deviation of the set
- Adding more numbers to the set → first find the mean
    - The closer to the mean the numbers are, the greater decrease in standard deviation
    - The further from the mean the numbers are, the greater increase in standard deviation
    - Same applies for removing… are you removing numbers that are closer to the mean or further from the mean?
- Tips
    - Faster way is just to use average diff. btwn elements and mean
    - When you need to find what set has the LARGEST STANDARD DEVIATION, look for set with the LARGEST RANGE as a general heuristic
    - For a set of consecutive even integers, you just need to know the number of elements of the set to know the standard deviation (standard deviation will be the same regardless of what the numbers are)

Table of Contents

Probability

- Independent events
  - P(A and B) = P(A) * P(B)
- Mutually exclusive events (cannot occur at same time)
  - P(A or B) = P(A) + P(B)
- Combination of independent and mutually exclusive events
  - $P = C(n/k) * p^k * (1-p)^{n-k}$
  - Example: Probability approach = P(1) * P(2 given 1 occurred) * P(3 give n1 and 2 occurred)
- Probability trees are helpful when problems include conditions or restrictions
- Symmetry can be helpful
  - E.g. there are 5 chairs. Matt / Rachel want to sit such that Matt is always left of Rachel. How many ways can this be done?
    - Because of symmetry, the number of ways that Bob is left to Rachel is exactly ½ of all possible ways.
    - Answer = 5!/2!3! * .5 = 5

Combinations & Permutations

- Arranging objects in a ROW
  - For situations where you have to arrange objects in a ROW and two objects can't be next each other (or some restriction like that)
    - For small number of arrangements: Write out all possible ways to arrange and then find arrangements that satisfy question's condition
    - For larger arrangements: Number of arrangements of n different objects in a row
      - N = n!
  - Combination of unordered collection of k objects taken from a set of n distinct objects
    - C= n!/k!(n-k)!
  - Permutation of ordered collection of k objects taken from a set of n distinct objects
    - P = n!/(n-k)!
- Arranging objects in a CIRCLE
  - R = (n-1)!

Sequences & Progressions

- Arithmetic, Geometric and Harmonic (take inverse of every term) Progressions
  - Arithmetic Mean = (a+b)/2 OR $(a_1 + \ldots + a_n) / n$
    - $a_n = a_1 + (n-1)*\text{difference}$
    - Sum of n term AP w/ common difference d: = (n/2)(2a+(n-1)d))
  - Geometric Mean = sqrt(ab) OR $(a_1 * \ldots * a_n)^{1/n}$
    - $a_n = a_1 * r^{n-1}$
    - Sum of n term GP with common ratio r: $= a_1*(1-r^n)/(1-r)$
    - Sum of infinite term GP with common ratio r = $a_1/(1-r)$
  - Harmonic Mean = 2ab / (a+b)

- For all non-negative real numbers, AM $\geq$ GM $\geq$ HM

3-D Geometry

- Cube
  - Volume = a^3
  - Surface Area = $6a^2$
  - Diagonal Length = sqrt(3) * a
- Cuboid
  - Volume = abc
  - Surface Area = 2(ab + bc + ca)
  - Diagonal = sqrt(a^2+b^2+c^2)
- Cylinder
  - Volume = pi * r^2 * h
  - Outer surface area w /o bases = 2pi * r * h
  - Outer surface area w bases = 2pi*r(r+h)
- Cone
  - Volume: ⅓ * pi * r^2 * h
  - Outer surface area w /o bases = pi * r * l = pi * r * sqrt(r^2+h^2)
    - L = lateral height (height moving up the side of the cone)
  - Outer surface area w bases = pi * r * (r+l) = pi * r * (r+ sqrt(r^2+h^2))
- Sphere
  - Volume = 4/3 * pi * r^3
  - Surface Area = 4 * pi * r^2
- Hemisphere (half of a sphere)
  - Volume = 2/3 * pi * r^3
  - Surface area w /o base = 2 * pi * r^2
  - Surface area w base = 3 * pi * r^2

Table of Contents

GMAT Club - YouTube Videos (GMAT Ninja & Others)

Quant: Developing a flexible mindset

- Don't jump right into brute force
  - Identify options to solve problems, then decide what is the best solution path
  - GMAT is testing if you can find the best solution path to engage in
- Estimation may be the fastest way to save time
  - For example, can you identify that the answer HAS to be negative, and right off the bat eliminate any positive answers?
  - Or can the denominator not be zero, eliminating any answers for x in the denominator that make the denominator zero?
- Triangles
  - If you find a 45-45-90 or 30-60-90 triangle, use the 30-60 or 45-45 sides as base and height
- Weighted Average Method (Teeter Totter)
  - Relative distance from weighted average will tell you the ratio of the two mixes
    - 3% (rum)------3.5% (mix)----------5.5% (vodka)
      - We know there is more rum (since weighted more towards rum)
      - Find the ratio → .5 / 2 = ¼ → 1 vodka: 4 rum, 5 total
      - Vodka makes up ⅕ of the total

Quant: Percents, Ratios & Gift of Direct Translation

- Always double check; very easy to mess up
- Wording
  - 200% greater than = 3x
  - 200% of = 2x
- Be wary of decrease by x%, this would mean OG * (1 - x/100)
- Ratios all have a common multiplier
  - if you have grapes, raspberries and cherries in the ratio 3g:5r:7c, then you have 3m:5m:7m (m is the common multiplier)
    - If you know that there are 60 more cherries than grapes
      - Then c = 60 + g, or 7m = 60 + 3m, m = 15
    - If asked for raspberries, you know multiplier is m = 15, so 15*5r = 75

Quant: Rates, Age, and Beyond

- 1/A + 1/B = 1/C → if 1/C = 14/45, then it takes 45/14 (reciprocal) hours for both to complete
- All about translating!
  - Car travels 462 miles per tank of gas on highway, and 336 miles per tank of gas in city
  - If car traveled 6 fewer miles per gallon in city on highway, how many miles per gallon did car travel in city?

    - Convert miles per tank into miles per gallon
        - Tank has x gallons
        - 462 miles / x gallons = mpg of highway
        - 336 miles / x gallons = mpg of city
        - 462/x = 336/x + 6 (since city is 6 fewer than highway)
        - Solve for x, then plug back into 336/x

Quant: Long, Weird & Intimidating Questions

- Correlation found in a study → longer the question is, the easier it is, on average
- Even products = has at least one even
- Odd products = both odd
- Even
    - O + O = E
    - O - O = E
    - E + E = E
    - E - E = E
- Odd
    - E + O = O
    - O - E = O

Quant: Counting, Sets and Series

- When given a sequence, pause for a moment and identify what the sequence is saying
    - For example, if given that $a_k = a_{k-1} + 2a_{k-2}$, don't just start plugging numbers in… realize that this is just saying that a_k = the previous term + 2 times the previous-previous term

Quant: Geometry

- For all PS in general, look at answer choices to get an idea of any cues
- Funky shapes → try to break into shapes that you do know
- Geometry questions are often algebra questions in disguise
    - Recognize difference of squares
- How to quickly solve for consecutive perfect squares
    - $21^2 = 20^2 + 20 + 21$
    - $19^2 = 20^2 - 20 - 19$

Quant: Permutations, Combinations & Probability

- 0! = 1
- How many ways to arrange the word "Circle" such that two occurrences of the letter C are separated by at least one other letter?
    - Total - CC (together)
    - CC (together) → glue method → Treat CC as one letter → 5!

- Answer = 6!/2! - 5! = 240
- Directional grid
  - Number of paths to get from point a to b (e.g. 8 steps to get from A to B, that will require 5 norths and 3 easts, number of ways would be 8! / 5!3!)

Quant: Inequalities Made Easy

- Always work with question stem first with DS
  - Plugging in values should be a fallback
- Never multiply or divide by a variable unless you know the sign of the values
- Multiple or divide by x^2 → a square of a variable is always NON-NEGATIVE (can be positive or 0) → don't assume a squared variables is always positive
- Squaring inequalities (or even exponent)
  - Picture on number line, and think about smallest and greatest value for squared
  - $x < 3$
    - x^2 → greatest = infinite, smallest = 0
    - $x^2 \geq 0$
- Cubing (odd exponent) does not change sign, so you can just cube both sides
  - $x < 3 \rightarrow x^3 < 27$
- Max/min concept used for when finding xy, x + y, x -y (not division, however)
  - Line up signs of two inequalities and perform operations on extremes
- Can ALWAYS add inequalities as long as the signs are the same
  - Cannot subtract, multiple or divide (always)
- Quadratic inequality with ONE variable
  - Always keep the RHS as 0
  - Move all to LHS, and factor into product of numbers (e.g. (x-2)(x-3) < 0)
    - Always maintain squared terms (b.c. sign will always be known as long as we know it cannot be 0)
  - Place roots on number line
    - Start w/ extreme right-most root, start w/ positive, move from right to left and alternate between signs
    - Then, depending on what RHS of inequality is, choose that portion of the number line
    - < 0 = choose negative portion
- If we know $(x-y)^2 / (x-y) < 0$, then we know (x-y) must be negative since the numerator will always be positive
  - $x - y < 0 \rightarrow x < y$

Quant: Go from Great to Incredible on DS

- Adding inequalities good for combining two statements - as long as signs are facing the same direction
  - S1) z > -2y
  - S2) -2z > y
  - Combined) -z > -y → z < y

- However, you can never subtract or multiply inequalities

Quant: Cutting Corners

- Percentage wording
    - 200% GREATER THAN x = x (1+2) = 3x
    - 200% OF x = x * 2 = 2x
- Read the question twice, may be missing really small wording and make sure you are answering the right question
- If a question is over your head, SKIP IT! Be prepared to miss 40% of quant questions…
    - Don’t spend more than 2 minutes unless you actually are on the right path; otherwise, much better for your score to move past and focus on nailing the easy questions
- DS
    - More work you do with the original question the easier it will be
    - Only look at one statement at a time; don’t look at S2 while looking at S1, can cloud judgement

Table of Contents

Quant: Absolute Value

- Basic info
  - Degree of a variable shows you how much roots it has
  - Sqrt(25) always just +5
    - Root of any number squared is ALWAYS positive (never negative)
  - $x^2 = 25 \rightarrow$ x can be + or - 5
    - Power of x → 2 means there are 2 possible solutions
    - $\text{sqrt}(x^2) \rightarrow$ + or - x
    - Root of any variable squared is always + or -
  - If ab = bc, we cannot cancel out b from both sides unless we know that b cannot be zero
    - Should move all to one side and factorize
  - Sign of $x^2 \rightarrow$ always NON negative
    - Can be 0 or positive
- $|x| \rightarrow$ 0 or +'ve
- |constant| = always +
- $|x| = \text{sqrt}(x^2) \rightarrow$ important property
- $|a+b| < |a| + |b|$
  - If different signs, this equation will hold
  - e.g. ab < 0
- -x = |-x|
  - |-x| = 0 or +
  - -x = 0 or +, if 0 then x = 0, if + then x must be negative
  - so $x \leq 0$
- If given |something| = something, make sure to cross verify solutions
- Absolute value = distance from the origin on the number line
  - |x+2| = 5
    - Origin = -2, distance = + - 5
    - Solutions = 3, -7
  - $|x| < a \rightarrow -a < x < +a$
  - $|x| > a \rightarrow x > a$, or $x < -a$
  - |2x+3| < 9
    - -9 < 2x + 3 < 9
    - -6 < x < 3
  - |2x + 3| > 9
    - 2x + 3 > 9
    - 2x + 3 < -9
- Type 1: |something| = something
  - Make sure to cross verify
- Type 3: |something| = |something|
  - If you have absolute value on both sides (completely), square both sides and remove the absolute value
  - b.c. $|x| = \text{sqrt}(x^2)$
  - |2x+3| = |x+1|

        - $(2x+3)^2 - (x+1)^2 = 0$
        - Now you have difference of squares
            - $(a+b)(a-b)=0$
            - $(2x+3+x+1)(2x+3-x-1)=0$
- Fallback approach = plug in values (-2.5 to 2.5 in half steps)
- Example
    - Is $\text{sqrt}[(x-5)^2] = 5 - x$?
        - $|x-5| = 5-x$?
        - In other words, is $5-x = 0$ or $+$, $\rightarrow 5-x \geq 0$, is $5 \geq x$?
- Example 2
    - If $|x+2| = |y+2|$, what is the value of $x+y$?
        - S1) $xy < 0$
        - S2) $x > 2$, $y < 2$
    - Can break down question stem into
        - $(x-y)(x+y+4) = 0$
        - So either $x = y$, or $x + y = -4$
        - Both statements tell us x !== y, so both statements alone are sufficient

Table of Contents

Wizako Notes

Number Properties

Recurring Decimals

- Only terminating if denominator in most reduced form has ONLY 2 and 5 as prime factors (doesn't need to be same powers)

Tests of Divisibility

- High level
  - 4 - last two digits divisible by 4 (45664 → 64 divisible by 4)
  - 6 - divisible by 2 and 3
  - 8 - last three digits divisible by 8 (45512 → 512 divisible by 8)
  - 11 - compute sum of alternate digits → if diff. is either a multiple of 11 or is 0, number if a multiple of 11
    - 14641 → 1+6+1 → 8, 4+4 = 8, diff = 0 → divisible by 11
  - 12 - divisible by 3 and 4
  - 25 - last 2 digits divisible by 25
- Break number into two parts that do not have any common factor other than 1
  - Checking if divisibility by X and Y is sufficient to test for divisibility by XY
  - 12
    - 3 and 4 (GCF = 1), checking for 3 and 4 is sufficient to test for 12
  - 75
    - 3 and 25 (GCF = 1), checking for 3 and 25 is sufficient to test for 75
    - 5 and 15 (GCF = 5), checking for 5 and 15 is NOT sufficient since they share a common factor other than 1

Prime Numbers

- Prime numbers have only two factors: itself and 1
- Stop at next approximate perfect square
  - If number is 132, check all numbers up until 12 (not including 12)
  - Check for PRIME numbers up until 12 (don't need to check for all numbers)
  - If none of the prime numbers divides the number, it is prime

Prime Factorize

- Factors of a number:
  - Any number that divides a number w/o leaving a remainder is a factor / divisor
  - Quotient of division should be an integer

Properties of Squares and Cubes

- Perfect Square:
  - Important: All powers of prime factors for a perfect square are even
    - Odd number of total factors

    - Smallest number to multiplied w/ 856 to make it into a perf. square?
        - $856 = 2^3 * 107 \rightarrow$ powers need to be even, so need 2 * 107 = 214
- Perfect Cube: All powers must be even

GCF and LCM

- Fractions
    - LCM of fractions: LCM(numerators) / GCF(denominators)
    - GCF of fractions: GCF(numerators) / LCM(denominators)
    - Examples
        - ⅔ and 4/9
        - LCM = LCM(2,4) / GCF(3,9) = 4 / 3
        - GCF = GCF(2,4) / LCM(3,9) = 2 / 9
- Product of exactly two numbers = GCF * LCM of two numbers
    - a*b = GCF(a,b) * LCM(a,b)
    - For more than 2 numbers, this may or may not hold
- Coprime numbers
    - Co-prime = no factor in common other than 1 (GCF of 2 co-prime numbers is 1)
    - LCM of 2 co-prime numbers = a * b
    - A = GCF * f_1, B = GCF*f_2, where $f_{1 \text{ and } 2}$ are co-prime
    - Any two consecutive integers WILL be coprime
- For any 2 numbers, LCM if ALWAYS a multiple of the GCF

When to use LCM or GCF? (hints in language to know when to use)

- If you are taking two numbers and dividing it into smaller parts, likely a GCF question
    - Hotel has A treadmills and B ellipticals and divides them into several identical rooms with no equipment left over. What is the greatest number of exercise rooms the hotel can establish?
- If two actions repeat at different intervals, and we want to find when these actions occur together, likely a LCM question
    - Multiples → actions REPEAT → if two events occurring multiple times, likely LCM question
- GCF Theory
    - A = GCF * f_1, B = GCF*f_2, where $f_{1 \text{ and } 2}$ are co-prime
- LCM Theory
    - LCM(A,B) = $f_1 * f_2$ * GCF, where $f_{1 \text{ and } 2}$ are co-prime

Express Number as Product of 2 Factors

- Number of ways of expressing an integer as a product of 2 positive integers
    - # of ways = # of Factors / 2
        - Unless it is a perfect square (odd number of factors), in which case do (# of Factors + 1) / 2
        - # of ways to express integer as product of 2 DISTINCT positive integers = (# of factors - 1)/2
    - # of factors = (x+1)(y+1)(z+1), where prime factorization = $a^x*b^y*c^z$

- Perfect Squares
    - Perfect squares have an odd number of factors (and each prime factor has an even power)
    - If a number has a odd number of factors, it will be a perfect square

Sum / Product of Factors using Prime Factorization

- Finding sum of factors of an integer
    - $n = a^p * b^q * c^r$ (a, b, c are prime factors; p, r, and r are powers)
        - Sum of factors of n = $[(a^{p+1}-1)*(b^{q+1}-1)*(c^r-1)] / [(a-1)*(b-1)*(c-1)]$
        - Number of factors = (p+1)(q+1)(r+1)
- Finding product of factors
    - Product of factors of $n = n^{(\text{Number of Factors} / 2)}$

Expressing # as Product of 2 Coprime numbers

- Depends only on number of prime factors
    - If number N has x prime factors, ways to express N as product of 2 coprimes
    - $= 2^{x-1}$

Remainders

- When two (or more) numbers have same divisor
    - Remainder of sum of 2 numbers = Sum of 2 remainders
    - Remainder of product of 2 numbers = Product of 2 remainders
    - Remainder of diff. of 2 numbers = Difference of 2 remainders
- Remainder Math
    - If remainder $\geq$ divisor, simplify by dividing again by divisor and compute the remainder
    - If remainder < 0, add divisor back and compute the remainder

Properties of Remainders and Divisions

- If N/18 has a remainder of 17, how many unique values of remainders are possible when N is divided by 54?
    - 54 is the 3rd multiple of 18, so 3 possible values exist
- Remainder Sum Rule
    - Remainders obtained from dividing two numbers by common divisor, d, are 33 and 47.
    - When the sum of the two numbers is divided by the same divisor, remainder is 30.
    - What is the value of the divisor?
    - $r_{sum} = r_1 + r_2 - d$, ONLY if $r_1 + r_2 \geq d$,
        - $30 = 80 - d$, $d = 50$
- Remainder of 144 / d = 46, what should d be?
    - $d = 144 - 46 = 98$
- Remember
    - Remainder of Product = Product of Remainders
    - Remainder of Sum = Sum of Remainders
    - Remainder of Difference = Difference of Remainders
    - Then,

        - If remainder ≥ divisor, simplify by dividing again by divisor and compute the remainder
        - If remainder < 0, add divisor back and compute the remainder
- EX: What is remainder when 1247 * 1246 * 1249 is divided by 12?
    - Find remainder of each 1247, 1246 and 1249 / 12, and multiply
    - 11 * 10 * 1 = 110, which is ≥ 12, so divide again
    - 110 / 12 → 12 * 9 = 108, so remainder = 2

Finding Remainder when Dividing x^n

- Remainder when 3^{29} divided by 8?
    - When any number divided by 8, values for remainder can be 0 to 7
        - Same with any number, values for remainder for 3 are 0 to 2
    - For a^b / c, remainders will ALWAYS repeat with a pattern
- How to find the pattern
    - Observe result of dividing successive powers of 7 by 5
    - Use the fact that remainder of product = product of remainders
        - Product, Remainder
        - 7, 7/5 = 2
        - 7 * 7, = 2 * 7 = 14/5 = 4
        - 7^3 = 4 * 7 = 28 / 5 = 3
        - 7^4 = 3 * 7 = 21 / 5 = 1
        - 7^5 = 1 * 7 = 7 /5 = 2
    - In the above, see how you DO NOT have to actually calculate the powers of 7
    - Also, the moment you see 1 as a remainder, the pattern repeats (for all numbers)
- Make it a practice to compute equivalent negative remainders (easier numbers to deal with)
    - Remainder when 7^{27} is divided by 50?
        - 7 →
        - 7 * 7 → 49/50 → 49, or 49-50 = -1
        - 7^3 → -1 * 7 = -7, or -7+50 = 43
        - 7^4 → 7^2 * 7^2 = -1 * -1 = 1 (STOP here, you know pattern repeats every 4th since this is a 1)
    - If you find the equivalent negative remainder is -1, then you know than double that will be when the remainder is 1
        - Remainder when 5^{60} divided by 17?
            - Remainders can be anything from 0 to 16
        - If you do the above work, you'll see
            - 5^8 = remainder of 16, or negative remainder of -1
            - Thus, 5^{16} has a remainder of 1, and the pattern repeats every 16
        - $5^{60} = 5^{48} * 5^8 * 5^4 = 1 * -1 * -4 = 4$
- Key Learnings
    - Look for remainder of 1 or -1
    - For large divisors, jump in steps of powers of 2 (compute a^2, a^4, a^8, a^{16} and so on)
        - This works b.c. any + integer can be expressed as sum of powers of 2
    - EX: 3^{48} divided by 19

        - 3^2, 9
        - 3^4, 81/19, 5
        - 3^8, 25/19, 6
        - 3^{16}, 36/19, 17 OR -2
        - $3^{48} = (3^{16})^3 = (-2)^3 = -8$, -8 + 19 = 11

Polynomial Remainder Theorem

- When a polynomial in x, f(x) is divided by (x-a), remainder is f(a)
    - Expression that comprises x, where x is raised to a power that is a whole number
- Ex:
    - What is the remainder when $5x^4 - 6x^3 + 3x - 2$ is divided by (x+1)?
        - Divided by (x-a), so a = -1 becomes (x+1)
    - a = -1 in our case, so remainder is f(-11)
        - 5(1) - 6(-1) - 3 - 2 = 6
- More GMAT-like ex:
    - $2x^4 + 3x^3 + ax^2 - bx + 3$ divided by (x-1), remainder = 11. When it is divided by (x+1), remainder = 13. What are a + b?
        - f(1) = 11, f(-1) = 13
        - Plug in and solve for a and b using system of equations

Division of $x^n + y^n$ and $x^n - y^n$

- 3 rules - take the time to remember these!
    - If n is odd, $x^n + y^n$ is divisible by (x+y)
    - If n is even, $x^n - y^n$ is divisible by (x-y)
    - $x^n - y^y$ is divisible by (x-y) for both odd and even n
- EX: What is the remainder when $5^{46} + 3^{46}$ is divided by 34?
    - Rewrite with odd powers
        - $x = 5^2$, $y = 3^2$
        - $5^{23} + 3^{23} \rightarrow x^n + y^n$ is divisible by (x+y) → divisible by (25+9) → divisible by 34 → remainder is

Fermat's Theorem

- For any prime number p and natural Number *N* that is not a multiple of *p*…
    - Remainder of $N^{p-1} / p = 1$
    - EX: Remainder of $15^{18} / 19 = 1$ (19 is prime, 15 is not a multiple of 19)
    - EX: Remainder of $52^{100} / 101 = 1$ (101 is prime, 52 is not a multiple of 101)

Highest Power of Composite Number that Divides Factorial of a Number

- EX: Highest power of 6 that divides 10!
    - First, prime factorize
        - 6 = 3 * 2

- If prime factors are the same power, just find the power of the highest prime (how many of the highest prime is in the factorial)
  - 3 → 4
- If prime factors are different powers, find number of each prime
  - EX: Highest power of 24 that divides 50!
    - $24 = 2^3 * 3$
      - 3 → 50/3 + 50/9 + 50/27 = 22
      - 2 → 50/2 + 50/4 + 50/8 + 50/16 + 50/32 = 47
    - $2^{47} * 3^{22}$ divided 50!
      - Since $24 = 2^3 * 3$, we need to have three 2s and a 3 to get 24
      - How many $2^3 * 3$ are contained in $2^{47} * 3^{22}$
      - Rewrite as $(2^3*3)^{15} * 2^2 * 3^7$ → highest power of 24 = 15
- Thus, highest power of 6 is 4

Number of Trailing Zeroes

- How many trailing zeroes in 50!
  - Same thing as asking how many 5's are in 50!
  - 50/5 + 50/25 = 10 + 2 = 12

Unit Digit of Higher Power of Numbers

- 3 Patterns (unit digit of numbers raised to positive integer powers)
  - If unit digit is 0, 1, 5 or 6, all powers of such numbers end in 0, 1, 5 or 6, respectively
    - $(441)^{85}$ → will end in 1
  - If unit digit is 4 or 9, odd powers have same unit digit, even powers have same unit digit
    - $4^1 = 4$, $4^2 = 6$, $4^3 = 4$, $4^4 = 6$
    - $9^1 = 9$, $9^2 = 1$, $9^3 = 9$, $9^4 = 1$
  - If unit digit is 2, 3, 7 or 8, unit digit pattern repeats every 4th power

Permutation / Combination Basics

- Order
  - Order matters if AB and BA are two distinct outcomes (more outcomes than when order doesn't matter)
    - n!/(n-k)!
  - Order does not matter if AB is the same as BA. We will count them as one outcome.
    - n!/k!(n-k)!
- How to determine whether order matters?
  - Is selection done for the same reason or different ones?
    - If reasons are different, order matters. If the reasons are the same, order matters.
- Number of ways of reordering
  - r distinct objects → r!
  - r similar objects → 1 way

    - r objects, of which x are alike and others are different → r!/x!
- Order
    - Scenario 1: n!/(n-k)!
        - If all scenarios are unique (AB and BA are different)
            - Selecting a President and a Vice President
        - Sampling where ORDER MATTERS and WITHOUT replacement
    - Scenario 2: n!/k!(n-k)!
        - If all scenarios are alike (AB and BA are the same)
            - Selecting 2 class representatives
        - Sample where ORDER DOESN'T MATTER and WITHOUT replacement
    - Scenario 3: n^k
        - Sample where ORDER MATTERS and WITH replacement
- Standard Framework - Without Replacement
    - Step 1) Choose r elements from n in nCr ways
        - n!/k!(n-k)!
    - Step 2) Check whether order matters → compute number of ways of reordering
        - r distinct objects → r!
        - r similar objects → 1 way
        - r objects, of which x are alike and others are different → r!/x!
    - Total outcomes = Multiple Step 1 * Step 2
- How many 3 digit positive integers exist, sum of whose digits is odd?
    - Method 1: list out and try to find pattern (fastest way on GMAT)
        - 100 (O), 101 (E), 102 (O), 103 (E), 104(O)... 140 (O), 141 (E)
        - Realize that half of the sums are odd, half are even
        - 900 3 digit #s, half odd, so 450
    - Method 2: Combinatorics Method
        - _ _ _
        - First slot can be 1 to 9 (9)
        - Second can be 0 to 9 (10)
        - Third slot can take 5 possibilities (first 2 slots will either add to even or odd, and the third slot will have to be the opposite)
- Examples
    - Sum of all 3 digit positive integers that can be formed by rearranging digits of 246?
        - Numbers add up to 12, each number appears (6/3) or 2 times (6 ways to re-arrange 3 numbers) → 24
        - 24 appears 100, 10 and 1 place. Answer is sum of 100(24), 10(24), 1(24) = 2664
    - Sum of all 4 digit positive integers that can be formed by rearranging digits of 2248?
        - Now, since a digit repeats, number of ways to re-arrange is not just 4!
            - 4! / 2! = 12 → 12 / 4 numbers = 3 times each number
            - Sum is 16 → 48 in each of 1000, 100, 10, 1 place
        - 53,328
- Rearrangement of Letters (don't confuse POSITION with ORDER)
    - Rearrange GRAPHITE such that positions of vowels do not change?
        - CCVCCVCV → 3! * 5! (3 vowels, 5 consonants)

    - Rearrange GRAPHITE such that the order of vowels does not change?
        - Must go A, followed by I, followed by E
            - AIEGRTHP, AGRTIHEP
        - Vowels
            - 8C3 * 1 (choose r elements from n in nCr ways and multiple by number of ways of reordering, which in this case you cannot reorder the vowels)
        - Consonants
            - 5C5 * 5! (choose r elements from n in nCr ways and multiple by number of ways of reordering, which in this case is 5!)
    - Rearrange GRAPHITE such that vowels appear together?
        - Group AIE as one letter
        - How many ways can XGRPHT be rearranged? 6!
        - How many ways can AIE be rearranged? 3!
        - Answer is 6! * 3!
    - Rearrange GRAPHITE such that vowels appear as one unit and consonants appear as one unit?
        - AIE = X
        - GRPHT = Y
        - Ways to arrange X and Y = 2!
        - Ways to arrange X = 3!
        - Ways to arrange Y = 5!
        - Answer is 2! * 3! * 5!

Rates - Speed, Time, Distance & Work

Formulas, Units and Conversions

- Basic
    - 1 km = 1000m
    - 1 mile = 1.6km = 1000m
    - 1 hr = 60 minutes = 3600 seconds
    - 1 mile = 5280 feet
- Fast conversions to commit to memory - super useful!
    - Convert km/hr to meter/second → 1km / hr = 5/18 m/s
        - E.g. 36 km/hr = 36 * 5/18 = 10m/s
    - Convert mph to meter/second → 4/9 m/s
        - E.g. 54 mph = 54 * 4/9 = 24 m/s

Simultaneous Travel

- 2 friends A and B leave Cambridge and Boston at same time and travel towards each other from their respective origins at constant speed. They meet at a point between the two cities and then proceed to reach their respective destinations in 32 minutes and 50 minutes, respectively. How long did B take to cover the entire journey?
    - A ----- C ---- B

    - AC took t mins, BC took t mins
    - CB took 32 mins, CA took 50 mins
    - A travels at *a* km/min, B travels at *b* km/min
    - AC = a * t = CA = 50 * b
    - CB = 32 * a = BC = b * t
    - at = 50b, bt = 32a, solve for t (t = 40, so 40 + 50 = 90 mins)

Average Speed w/ Equal Distances

- Use Xiggi's Formula → 2ab/(a+b)

Relative Speed

- → ← = distance reduces at a + b (towards each other)
- ← → = distance increases at a+b (away from each other)
- ← ← = distance reduces at a-b (same direction)
- Relative Distance = Relative Speed * Time

Races (Ratios with DST)

- A gives B a start of 10m in a 100m race
    - Ratio of distances covered = 100:90 (A:B)
    - Time that A runs 1000m, B will run 900m
- A gives B a 5s start in a 100m race:

Mixtures

Basic Concepts

- 2 types of mixture questions
    - Type 1: Simple weighted average
    - Type 2: Find ratio given the average
        - Po bought 10kg of rice from 2 varieties, one costing $4 per kg and other costing $5 per kg. If the overall cost is $44, how much of the $4 per kg did he buy?
        - Algebra Method
            - A+B = 10 → 5A +5B = 50
            - 4A + 5B = 44
            - A = 6 (subtract two equations)
        - Alligation Method for Type 2
            - $44 / 10 = 4.4
            - 4 -- 4.4 --- 5
                - 5 - 4.4 = .6
                - 4.4 - 4 = .4
                - Ratio is in 6/4 for $4 to $5 (you know $4 is the higher number since 4.4 is closer to 4 than it is to 5)
- EX: On an average, 2 liters of milk and 1 liter of water are needed to be mixed to make 1 kg of Sweet A and 3 liters of milk and 2 liters of water are to be mixed to make 1 kg of Sweet B. If 130 liters of milk and 80 liters of water were used, how many kg of each sweet was made?

- Let kg of Sweet A = x
- Let kg of Sweet B = y
- To make 1kg of x, need 2x milk and x water
- To make 1 kg of y, need 3y milk and 2y water
- Total milk = 2x + 3y
  - 2x + 3y = 130
- Total Water = x + 2y
  - x + 2y = 80, or 2x + 4y = 160
- Subtract equations
  - y = 30, x = 20

Average / Statistics

Weighted Avg.

- Calculate with ease with large numbers
  - The average salary of a graduating class of 30 students is $8420 per month and that of another class comprising 20 students is $8438 per month. What is the average monthly salary of the students of the two classes taken together?
    - Strategy 1: Answer: 30(8420) + 20(8438) / 50 → hard to compute
      - Rewrite as 30(8420) + 20(8420+18), or 30(8420)+20(8420)+20(18), or…
      - 50(8420)+360) / 50
      - 50(8420)/50 + 360/50, or 8420 + 360/50, or 8420 + 7.2 = 8427.2
    - Strategy 2: Standard Framework (easier method)
      - Subtract smaller average from both averages and compute weighted average
        - Smaller average = 8420
        - Class 1: 8420 - 8420 = 0 → S = 30 * 0 = 0
        - Class 2: 8438-8420 = 18 → S = 20 * 18 = 360
        - Class 1 and 2: Sum of 2 classes = 0 + 360
          - Intermediate Weighted Avg. = 360/50 = 7.2
      - Add smaller average to weighted average to get final answer
        - 8420 + 7.2 = 8427.2

Combining Median / Range - Problems

- Odd number of items → middle number is Median
  - Which number is the middle number? N / 2, then round up
    - 3 numbers / 2 = 1.5, 2nd number is the Median
    - 11 numbers / 2 = 5.5, round up = 6th number is the Median
- Even number of items → average of middle two numbers is Median
- EX: Consider 11 distinct integers whose median is 90 and range is 60. The median for the five smallest integers is 65. What is the maximum range for the five largest integers?
  - x11 - x1 = 60

    - We want to find max value for x11 - x7, which is same as maximizing x11 and minimizing x7
        - Min. value for x7 = 91 (distinct, greater than x6 which is 90)
        - To max x11, max x1 (maxing x1 will push x11 out as far as possible)
        - Max value for x1 = 63 (since x3 = 65, then x2 = 64, then x1 = 63)
        - x11 = 60+x1 = 123
        - x11 - x7 = 32

Maximum / Medium Median - Framework

- Maximizing 1 number in a set = minimizing all other numbers
- Vice versa, minimizing a number = maximizing all other numbers
- EX: The arithmetic mean of five positive integers a, b, c, d, and e is 22, and a < b < c < d < e.
    - If e is 40:
        - We know Sum is 22*5 = 110, minus 40 → a+b+c+d = 70
        - Part 1: What is the greatest possible value of the median of the 5 integers? (aka c)
            - Minimize other terms: a = 1, b = 2, d = c+1 (since all are distinct)
                - Rewrite a+b+c+d = 70 → as 1 + 2 + c + c + 1 = 70
                - 2c = 66, c = 33
        - Part 2: What is the least possible value of the median of the 5 integers?
            - Maximize d = 39
            - Maximize a and b
                - Given a < b < c… trying to max a and b is basically trying to push them out right as far as possible while staying less than c…
                - AKA, b = c - 1, and then a = c -2
            - Rewrite a+b+c+d = 70
                - (c-2) + (c-1) + c + 39 = 70
                - Solving we get c = 34/3, or 11.33
                - Need to round up to get least integer value

Overview of Concepts

- Avg = S/N
    - Use standard framework almost like a DST table, but for NA = S
        - Number * Average = Sum
            - Assign variables to unknown numbers; can add and subtract number / sum columns, but NOT average
- Weighted average can be solved for given either Numbers OR Ratios
- Manipulating Median
    - If equal # of observations added to either side of median, median remains unchanged
    - If more observations added to left of median, median shifts to the left, and vice versa
- Manipulating SD
    - Adding or subtracting constant ‘k’ to each term does not alter SD of the group
    - Multiplying or dividing a constant ‘k’ to each term changes relative diff. by the same factor. So SD becomes k times the original SD.
- Mode = most frequent → if no number occurs more than once, there is no mode

Table of Contents

Interchanging digits of a 2-digit number

- Number AB (where a and b are digits) can be written as 10A+B → ex: 21 can be written as 10(2)+1
- Difference between 'ba' and 'ab = 9(b-a)
- Sum of ba and ab = 11(a+b)

Ratios

Basic Concepts

- Given two quantities, we can find the ratio
- Given the ratio, we cannot find the quantities

Combine Two Ratios using LCM of Common Term

- If given x:y = 3:4, and y:z = 5:6
    - Common term is y → rewrite both ratios using LCM of 4 and 5 (the two y values)
    - 3:4 → 15:20, 5:6 → 20:24
- Now, combined ratio of x:y:z = 15:20:24

Direct Proportion / Linked Proportion / Inverse Proportion

- Linked = relationships are linked, but not in direct portion
- Inverse Proportion

Percentages

Basic Concepts

- Percentage to fractions equivalents - common values
    - ⅙ = half of ⅓ = 16.66%
    - 1/7 = 14.3%
    - ⅛ = half of ¼ = .125
    - 1/9 = .11
    - 1/11 = .09
    - 1/12 = .083

Percentages & Mixtures

- EX: Fresh grapes contain 80% water by weight and raisins obtained by drying fresh grapes contain 25% water by weight. How many kg of fresh grapes are required to get 20 kg of raisins?
    - 20kg raisins → 5kg water, 15kg solids
    - 20% of fresh grapes is solids → 20% of Xkg of fresh grapes = 15? Find x
        - X = 75kg

Table of Contents

Manhattan Prep - Advanced Quant

Chapter 1: Advanced Principles

- When dividing, for the quotient to be odd, you must get rid of the even factors in the numerator (the denominator must cancel out all the even factors in the numerator)

Chapter 2: Strategies & Tactics

- Anytime you see the word "approximately" or a synonym, don't try to solve for the exact answer: estimate
- If a PS question does NOT say "the diagram is *not* drawn to scale", then it IS drawn to scale
- When working from the answer choices, start with B and then D (can give you a good direction of which answers to go and prevent you from wasting time checking out every single answer)
- Try to eliminate answers right off the bat based on whether it can be positive / negative, number of decimals, etc…

Chapter 3: DS Principles

- Assume nothing
    - Be wary of zero values, integers vs. fractions, negative values
    - In two way matrices, check if they implicitly give you information on how much "Neither" cell is
        - If they say that everyone has either one or both, then you can fill in 0 for neither
- If asking if x is odd, and statement gives something about x+2, also asking if x+2 is odd
    - (y+2)! / x! = odd positive integer
    - If a factorial divided by a factorial is an odd integer, all of the terms in the denominator cancel out and only one term can remain in the numerator, which must be odd

Chapter 4: DS Strategies & Tactics

- Check for constraints (checklist below):
    - Zero values?
    - Integers vs. Fractions?
    - Negative values?
- Hidden constraints
    - Countable items must be non-negative integer
        - Zero is only possible if it is possible for items not to exist at all
    - Many non-countable quantities must be non-negative numbers, though not necessarily integers
        - Side of triangles
        - Height of person
    - Some other non-countables are able to be negative
        - Profit of company

        - Growth rate of population
- Although testing numbers shouldn't be first resort, it may be necessary
    - Test the following types:
        - Negative, Zero, Positive
        - Odd, Even, Proper Fraction, Improper Fraction
    - Every integer and half integer from -2 and 2 would test all of the above cases
        - {-2, -3/2, -1, -½, 0, ½, 1, 3/2, 2)
        - May not need to test all of the 9 values… if problem says value must be positive, then 5 of 9 values can be ignored
    - Number line method and inequalities
        - Remember that solution set is an OR statement, not an AND statement
        - If given $a^2(a-1) < 0$, ignore the a^2 since it doesn't change the sign
            - $(a-1) < 0$ will have same sign as $a^2(a-1) < 0$, so use number line method with just $(a-1) < 0$ or $a = 1$
        - Alternatively, if given $a^2(a-1) < 0$, you can divide both sides by a^2 because you know a cannot be 0 and the sign won't flip because it is a positive value
            - Then will just be left with $(a-1) < 0$
- Advanced Guessing Techniques (from highest certainty to lowest)
    - If two statements tell you exactly same thing (after rephrasing), answer is either D or E
    - If two statements are clearly sufficient together, then eliminate E
    - If one statement is contained within (i.e. is a subset of) the other, then eliminate C and "broader statement" only
        - S1) X > 50!
        - S2) X > 10!
        - Any value that satisfies S1 also satisfies S2, but not other way around
        - Eliminate B (S2) and E
    - Spot a C trap
        - If it is very obvious that combined statements would be sufficient, but you can't eliminate possibility that one statement alone is sufficient, then eliminate C and E
    - Spot cross multiply inequalities
        - Be careful for signs, but sometimes you will be able to know the sign of the variable based on whether it is a countable or non-countable quantity
    - Judge by appearance
        - If information in statement has a structure and complexity similar to the question, and has the right ingredients (variables, coefficients), it's more likely to be sufficient than otherwise
            - Won't crack every case, but surprising how far this will get you
- For $y^3 < |y|$, any value less than 1 will satisfy the equation
- If asked whether product of 3 integers is even, only one of the variables has to be even for the product to be even

Chapter 5: Pattern Recognition

- Sequences

    - Some sums involve matching pairs that sum to same number (or even cancel each other out)
        - Sum of 1, 5, 8, 10, 11, 11, 12, 14, 17, 21
            - Start from 2 11's and move outwards, each pair sums up to 22
            - 5 pairs of 22 = 110
    - When asked for sum of first X elements in a non-arithmetic or non-geometric sequence, keep track of cumulative sum
- Remainder problems
    - Remainder whenever an integer is divided by 10 will always be the same as the units digit of the original number
    - Remainders can disguise an underlying pattern
        - Ex: A repeating cycle of 1,2,3,0 emerges from remainders when dividing numbers by 4

Chapter 6: Common Terms & Quadratic Templates

- Algebra
    - If a/b = ⅗, then (b+a)/a → b/a + a/a, where b/a is reciprocal of a/b or 5/3
    - If $1/(a^2-b^2) < b^2-a^2$ → b^2-a^2 is negative a^2-b → asking is $1/x < -x$?
- Perfect Squares to memorize (seeing these numbers should jump out)
    - $20^2 = 400$
    - $25^2 = 625$
    - $30^2 = 900$
- Perfect cubes (up to 10) - seeing these numbers should jump out
    - $3^3 = 27$
    - $4^3 = 64$
    - $5^3 = 125$
    - $6^3 = 216$
    - $7^3 = 343$
    - $8^3 = 512$
    - $9^3 = 729$
    - $10^3 = 1000$
- Common Powers
    - 2
        - 3 - 8
        - 4 - 16
        - 5 - 32
        - 6 - 64
        - 7 - 128
    - 3
        - 3 - 27
        - 4 - 81
        - 5 - 243
    - 4

        - 3 - 64
        - 4 - 256
        - 5 - 1024
- Common factors
    - If you have $7^{n+1} - 7^{n-1}$, you can pull out 7^{n-1} since $7^{n+1} = (7^{n-1}) * 7^2 = 7^{n-1+2}$
- Quadratic Templates
    - Many times difficult looking sums and differences are just difference or sum of squares
    - Square of a sum
        - $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
        - Ex: Sum of 9999^2 and $10{,}001^2 = (10^4 - 1)^2 + (10^4+1)^2 = 2(10^8 + 1^2) = 200{,}000{,}002$
    - Square of a difference
        - $(a+b)^2 - (a-b)^2 = 4ab$
- Quadratic Templates in Disguise
    - $198*202 \rightarrow (200-2)(200+2) = a^2-b^2 = 200^2 - 4 = 40000 - 4 = 39996$

Chapter 7: Visual Solutions

- Rubber Band Geometry
    - Figure out what combination of information cements the problem in place
        - What combination of information removes all flexibility (e.g. knowing point but not slope, knowing slope but not point = infinite number of lines, but if given slope and point, you know the exact line)
        - No flexibility = sufficiency
    - Typically, for rectangles , length is the longer side (vs. width) by convention
- Number Line Techniques
    - Changes in Sd
        - Moving terms away from mean increases Std. Dev. of list
        - Moving terms closer to mean increases Std. Dev. of list
- Maximizing or minimizing one term
    - Maximize one term by minimizing the other terms (and vice versa for minimizing)
    - Pay attention to whether terms must be different or terms can be equal
    - If you want to minimize the final term / largest term in a list, you want to maximize earlier terms, but in a way that doesn't make the final term too large (e.g. can make last two terms equal to each other)
- If you know the diagonal of a rectangle or square, you know the sides must be less than the diagonal
- Divisibility, Primes & Consecutive Integers
    - If a and b are consecutive positive integers, and ab=30x, is x an integer?
        - Rephase into $\rightarrow$ x = ab/30
        - For x to be an integer, *ab* must be divisible by 30
        - For a number to be divisible by 30, it must have 2, 3, and 5 as prime factors
        - Thus, the question becomes does ab have 2, 3 and 5 as prime factors?

        - Since a and b are consecutive positive integers, the product is automatically divisible by 2, so the question can be further simplified into does ab have 3 and 5 as prime factors?
            - S1) a^2 is divisible by 25
                - a^2 has 5 and 5 as prime factors
                - a has 5 as prime factor, but nothing about whether it has 3 as a prime factor
            - S2) 63 is a factor of b^2
                - b^2 has 7 and 3^2 as prime factors
                - b has 3 as prime factor, but nothing about whether it has 5 as a prime factor
            - C) Taken together, you know a has 5 as a prime factor and b has 3 as a prime factor

Personal Quant Notes / Error Log (Contains Questions from OG Problems/Tests)

- When starting any question… FIRST thing to do is look out for
    - FINDZ → which values are allowed?
        - Fraction
        - Integer
        - Negative
        - Distinct
        - Zero
    - These should jump out at you first
- It's not what it looks like!
    - Never, ever do hard arithmetic
        - Try to think like the testmaker… why would they write a problem a specific way?
        - Think about different perspectives (with geometry, literally look from different perspectives… turn figures around)
    - Key is to find the rule being tested
        - Find sum of all possible values of x for which $4^x - 2^{x+3} - 2^{x+2} + 32 = 0$
            - $2^{2x} - 2^x(2^3+2^4)+32 = 0$
            - $2^{2x}-2^x(12)+32 = 0$
            - Let $A = 2^x$
            - Becomes: $A^2 -12A + 32$ → quadratic, A = 8 or 4
                - $2^x = 8$ or 4 → x = 3 or 2, so sum of all possible values for x is 5
        - Greatest prime factor of $25^2 + 44^2 + 2200$?
            - Very rarely does the GMAT make you do such brute math to find the answer
            - Realize that the statement is written as $a^2+2ab+b^2$!!
                - $a^2+b^2+2ab = a^2+2ab+b^2 = (a+b)^2$
                - To quickly check if 2ab = 2200

        - 2*25*44 → 50 * 44 = 100 * 22 = 2200
        - THIS TYPE OF MATH WILL SAVE TIME AND EFFORT
    - $(25+44)^2$ → 69^2 → 69 is a factor → prime factorize → 23*3
- DS
    - When combining statements, plug in values is absolute last approach
        - Substitute one statement into the other OR use math. ops. btwn 2 statements
        - EX
            - Question: is x/18 an integer?
            - S1) 3x/18 = integer
            - S2) 5x/18 = integer
            - C)
                - Multiply S1 by 2 → 6x/18 = integer
                - Subtract statements → x/18 = integer → this is exactly what the question is asking!
    - Never discard a statement until proven insufficient
- Average Speed
    - AS = 2ab/a+b → Xiggi's formula (a and b are the two speeds, distance is the same in both directions)
    - EX:
        - Asked: is a > 40? (given a and b are positive)
        - If given: Average Speed = 80
        - AS = 80 = 2ab/a+b → ab/(a+b) = 40
            - ab = 40(a+b)/b → a = 40(a+b)/b → 40 * (a+b)/b
                - (a+b) / b > 1
- If DS asks is x > 0, all you need to find out from a statement is that x !== 0
    - If the statement shows that x cannot be zero, then it is sufficient (b.c. then x is either positive or negative)
    - Don't try to find the exact answer
    - For example
        - Given |x+3| = 4x - 3
            - You know that x != 0 and that the answer will either be +ve or -ve, but you don't need to go further to find the answer
            - You could also check → 4x-3 $\geq$ 0, so x $\geq$ ¾, so you know for sure x > 0 without solving for actual solutions
- Place value
    - Sum of all 3 digit numbers that can be constructed using 3, 4, 5 if each digit is used once?
        - 3*2*1 = 6 combinations, each number appears 6/3 or two times in the 100s, 10s and 1s place
        - 3 + 4 + 5 = 12 * 2 times = 24
        - 111*24
        - 100(24)+10(24)=1(24)=2400+240+24=2664
    - If numbers are distinct:

        - 1. Sum of all the numbers which can be formed by using the n digits without repetition is: (n-1)!*(sum of the digits)*(111…..n times)
        - 2. Sum of all the numbers which can be formed by using the n digits (repetition being allowed) is: n^{n-1}*(sum of the digits)*(111…..n times)
- If asked whether (x+y)(x-y) > 0, need to know signs of both x+y and x-y to know the sign of the product
    - Not enough to just know that x > y (which would give sign of RHS)
- Cardinality of a set is the number of distinct elements
    - Number of subsets possible for a set of n elements = 2^n (including the empty set & the set itself)
- For two-way tables, beware of overlaps / don't double count
- Number theory
    - If given 5x/18 = integer
        - Unless specified, don't assume X has to be an integer!
        - x could be 18
        - x could be 18/5
    - Sum of 3 diff. numbers = 3 * (x), then x = the mean,
        - Largest or smallest number can't be the mean, so median must = mean
    - If x is an integer, x^2 is a perfect square, x^3 will be a perfect cube
    - The sum of the reciprocals of consecutive integers 21 to 30 falls between what range?
        - Upper bound = 10 * 1/21 = 10/21 (if all 10 numbers were 1/21)
        - Lower bound = 10 * 1/30 = 10/30 = ⅓ (if all 10 numbers were 1/30)
        - So the sum, S falls between ⅓ < s < 10/21
- Precision of language
    - If given "the difference btwn M and J is 20", it could be EITHER (M-J) or (J-M) = 20
        - Unless you know that M is larger, don't assume this means (M-J) = 20
- Ratios
    - If ratio of X:Y is 3:4, then you know X is a multiple of 3 and Y is a multiple of 4
        - If question asks you to find Y, eliminate any answer choices that aren't a multiple of 4
        - START HERE with Ratio problems.. can quickly eliminate answers
    - EX: Po and Shifu had stamps in ratio 5:3. After Po gave 10 stamps to Shifu, the ratio became 7:5. As a result of the gift, how many more stamps does Po have than Shifu?
        - When you solve for the common multiplier in these scenarios, apply it to the ORIGINAL ratio, not the new one.
        - In this case, 5m-10/3m+10 = 7m/5m → m = 30
        - Po had 5*30 or 150 stamps, Shifu had 3*30 or 90 stamps originally
            - Don't forget to apply the transaction of 10 stamps: trap answer would be 60 (just saying 150 - 90)
            - Now, Po has 150-10 or 140, and Shifu has 90+10 or 100 stamps
        - It would be WRONG to say Po has 30*7 and Shifu has 30*5 stamps.
- Probability:
    - P(A U B) = P(A) + P(B) - P(A ∩ B)
        - Probability of A OR B if A and B are NOT mutually exclusive

        - a+b-(a*b)
    - If they are mutually exclusive, would just be P(A) + P(B)
  - $P(A \cap B) = P(A) * P(B)$
    - Probability of A AND B
- Teams playing games
  - Games played = n(n-1) / 2
  - # of games each team plays = n-1
- For absolute value equations, plug solutions back in to test!
- Reciprocals of inequalities
  - $x > y$, $1/x < 1/y$ if same sign (flip inequality), $1/x > 1/y$ if opposite signs
- Right triangles
  - 3, 4, 5 → 6, 8 ,10
  - 5, 12, 13 → 10, 24, 26
  - 8, 15, 17 → 16, 30, 34
  - 7, 24, 25 → 14, 48, 50
  - 9, 40, 41 → 18, 80, 82
- Divisibility rules
  - 4 → last 2 digits divisible by 4
  - 8 → last 3 digits divisible by 8
  - 12 → divisible by 3 and 4
- Ways to arrange ABCDE but A and C are separated by at least 1 letter? → 5! - 2(4!)
  - Multiply the second because AC and CA are separate
- Ways to arrange CIRCLE such that the two Cs are separated by at least 1 letter? → 6!/2! - 5!
  - Don't multiply second by 2 because the two letters are the same (diff. than example above with AC and CA)
- If given 13n/m is an integer, and m has to be between 4-12, you know that n must be divisible by m
  - If m is a value between 4 and 12, 13/m cannot be an integer (13 is prime, so only thing that divides it is 1 and 13), so n/m must be an integer
- If asked whether median = mean of a set, it is also asking whether the set is evenly spaced
- If given $y^2 = (x+1)^2$ → solutions are $y = (x+1)$ and $y = -(x+1)$
  - Don't forget that there are two solutions here!
- If given d is odd and that (x+y)/d and (x-y)/d are both integers → combining statements → 2x/d and 2y/d are both integers
  - For 2x/d to be an integer, x must be a multiple of d, since d is odd and it can't cancel out with the 2, so x must be divisible by d (same holds for y)
- If given that 360 chairs are to be set up in a rectangular arrangement with x rows of exactly y chairs each and the only other restriction is that $10 < x < 25$,
  - $360 = 2^3 * 3^2 * 5$
  - x is $\geq$ 11 and $\leq$ 24, so write out all the numbers and cancel anything that has anything other than 2, 3 and 5 as a factor
- If a sprinkler sprays in a circle with radius of 2
  - Diameter = 4
  - Greatest rectangular area that a sprinkler can cover = square area (square maximizes area)

        - Square in a circle → diameter of circle = diagonal of square = 4 = s * sqrt(2)
        - Side of square = 4/sqrt(2) → area of square = 16/2 = 8
    - Max rectangular area that 4 sprinklers can cover? = 8 * 4 = 32
    - If area of rectangle is 32, it could or could not cover the entire rectangle
        - If measures were 32 * 1, it could
        - If measures were 64 and .5, it could not
- In data sufficiency problems it is often helpful to consider contrasting extreme scenarios when they exist
    - If a variable can be any positive real number, then consider the scenario when the variable is very close to 0 and the scenario when the variable is very large
    - If a quadrilateral can be any rectangle, then consider the scenario when the rectangle is a square and the scenario when the rectangle is very long with a small width
- If nothing is given about a variable, only assumption we can make is that they are real numbers
    - E.g. they can be zero, decimals, fractions, negative
- If given x is divisible by all the factors of 27, then x = 27k, where k is an integer
- Given a quadratic equation: $ax^2 + by + c = 0$
    - The sum of the roots of a quadratic equation = $-b/a$
    - The product of the roots of a quadratic equation = c/a
- Triangle rules
    - Triangle side rule
        - Third side must be between sum and diff. of other two sides
    - Obtuse vs. Acute
        - If 'c' is the longest side, for an obtuse triangle, $c^2 > a^2 + b^2$
        - Conversely, if $c^2 < a^2 + b^2$, the triangle is acute
- If asked to compare square roots or fractions, often easier to compare their squared values
    - 3*sqrt(2) vs. 7/sqrt(3) → which is larger?
        - Compare squared terms
        - 18 vs. 49/3 → so 3*sqrt(2) is larger
- Hexagon
    - Diagonal = 2s
    - Diagonals break hexagon into 6 equilateral triangles
    - Given the side of each hexagon, can find the height of each equilateral triangle
- If given: A box of light bulbs contains exactly 3 light bulbs that are defective. What is the probability that a sample of light bulbs picked at random from this box will contain at least 1 defective light bulb?
    - Not enough to know what size the sample was → need to know the total number of light bulbs
- Sum of 1 to 4 digit numbers using only digits 0 and 1?
    - From 0000 to 1111, there are 4 places and each place can have digits 0 or 1.
    - So, there will be 16 integers in all (4 places, 2 choices per place = 2^4) and each digit will be repeated 8 times per place (16 integers / 2 choices = 8 times)
    - Hence, sum of digits per place is 8 and total sum of all the integers is 8888 (8 * 1111, or 8*(1000+100+10+1))

- If given radius = 7/16 inch, and that 1 inch corresponds to 2 miles in a photograph, and asked to find the surface area in square miles
  - 1 inch = 2 miles → 1 $inch^2$ = 4 $miles^2$
  - Or, first convert radius to miles → 14/16 = ⅞, then find area
- If given $(-2^n)^{-2} + (2^{-n})^2$ → this does not equal zero
  - Given that n is a positive integer, plug in 1 and find which answer choices matches the result… this will be the fastest way rather than trying to actually solve for what this equals
- If asked to find the roots given ax(cx+d)=−b(cx+d) is a quadratic, and a, b, c, d are nonzero real numbers
  - Realize that you are already given the roots!
    - ax(cx+d) + b(cx+d) = 0
    - (ax+b)(cx+d) = 0
    - x = -b/a, x = -d/c
- When order matters (ABC is different from CBA)
  - Number of ways to pick 3 out of 8 = 8!/3!
    - = n! / (n-k)!
- When order doesn't matter (ABC is same as CBA)
  - Number of ways to pick 3 out of 8 = 8!/3!5!
    - = n! / k!(n-k)!
- If given "a" and "d" are integers, and 2a = d + 10
  - Since 2a = even, d must also be an even integer
- To find # of factors less than a #, find number of factors of that # and subtract 1
  - x^2 divisible by exactly 4 positive integers < x^2
    - Means that x^2 → 5 total factors
    - Number of factors = $a^x b^y c^z$ → (x+1)(y+1)(z+1) → 5 cannot be product of 2 numbers greater than 1 since it is a prime number = 5*1
    - $x^2 = y^4$ → x = y^2 → 3 total factors, 2 less than x
  - 2x is divisible by exactly 3 positive integers less than 2x
    - Means that 2x → 4 total factors = 4*1 = 2*2
    - $2^1 * x^y$ = (2)(y+1) = 4
- Similar triangles
  - If two triangles have two angles with the same measure and share a common angle, the corresponding sides are proportional / in same ratio
- If given $(m/n)^{12}$ = an integer, and m and n are both integers
  - You know m/n must be an integer
  - No fraction, like 1/2 or 3/2, when raised to some positive integer power can give an integer
- Given n(n+1)(n+2), how many n's are divisible by 8 from 1 to 96?
  - Any integer n(n+1)(n+2) will be divisible by 8 if n is a multiple of 2 → 48
  - Will also be divisible by 8 if (n+1) is a multiple of 8 (n = 7, 15) → 12 multiples of 8
  - 60/96 = ⅝
- Given sum of reciprocals of consecutive integers, consider upper and lower bounds
  - Sum of reciprocals of consecutive integers 43 through 48

    - Upper bound = 6*1/43 = 6/43 ~ 1/7
    - Lower bound = 6/48 = ⅛
    - ⅛ < Sum < 1/7
- Divisibility by 3 can be easily checked without adding all the digits of a large number:
    - Ignore any digits that are a multiple of 3, or whose sums are a multiple of 3
    - For the number: 1k2k24 → how many values of k are possible?
        - Ignore 1,2
        - Ignore 2,4 (add up to 6)
        - So if k is a multiple of 3, then the number will be divisible by 3, else it will not be
        - There are 4 single digit multiples of 3: 0, 3, 6, 9
- Interest
    - In twelve years, the principle of 1,000 was multiplied by 4, so in twelve more years (24 years total) it would be multiplied by four again --- after 24 years, there will be $16,000.
    - To find how many years to get to 8,000
        - From 1,000 to 4,000 (4x) = 12 years
        - From 4,000 to 8,000 (2x) = 6 years
        - Thus, should take 18 years
- Tricky wording
    - If asked: How many people in Town X read neither the World newspaper nor the Globe newspaper?
        - S1) Of the 2,500 people in Town X, 1,000 read no newspaper.
            - DON'T assume that the World and Globe newspapers are the only newspapers!
            - It could be that the 1,500 people who read at least 1 newspaper all don't read either the World or the Globe (but read other newspapers), then the answer would be 2,500
            - Or, it could be that the 1,500 people who read at least 1 newspaper all read the Word and the Globe, then the answer would be 1,500
- Upstream/downstream
    - If given upstream took .5 hours longer than downstream
    - Set $t_{upstream} - t_{downstream} = .5 \rightarrow d/r - d/r = .5$
- $5^x - 5^{x-3} = (124)(5^y)$ → what is y in terms of x?
    - Factor out the 5^{x-3}, not the 5^x
    - $5^{x-3}(5^3-1) = 124(5^y)$
    - x-3 = y
- Area of parallelogram = base * height (bh) → not enough to know all the sides of the parallelogram!
    - Base can be taken either of the parallel sides
- Probability of getting the same outcome on 3 coin flips
    - NOT ½*½*½
    - CORRECT: 1 * ½ * ½ (e.g. first one can be either heads or tails, and then second and third have to be the same)
- From ft^2 and in^2 to inches per foot

    - If a room that has an area 576ft^2 is represented by a model that has an area of 2,304inch^2
        - 576ft^2/2304in^2 = 1ft^2/4in^2 = 1 ft/2inches = 2 inches/1ft (scale of model = 2 * actual room)
- For rectangles and squares, diagonals are congruent. However, diagonals of Rhombus are NOT congruent
- Overlapping Sets
    - Total = A+B+C - (sum of 2 group overlaps) + all three + Neither
        - A+B+C+-(all2)+a3+n
        - different than EXACTLY 2 group overlaps since it will include the overlap for all 3 groups
    - Total = A+B+C - (sum of EXACTLY 2 group overlaps) - (2 * all 3) + Neither
        - A+B+C-(ex2)-(2*a3)+n
        - Only use when given or asked for information regarding EXACTLY 2 group overlaps
    - With two groups
        - Total = A + B - Both + Neither
- Linear Diophantine → Ax+By = C
    - If AB < C, at least 1 solution
        - C/AB = number of solutions (if C/AB is btwn 1 and 2, then there is either 1 or 2 solutions)
    - If AB > C, exactly 1 solution
- AB = 10A + B, BA = 10B + A
    - Original - Reversed = 9(a-b) → AKA difference between AB and BA
    - Original + Reversed = 11(a+b) → AKA sum of AB and BA
- When giving ratios and changes, apply multiplier to original ratio and don't forget to apply any transformations/transactions
- To check whether a number is prime, check up to the closest perfect square
    - E.g. to see whether 143 prime, check numbers < 12 (e.g. starting with 11, since 144 is the closest perfect square > 143)
- GCF and LCM
    - A*B = GCF*LCM
    - A = GCF * f_1, B = GCF * f_2, where f_1 and f_2 are co-prime
    - Two numbers, dividing into smaller parts → GCF question
    - Two actions at diff. intervals, and want to find when they occur together → LCM question
- Max/Min concept of inequalities → can use for xy, x+y, or x-y, but NOT division
- Perfect squares: 2 properties
    - Even powers of all prime factors
    - Odd number of total factors
- Number line → if |C-A| and |C-B| are equal, then C is in the middle of A and B
- Changing Std. Dev.
    - Adding/subtracting constant 'k' to each number in set does nothing to standard deviation
    - Multiplying/dividing by constant 'k' for each number in set → SD becomes k times the original SD

- If given
  - Is $1/(a^2-b^2) < b^2-a^2$?
    - b^2-a^2 is just $(a^2-b^2) * -1$ → so question is really asking,
    - Is $1/x < -x$? If there is a current *c* in the stream flowing *downstream* and the boat moves at *k* mph in still water
- Current problems
  - Upstream rate = k - c
  - Downstream rate = k + c
  - Current is the same in up/downhill… will just either slow down boat or speed it up based on whether the current is flowing *downstream* or *upstream*
  - Sometimes, won't be given flat out which direction current is flowing, but other information will act as a clue
    - EX: It takes the same amount of time for the boat to travel 4 kilometers directly downstream as it takes for it to travel 3 kilometers directly upstream.
      - Notice it takes the same time to travel longer downstream than upstream = downstream is faster, so current must be flowing downstream
- If a DS question gives you two values for the same variable, try to set those 2 values equal to each other
  - EX:
    - (1) z = 32x
    - (2) z = 4y
    - When combining, set 32x = 4y, to find that y = 8x (which ended up being what you needed to find the answer)
- Quadrilaterals and rectangles
  - If opposite angles of a quadrilateral are 90 degrees, it does NOT mean its a rectangle (think of a kite)
  - To know its a rectangle, we need
    - All angles to be equal (90 degrees) and
    - The sides to be parallel with equal diagonals
  - Remember, squares are rectangles, but not all rectangles are squares
- Properties of different shapes
  - 3 Properties of rectangle
    - ALL angles are equal (and 90 degrees) (not just opposite angles)
    - OPPOSITE sides are parallel and equal
    - Diagonals bisect each other
  - 3 Properties of square
    - All angles are 90 degrees
    - ALL sides are parallel and equal
    - Diagonals bisect each other PERPENDICULARLY
  - 4 Properties of Parallelogram → AREA = b*h
    - Opposite angles are equal
    - OPPOSITE sides are equal and parallel
    - Diagonals bisect each other
    - Sum of any two adjacent angles is 180 degrees

    - 4 Properties of Rhombus (e.g. diamond; would be a square if all internal angles were 90 degrees) → AREA = (D1 * D2) / 2 = .5 * d1 * d2 = b * h
        - Opposite angles equal
        - ALL sides equal and parallel
        - Diagonals bisect each other perpendicularly
        - Sum of any two adjacent angles is 180 degrees
- Rotating lines
    - If two lines intersect and there angle is *theta* and you want to rotate them in opposite directions until they are perpendicular
        - First, rotate each line by angle *theta/2* to get them to be coincident
        - Then, rotate each line by 90/2 degrees in opposite directions (or 45) to get them to be perpendicular
    - Essentially, both lines combined have to rotate by *theta* then by 90 to get perpendicular, so each line is (90+theta)/2
- If something costs X dollars, it costs 100 * X cents
    - If something costs C cents, it costs C/100 dollars
- Sum of consecutive integers
    - Average * # of terms
        - (first + last / 2) * (last - first + 1)
- Sum of first N consecutive integers (e.g. first 200 integers)
    - n(n+1)/2, where n is the number of terms
- If after being fully reduced, a fraction's denominator has any prime factors OTHER than 2 or 5, it won't terminate
- Reciprocals of inequalities
    - If same signs, flip ($x < y$, then $1/x > 1/y$)
    - If diff. signs, don't flip ($x < y$, then $1/x < 1/y$)
- ALL Evenly spaced sets (not just consecutive integers) → average = first + last / 2
- 3 consecutive integers → one will be a multiple of 3, so whole thing will be a multiple of 3
- Common right triangles
    - Most common
        - 3-4-5
        - 5-12-13
        - 8-15-17
    - 7-24-25
    - 9-40-41
- Use central angle, not inscribed angle, when finding area of a sector
    - Central angle (from center / radius) is 2x inscribed angle (from diameter)
- Discriminant rule (b^2 - 4ac) → never really comes up in my experience
    - Discriminant > 0, 2 solutions
    - = 0, 1 solution
    - < 0, no solutions
- All perfect squares
    - Odd number of total factors
    - Sum of distinct factors is odd

    - Only even powers of primes
- Exponents and number of solutions
    - Even exponents = 2 solutions
    - Odd exponents = 1 solution
- Standard Deviation
    - Decrease/increase of all elements in set by
        - CONSTANT PERCENTAGE = change of Std. Dev. by that same percentage
        - CONSTANT VALUE = will NOT change Std. Dev. of set (will stay the same)
    - For a set of consecutive even integers, you ONLY need to know the NUMBER of elements in the set to find the standard deviation (doesn't matter what the numbers are)
    - Adding more numbers to a set
        - If closer to mean, greater decrease in Std. Dev.
        - If further than mean, greater increase in Std. Dev.
- Combinatorics
    - n!/k!(n-k)! for UNORDERED (majority of problems)
    - n!(n-k)! for ORDERED
    - (n-!)! for arranging in a CIRCLE
    - Arrange CIRCLE so that two C's are separated by 1 letter
        - 6!/2! - 5!
    - Arrange 6 people so that 2 don't sit next to each other
        - 6! - 2(5!)
- Median and Mean
    - If MEDIAN > MEAN, more than half the numbers are GREATER than the MEAN
    - If MEDIAN < MEAN, more than half the numbers are LESS than the MEAN
    - If MEDIAN = MEAN, the set is equally spaced, but we don't know anything about Std. Dev. unless we know they are consecutive integers and we know the number of terms
- x-y = diff. of squares = (sqrt(x) + sqrt(y))(sqrt(x)-sqrt(y))
- Remember that two-digit number AB can be re-written as 10A + B
- Median = mean = evenly spaced set
    - If 3 numbers, if smallest number same distance from middle number than largest number, it is evenly spaced, and median = mean
- Prime factorization of hard number → 143
    - If no easy divisibility rules are found, check every prime up to square root of the number (in this case, would check every number up until 12 b.c. $12^2 = 144$, which is larger than 143)
- If given two numbers and given their GCF and LCM
    - A * B = GCF * LCM
    - A = GCF * f_1, B = GCF * f_2, where the $f_{1 \text{ and } 2}$ are unique, co-primes
- 5! and above, last digit will always be 0 (starting with 120…)

Table of Contents

Question

Is 3x + 7y an integer?

I. $(x+y)^3$ is an even integer
II. $(x-y)^3$ is an even integer

-
    - Doesn't say that x and y have to be integers in problem stem
        - S1) if (x+y)3 = even integer, than x+y has to be even
        - S2) if (x-y)3 = even integer, than x-y has to be even
        - Combining (add /subtract two statements)
            - x+y = even
            - x-y = even
            - 2x = even → x has to be integer
            - 2y = even → y has to be integer
        - Therefore, 3x + 7y is definitely an integer
- Linear Diophantine Equations: Ax+By = C
    - Watch this video from GMAT Club; super niche concept, but can help save a lot of time
    - First, DOUBLE CHECK that values can only be integers… if not, then this statement doesn't work
        - But also, pay attention to whether they have to be positive or negative integers… both are Diophantine equations, but the exchange rate method will give you diff. answer
    - Ax + By = C AND all are restricted to integer values → can solve for number of combinations (whether it will be 0, 1 or > 1)
        - Given, 91y + 28z and that y and z must be integers, how many combinations of y and z are equal to 703?
            - 91y + 28z = 703
            - 7(13y + 4z) = 703 (703 is not a multiple of 7)
            - 7*(some number) = not a multiple of 7 → 0 combinations are possible
    - Takeaway
        - If we know there is not 0 solutions from the above, then check if AB < C
            - If AB < C, we know the equation must have at least one solution
                - To get number of solutions when AB is < C, take C/AB
                    - If C/AB is between 1 and 2, then there are 1 or 2 solutions
                    - If C/AB is between 2 and 3, then there are 2 or 3 solutions
                - To get exact solutions, use "Exchange rate" algo
                    - Begin by finding single easy solution
                        - if 7x + 5y = 63 →
                            - 63 / 35 = btwn 1 and 2 → either 1 or 2 solutions

- 63 is multiple of 7, so easy equation would be (9, 0)
- Exchange B copies of x for A copies of y
  - (9-5, 0+7)
  - Solutions are (x-nB, y+nA)
- Only stop if the statement is restricted to positive numbers of x and y (which if we are talking about people, toys, etc… it would)
- Example where x and y are integers, but don't have to be positive:
  - Given x and y are integers and 2x+9y = 57, what is the smallest possible value of $x^2 + 2xy + y^2$ (e.g. $(x+y)^2$)?
    - 24, 1 = one solution
    - 15, 3
    - 6, 5
    - -3, 7
    - -12, 9 → if you plug into $(x+y)^2$, will give smallest value from the above

- If AB > C, then there is exactly one solution

- To combine inequalities on DS, solve for range of one variable, then repeat, solving for the other variable
- a, b, c and d are consecutive integers such that product abcd = 5040. What is the value of d?
  - S1) d is prime
  - S2) a > b > c > d
  - 5040 → divisible by 9 and by 10 (2 of the consecutive integers)
    - 5040 = 9 * 560 = 9 * 56 * 10 = 9 * 7 * 8 * 10
  - So we know a, b, c and are the consecutive integers 7-10 (either all negative or all positive)
  - S1) Sufficient → primes can only be positive
  - S2) Insufficient (TRAP ANSWER) → don't forget that abcd could either be
    - 7,8,9,10, OR (d would be 7)
    - -7,-8,-9,-10 (d would be -10)
    - S2 would give you 2 different answers for d
- Two cards from a pile of 1-6 are selected with replacement. What is the odds that one of the two cards is 5?
  - Possibilities that add up to 8
    - (6,2) (2,6) (5,3) (3,5) (4,4) → (4,4) is ONLY ONE possible scenario, whereas (5,3) and (3,5) are considered two unique scenarios
    - You can't pick (4,4) 2 different ways in the GMAT (only 1 way)
    - So there are 5 possibilities, not 6 → answer is thus ⅖
- If a line is tangent to a circle at a specific point
  - There is only one line of tangency at any given point

        - If you are given that a line is tangent to a circle at point (X,Y) on DS, you know there is only one equation for that line
        - That line will be perpendicular to the radius / diameter
    - From any external point, we can draw two tangents to a circle
- Median = mean when set is evenly spaced
    - In set of 3 numbers, When smallest number is same distance from middle number than largest number
- Don't forget that numbers can be written as:
    - If given digits ABC → 100A + 10B + C
    - Example: If a 2 digit positive integer has its digits reversed, the resulting integer differs from the original by 27. By how much do the two digits differ?
        - Let AB be 2 digit positive integer
        - 10A + B = original
        - 10B + A = reversed
        - original - reversed = 9(A-B) = 27
            - A-B=3
- Max / min values
    - When you want to max one value, you want to minimize all other values
- For ratio problems, if given ratio of people, can only have whole people (no fraction of people) → may be that case that common multiplier has to be an integer (unless common multiplier is a fraction that still causes people to be even)
    - 8:6:2, given m < 2, m can = 1.5 → 12, 9, 3 still works…
    - 5:2:7, given m < 2, only m that guarantees all people will be a whole number is 1
- Working on the question stem in DS
    - If given x+y = 10 and asked is x > y,
        - Rewrite as
            - x = 10 - y
                - 10-y > y → 10 > 2y → y < 5
            - y = 10 - x
                - x > 10 - x → 2x > 10 → x > 5?
        - So this question is asking us whether y < 5 and x > 5?
- $\frac{0.99999999}{1.0001} - \frac{0.99999991}{1.0003} =$
    - Problems like these are often just using the concept of difference of squares and/or powers of 10
    - Try to rewrite as diff. of squares
        - E.g. → LHS side becomes $1 - 10^{-8} / 1 + 10^{-4}$, which becomes $(1 + 10^{-4})(1 - 10^{-4})/(1 + 10^{-4})$, top part cancels out and left with $1-10^{-4}$, and so forth with the RHS…
- 3D Objects
    - When fitting 3d objects into one another, it is not enough to know there respective volumes (must know l, w and h for each object)
- Average Speed WITHOUT Xiggi's Formula
    - Make a RTW table, calculate total time and total distance

    - AS = Total Distance / Total Time
- Border placed around photo
    - Area of border = just the border (not the entire border + photo)
    - Area of full thing = length of photo + 2(border width) * width of photo + 2(border width)
- Fractions and terminating decimals
    - If you have 2^x and 5^y in the denominator, turn them into pairs of 10 (same number of x's and y's… e.g 2^3 * 5^3)
    - Convert .2^4 into power of 10 → $(10^{-1} * 2)^4$
- Factoring hard numbers
    - Prime factorization of → 143
        - If no easy divisibility rules are found (e.g. divis. by 2, 3, 6, 8, 9)
        - Check every prime up to square root of the number (in this case, up to < 12)
            - 11 * 13
- GCF and LCM
    - If given exactly 2 numbers (needs to be a pair), and you are given both the GCF and LCM, then the product of the two numbers is the product of the GCF and LCM
        - A * B = GCF * LCM
            - GCF * f_1 * GCF * f_2 = GCF * LCM
    - If given the GCF between 2 numbers, A and B, you know that
        - A = GCF * f_1
        - B = GCF * f_2
        - Where f_1 and f_2 are unique, co-primes (only integer that evenly divides them both is 1)
- Ex: Trap with combining algebraic statements
    - a and b are positive integers, what is the value of a?
    - s1) 4a-3b = 6
        - Not sufficient → 4(3) - 3(2) = 6 and 4(6) - 3(6) = 6
    - s2) 3a+5b = 22
        - Sufficient → 5b will be 5, 10, 15 or 20
            - 22 - 5 = 17 → not divisible by 3
            - 22 - 10 = 12 → divisible by 3 (only value of a that works)
            - 22 - 15 = 7 → not divisible by 3
            - 22 - 20 = 2 → not divisible by 3
    - Trap answer is thinking that you need to combine the statements
- Thinking like a number theorist
    - Smallest positive integer n for which n! / 18^8 is an integer
        - Factor 18^8 into (2^8 and 3^16)
        - How many factors of prime number p does n! have?
            - n / p + n/p^2… until p < n
    - If x is the smallest positive integer not prime and not a factor of 50!
        - 51 IS a factor of 50! (since 51 = 17*13, and 17 and 13 is in 50!)
        - First non-factor of 50! after 50 is 53 (first prime)

            - While we can't use 53 (since it's prime), we CAN use anything contains 53 (e.g. 53 * 2 = 106) → this is the smallest number that is not prime and not a factor of 50
    - 0 is an integer and is even → however, it is neither positive nor negative
        - 0 is also a multiple of everything
    - Perfect squares → prime factorization must contain only even powers
    - Perfect fourth power → prime factorization must contain only powers that are multiples of 4
- Fractions / percentages are not even numbers
    - Only integers can be odd or even
- 5! and above → last digit is always going to be 0
    - 120, 720, 5040, etc...
- How many ways can you pick 2 people out of x people?
    - x! / 2!(x-2)! → (x)(x-1)/2!
    - Simplified: x(x-1) / 2!
    - Pick 3 people out of x would be → (x)(x-1)(x-2) / 3!
        - x! / 3!(x-3)! → (x)(x-1)(x-2) / 3!
- Backsolving is the best way forward on a lot of 700-800 level questions
    - Start with Option C, and then move towards easy numbers to work with
- For number line problems, if given that the distance between points C and A is the same as the distance between C and B, that means that C is in the middle of A and B
- Be more careful of what language is used
    - EX: A circular jogging track forms the edge of a circular lake that has a diameter of 2 miles. Johanna walked once around the track at the average rate of 3 miles per hour. If t represents the number of hours it took Johanna to walk completely around the lake, which of the following is a correct statement?
        - The total distance travelled is not 2 miles (this was the mistake that I made) → 2 miles is the diameter of the circle. Total distance is the circumference of the circle, or 22/7 * 2
- $|x - y| > |x| - |y|$ is true if x and y have the opposite signs
    - However, $|x| + |y| \geq |x+y|$ ALWAYS (always equal, sometimes greater)
- Sum of infinite term GP with common ratio $r = a_1/(1-r)$
- Sum of n terms in AP
    - Sum of n term AP w/ common difference d: = (n/2)(2a+(n-1)d))
    - Sum of terms a_{10} through a_{18} = sum of a_{18} - sum of a_9
- Work
    - General formula for calculating the time needed for THREE workers working simultaneously to complete one job is the reciprocal of their respective rates
    - Take reciprocal of 1/A + 1/B + 1/C
- Median / Mean
    - If median > mean, then more than half the numbers are greater than the mean
        - This means that on average they must be closer to the mean than the numbers below the mean in order for everything to balance at the mean.

    - If Median = Mean, set is equally spaced, but that doesn't mean they are consecutive integers, so we know nothing about the std. dev. just from this fact
    - Adding two means together
        - Adding a number greater than the mean will increase the overall mean
        - Subtracting a number lesser than the mean will reduce the overall mean
    - Simple vs. Actual Average
        - Average people/branch * Average AUM/branch != Total AUM
            - These are simple averages… need to use a weighted average that takes into account the actual people per branch
                - Actual people per branch * Avg. AUM at that branch
            - To get actual AUM, would need to weigh each group by number of people in each branch
                - EX: (Actual People Branch A * Avg. Branch A) + (Actual People Branch B * Avg. Branch B) / # of people in A and B
        - Average AUM/branch = Total AUM/# branches
            - If we know the # of branches, we know the total AUM
- 200% increase from 200 is 600
    - % increase = Change in Value / OG
    - 2 = Change in Value / 200
    - Change in Value = 400 → new value = 200 + 400 = 600
    - Or, New Val = 200*(1+2) = 600
- If you have a set [3, 6, 9, 12, 15, 18 . . .] for each day
    - And you know that on exactly 9 days, the cars is a multiple of 9
    - You know that multiples of 9 occur every 3 days… that happens 9 times
    - So if that happens, there is a minimum of 27 days and it has to be < 30 days (which would be 10 days where there is a multiple of 9)
    - Thus, answer could be 27, 28, or 29 days
- If annual interest is 8% compounded quarterly, quarterly interest is 2% and half-year interest is 4%
    - We could use compound interest formula to find minimum principal he needs to invest to make $100 of interest after 6 months, but to save time
        - If we know that after 6 months, interest is 100…
        - Then 4% (half-year interest rate) * Principal = 100
        - Principal = 2500
- Smart numbers
    - EX: Since we are looking for when ¼ of X occurs, use X = 4 and we are looking for when X < 1
        - Each year, X has X * $5/7^n$ left (loses 2/7 per year)
        - Look for when new X is < 1
- Subsets
    - 100 Married couples
    - If given: 75 have > 1 child AND 40 have > 3 children
        - Then → Couples that have 2 or 3 children = 75 - 40 = 35
        - P(X > 1) = 75 and P(X > 3) = 40

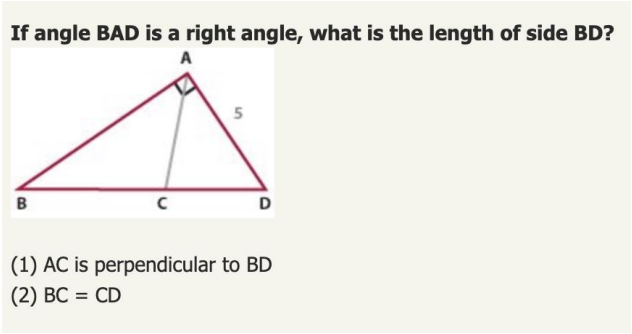
        - P(X > 3) is a subset of the 75
        - So the difference is how many people have > 1 but <=3 (i.e. 2 or 3) = 75-40 = 35
- Ratios
    - All ratios employ a common multiplier
        - The ratio by weight, measured in pounds, of books to clothes to electronics in Jorge's suitcase initially stands at 8 to 5 to 3.
            - 8:5:3 can be written as 8x/5x and 5x/3x
        - If removing 4 pounds of clothing doubles the ratio of books to clothes, then
            - 8x / 5x-4 = 16x / 5x → solving for X (and assuming it can't be 0), X = 1.6
            - Then we know that electronics weigh about 4.8 pounds, or 3 * 1.6 (1.6 is the common multiplier)
- In year x, it rained on 40% of all Mondays and 20% of all Tuesdays. On what percentage of all the weekdays in year x did it NOT rain?
    - (1) During year x, it rained on 10% of all Wednesdays.
    - (2) During year x, it did not rain on 70% of Thursdays and it did not rain on 95% of all Fridays.
    - Combined
        - Even though combining we get the annual rain percentages for each rain day, we don't know how many of each week day there was in a year (there are not an equal number of each weekday in a year)
- Geometry

If $\frac{1}{a^2} + a^2$ represents the diameter of circle O and $\frac{1}{a} + a = 3$, which of the following best approximates the circumference of circle O?

- 28
- 22
- 20
- 16
- 12

    -
        - When you see a question like this, try to put 1/a + a into the form of $1/a^2 + a^2$ by squaring both sides
    - Distance a car travels = Circumference per tire * # of rotations
    - Just knowing the height of an equilateral triangle, you can solve for the side length, and thus the area
        - Break triangle into 2 30 60 90 triangles and solve in ratio: x : sqrt(3) *x : 2x for 30 : 60: 90
            - REMEMBER: x would give you the length of half the equilateral triangle, so side length is x * 2
        - $A = s^2 * \sqrt{3}/4$ or A = ½ * b * h

- To double volume of a cylinder, since V = pi * r^2 * h, 2V = 2pi * r^2 * h… want to double either the r^2 term of the h term
  - Either double height (100% increase) → 2h
  - Or double r^2 → 41% increase of radius → 2*r^2 = sqrt(2) * r = 1.41r (41% increase)
- If a line from a vertex to the opposite side is both perpendicular to it and bisects it then this side is a base of an isosceles triangle



  - Statement 2 means that AC bisects BD
  - Statement 1 shows that AC is perpendicular to BD
  - Thus, AC is a perpendicular bisector and ABD is an isosceles triangle
    - BD is hypotenuse, since it is opposite the right angle
    - BD = x * sqrt(2) (45-45-90 triangle)
- Definitions
  - Is quadrilateral ABCD a rhombus?
    - Diagonals are perpendicular bisectors
    - All sides are equal
  - Is quadrilateral a rectangle?
    - Knowing that the diagonals of quadrilateral ABCD (i.e. AC and BD) bisect one another establishes that ABCD is a parallelogram, but not necessarily a rectangle
    - If a parallelogram has one right angle, all of its angles are right angles (in a parallelogram opposite angles are equal and adjacent angles add up to 180)

- Number Properties
  - Is |x| < 1 ?
    - Can be rewritten as is -1 < x < 1
  - Difference between sales / unit and cost / unit = profit per unit = total profit / # units
    - # units = total profit / profit per units (and you know # units is an integer), so profit total profit has to be divisible by profit per unit
  - Never multiply or divide inequality by a variable (or by an expression with variable) unless you are sure of its sign since you do not know whether you must flip the sign of the inequality.
  - Can only square both sides of an inequality if you KNOW for sure that both sides have the same signs
    - If both sides positive, don't flip
    - If both sides negative, flip signs

    - |a| + |b| > |a+b| holds true ONLY when and b have opposite signs
        - In all other cases, |a| + |b| = |a+b|
    - Note that in some cases we'll be able to find x, as there will be only one solution for it, but generally when we are told that there are n ways to choose x out of m there will be (in most cases) two solutions of x possible.
- If there had been x + 1 individuals in the group, exactly 56 different 3-person teams could have been formed.
    - Translation: There are 56 ways to choose 3 people out of x+1 people
    - (x+1)! / (x+1-3)! * 3! = 56
        - (x+1)! / (x-2)! * 3! = 56
            - (x+1)! = (x-2)! * (x-1) * x * (x+1)
            - (x-1)(x)(x+1) = 56 * 3! = 6 * 7 * 8
            - x = 7
    - Suppose we are told that there are 10 ways to choose x people out of 5 → there are 1 or 2 solutions
    - Suppose we are told there are 10 ways to choose 2 people out of x → there is ONLY 1 solution
        - x!
        - x!(x-2)!
- Which of the following is the lowest positive integer that is divisible by the first 7 positive integer multiples of 5?
    - Lowest positive integer that is divisible = what is the LCM of first 7 positive integer multiples of 5?

Table of Contents

Verbal

SC: Common Idioms

If written in the form UPPERCASE vs. normal case vs. normal case, then the uppercase is the correct usage and the alternatives are common trap answers

For example:
CONSIDER vs. Consider to be vs. Consider As
This signals that "Consider" is the correct usage, without "to be" and without "as" attached to it

- Such as vs. Like
  - Such as (examples)
  - Like = comparison (similar to)
  - Never use "like" to introduce examples... use "Such as" or "Including"
- Like vs. As
  - Like (comparison, used to compare nouns)
    - - 2 things are similar
  - As (comparison, used to compare verbs)
    - - 2 actions are similar
- Due to vs. Because Of
  - TRICK: Replace with "caused by" and "as a result of" for due to vs. because of:
  - Due to (caused by) = modifies nouns (more likely to be incorrect)
  - Because of (as a result) = modifies verbs (more likely to be correct)
- Whether vs. If
  - Whether: Alternatives (I am not sure whether the oven is off).
  - If: Conditionals (I will turn it off IF it is on)
  - WRONG: I am not sure if the oven is off (is wrong, since it means if the oven is off, then I am not sure)
- Between vs. Among
  - Between: Groups of 2
  - Among: Groups of 3+
- The more so
  - The more so = even more so
  - Examples:
  - 1. The play was impressive, [all] the more so because the students had written it themselves. (all is not necessary)
  - Same as saying: The play was impressive, even more so because the students had written it themselves.
  - 2. Like the Dreyfus affair at the turn of the century and the Sacco-Vanzetti trial in 1921, the Hiss case became the political cause of an era, the more so because it was the first time a congressional hearing was shown on television.
- CONSIDER vs. Consider to be vs. Consider As
  - RIGHT: I consider Karen a friend.

    - WRONG (Consider as / to be): I consider Karen as a friend / I consider Karen to be a friend.
- REGARD AS vs. Regard to be
    - Regard AS: The park is regarded as a landmark.
    - WRONG (Regard to be): Regard to be (The park is regarded to be a landmark)
- VIEW AS vs. View to be
    - View AS: Many view the rule as unfair.
    - WRONG (View to be): Many view the rule to be unfair.
- PREFER TO vs. Prefer Over
    - Prefer to: I prefer pasta to pizza.
    - WRONG (Prefer over): I prefer pasta over pizza.
- ESTIMATE TO BE vs. Estimate at
    - Estimate at: MY dog was estimated to be 4 years old.
    - Wrong: The shelter estimated my dog at 4 years old.
- PROHIBIT FROM vs. Prohibit that vs. Prohibit to
    - Prohibit from: I was prohibited from X.
    - WRONG: The bar prohibits that anyone under 21 sits at the bar.
    - WRONG: The bar prohibits anyone under the age of 21 to sit in the area.
- FORBID TO vs. Forbid from vs. Forbit that
    - RIGHT = Forbid to
    - WRONG: The test forbids students from using mechanical pencils.
    - WRONG: The test forbids that students use their own mechanical pencils.
- DIFFERENT FROM vs. Different than
    - DIFFERENT FROM is always the right usage in comparisons
    - EX: My shoes are different FROM Deven's.
- DISTINGUISH X FROM y vs. Distinguish between x and y
    - Distinguish... from = right
- WHETHER OR vs. Whether X or also Y vs. Whether they be x or y
    - WHETHER OR = right
    - E.g. -> Whether X or Y -> RIGHT
    - (not "whether X or also y", not "whether they be x or y")
    - Whether or not is also INCORRECT. (considered redundant)
- PRODUCTION AMONG vs. Production by
    - Production among = right
    - WRONG: Production of clothing by women increased.
- BOTH X AND Y vs. Both X as well as Y
    - Both X and Y
    - Both X as well as Y is WRONG
- INSTEAD OF vs. Instead of with
    - Instead of = right
    - Instead of with = wrong
- USED TO vs. Use to
    - USED To = right
    - Use to = wrong

Table of Contents

- Countable vs. Non-countable
    - MFN = countable
        - Many, fewer (than), number of (MFN)
    - MLA = uncountable
        - Much, less (than), amount (MLA)
    - Little (can be used for both, but means different things for each)
        - Countable items (size)
        - Uncountable items (quantity)
- AN AMOUNT AS MUCH AS vs. An amount equal to
    - An amount as much as = correct
    - Equal = characteristics and traits (e.g. wrong in this case)
- NUMBERS GREATER THAN vs. Numbers more than
    - Numbers = greater than
    - Would not use "more than" with the word "Numbers"
    - Correct Usage: The numbers from this year are greater than those from last year.
- INCREASED DEMAND FOR vs. increased demand in
    - INCREASED DEMAND FOR = correct
- DECREASED AVAILABILITY vs. less availability
    - Less availability would only be correct if there was a "than" -> less availability than
- ADVOCATE FOR or ADVOCATE X AS THE MEANS [of/to] Y vs. Advocate for the means of X
    - RIGHT:
        - - ADVOCATE FOR
        - - ADVOCATE X AS THE MEANS [of/to] Y
    - WRONG:
        - - Advocate for the means of x
- APPEAR AS vs. APPEAR TO
    - Appear as = followed by noun
    - Appear to = followed by verb
- TEN TIMES AS MUCH/MANY AS vs. Ten times more than
    - TEN TIMES AS MUCH/MANY AS = correct
    - INCORRECT to use "MORE THAN" with numbered comparatives
    - 5 times as much as (correct)
    - 5 times as many as (correct)
    - 5 times more than (incorrect)
- MORE LIKELY THAN vs. more likely that
    - More likely than = correct
- AS LIKELY AS vs. as likely for
    - As likely as = correct
- TO TRY TO vs. to try and
    - "To try to" = correct
- PREJUDICE AGAINST vs. prejudice towards
    - Prejudice "against" = correct
- NOT SO MUCH... AS vs. Not so much... but

    - Must be parallel
    - RIGHT: Not so much "because of" X as "because of" Y
    - WRONG: Not so much "because of" X as "the results of" Y
    - WRONG: Not so much... but =
- The word "Hopefully" on the GMAT
    - Must be used at beginning of sentence or beginning of an independent clause and be set off by the use of commas
- Each other vs. one another
    - Each other = 2 things
    - One another = 3+ things
    - However, if meaning of sentence is not to show reciprocal actions, than cannot use "each other" or "one another"
    - MORE EXPLANATION:
        - Can only use these two when you are trying to show reciprocal actions.
        - RIGHT: Each company seeks to meet consumer needs and wants more successfully than the others
        - WRONG: Each company seeks to meet consumer needs and wants more successfully than one another
        - The latter is wrong because meeting needs is not something that they are doing with each other.
- WITHOUT CONSIDERING THAT vs. overlooking that
    - Without considering that = right
    - Overlooking that = wrong

Table of Contents

Example Verbal Scratch Pad Setup

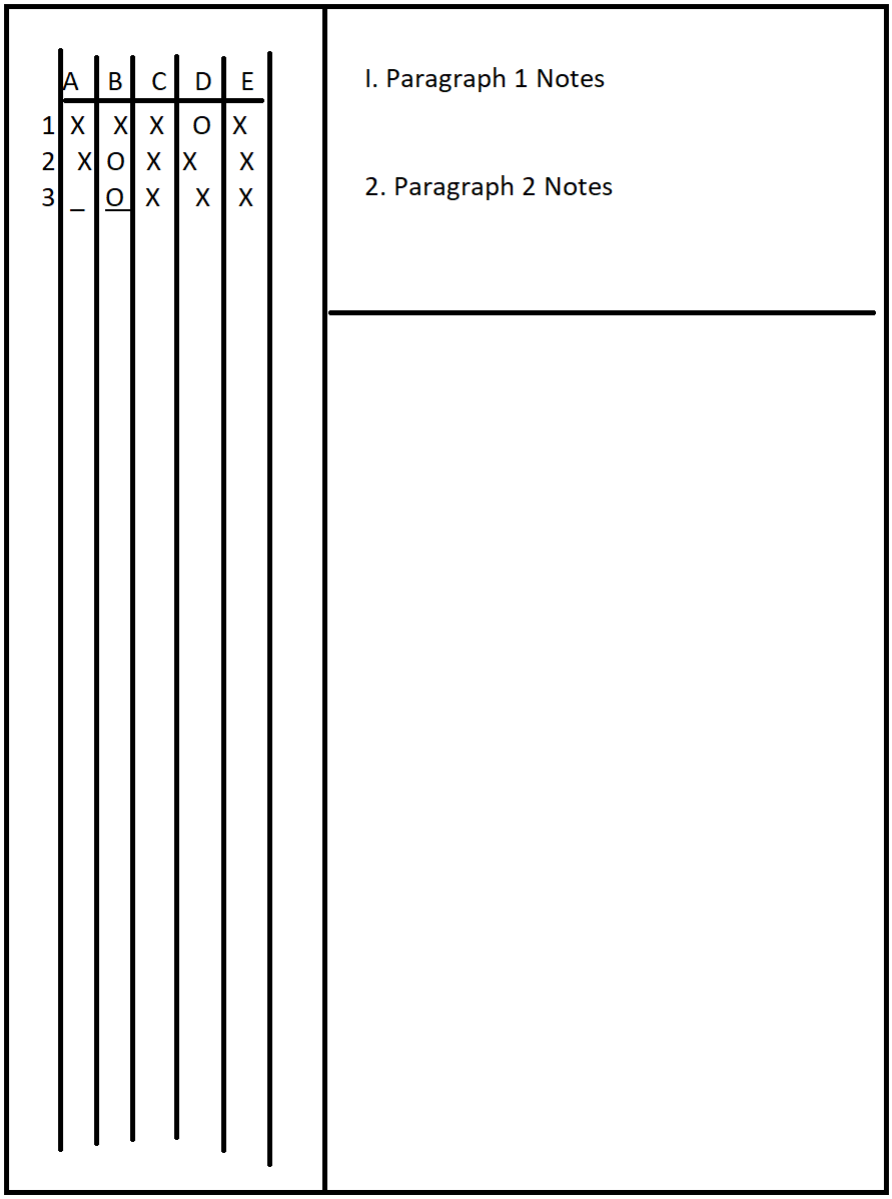


Table of Contents

Manhattan Prep - Strategy Set - Verbal

MGMAT Strategy Set - Guide 6 - CR

- Argument Structure
  - Pay attention to signal words (cause / effect, concession, contrast, premise)
  - When there is more than one conclusion or claim, use the THEREFORE test (because = intermediate conclusion, supports the actual conclusion, i.e. therefore)
    - Because A, Therefore B… vs.
    - Because B, Therefore A
  - 5 components of an argument
    - Premise = piece of evidence (fact or claim) that supports conclusion
    - Final conclusion = main claim
    - Intermediate conclusion = claim and a premise; supports final conclusion
    - Background Info
    - Counterpoint / counter premise: goes against conclusion
- Methodology
  - 3 types of CR questions
    - Structure-based
    - Assumption-based
    - Evidence-based
  - Work from wrong to right (dump “No way!” answers first)
- Structure-based family
  - “Describe the Role” of boldface font
    - Classify statement in boldface as
      - Conclusion (main claim)
      - Premise (support’s conclusion)
      - Something else (counter premise, background info, acknowledgement of weakness)
    - If there are two statements; ask whether are they on the same side of the fence, or opposite sides?
    - Is statement fact or opinion? (“evidence” = fact)
    - Common Trap
      - “One Word Off” → Tempting trap answers tend to be “off” by just one word, typically at end of sentence
        - E.g. supports a claim that the author as a whole argues against (for) → against = support a counter conclusion, for = support a conclusion
      - “Half Right” → if two statements, description of first one is correct but second is incorrect
  - “Describe the Argument”
    - Common Trap = “One Word Off”
- Assumption-based Family
  - Find the Assumption

        - If the correct answer were not true, the argument would not be valid
        - Assumption doesn't have to be true in the real world; only has to be true in the mind of the author
        - When stuck on a few choices, try negating answer choices and see what breaks down the author's argument
        - Common Traps
            - Answers that have no tie to the conclusion
            - Answers that use reverse logic (does the opposite of what you want; e.g. makes an argument worse when you are looking for a strengthener)
            - Answers that make an irrelevant distinction / comparison
    - Strengthen and Weaken
        - Strengthen
            - New piece of info that if added will make argument somewhat more likely to be true
        - Weaken
            - New piece of info that if added will make argument somewhat less likely to be valid
        - Common Traps
            - Reverse logic (strengthens instead of weakens)
            - No tie to the argument
            - Be wary of EXCEPT questions (e.g. all of the following weaken the conclusion EXCEPT)
                - Does not have to do exact opposite; in example above, the "odd one out" does not have to necessarily *strengthen* the argument, it could simply have no impact on the argument
                - For these, label each response as either W, S, or N (weaken, strengthen, neutral / does nothing)
    - Evaluate Argument / Find the Flaw
        - Evaluate questions (e.g. "in evaluating the claim", or "in evaluating the recommendation") → find an answer that will help determine whether or not the conclusion is likely to be valid
            - Correct answer has 2 paths:
                - one will make conclusion a little more likely to be valid,
                - the other will make conclusion a little less likely to be valid
            - Exception to above, is questions with EXCEPT
                - All wrong answers will take down two "paths", except the odd one out will NOT take me down two paths
            - Watch out for extremes in arguments: solely, exclusively, only, etc…
- Evidence Family
    - Inferences (can also be worded as "what can be logically concluded)
        - On the GMAT, inference is something that <u>absolutely must be true</u> according to evidence given in argument
            - Look out for extremes ("can" happen vs. "must" happen)... <u>extremes are less likely to be true</u>

                - Be wary of generalizations (e.g. generalizing the “flu” to “all illnesses”)
                - Don’t be distracted by “real-word” statements
            - EXCEPT questions → correct answer will be one that does NOT have to be true
        - Explain a Discrepancy
            - Answer should resolve / fix the discrepancy and make all the information fit together → correct answer will be one that does NOT resolve / fix the discrepancy
- Wrong Answer Analysis
    - No Tie (most common in assumption questions) → no tie to argument or conclusion (doesn’t affect either)
    - Reverse Logic (does the opposite of what you want; e.g. strengthens instead of weakens)
    - Diversion (irrelevant distinctions, real-world distractions, or switching terms)
    - Close but No Cigar (One Word Off or only “Half Right”)

MGMAT Strategy Set - Guide 7 - RC

- The foundation
    - First / last paragraph matter the most
        - For each paragraph, first and last sentence matter the most
        - Try to keep track of main idea of each paragraph
    - Pay less attention to specific details, focus more on WHY info is present
    - Common Traps
        - Question is asking what does FIRST paragraph show, and an answer choice—while correct—is not discussed until the THIRD paragraph… don’t fall for the trap, stay in scope
- RC grouped into two major categories: general questions (primary purpose / paragraph) and specific questions (detail / inference / specific purpose)
    - General Questions
        - Direct contradictions
        - Extreme words *without support* in passage
        - One word off
        - Out of scope (not mentioned in the passage)
        - True but not right (e.g. passage may say this, but doesn’t answer question asked)
- General Notes
    - While correlation !== causation, correlation can still mean that something is more or less likely to occur

MGMAT Strategy Set - Guide 8 - SC

- SC Process
    - SC tests for both grammar AND meaning… don’t forget about meaning!
        - A sentence can be grammatically correct and yet illogical or ambiguous
    - First word of five answers will always contain at least one difference: this is known as the “split” among answers

    - "Best Answer" may not be ideal… will be correct grammatically, but may feel overly formal or even awkward… expect some correct answers to not sound that good
- Grammar & Meaning
    - 5 Grammar Terms
        - Clause: set of words that contains a subject and a working verb
        - Modifier: provides additional info in a sentence beyond core subject / verb
        - Sentence Core: bare minimum needed to have a coherent sentence (any independent clauses with some essential modifiers)
        - Conjunction: words that help stick parts of sentences together
        - Markers (not official grammar term, but important for GMAT): words that flag or clue a certain kind of issue is being tested
            - *Unlike*: think about comparisons
    - Meaning
        - Be wary of "cousin" words; words that sound similar, but have diff. meanings (e.g. economic means monetary, economical means thrifty / efficient)
        - Another example: the course ruled the plaintiff SHOULD pay damages (court cannot impose moral obligations…)
        - Match your words → certain parts of the sentence must match up
        - Avoid redundancy → no right answer on the GMAT will contain redundant words (e.g. *rose* by a 10% *increase*, the prices *sum* to a *total* of $100)
- Sentence Structure
    - Subject and Verb must agree in number
    - The importance of THAT
        - WRONG: I know Rick is an actor. (missing *that* can make the sentence ambiguous; do you actually know Rick himself, or do you know something about Rick?)
        - RIGHT: I know that Rick is an actor.
    - Answers that connect two independent clauses using only a comma are wrong…
    - Semi-Colons
        - Semicolons connect two closely related statement; each statement must be able to stand alone as an independent sentence
        - Often followed by transition expression (e.g. however, therefore)... still need a semicolon, commas are incorrect
            - WRONG: Bo and Django are inseparable, therefore, we never see them apart.
            - RIGHT: Bo and Django are inseparable; therefore, we never see them apart. (comma after therefore is optional)
    - Non-essential vs. essential clauses
        - Commas that enclose a dependent clause make it a nonessential clause… e.g. if the clause was removed, the meaning of the sentence wouldn't change
            - WRONG: People, who talk loudly on their cell phones in crowded trains, show little respect for other passengers. (the clause enclosed in commas is essential!)

- RIGHT: People who talk loudly on their cell phones in crowded trains show little respect for other passengers.

- Modifiers
  - Modifiers should be as close as possible to the nouns they modify
  - Noun Modifiers w/ Relative Pronouns
    - *Who* and *Whom* must modify people; nothing else can (e.g. not *which*)
      - WRONG: People that are well-informed do well.
      - RIGHT: People who/whom are well-informed do well.
    - *Who vs. Whom*
      - When to Use *Who/whoever*
        - In a sentence, who is used as a subject. If the subject of the sentence is performing the action, use who.
          - RIGHT: Who would like to go on vacation?
          - RIGHT: Who made these awesome quesadillas?
      - When to Use *Whom/whomever*
        - Whom is used as the object of a verb or preposition. "HIM/HER/THEM"? = whom.
          - RIGHT: To whom was the letter addressed?
          - RIGHT: Whom do you believe?
          - RIGHT: I do not know with whom I will go to the prom.
      - Ask yourself whether the answer to the relevant part of the question he/she/they or him/her/them
        - Who vs. Whom (An easy way to remember the difference is that the two words that end in -m go together)
          - He = Who
          - Him = Whom
        - Whoever vs. Whomever (Just like above, the two words ending in -m correspond with whomever)
          - Him + He = Whoever
          - Him + Him = Whomever
    - *Which* cannot modify people, but use *Which* only to refer to nouns, never to refer to an entire clause
      - *Which* clauses must refer to the closest preceding main noun and not the whole clause
    - *Whose* can modify either people or things
    - *Where* can be used to modify a noun place; CANNOT modify a "metaphorical" place (e.g. condition, situation, circumstance, arrangement)… in these cases, use *In Which* instead
  - Adverbial Modifiers must be unambiguous
    - WRONG: He caught up with his sister more rapidly… (more rapidly *than what? than before? than someone else?*)+
  - Check "comma-ing" modifiers and see what clause it refers / is attached to
- Quantity
  - Do not use the word *less* with countable items

    - Use *less* with unit nouns / uncountable items (e.g. money and volume are not countable, e.g. one money, two money doesn't make sense)
        - Correct: I have less than 20 dollars
        - Incorrect: I have fewer than 20 dollars.
    - Should use *NUMBER* with countable substances; use *AMOUNT* or *QUANTITY* with uncountable substances
    - Use *Between* only with two things or people; use *Among* with 3+ things or people
    - The word *Numbers* → compare using *Greater than*, not *More than*
        - Correct: Its numbers are now suspected to be much greater than before.
    - Increase / Decrease vs. Greater / Less
        - I / D express change of a thing over time
        - G / L signal comparison btwn 2 things
- Parallelism
    - Comparable sentence parts must be structurally AND logically similar
    - Signal words do not have to be the same word
        - CORRECT: There are many dogs WHO eat gourmet food but WHOSE parents never did.
    - Don't always have to repeat
        - CORRECT: She WILL WALK to school in the morning and RUN home in the afternoon. (*will* is understood to apply to *run*)
    - Gerunds (Gerunds require repetition)
        - WRONG: the rising and running / the uprising and escape
        - CORRECT: the rising and the running / the uprising and the escape
    - You can pair working verbs in different tenses as long as the meaning of the sentence supports the different tenses
        - CORRECT: She eats fruit all the time and drank some watermelon juice yesterday.
    - If a list has at least three items, must use a comma + *And*
        - WRONG: Life is cool and fun and neat.
        - RIGHT: Life is cool, fun, and neat.
        - RIGHT: She argues that the agency acts with reckless abandon and with disregard for human life and property. (only two items here, so it's correct even though there are 3 *and's*; reckless abandon, + disregard for human life and property)
    - Parallel Meaning: subject / object have to be parallel
        - WRONG: The bouquet of roses was a giving of life.
        - RIGHT: The bouquet of roses was a gift of life.
    - Common Idioms w/ built-in parallel structure
        - Between x AND y
        - Distinguish x FROM y
        - Estimate x TO BE y
        - Whether x OR y
        - View x AS y
        - Think of x AS y

        - Mistake x FOR y
        - x, like y, has / like x, y has
        - x is __, AS is y
        - Numbers… GREATER THAN
- Comparisons
    - Omitted Words [x]
        - Can omit units, verbs, and even whole clauses from the second term, as long as there is no ambiguity in the comparison
            - RIGHT: My car is bigger than Brian's [car]
            - WRONG: I like cheese more than Julie (do you like cheese more than Julie does, or do you like cheese more than you like Julie?)
        - GMAT allows unnecessary helping verbs, but both versions are correct
            - RIGHT: Apples are more healthy to eat than burgers.
            - RIGHT: Apples are more healthy to eat than burgers ARE.
    - Like vs. As
        - Use *Like* to compare nouns, pronouns or noun phrases (can also be followed by gerunds, or -*ing* forms used as nouns)
            - *Like* cannot be followed by verbs (but can be followed by gerunds, which are nouns that look like verbs)
        - *As* can be used to compare two clauses
            - RIGHT: LIKE her brother, Rhea aced the exam.
            - WRONG: Like her brother did, Rhea aced the exam.
            - RIGHT: As her brother did, Rhea aced the exam.
            - WRONG: Law students learn to think like a lawyer does. (either *as a lawyer does*, or *like a lawyer*)
    - Comparative vs. Superlative
        - When comparing only two items, use the comparative (i.e. *taller*) rather than the superlative (i.e. *tallest*); when singled out from a group, use the superlative
            - WRONG: Although the two towers appear identical, the west tower is the tallest, standing 15 inches taller than the east tower.
            - RIGHT: Although the two towers appear identical, the west tower stands 16 feet taller than the east tower.
- Pronoun
    - *It, Its, They, Them and Their*
        - *Their* should NOT refer to singular objects
            - WRONG: whenever a STUDENT rings, take down their information.
            - RIGHT: whenever a STUDENT rings, take down his or her information.
            - RIGHT: whenever STUDENTS ring, take down their information.
        - Do NOT use *this* or *these* in place of nouns
        - Any new copy *that* or *those* must agree in number w/ previous version; if you must change number, repeat the noun
            - WRONG: Her company is outperforming those of her competitors.
            - RIGHT: Her company is outperforming the companies of her competitors.

Table of Contents

- Verbs
    - Present Perfect = Have/has + Past Participle
        - Means that the action is definitely over, but its effect is still relevant to the present moment
    - Present to Future or Past to Conditional
        - RIGHT: I believe the machine will be wonderful.
        - WRONG: I believe the machine would be wonderful.
        - RIGHT: I believed the machine would be wonderful.
- Idioms
    - Common Idiom List
        - Ability TO
        - Allows FOR
        - And can separate two items and be preceded by a comma (We work all night, AND we sleep all day).
        - As… AS
            - e.g. As many… AS
        - Being (appears in more wrong answers than right ones, but it can be correctly used as a gerund or a participle; *Being infected makes you sick* is correct grammatically)
        - Both… AND
        - Consider
            - RIGHT: I *consider* illegal the law passed last week.
        - Either… OR
        - From… TO
        - No sooner… THAN
        - Not… BUT
        - Not only… BUT ALSO (includes comma)
            - Not only… BUT (also right, but has no comma)
        - Rather… THAN (*instead of with* is wrong)
        - So… AS TO
        - Whether
            - RIGHT: I do not know whether I will go.
            - WRONG: I do not know if I will go (IF requires a consequence)
        - Whether… OR (not "whether X or also y", not "whether they be x or y")
    - Top 10 most frequent GMAT idioms (by Magoosh)
        - Require that X be Y
        - Estimate to be
        - Prohibit X from Y
        - Believe X to be Y
        - Consider X Y (no 'to be')
        - X expected to Y
        - Not only…but also…
        - Neither…nor…
        - Just as…so too… (can also be used without the "too")

        - Prefer X to Y
- Meaning, Structure & Modifiers: Extra
    - Concision: don't make it too short
        - Pattern #1: Keep the prepositional phrase if you need to (e.g. *the Boston soldier* vs. *the soldier FROM Boston; the oxygen amount* vs. *the amount OF oxygen*)
        - Pattern #2: Keep *that of* or *those of* if you need to
    - Compound Subjects
        - *Or* = singular
            - RIGHT: Linda or Guy has a red car.
            - WRONG: Linda or Guy have a red car.
        - With *Either or* / *Neither nor* / *Not only… but also…*, if the two nouns disagree in number, use the noun closest to the verb to determine agreement
            - RIGHT: Either the boss or the employees take a break.
            - RIGHT: Either the employees or the boss takes a break.
    - Connecting Punctuation
        - Colon: used to provide further explanation for what comes before it
            - What comes before the colon must be able to stand alone as a sentence
                - What comes after the colon doesn't have to be able to stand alone as a sentence
            - Whatever needs explanation should be placed as close to the colon as possible
            - Semicolon vs. Colon
                - Semicolon = connects 2 related ind. clauses, but 2nd doesn't necessarily explain the 1st
                - Colon = always connects a sentence with examples or a further explanation
        - Dash: can be used as an emphatic comma, semicolon, or colon
            - Can be used to help maintain an unambiguous meaning
                - WRONG: My 3 best friends, Po, Oogway, and Rhino, and I went skiing.
                - RIGHT: My 3 best friends—Po, Oogway, and Rhino—and I went skiing.
            - Can be used to restate or explain an earlier part of the sentence
                - Unlike the colon, the dash does not need to be immediately preceded by the part needing explanation
    - Collective Nouns: some words can be singular or plural based on context
        - WRONG: The DATA collected by the researchers CONFIRM that heart disease is congenital; it also INDICATES that certain genes are sex linked. (has to be uniform)
    - SANAM Pronouns (Some, Any, None, All, More/Most) can be singular OR plural depending on the context of the sentence
        - However, *Not one* is always singular
    - Each and Every
        - *Each* or *every* requires a singular verb form

- RIGHT: Every dog, cat and pig HAS feet.

- Quantity Words / Phrases
  - *THE number of* = singular, *A number of* = plural
  - *Majority, minority and plurality* can be either singular or plural
- Subgroup modifiers
  - RIGHT: This model explains all known subatomic particles, [SOME OF WHICH WERE, SOME OF THEM, SOME] only recently discovered.
  - WRONG: This model explains all known subatomic particles, [OF WHICH SOME WERE, SOME OF THEM WHICH WERE, SOME OF WHICH] only recently discovered.

- Parallelism & Comparisons: Extra
  - More on *Like*
    - Only nouns or pronouns can follow *Like*
    - A *Like* comparison can be metaphorical
      - RIGHT: He ran *like* the wind. (doesn't mean the wind ran, only that the wind moves fast)
    - Be careful about ambiguity with *Like* at the end of sentences
      - WRONG: I want to coach dogs LIKE Po.
      - RIGHT: I want to coach dogs, LIKE Po. (notice comma)
    - Do NOT use *Like* to introduce examples; on the GMAT, *Such as* is the right way to introduce examples
  - Numbers
    - Use *times* and *as … as* to relate quantities by multiplication (not *5 times older than*)
    - Use *times* without *as* or *than* to indicate direct multiplication
  - Other Comparison Constructors
    - Ambiguity and *More* / *Less* (watch out when *more* comes before an adj. + noun)
      - WRONG: We have even MORE efficient engines than before. (do we have more efficient engines, quantity wise? or are the existing machines just more efficient?)
- Pronouns & Verbs: Extra
  - Other Pronouns
    - *There*: "in that place"; should be a noun, not an adjective
      - WRONG: At current prices, <u>American oil</u> may be worth investing in, if wells can be dug THERE.
      - RIGHT: At current prices, oil <u>in America</u> may be worth investing in, if wells can be dug THERE.
    - Reciprocal Pronouns *One Another* and *Each Other* should be used to indicate <u>interaction btwn parties</u>; they are NOT interchangeable with *Themselves*
  - *Do so* vs. *Do it*
    - *Do so* can refer to an entire action
    - *Do it*; the pronoun *it* must refer to a noun, NOT a verb (verb does not have to be *Do, e.g. Ate it, Studied it*)

    - It is often smoother to use a generic synonym for antecedent than to repeat the noun exactly
        - RIGHT: New "NANO-PAPERS" incorporate fibers that give these materials strength.
    - Progressive Tenses
        - Present Prog. = *is playing*
            - Don't use for general definitions (instead, use simple present)
            - Don't use to indicate future actions (instead, use the simple future)
        - Past Prog. = *was playing*
        - Future Prog. = *will be playing*
    - Verbals
        - Infinitives: *to watch*
        - Gerunds: *watching*
            - Simple vs. Complex
                - Simple: EATING apples quickly. (more verb-like)
                - Complex: The quick EATING of apples. (more noun-like)
                - Simple and complex gerunds should NOT be made parallel to each other.
        - Participles: *watching* (used as adj. or adv.) or *watched* (used as adj.)
- General Notes
    - Read entire original sentence and note possible grammar and/or meaning issues
    - Examples should be introduced using *including*, not *like*
    - *From which success stems in applications* and *from which success in applications stems* are both correct (even though the first one sounds weird)
    - Check for sentences with words that are meant to show contrasts and make sure they are actually contrasts (e.g. *although X is strong, Y is strong* doesn't make sense)
    - *Because* can be separated from the main clause by a comma
    - Do not use a comparative adjective unless you have a *Than* in the sentence (e.g. *Higher [than]*)
        - RIGHT: The ties looked more appealing inside the store than [they did] on the racks outside. (*they did* is also correct, but not necessary)
    - *Ultimately* indicates that the action is meant to happen in the future
        - WRONG: The rate of language extinction is accelerating, a tendency ultimately culminating in the survival of just a few languages, according to some. (is accelerating = present tense, should be *that will ultimately culminate*)
    - NEVER use *Would* in an *if-clause*
        - WRONG: She may feel better if she would eat the medicine.
        - RIGHT: She may feel better if she eats the medicine.
    - When bossy verbs (e.g. recommended, suggested) are used, it must be followed by the command subjective
        - WRONG: Rachel suggested Patrick should make a salad.
        - RIGHT: Rachel suggested that Patrick make a salad.
    - (Surprisingly?) right

        - RIGHT: Kelp has become popular among growers of tomatoes, who generally are willing to pay a premium for organic products. (*Who* correctly modifies the noun phrase *growers of tomatoes*, not just *tomatoes*)
    - Redundancy Flags
        - *Have to* or *Must* may be used in a redundant matter
            - WRONG: The plan ensures that action MUST be taken. (ensured / must = redundancy)
            - RIGHT: The plan ensures that action WILL be taken.

Table of Contents

GMAT Club - SC Guide

- Part I: Fundamentals
  - Verb tense usage
    - Do NOT use simple form of verb tense when words such as *before, when, after, since, by the time* are included
  - There + Be
    - The expression of place can be omitted when the meaning is clear
      - RIGHT: There are seven continents. (implied expression of place is clearly *in the world*)
  - Other vs. Another
    - Another / other = not specific
    - The other(s) = specific
  - Like (things; 1 word) vs. As (actions; 2 verbs)
    - Use like to say two things are similar (one verb in a sentence w/ a like comparison)
    - Use as to say two actions are similar (two verbs in a sentence w/ an as comparison)
      - No difference in meaning btwn *as if / as though*; can be used interchangeably
  - Between vs. Among
    - Between = talking about distinct relationship
    - Among = talking about items, groups, or people in general
  - Proper use of problem verbs (raise/rise, set/sit, lay/lie) - TODO review
    - Raise, set and lay are followed by an object
    - Rise, sit, and lie are NOT followed by an object
  - Pronouns
    - *We, you,* and *us* can be directly followed by a noun to make it clearer to whom is being referred (e.g. We parents are…)
    - Do not use apostrophes with possessive pronouns (e.g. the Bird is grooming its wings...)
    - English NEVER uses *hisself* or *thierselves*
  - There vs. Their
    - *There* is an adverb meaning that place, but it is also used as a pronoun introducing a clause or sentence
    - *Their* is a possessive pronoun and it is used to show ownership of a thing or concept
  - Tag Questions
    - Placed at end of sentence clause which speaker is uncertain about
    - Negative forms are usually contracted
      - (e.g. She *was* happy, *wasn't* she?)
    - Don't change the tense of the verb from main clause to the tag
  - Adverbs

        - In a question, an adverb is placed directly after the subject (e.g. Does the bus always come on time)
    - Prepositional Phrases
        - If appears at beginning of sentence, must be followed by a comma
    - Comparisons
        - NEVER use *different than*; ALWAYS use *different from* (which is the opposite of *the same as*)
        - Different than = wrong in comparisons!
    - Comparatives
        - Numbered Comparatives (e.g. *half, twice, three times, etc.*)
            - The phrase *More than* is NOT used with numbered comparatives
                - WRONG: The jacket costs four times more than the shoes.
        - Double Comparatives
            - When a sentence begins with comparative structure (e.g. *The harder you study*), second clause must also begin w/ a comparative (e.g. *the easier the class will be.*)
        - Either or / Neither nor → always use a singular verb when used as the subject
            - RIGHT: Either of the cars is a good choice for the race.
    - Negation
        - None / no
            - None = plural count or non-count noun; CANNOT be used with a single count noun
            - No = can be used with all nouns
        - Watch out for double negation
- Part II: Intermediate
    - Coordinating Conjunctions
        - Conjunctions such as *if, and, so, and, but* CAN be used to start sentences. But they still need to make sense (<-- *like that*).
            - However, adverb clauses are dependent clauses and cannot stand alone as a complete sentence
                - WRONG: John went to bed. Because he was sleepy.
        - *So* can be used in comparisons (e.g. *She is not so old as her sister*) or as an adjective that means "very"
    - Who / Whom / Whose
        - *Who* is followed by a verb and replaces the subject noun/phrase
        - *Whom* is followed by a noun/phrase and replaces the object noun/phrase
            - Whom can also be used with a preposition; Whom will always follow the preposition when used this way
            - E.g. Whom is either followed by a noun/phrase or preceded by a proposition
        - *Whose* is a relative pronoun that indicates possession
    - Restrictive vs. Nonrestrictive clauses
        - Nonrestrictive clause = additional info and can be removed w/o disrupting meaning

- Separated from original sentence by commas
- Restrictive clause cannot be omitted from sentence w/o disrupting meaning
  - Not separated from original sentence by commas
- Ultimately, remember to add commas before and after a nonrestrictive clause
- *That* vs. *Which*
  - *Who, whom, which and whose* can be used in either restrictive or nonrestrictive clauses
  - *That* can ONLY be used in restrictive (NO COMMA)

- *Is the* = singular; *Is one of the* = plural
- Gerunds & Infinitives
  - Whenever a preposition is followed directly by a verb, the verb will be in gerund form
- Hardly, barely, rarely, seldom
  - Incorrect to use two negatives together (e.g. *hardly no bread left*)
- Conditional sentences
  - Unreal conditionals convey situations that would take place if circumstances were different
    - Past tense form of verb *be* is always *were* in these cases; it can NEVER be *was*
  - Real conditionals: when placing the *if* clause first, a comma is used after it
- Whether vs. If
  - Generally, *whether* indicates a choice btwn 2 possibilities, whereas *if* is based on a condition of something happening or not
  - Can be used interchangeably sometimes
  - However, if sentence gives 2 DISTINCT possibilities, then *Whether* should be used
- Because vs. because of
  - *Because of* is immediately followed by a noun or noun phrase; NOT a verb

- Part III: Advanced
  - *Must* CANNOT be used for a past obligation
  - *Use to* is incorrect; always use *Used to*
  - *Would rather*
    - *Would rather* must always be followed by a verb; *Prefer* may or may not be followed by a verb
    - When 2 things are compared, *Would rather* is followed by *Than*, whereas *Prefer* is followed by *to*
    - If talking about past tense / completed action, use *Would rather + had (simple past tense)*
      - e.g. Po would rather they had served pizza than burgers last night.
  - Past tense of the verb *Be* must always be *Were*; NEVER *was* in a contrary to fact sentence
    - WRONG: The boy acts as if he was stupid.
    - RIGHT: The boy acts as if (though) he were stupid.
  - *That* vs. *Which*

        - When the information in the relative clause (anything underlined above) is essential to the meaning of the entire sentence, use “that”
        - When the information following a relative pronoun can be removed without changing the meaning of the sentence, use “which” and separate it using a comma
        - If info following relative pronoun is not that meaningful, use which
    - When *That* is optional vs. mandatory
        - *That* is usually optional after the following verbs when introducing another clause: *Say, Tell, Think, Believe*
        - *That* is usually mandatory after the following verbs when introducing another clause: *Mention, Declare, Report, State*
    - Subjunctive
        - Ex: The officer insisted that he turn (not *turned*) around
    - Inclusives
        - Not only… but also (also is sometimes omitted, but it is best to include it)
            - Must be placed immediately before the phrase to which it refers
                - WRONG: I am not only good at baseball but also soccer.
                - RIGHT: I am good not only at baseball but also soccer.
                - RIGHT: I am not only good at baseball but also good at soccer.
        - As well as
        - Both… and
    - Clauses of concession
        - With any prepositional phrase which begins a sentence, a comma is used to separate it from the main sentence clause

Table of Contents

GMAT Club - YouTube Videos - Verbal (GMAT Ninja & Others)

SC: Setting Priorities on GMAT SC

- Eliminate DEFINITE errors, then compare remaining choices, focusing on MEANING
- Priorities with definite errors
    - Pronouns: if, they, that / those
    - Modifiers: that, which
    - Parallelism: and, or
    - Verb tenses: had + verb
    - Comparisons: un(like)
    - Subject-verb
    - Comma-splices: two independent clauses separated by a comma = wrong
- Starfish, data, media… what they have in common is that they could all either be singular or plural
    - However, other parts of the question stem will probably indicate what the GMAT wants you to know as to whether it is singular or plural (e.g. has or have)
- Past perfect → has / had + verb
    - First of two actions that happened in the past
- Present perfect → Has or have + verb
    - Happened in past and still happening today
- Equivalent / equal compares qualities or characteristics; as many as compares numbers or people

SC: Pronouns are NOT always ambiguous

- Ambiguous pronouns: Subject pronouns (they, it, she, etc...) cannot refer back to possessive nouns
    - EX: Investment officers' fees are based on how the funds they manage perform.
        - 'They' can't refer to investment officers', since it is a possessive noun
        - In this sentence, 'they' refers to the fees, which would be incorrect since fee's can't manage funds
    - EX: The girl's husband eats so much that she calls him a pig.
        - 'She' can't refer to the girl since it is a possessive noun
- If you have two clauses, While x's can hurt y's, they can also hurt z's.
    - Subject of the second clause unambiguously refers to the subject of the first clause ('they' refers to x's, not y's)
- If 'which' is after a comma, it generally refers immediately to what came before it, but not always

SC: "That" in my sentence

- That can be used as
    - Modifier
    - Singular pronoun

    - Subordinating a clause
        - I believe that X and that Y. (need the second that, otherwise not linked to the "I believe")
- Touch rule is not an absolute rule
    - He personifies the devastation and enslavement in the name of progress that have decimated native people.
        - The devastation and enslavement is what decimated people, not progress
        - So that must refer to the devastation and enslavement, and it is ok that it does not directly refer to the noun preceding it (progress)
    - Emily Dickinson's letters to Susan Huntington Dickinson, which were written over a period beginning a few years before Susan's marriage to Emily's brother and ending shortly before Emily's death in 1886, outnumber her letters to anyone else.
        - "which" refers to the subject "letters"… if there is a prepositional phrase that is in between "which" and the subject it wants to modify, and the logical antecedent is clear, then "which" doesn't have to be right next to its antecedent
        - In this case, which CANNOT modify Susan (a person), so we know that which must refer to the letters

SC: Parallelism & Meaning

- Don't use your ear! Parallelism has nothing to do with sound
- Verb tense has nothing to do with parallelism
- Find the stem → unlocks the parallelism
- Look out for words that look like verbs but aren't verbs! (-ing and -ed words that aren't actually verbs)
    - The sunbird, an animal found in the Philippines and resembling a hummingbird, has metallic colors on its head.
        - "Found" and "Resembling" are both adjectives, so this sentence is correctly parallel
    - The angry politician, frustrated by the opposition's parliamentary tactics and screaming about the other parties' unconstitutional behavior, is both a hypocrite and a narcissist.
        - "Frustrated" and "Screaming" are parallel to each other; both are adjectives modifying the angry politician

SC: Simplifying Verb Tenses

- Present perfect → occurred in the past and continues in the present
    - Has or have + participle
    - Since 1996, I have eaten breakfast daily.
- Past perfect → first of two past actions
    - Had + participle
    - I had been a real jerk until I ate breakfast.

Table of Contents

SC: Comparisons

- Do → look for verb phrase
  - Dirt roads cost twice as much to maintain as paved roads do.
    - Do = cost to main
    - Dirt roads cost twice as much to maintain as paved roads cost to maintain. (logical, comparing cost of maintaining dirt roads to cost of maintaining paved roads)
  - Maintaining dirt roads costs twice as much as paved roads do.
    - Do = costs
    - Maintaining dirt roads costs twice as much as paved roads costs. (illogical, comparing maintenance to costs)
- In 1998, more babies were born to women over the age of 30 THAN under it.
  - "It" refers to "the age of 30"
    - more babies were born to women over the age of 30 THAN under the age of 30.
  - Would be illogical to say
    - more babies were born to women over the age of 30 than were born under it. (illogical since this implies that babies were born under the age of 30)
- In no other historical sighting did the comet cause such a worldwide sensation as in its return.
  - We want to compare "In no other historical sighting" to as "in another historical sighting"
- Ten times as much/many AS vs. Ten times more THAN
  - Less = uncountable (money)
  - Fewer = countable (dollars)
- Use "that of" and "those of" to quickly eliminate answer choices

SC: Punctuation

- Whether or not is considered redundant on the GMAT → usually always wrong!
  - Just use whether...
- Other than semicolons, usually punctuation is not the deciding factor of a right or wrong answer
- Two independent clause cannot be separated by a comma
  - Semi-colon separates two independent clauses

SC: 4 Modifier Rules Most People Get Wrong

- 4 types of modifiers
  - Participial phrases (-ing/-ed)
  - Relative causes (wh-/that)
  - Appositives
  - Prepositional phrases

- 4 types of modifiers
    - Participial phrases
        - Beginning of sentence, describe noun immediately after
            - Bombarded by bullets, the troops retreated
        - In middle, where they describe noun immediately before
            - Dogs trained by professionals are much more obedient.
        - At end after a comma, where they describe the whole clause before (NO longer describes noun right before it, but the whole clause)
            - Kit Carson roamed the Rockies, working as a trapper and establishing a reputation as one of the most able mountain men of his time.
    - ONLY applies when its an -ing/-ed words (otherwise should describe closest noun)
        - IF which / that, needs to modify a noun
            - Can't jump a verb to modify a noun
    - Relative clauses or appositives describe the closest noun OR noun phrase
        - The bench by the pond, which was recently painted, is my favorite place.
            - "Which" describes the "bench by the pond" → by the pond is a prepositional phrase, so we can ignore it
        - Prepositional phrases can act either as adjectives or adverbs
            - When they act as adjectives, they describe noun immediately before
            - When they act as adverbs, they describe the whole clause before or after
- Parallelism with Modifiers
    - Modifiers are only parallel to other modifiers of the same type
        - Participial phrases (-ind/-ed)
        - Relative clause (wh-/that)
        - Appositive (noun phrase)
        - Prepositional phrase (starts with preposition)
    - Can mix -ed and -ing (they are both participial phrases, and thus are parallel)
        - The oldest ocean crust is thought to date from the Jurassic period, formed from huge fragments of lithosphere and lasting 200m years.
        - "IS" thought to date → still is oldest
        - Formed (instead of forming) b.c. it is not still forming (to form is a one time action) and lasting b.c. it still is the oldest
        - Formed and lasting and parallel!

SC: Countable & Non-Countable

- MFN - Countable (dollars)
    - Many
    - Few(er)
    - Number
    - Little (size)
- MLA - Non-countable (money)
    - Much
    - Less

    - Amount
    - Little (quantity)
- If a noun is plural, the GMAT may be hinting that it is countable (e.g. dioxins is plural / countable, you wouldn't say moneys)
- If not modifying a noun, don't have to worry about counting (e.g. there will be other errors)
    - … whose access to water was less limited (less is modifying limited, not a noun)
    - However much voters may agree that… (much is modifying the agreement)
- Not always absolute rule;
    - If all 5 answer choices have something that can only refer to one of non-countable or countable items, even if it seems like it shouldn't work, this flags that the answer choice has to do with other errors

SC: Special Parallelism Triggers

- Not (only)… but (also)
    - Don't need only AND also
    - Just because there is a not only, you don't need a but also (don't even need a but)
- More important is that what follows the Not (only) & the but (also) are parallel
    - Same with both… and
    - Same with either… or
- Not only are thieves able to do X, but also pilfer info such as Y… (are and pilfer are both verbs, so it's fine)
- Verb tense doesn't have anything to do with parallelism
    - If verb tenses switch, just need to make sure that it is expressing the right logic (e.g. it makes sense to switch the verb tense)
    - The Incas could not have come about without corn, which not only WAS nutritious but also COULD BE stored for long periods (totally fine!)
- Whether or not is always wrong → considered redundant on the gmat

CR: 7 most common GMAT CR / RC Mistakes

- Inference = not directly mentioned in the passage
- EXCEPT
    - Cross out anything that _____
- Always find 4 wrong answers, not the one right answer → don't fall in love with one answer and then not look at other answers!

CR: Boldface & Fill-in-the-blank

- Invest in understanding and understanding the overall conclusion before trying to understand boldface / fill in the blank
- Conclusion is not always the last sentence, not always preceded by "Therefore"
- Think of conclusion as "what is the main thing that the author wants me to get?", instead of looking for words like "therefore" or "clearly"

Table of Contents

CR: Strengthen, Weaken and Assumption

- Thing that will strengthen / weaken is likely something external that wasn't explicitly mentioned in the passage, but nonetheless is relevant to the conclusion
- Assumption problems → you will NEED the answer for the author to be able state the conclusion (otherwise, the conclusion would be weakened / false)
- Evaluate the argument → might be something that strengthens or weakens

CR: Evaluate & Assumption

- Pre-thinking Assumption
    - UNSTATED idea that MUST BE TRUE for the conclusion to be valid
        - Without the assumption, conclusion is shattered / invalidated
        - Unstated = new information (not in passage explicitly)
    - Strengtheners are NOT assumptions
    - Correct answer → Negated version will break down the conclusion
    - Incorrect answer → Negated version will not break down conclusion
- Pre-thinking → one potential assumption before going to answer choices (prethinking !== prediction)

CR: Assumption Questions without Negation

- Conclusions not only last sentence
    - Calls for action (should, needs to)
    - Needs to cite a reason why (since, however)
    - If a sentence states something without a reason why, then it is a premise (not a conclusion)
- Look out for
    - Generalization (since it happened in past, must happen in the future)
    - Gaps in logic (conflating two things are equal)
    - Precision in language (numbers vs. percents)
- Find the gap → Look out for two elements that are “conflated” as equal → look out for possible differences in meaning
    - Committing crimes is NOT the same as being arrested for committing crimes
    - Cheating on a test is NOT the same as being caught for cheating
    - Increase in TAXES for company is NOT the same as increase in cost to consumer (e.g. assumes that the company passes on increase in tax to consumers)
- Assumptions does NOT guarantee the validity of the argument
    - However, assumptions must be true for the argument to make sense
- ASSUMPTIONS != STRENGTHENERS
    - If something just strengthens the argument, but does not HAVE to be true, then it is not an assumption
- Be careful when numeric values are involved
    - Equating numbers to percentages is suspect

        - If comparing raw numbers, then it could be wrong
        - If already comparing PERCENTAGES or PROPORTIONS, that is actually a good thing because it takes into account the difference in numbers
            - Percentages and proportions are ideal → we don't need to know that the raw numbers in each group were equivalent (tempting wrong answer)
    - However, don't immediately jump into a numbers answer… it could be another more relevant gap (see above)

Table of Contents

Powerscore - CR Bible

- Read the fine print: if the plan can achieve greater efficiency, don't assume it WILL or MUST achieve it
    - If it can improve retention… don't think it will eliminate attrition completely
- Types
    - Must be True
        - Don't dismiss paraphrased answers because you think "this is too easy"
        - Correct answers
            - Paraphrased answers that restate portion of stimulus in diff. terms
            - Combination answers (combine 2+ statements from stimulus)
        - Incorrect traps
            - Could be / likely to be true (correct answers MUST occur based on what you read)
            - Exaggerated answers (likely to improve vs. must improve)
            - Reversed answers (many people have some security system in their house → some people have many security systems in their house)
        - Opinions vs. Assertions
            - If stimulus only contains opinions, then any answer that is presented as a fact is incorrect
    - Main Point
        - Correct answer
            - Paraphrase of the conclusion of the argument
            - Think about "what does the author want me to believe / takeaway / learn from this passage?"
        - Incorrect traps
            - True but do not encapsulate main point
            - Solely repeat premises of the argument
    - Weaken Questions
        - Most frequent question type on CR
        - Works really well with pre-thinking approach since there must be a gap of logic
        - Correct answer attacks conclusion, accepting premises as fact
            - Shows that conclusion fails to account for some other element / possibility, or that conclusion does not necessarily follow from premises
        - Almost all (but not 100%) of correct answers on weaken questions attack the conclusion and leave the premises unaddressed
        - Common weakeners
            - Incomplete info → fails to consider all possibilities or relies on incomplete evidence → attack by bringing up new possibilities / info
            - Improper comparison → tries to compare two items that are different
            - Overly broad conclusion → broader conclusion is drawn than what the premise(s) support(s)
        - Incorrect traps
            - Opposite answers

- Attack conclusion that is similar, but different, from one presented in stimulus

- Cause and Effect Reasoning
  - Identify where in argument causal reasoning is presented
    - Causal conclusion = argument is flawed (focus of GMAT)
    - Causal premise = argument may be flawed, but not b.c. of causal statement
      - Generally, premises are meant to be unchallenged → GMAT is more concerned with reasoning that follows from a premise
  - Alternate explanations
    - 2 events could be caused by a 3rd factor
    - Reverse causality (y actually causes x, x doesn't cause y)
    - 2 events are related, but not causally
    - 2 events occurred by chance
  - How to attack "X causes Y"
    - Find an alternate cause for stated effect
      - Z causes Y
    - Show that even when cause occurs, effect doesn't occur
      - X happens, but Y doesn't
    - Show that while effect occurs, X doesn't
      - Y happens, but X doesn't
    - Reverse
      - Y causes X
    - Show that statistical problem exists with data used to make causal statement
- Strengthen Questions
  - Focus on the conclusion
  - Look for potential weaknesses and answer choices that close the gap
  - Many require students to find the missing link btwn the premise / conclusion
  - Arguments that contain analogies / surveys rely upon the validity of those analogies / surveys
  - When strengthening a proposal
    - Look for answers that shows the plan is practical
  - Supporting a cause / effect relationship
    - Eliminate any alternate causes
    - Show that when cause occurs, effect occurs
    - Show that when cause does not occur, effect does not occur
    - Eliminate possibility that the relationship is reversed
    - Eliminate possible problems with the data
- Assumption Questions
  - 2 roles
    - Supporter: link together new or rouge elements / fill logical gaps in argument
    - Defender: eliminate possible sources of attacks on the argument

        - 3 quirks
            - Watch out for answers starting with phrases “at least one” or “at least some” → likely to be incorrect (but don’t automatically assume, just be aware)
            - Avoid answers that claim an idea was most important consideration for the author
        - Assumption and causality (same as strengthen + causality)
            - Supporting a cause / effect relationship
            - Eliminate any alternate causes
            - Show that when cause occurs, effect occurs
            - Show that when cause does not occur, effect does not occur
            - Eliminate possibility that the relationship is reversed
            - Eliminate possible problems with the data
    - Resolve the Paradox
        - Correct answer will explain both sides of the opposition
        - Ignore answers that address neither side of the cause or just one side of the cause
        - Trap answer
            - Explains only one side
            - Similarities and differences
                - A similarity cannot explain a difference, and vice versa
    - Flaw in the Reasoning / Method of Reasoning
        - If an answer choice describes an event that didn’t occur in the stimulus, the answer is INCORRECT
            - EX: Argument accepts a claim on the basis of public opinion of the claim
                - Must identify whether
                    - Author accepts a claim
                    - Acceptance is done on basis of public opinion
        - Watch out for answers that are only partially true (half wrong = all wrong)
        - Common Errors of Reasoning
            - Errors in Use of Evidence
                - Citing facts that such a situation has always existed does not help disprove that something else has increased that situation
                - “Author cites irrelevant data; fails to give any reason for judgement it reaches”
            - Internal Contradiction
                - “Bases a conclusion on claims that are inconsistent with each other”
            - Exceptional Case / Overgeneralization
                - Supports a general claim on the basis of a single example
                - Draws a broad conclusion from a small sample of instances
            - Errors in assessing force of evidence
                - Lack of evidence = position is false
                - Lack of contra-evidence = position is true
                - Little evidence = position is false

        - Source Argument
            - Attacks the person / source instead of the argument (ad hominem)
        - Circular Reasoning
            - Assumes what it attempts to demonstrate
    - Errors of Conditional Reasoning
        - Mistakes necessary condition for sufficient condition, or mistakes a sufficient condition for a necessary condition
    - Mistaken Cause / Effect
        - Mistakes an effect for a cause
        - Fails to exclude an alternative explanation for observed effect
        - Confuses coincidence of 2 events with a causal relation btwn the two
    - Straw Man
        - Recasting an argument unfairly → "what you're saying is __" + refashioned / weakened statement
        - Portrays another's views as more extreme than they really are
    - Appeal fallacies
        - Appeal to authority
        - Appeal to popular opinion / numbers
        - Appeal to emotion
    - Survey errors
        - Biased sample
        - Questions are improperly constructed or misleading
        - Inaccurate responses (people do not always tell the truth)
        - "Generalizes from an unrepresentative sample"
    - Errors of Composition / Division
        - "Assumes that what is true of a whole must also be true of each of its parts"
        - "Assumes that b.c. something is true of each of the parts of a whole, it is true of the whole itself"
    - False analogy
    - False dilemma
    - Time shift errors (what happened in past must happen in present / future)
    - Relativity flaw
        - Relative relationship premise used to draw absolute conclusion (or vice versa)
    - Sunk cost / concorde fallacy
    - Numbers / percentages errors
        - Improperly equating percentage w/ a definite quantity, or using quantity info to make judgement about the percentage represented by that quantity
- Boldfaced portions
    - Common wrong answer = correctly describing part of the argument, just not the part referenced in the question stem
- Parallel Reasoning

    - Irrelevant
        - Topic of stimulus
        - Order of presentation of premises / conclusion
    - Relevant
        - Method of reasoning
        - Validity of argument
        - Conclusion
            - Must match certainty level / intent
                - If conclusion uses
                    - Absolutes (must, never, always) → correct answer will use similar absolutes
                    - Opinions (should) → correct answer will match
                    - Conditional conclusion
            - Don't look for necessarily identical wording
        - Premises
            - Premise Test: If in argument there are two premises that independently prove the conclusion, correct answer choice will have two independent premises
                - If an answer choice has two premises that work TOGETHER to prove the conclusion (i.e. neither premise proves the conclusion alone, only together), in this case, that would be incorrect
- Numbers and Percentages
    - First, make sure number or percentage is the focus of the problem and not merely a distractor element (e.g. example about crimes committed vs. arrests)
    - Figure out what of 3 elements are missing (given 2, you can figure out the 3rd)
        - Percentage, Number and Total
        - If given just a number and the total, but nothing on the percentage, reasonable chance that the answer will revolve around or reference a percentage
        - If given a number and a conclusion is drawn about the percentage, there is likely missing information about the total population (is it changing / staying the same?)
    - If average price goes down, either—relative to the original average—more cheaper ones were added or more expensive ones were removed
    - Misconceptions (same with decreasing)
        - Misconception #1: Increasing percentages = increasing numbers (only true if overall total remains constant)
        - Misconception #2: Increasing numbers = increasing percentages
    - Must be true - Numbers and Percentages
        - If stimulus contains percentage or proportion info only, avoid answers that contain hard numbers
        - If stimulus contains only numerical info, avoid answers that contain percentage or proportion info
    - Markets and market share

        - Company can gain market share if market shrinks and they maintain constant size, or if they grow in an unchanging market
        - Company can lose market share even if their sales stay the same, but the market grew
        - Company could close sales and still gain market share if overall market became smaller
    - What would be most useful in evaluating the argument?
        - Variance Test: only use once you've already narrowed down answer choices
            - supply polar opposite responses and see how they affect conclusion → different responses should be produced
            - EX: If the answer choice says "What percentage of people ..?" → immediately test out two extremes: 0% and 100%
                - If it is correct, then one extreme should strengthen the argument and the other should weak the argument
- Must be true
    - If only opinions are in passage, then any choice that talks about facts is incorrect (and vice versa)
- Weak causal relationships
    - Show that when X happens, Y doesn't (or vice versa)
    - Alternate causes
    - Reverse causality
- For assumption questions
    - Watch out for "at least one" or "at least some", more likely to be incorrect (not always true though)

Table of Contents

Personal Verbal Notes / Error Log (Contains Questions from OG Problems/Tests)

SC

- Who vs. Whom vs. Whose
    - 'Who' and 'whom' can refer only to people, 'whose' can refer to ANYTHING (even people)
- When you want to express a general idea you must use the simple present tense (e.g. have) not present participle (e.g. having)
- A present participle (such as RAISING) implies an action happening at the same time as the main action (Has proposed)
    - EX (WRONG): The city has proposed a number of water treatment and conservation projects at a cost raising water bills so high that enough so that even environmentalists are beginning to raise alarms
        - Since the city has only PROPOSED the projects -- and these projects might not actually happen -- the sentence cannot say that their cost is raising water bills (in the present)
- When which is used as a classical relative pronoun, there should be a comma before which
    - But not a 100% rule, usually if no comma is before which, there will be another error in the sentence
- Passive voice not favored on the GMAT (not absolute rule, but a good general rule of thumb)
    - Active: Po ate breakfast.
    - Passive: The breakfast was eaten by Po.
- That vs. which
    - That = restricting = narrows scope = vital
    - Which = non-restrictive = adds extra info = not vital
        - Can remove and not change meaning of sentence
    - Use of commas
        - Use comma with nonrestrictive
- If both simple present and present perfect (has / have + verb) make sense, then usually the simple perfect is preferred
    - Only use present perfect (or any tense other than perfect) when it wouldn't make sense to use simple tense
- Not only → doesn't require a but, or but also → can just be Not only x, also y
    - RIGHT: not only are thieves able to divert cash, they also pilfer valuable information…
        - Are thieves able = subject + verb
        - they also pilfer = subject + verb
    - WRONG: not only are thieves able to divert cash… but also pilfer valuable information…
        - Are thieves able to = subject + verb
        - Pilfer valuable information = just verb (no subject; not parallel)
- Each / every + Each other vs. one another

    - Each and every are singular and thus require a singular verb
    - **Each other** is used to refer to two things, and **one another** for three or more
- Being
    - If **being** is used as a verb (preceded by some form of "to be") or a noun, it is acceptable.
        - RIGHT (noun / gerund): Being a GMAT tutor makes Mike happy.
        - RIGHT (verb): Charles is being cruel to his GMAT students today.
    - However, **being** is never acceptable as a modifier
        - WRONG: Being born in Italy, Domenico has a deep understanding of outstanding cuisine.
- Countable / Non-countable
    - Ex: However much United States voters may agree that
    - However much is modifying the verb phrase “US voters may agree”, so don’t have to worry about countable / non-countable in this case
- Correct / Incorrect Pairs of Idioms
    - 1
        - Correct: "more likely than" and "as likely as"
        - Incorrect: “more likely that" and "as likely for”
- Examples
    - 1
        - WRONG: It is nearly four times as likely <u>for minority graduates than other graduates</u> to plan to practice law (underlined portion is wrong, in correct comparison comparison)
        - RIGHT: Minority graduates are nearly four times as likely as other graduates to plan on practicing law (correct)
        - Idioms
            - You could say "as many / likely ... as" or "more than" -- but "as many... than" or “as likely than” is simply wrong
- Present participle implies an action happening at the same time as the main action
    - WRONG: The city has proposed projects at a cost raising bills very high. (since city has only proposed the projects, cannot say that their cost is raising water bills high—which implies is happening right now)
- Another reminder: Who/Whom are the only things that can modify people!
- List with 3 items, use comma + and (comma is required)
- When comparing only two items, use comparative (i.e. taller) rather than superlative (i.e. tallest)
- Bossy verbs → followed by command subjunctive (suggested that Po make a salad, not suggested that Po *should* make a salad)
- Redundancy
    - Ensures that plan must achieve its goals = redundant
    - Remains at a consistently high price = redundant
- That vs. Which
    - That can ONLY be used in restrictive (no comma)
    - Which can be used in either
- Distance to vs. Distance from

    - Both “Distance to” and “Distance from” are idiomatically correct; usage of one or the other will differ based on meaning
- Possessive pronoun
    - Proximity: not an absolute rule, but a good yardstick
        - My wife and I went to Julia’s house for her birthday
            - Would be awkward to say: “My wife and I went to Julia’s house for Julia’s birthday”
            - Yes, there are two females, but because her is closest to Julia, it is clear who “her” is referring to
    - “ITS” is a possessive pronoun that can correctly refer to possessive
        - The dog’s fur was as dark as its eyes. → correct
        - The man’s hair was so dark that he dyed it blond. → incorrect
        - You can’t use a SUBJECT pronoun to refer back to a possessive, only a POSSESSIVE pronoun
            - You CAN use a POSSESSIVE PRONOUN (such as “its”) to refer back to a possessive
    - Irrigation projects have enlarged the habitat of the freshwater snails that are the parasite’s hosts for part of its life cycle.
        - Parasite’s is possessive and “its” is possessive, so no problem here
        - “its” would have to refer to the nearest singular, which is “the parasite’s”
- Were = past tense of *be*
- Do not use simple form of verb tense when words such as *before, when, after, since, by the time,* are included
- MORE THAN is NOT used in NUMBERED comparisons
    - The jacket costs four times ~~more than~~ the shoes.
- Commas inclosing a dependent clause make it nonessential
- Meaning (super important for 700+)
    - Need to pay attention to meaning of the ORIGINAL sentence
    - EX:
        - RIGHT: The drawings show the buildings in the imaginary place.
        - WRONG: The drawings show that the buildings were in the imaginary place.
            - Drawings cannot prove that a building is in an imaginary place
    - If given “Z happened because X, and Y”
        - The comma makes it such that X is the only reason for the “because”, and Y is just another fact
        - Remove the comma to show that both X and Y caused Z
    - EX:
        - RIGHT: People’s hope that the economy will improve lies in the increase in spending that is projected.
        - WRONG: People’s hope that the economy will improve lies in the projection of increased spending.
            - Projection may cause people to hope, but their hope can’t lie in the projection… hope lies in what the projection is projecting!

            - LOGIC: people are hoping that the spending will improve the economy, not the projection
    - EX:
        - OQ: A star will compress itself into a white dwarf, a neutron star, or a black hole after it passes through a red giant stage, depending on mass.
        - RIGHT: Mass determines whether a star, after passing through the red giant stage, will compress itself into a white dwarf, a neutron star, or a black hole.
        - "A red giant stage" vs. "The red giant stage" not necessarily a deal breaker in answer choices
            - The policy helps the elderly patient of America. (even though there is not one singular elderly patient, this type of language is acceptable, and used in real life)
- Comparisons
    - EX:
        - WRONG: Technological advances may make it possible to build robots that resemble human dexterity, adaptability, and sensory capabilities.
            - Illogical comparison: robots can be compared with humans, capabilities/characteristics of robots can be compared with capabilities/characteristics of robots, but we can't mix and match, because it is not an apples to apples comparison.
- Idioms (Correct / Incorrect)
    - To try to vs. To try and
        - RIGHT: It may someday be worthwhile to try to recover uranium from seawater
        - WRONG: Someday, it may be worthwhile to try and recover uranium from seawater
    - Not so much… as
        - Must be parallel
            - RIGHT: Not so much "because of" X as "because of" Y
            - WRONG: Not so much "because of" X as "the results of" Y
        - Not so much… but = wrong
    - The use of hopefully on the GMAT
        - Must be used at beginning of sentence or beginning of an independent clause and be set off by the use of commas
- Parallelism
    - I like foods that "ARE adjective and adjective" and that "verb"
    - EX:
        - RIGHT: Psychiatrists are seeking to determine when it becomes destructive and which kinds of mental problems it can signal
            - Parallel: trying to determine "When…" and "Which"...
        - WRONG: Psychiatrists are seeking to determine when it becomes destructive and the mental problems that are signaled by it
            - Not parallel: trying to determine "when" and "the mental problems"
    - Weird things

        - EX: When Medicare was enacted in 1965, it was aimed at preventing a catastrophic illness from financially destroying the elderly patient.
            - Even though it wasn't enacted for a single patient, this type of usage is something we use in everyday speech (meant to refer to the group as a whole)
            - Another EX: "Mike advocates for the common man."
- Pronoun Ambiguity
    - **Resumptive modifiers**: Resumptive modifier is nothing but a modifier that is restated such that it avoids any ambiguities. → comes up pretty frequently for 700+ questions
        - OQ (wrong): Since the 1930's aircraft manufacturers have tried to build airplanes with frictionless wings, shaped so smoothly and perfectly that the air passing over them would not become turbulent.
            - Need to repeat "wings" after comma so we know what "shaped" and "them" refer to… the airplanes or the wings? (wings in this case)
- Like vs. As
    - Like = compare only nouns (say two things are similar) → 1 verb
    - As = compare clauses (say two actions are similar) → two verbs
    - Rules of thumb
        - *Like* is used mostly at the beginning of sentences… it is tough to put like at the end of a sentence and get the meaning right
    - EX: During an ice age, the buildup of ice at the poles and the drop in water levels near the equator speed up the earth's rotation, (like/just as) → would use *as* since we want to compare "speed up" with another verb
- One another vs. the others
    - RIGHT: Each company seeks to meet consumer needs and wants more successfully than the others
    - WRONG: Each company seeks to meet consumer needs and wants more successfully than one another
- "Each other" & "one another" used to show reciprocal actions
    - Laurel and Harry love each other.
    - The characters fight with one another.
    - Diff.
        - Each other = only two entities, whereas one another is > 2 entities
    - If meaning of sentence is not to show reciprocal actions, than cannot use "each other" or "one another"
        - WRONG: Each company seeks to meet consumer needs and wants more successfully than one another
- EX:
    - Ambiguity: Financial uncertainties resulting from the accident at Three Mile Island may prove even more deterring to the nuclear industry than political opposition
        - Are the financial uncertainties deterring the nuclear industry or the political opposition?
        - We are actually trying to compare financial uncertainties to political opposition

- So we need to add an “than is political opposition” (‘is’ removes ambiguity’)

- Change in tenses → acceptable
  - Financial uncertainties resulting from the accident at Three Mile Island may prove even more deterring to the nuclear industry than is political opposition
- Past perfect (we don’t need to repeat the past perfect and they don’t need to be the same verb tenses); all of the following are correct
  - Had resumed nesting or begun
  - Had resumed nesting or had begun
  - Had resumed or were investigating
- Parallelism has nothing to do with verb tenses!
- Concision
  - “Because” better than “for the reason that”
- Parallelism has NOTHING to do with verb tense
  - Example: “X could be more deterring to Z than is Y.”
    - Basically, X could be more deterring than Y currently is
- New Idioms
  - Possible
    - It is generally best not to use abstract modifiers like "possible" with concrete nouns like "workshop" or "building"
      - WRONG: An excavation occurred at the site of a possible workshop.
      - RIGHT: An excavation occurred at what might have been a workshop.
  - “The more so” → basically means “even more so”
    - The play was impressive, [all] the more so because the students had written it themselves. (all is not necessary)
    - EX2
      - RIGHT: Like the Dreyfus affair at the turn of the century and the Sacco-Vanzetti trial in 1921, the Hiss case became the political cause of an era, the more so because it was the first time a congressional hearing was shown on television.
      - WRONG: Like the Dreyfus affair at the turn of the century and the Sacco-Vanzetti trial in 1921, the Hiss case became the political cause of an era, more for the reason that it was the first time a congressional hearing was shown on television.
        - Wrong b.c. it needs to repeat “so” so that “more” can refer back to a clause (in this case, so refers back to “the Hiss case became the political cause of an era”)
        - In the absence of “so”, comparative adjective “more” would require a “than”
- Only who/whom/whose can refer to people
- Look out for -ed and -ing words are not verbs, but adjectives
  - Anytime you see an -ed or -ing word, think about this
- Had + Verb = past perfect → first of two past actions
- Whether or not = wrong on GMAT (redundant)

- Countable/uncountable only relevant when describing a noun
    - The access to water is less limited in NJ than in NYC. (less is modifying limited, not a noun, so the countable/uncountable debate is not a point of concern)
- Present participle (such as raising) implies an action happening at same time as main action
- Each Other vs. One Another (only use when parties are reciprocating actions with each other → e.g. Jenny and Jack love each other. NOT Each company works as hard as each other. (would be "as hard as the other companies"))
    - Each other = two things
    - One Another = 3+ things
- Each and every = singular (each and every dog is cute)
- Idioms
    - Without considering that = right
        - Overlooking that = wrong
    - The same ___ as (not "the same __ that")
    - "Even"
        - Idiomatically, we can only put the word "even" before a word without an article, not the other way around
        - RIGHT: "The plan could have an even better effect than anticipated."
        - WRONG: "The plan could have even a better effect than anticipated."
    - "As much X as Y"
        - The centers were as much X as Y
            - You can omit the [they were] → "were as much X as [they were] Y"
    - Unambiguous pronouns
        - Right: The buildings were as much community centers as purely religious edifices, structures that represented a city's commitment
            - In this case, since there is no pronoun, structures refers back to all 3 (buildings, centers, edifices)
        - Wrong:
            - The buildings were as much community centers as purely religious edifices; they were structures that…
                - Here, they is no ambiguous; does it refer to centers, edifices, or buildings?
- X in Five…
    - One in five Americans saves → singular
    - Two in five Americans save → plural
- More Examples
    - Example 1
        - Right: Confinement buildings must be adapted to animals, rather than animals to buildings.
            - … rather than animals [adapted] to buildings (adapted is implied here)
        - Wrong: Confinement buildings must be adapted to animals, rather than adapting animals to buildings.
            - Missing parallel verb structure
    - Example 2

        - Right: Dr. Hegsted argues that just as polio vaccine is given to ____, mass dietary change is needed to ___
            - Parallel
        - Wrong: Dr. Hegsted argues that like polio vaccine, which is given to accomplish X, mass dietary change is needed to accomplish Y
            - This suggests that polio vaccine and mass dietary change both achieve Y
    - Example 3
        - Right: Those Americans, approximately one-fifth of all employees, who work shorter hours at a job in order to care for an elderly relative save society millions of dollars.
        - Wrong: One in five Americans working shorter hours at their jobs in order to provide care for an elderly relative saves society millions of dollars.
            - "working" modifies Americans… this makes it seem that "of the Americans who work shorter hours… relative", ⅕ save society millions
            - The intended meaning of the sentence is that ⅕ of all American employees work shorter hours to care for their relative and thus save society money
    - Example 4
        - Right: The principles of the plan released by Congress could have even greater significance for the economy than do the particulars of the plan.
            - "Do" b.c. particulars of plan = current/immediate (plans have been released)
        - Wrong: The principles of the plan released by Congress could have even a great significance for the economy than the particulars of the plan.
            - Idiomatically, we can only put the word "even" before a word without an article, not the other way around → ("An even better trap", NOT "even a better trap")
        - "Do" vs. "Have"
            - I have more love for pies than my brothers do. (present tense, "do" preferable to stand in for "have").
                - "Do" refers to "have love for pies"
            - I have been going to the gym more than my brothers have. (in present perfect, "have" is preferable).
                - Second "have" refers to "have been going"
    - Example 5
        - Wrong: The asteroid is on an elliptical path that orbits the Sun.
            - This makes it seem like the path, rather than the asteroid, is the thing orbiting the sun
    - Example 6
        - Right: His periods of alertness alternated with periods of lethargy and withdrawal.
            - Correctly contrasts 2 periods: 1 of [alertness] and 1 of [lethargy and withdrawal]

        - Wrong: His periods of alertness alternated between periods of being lethargic and withdrawn
            - Illogical: claims that periods of focus went from lethargy to withdrawal… which are not traits of focus
- Using adjectives as adverb
    - Some words can be used as both adjectives and adverbs; flat is one among them.
        - A few more are late, fast, hard, close, deep; these do not need to be suffixed with the 'ly' tag to denote that they are adverbs
        - Flat vs. Flatly
            - Right: The clothing was packed flat.
                - Flat is describing the clothing
            - Right: He flatly denied it.
            - Wrong: The clothing was packed flatly.
        - The ball was thrown high. (you wouldn't say "thrown highly")
    - Ultimately, adverbs (when describing adjectives) should be in front of adjectives

RC

- Watch out for irony (and ultimately, tone) in author's voice
    - One example was the term "brilliant" being used ironically
        - When asked for a term that could be substituted without changing meaning of statement, most important thing is author's tone, not the literal meaning of the term
- First and Last sentence contains main idea and sets the tone
- Watch for trigger words (but, however, still, although, regardless)
- If a passage discusses a specific study for a portion of the passage, it is more likely that there is a broader point being made than that study is the main purpose of the passage
- Primary purpose of passage = correct answer will likely be less specific than an answer choice that seems very specific
- In RC, assert = explicitly in passage, whereas inferred = not in passage
- Veritas Prep videos
    - Trap answer = occurs in passage, but not answering the right question!
    - Precision in language… be on the look out
        - Disliked "strong socialist messaging" vs. disliked "all socialist messaging"
    - Inference questions
        - Just like CR, right answer MUST BE TRUE
        - Wrong answer
            - Could be true or likely to be true = likely incorrect
            - Generalizations (some is easier to prove than all)
- Look for wrong answers
    - Mindset should be finding 4 wrong answers (not finding the right answer necessarily)
- Notes
    - Main ideas
    - Tone shifts

    - Major transitions

CR

- Necessary & Sufficient
    - Necessary vs. sufficient
        - To get on the plane, you need a boarding pass
            - Does not mean that a pass is sufficient, just one thing that is necessary (may be others)
- Most people who took a poll say they voted in favor because it will reduce congestion on highways, and they drive on those highways
    - This shows that most people who voted are planning to continue using the highway, so it may not achieve its goal
    - Railroads would only help if less people took highway and more people took railroads…
- If CR stem gives you info about rates, and asks what can be drawn / concluded from the info / statistics…
    - CANNOT draw any conclusion about raw numbers just based off of rates
    - Simpson's paradox: overall rate different from in-category rates --> due to diff. distribution across categories and diff. criteria within categories
- If asked in CR what is the main point of the argument, the question is asking about the conclusion, not the premises used
- Unemployment rates
    - Voluntary retirement of workers does not affect unemployment rate
- Pay attention to small modifiers
    - Planets in OUR galaxy
    - Answer choice about all the planets that exist would be out of scope
- Do not assume anything!
    - EX: If the question asks how farmer's can maintain as much profit as possible, and the answer choice is "Seek long-term contracts to sell grain at a fixed price" → don't assume that the fixed price is profitable!
    - EX: If stem says "planets in our galaxy" and answer choice says "All planets"... they do not mean the same thing → pay attention to modifier words like "our galaxy"
- Gaps (modifier words)
    - Severe accidents vs. involved in accidents
    - Serious crimes vs. petty crimes
    - Arrested vs. committing crime
    - Cheating vs. caught for cheating
- Examine the paradox → still about bridging the gap
    - Not strengthening or weakening existing evidence / information
    - For two groups, same rate within categories, but different rates overall
        - If already given rates, then the size of each group does not matter!
        - Distribution of groups across categories have to be different, and within each category, has to be different at determining rate
- Would be most helpful in evaluating

    - Doesn't have to resolve the question, but should make the answer more compelling
    - Correct answer should be more compelling than the wrong answers in making the argument seem more valid
- Examples of gap
    - A smaller percentage of people will eat at fast food restaurants, therefore number of fast food restaurants will decrease
        - Well, even if smaller percentage will eat there, if the population is much larger, than the number of people who will eat there could be larger too
- Conclusions must have a "why" (facts / principles are not conclusion → can be used to support conclusion)
- Be patient if the first few words are "out of scope" (watch the curveball) + beware of mental inertia (if the goal is accuracy of allergy tests, don't worry about safety of allergy tests)
- From hating to loving critical reasoning
    - Strengthen / weaken / assumption / useful to evaluate / most likely to suggest that goal won't meet objective (latter uses same method of finding the gap)
        - NOT process of elimination → find the gap
        - Spend more time on the argument than the answer choices (similar to DS)
        - If there are extra words / modifiers in the conclusion, pay attention to if there is a gap
            - Wordplay (arrests vs. crimes committed, reported vs. had)
            - Correlation vs. Causation (instead of x causing y, maybe y causes x, or a third factor causes both of them)
            - Generalization (is sample indicative of the whole?)
            - Statistics & Data Flaws (# vs. %?)
    - Inference / boldface = process of elimination
- When an analogy is used as evidence, the assumption is that the analogy does apply
- With plan / strategies (e.g. why the suggested plan won't achieve it's goal)
    - Objective = conclusion
    - A better plan does not weaken the plan… must actually address the specific plan, not just offer a better plan
- Precision of language
    - Combat / reduce the problem → does not mean that you want to completely eradicate the problem, just reduce it
- Find the gap
    - Strengthen/weaken, assumption, evaluate, resolve the paradox
    - Strengthen
        - Narrow the gap
    - Weaken
        - Broaden the gap
    - Find the gap does not apply as much to inference questions
- Data-driven problems
    - General things to watch out for
        - Percent vs. Hard Value
        - Average !== Typical

        - Percent of the wrong thing
        - Irrelevant Stats
        - Hanging statistic without reference point
        - Incongruent samples / population sizes
            - When given statistics about a sample to draw a conclusion about the broader population… must address if the sample is representative of the population
            - Similarly, if an analogy is drawn between two situations, have to address whether those analogies are actually comparable
    - Average does not necessarily mean the normal case
        - If average increased, may be just that low numbers were removed or high numbers were added
        - If average decreased, may be just that high numbers were removed or low numbers were added
    - Any statistic in a vacuum is not that helpful
    - Many times we are not trying to prove the evidence is true; just whether the evidence is relevant
    - Don't eliminate answer choices just b.c. it seems out of scope
- Useful for evaluating ___ questions
    - Correct answer → one way should strengthen the argument, another way would weaken the argument… incorrect answers will only take you down one path both ways (only strengthen, only weaken, or only neutral)
- Don't let previous bias sway away from what the argument is trying to address
    - To most accurately determine food allergies, a physician should use the test that reduces false negatives because ____
        - WRONG: Some food allergies cause reactions that could be life threatening (may be true, but this doesn't address the "accuracy" of tests
        - RIGHT: All tests for food allergies have the same proportion of false positives
            - We can increase accuracy by either reducing false positives or false negatives
            - If false positives are the same across all tests, then it makes sense that we should choose the tests that reduce the false negatives
- Example
    - All people who are successful in business are invited to join the club.
        - This tells us nothing about who is NOT invited… maybe there are also other types of people who are invited
- Assumption
    - Watch out for distinction btwn rates/percentages and conclusions drawn about actual numbers
    - If given rates or percentages of two groups, then numbers don't matter
- Inference
    - Likely to be true does not mean right → as long as it COULD be false, it is eliminated
    - Inference = must be true
- Look out for extreme modifiers

    - e.g. if passage says people are concerned about teenagers committing SERIOUS crimes, and answer choice may say “most crimes teenagers commit are petty”
- CR Example
    - Argument → gov’t should devalue currency b.c. in past, depressed currency increased exports for manufacturing
    - Weakener → after several decades of operating below peak capacity, the country’s manufacturing sector is operating at peak levels → this shows that depressed currency won’t lead to increased exports b.c. the sector is already operating at peak levels / cannot increase manufacturing levels to handle increased demand
        - TL;DR → If your country is already operating at near peak production capacity, what's the point of trying to raise exports if you can't produce anymore?
- Supply/demand
    - Oversupply drives prices down
    - Overdemand drives prices up
- Super niche: for an animal to be weaned means to move away from reliance on mother’s milk to other sources of nutrition
- Focus on attacking the conclusion, not the premise!
- Circular reasoning
    - A is B. Thus, A causes C. After all, if A did not cause C, then A would not be B.
    - Only comes up in harder problems
- Statistics
    - If given rates/percentages, numbers in each group don’t matter
- Using an analogy assumes that the analogy is relevant
    - Example: If we are using the modern day Incan sheep rather than other modern day sheep to learn about the first domesticated sheep, the assumption (for the analogy to hold) is that modern day Incan sheep more closely resemble their forebears than other modern day sheep
- Which of the following would be helpful in evaluating?
    - One way would strengthen the argument, another way would weaken / negate the argument

Table of Contents

Integrated Reasoning

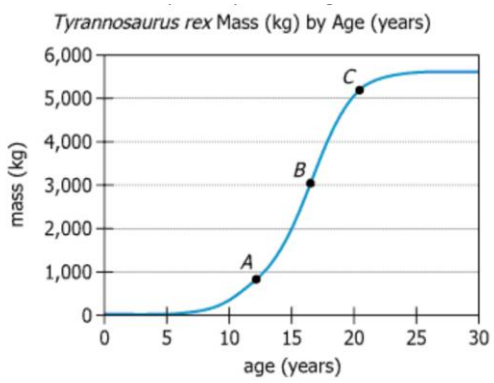
MGMAT Strategy Set - Guide 9 - Integrated Reasoning

- 12 questions, 30 minutes → 2.5 minutes per question, on average
    - Plan to guess on 2-3 questions = score of 5-6 → gives more time for other questions
    - <= 3 and >=7 scores are significant, but in the middle doesn't really matter
- Types of questions
    - Multi-source reasoning Prompts (multiple tabs)
    - Table Prompts (sortable table)
    - Graphical Interpretation
    - 2-part Analysis
- Type 1: Multi-Source Reasoning
    - Read through each tab, jot down notes BEFORE looking at the answer choices
    - Testing whether you can read around the scary technical language and still process the high-level info
- Type 2: Table Analysis
    - Can only sort by 1 column → Sort by the column that the question asks about
- Type 3: Graphical Interp.
    - Percent increase = Change in Value / Original Value
        - New Value = Original Value * (1 + percent increase)
        - 300% increase from 200
            - 200*(1+3) = 800
    - For line charts with two companies / parties, use the increment method to calculate the cumulative difference for each month between the parties
- Type 4: 2-Part Analysis
- IR Strategies
    - For MSR, read all tabs before looking at questions
    - For tables, sort by column that is relevant to the question
    - For graphical, look at the questions before looking at data
    - For two-part, glance at answer choices and determine whether it is a quant-focused question or verbal/logic-focused

Personal IR Notes / Error Log (Contains Questions from OG Problems/Tests)

- For a s-shaped graph, you can take midpoint of linear parts of the S curve as a proxy for that region's average

Table of Contents



- To find the average mass from point A until the end of the chart (around 12 years to 30 years)
  - Point A to Point C (from 12 years to 20 years, or 9 years total) is roughly linear, so midpoint is average → 9 years at 3,000
  - From 21 years to 30 years (10 years) is roughly an average of 5500 for 10 years
  - Weighted Average = (9*3000)+(10*5500) / (19)

- If question asks "Which of the following meet the criteria" and part of the criteria says "Definitely Y, and preferably X" → X is NOT required, but Y is (correct answer may not have X but will have Y, trap answer will have X but not Y)
- Probability → 2 way matrices
  - If the question asks you "How much are X or Y or both"? → Add P(X) + P(Y) - P(X and Y)
    - X+Y-Both
- True or false - Trap answers have "half right"
  - Gold production from placer deposits makes up only a few percent of total production <u>primarily due to increased quantities of gold being produced from lode deposits</u>.
    - Bold portion is supported in passage, but underlined statement is not mentioned as the cause of the bold statement
    - Don't just check facts; check the stated reasoning for the facts and whether that reasoning is supported by the passage
- If passage states "On average, companies make $100 a year in revenue"

If question asks what info can be extrapolated about Company ABC given the info in passage, unless explicitly stated, we CANNOT assume that Company ABC makes $100 a year in revenue

Table of Contents

AWA

chineseburned's AWA Guide (all you need)