Zoltan's solutions to GMAT Official Guide 2018 Problem Solving

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This book is dedicated to Klaudia.

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The solutions in this book are my own, favorite solutions to all Problem Solving practice questions in GMAT® Official Guide 2018. With a Master's Degree in Economics, two decades of private tutoring experience, and a 760 score on the GMAT, I hope that I can help you strengthen your GMAT skills.

I'd like to thank Jeeva Venkatraman, my student from India. He patiently listened to my explanations while I was learning how to teach GMAT quant in English.

1 Solutions to Problem Solving practice questions

PS1

To get the percent discount, we can simply determine the ratio of the discount value to the regular price of the coat.

$$\frac{150}{500} = \frac{3}{10} = 30\%$$

Answer: E

PS2

Imagine her story as a sequence of events, each event having its own growth factor. To determine the cumulative effect of these growth factors, we multiply them together. Thus, the story of her \$500 is

$$500 \cdot \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{6}{5} = \frac{100 \cdot 6}{5} = 20 \cdot 6 = \$120$$

Answer: D

PS3

8 squares are labeled x or y, and 4 squares are labeled v or w. Thus, the ratio in question is

$$\frac{8}{4} = \frac{2}{1} = 2:1$$

Answer: E

PS4

The least common denominator of these fractions is 60. Thus, the value of the given expression is

$$\frac{20 + 30 - 50 + 12 + 15 - 27}{60} = \frac{0 + 27 - 27}{60} = \frac{0}{60} = 0$$

Answer: A

PS5

Let w be the number of white tulips in each bouquet, let r be the number of red tulips in each bouquet, and let b be the number of bouquets to be made. We can set up two equations about these variables.

$$15 = w \cdot b$$
$$85 = r \cdot b$$

Since all variables are positive integers, b is a common factor of both 15 and 85. Thus, the greatest possible value of b must be the greatest common factor of $15 = 3 \cdot 5$ and $85 = 5 \cdot 17$.

$$GCF(15, 85) = 5$$

Answer: B

PS6

We can simply determine 5/4 of 5 as

$$\frac{5}{4} \cdot 5 = \frac{25}{4} = 6.25$$

Answer: E

PS7

x years from now, Rebecca will be $34+x$ years old, and her daughter will be $8+x$ years old. To determine x, we can set up an equation based on the relationship between their ages.

$$34 + x = 2(8 + x)$$
$$34 + x = 16 + 2x$$
$$x = 18$$

Answer: C

PS8

This is a unit conversion factor problem. First, we write down the question: mi/gal =? Then, we begin with the factor that has the same type of unit in its numerator. Finally, we simply multiply the first factor by other factors until all unnecessary units cancel out and we reach the required form.

$$\frac{32 \text{ mi}}{1 \not{h}} \cdot \frac{1 \not{h}}{24 \text{ gal}} = \frac{32 \text{ mi}}{24 \text{ gal}} = \frac{4}{3} \ \frac{\text{mi}}{\text{gal}}$$

Answer: D

PS9

$$2 \cdot c \cdot b \cdot 100 = 200bc$$

Answer: C

PS10

The road there is the first 50% of the total distance to be driven, and the road back is the second 50% of it. We can simply add together the distances he has already driven.

$$50\% + 50\% \cdot \frac{1}{10} = 50\% + 5\% = 55\%$$

Answer: E

PS11

A quick refresher: The classical probability formula:

$$P(A) = \frac{Favorable}{Total}$$

Another refresher: The number of terms of an arithmetic sequence:

$$n = \frac{a_n - a_1}{d} + 1$$

Consecutive integers form an arithmetic sequence with a difference of 1. To solve this probability problem, we can use the classical probability formula.

$$\frac{299 - 200 + 1}{350 - 101 + 1} = \frac{100}{250} = \frac{2}{5}$$

Answer: A

PS12

Let x be the total value of the item. We can set up an equation about the amount of the import tax he paid.

$$(x - 1{,}000) \cdot \frac{7}{100} = 87.5$$

$$x - 1{,}000 = \frac{8{,}750}{7}$$

$$x = 1{,}250 + 1{,}000 = \$2{,}250$$

Answer: C

PS13

Let x be the number 10-cent coins and let y be the number of 25-cent coins. We can set up the following system of equations.

$$x + y = 16$$
$$0.1x + 0.25y = 2.35$$

Using the *transformation & elimination* method, we can solve the system of equations for y.

$$10x + 25y = 235$$
$$10x + 10y = 160$$

If we subtract the second equation from the first equation, we can eliminate x.

$$15y = 75$$
$$y = 5$$

Answer: B

PS14

The six known sales volumes in increasing order are 2, 3, 4, 6, 7, and 8. Their sum is 30, so the daily average sales volume for the seven days is $A = (30+x)/7$, where x is the sales volume for the seventh day. Since the ordinal number for the median is $(7 + 1)/2 = 4$, the median is the 4th term in the increasingly ordered list of the seven daily sales volumes.

I. (X) $Me = 4$, and it is not equal to $A = \dfrac{30+2}{7} =$ non-integer.

II. (X) $Me = 4$, and it is not equal to $A = \dfrac{30+4}{7} =$ non-integer.

III. $(\checkmark)$ No need to check. [$Me = 5$, and it is equal to $A = \dfrac{30+5}{7} = 5$.]

Answer: B

PS15

Let x be the number of questionnaires mailed. We can set up an inequality about the number of responses needed.

$$x \cdot \frac{3}{5} \geq 300$$
$$x \geq 300 \cdot \frac{5}{3}$$
$$x \geq 500$$

Answer: D

PS16

We can choose possible values for the variables and evaluate the answer choices. For example, if $x = 2$, $y = 3$, and $z = 4$, then

	Evaluation
(A)	$4 \cdot 3 = 12$
(B)	$4 \cdot 4 = 16$
(C)	$2 \cdot 7 = 14$
(D)	$3 \cdot 6 = 18$
(E)	$4 \cdot 5 = 20$

Answer: E

PS17

A quick refresher: The formula for the perimeter of a rectangle is $P = 2(l + w)$.

We can set up a system of equations about the unknown dimensions of the garden.

$$l = 2w$$
$$360 = 2(l + w)$$

Substituting l in the second equation, we get that

$$\begin{aligned} 360 &= 2(2w + w) \\ 360 &= 6w \\ w &= 60 \implies l = 2 \cdot 60 = 120 \end{aligned}$$

Answer: A

PS18

Each dimension of the floor is a multiple of the side length of a carpet square, so the number of carpet squares needed is the ratio of the floor area to the area of a carpet square. Thus, the total cost is

$$\frac{8 \cdot 10}{2 \cdot 2} \cdot 12 = 2 \cdot 10 \cdot 12 = \$240$$

Answer: B

PS19

$$893 \cdot 79 = 893 \cdot (78 + 1) = 893 \cdot 78 + 893 = p + 893$$

Answer: D

PS20

We can organize the given data into a cross table. The value in cell H&F is 0 because the original information doesn't allow this category of books. Let x be the number of hardcover nonfiction books.

	F	NF	
P	$2x + 40$	$x + 20$	
H	0	x	
			140

We can set up an equation about the total number of books.

$$\begin{aligned} 4x + 60 &= 140 \\ 4x &= 80 \\ x &= 20 \end{aligned}$$

Answer: B

PS21

A quick refresher: The arithmetic average is the ratio of the sum of terms to the number of terms.

$$\begin{aligned} \frac{51 + N}{4} &= 18 \\ 51 + N &= 72 \\ N &= 21 \end{aligned}$$

Answer: C

PS22

If we draw a figure, we can quickly solve this problem.

$$A\text{—}S\text{—}B\text{—}C$$

We know that $AS = SB = BC$ and $SC = 4$, so

$$AC = \frac{4}{2} \cdot 3 = 2 \cdot 3 = 6$$

Answer: D

PS23

This is a unit conversion factor problem. First, we write down the question: mi =? Then, we begin with the factor that has the same type of unit in its numerator. Finally, we simply multiply the first factor by other factors until all unnecessary units cancel out and we reach the required form.

$$\frac{1 \text{ mi}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \cdot \frac{2 \cancel{\text{h}}}{5 \cancel{\text{gal}}} \cdot 3.75 \cancel{\text{gal}} = 3.75 \cdot 24 \text{ mi} = 90 \text{ mi}$$

Answer: E

PS24

We can organize the given data into the following table.

	Effective price	Quantity	Cost
Hollow pine	\$40	5	$40 \cdot 5 =$ \$200
Solid oak	$2 \cdot 40 \cdot \frac{3}{4} =$ \$60	6	$60 \cdot 6 =$ \$360
Total			\$560

Answer: C

PS25

$$-\frac{11}{2} < y - \frac{1}{2} < \frac{11}{2}$$
$$-\frac{10}{2} < y < \frac{12}{2}$$
$$-5 < y < 6$$

The only answer choice in the above interval is C.

Answer: C

PS26

Triangle ABD is isosceles because $AD = AC - DC = 2 - 1 = 1$ and consequently $AD = BD$. Angle ABD is one of its base angles, so the measure of angle ABD must be

$$\frac{180° - 120°}{2} = \frac{60°}{2} = 30°$$

Answer: C

PS27

A quick refresher: $\sqrt{x^2} = |x|$

If we take the square root of both sides of the given equation, we get that

$$|k| = |m|$$

Answer: E

PS28

Let x be the common multiplier for the parts of the ratio. For the actual amounts, the following must be true.

$$M : N : O : Total = 15x : 20x : 30x : 65x$$

If we determine the common multiplier, we can calculate the amount paid to Makoto.

$$\begin{aligned} 65x &= 780 \\ x &= \frac{780}{65} \implies M = 15x = 15 \cdot \frac{780}{65} = 3 \cdot \frac{780}{13} = 3 \cdot 60 = \$180 \end{aligned}$$

Answer: D

PS29

Using the Pythagorean triple $3 - 4 - 5$, we can get that $PR = 10$. To answer the question, we have to determine the ratio of the additional distance Mary walked to the distance Ted walked.

$$\frac{8 + 6 - 10}{10} = \frac{4}{10} = 40\%$$

Answer: B

PS30

Let x be the total amount of money he spent at the supermarket. We can set up an equation based on his shopping list.

$$x = \frac{1}{2}x + \frac{1}{3}x + \frac{1}{10}x + 6$$
$$x = \frac{15+10+3}{30}x + 6$$
$$x = \frac{14}{15}x + 6$$
$$\frac{x}{15} = 6$$
$$x = \$90$$

Answer: C

PS31

$$N = \frac{1}{1-1.25} = \frac{1}{-0.25} = -4$$

Answer: C

PS32

A quick refresher: For similar 3D objects:

$$\text{Ratio of volumes} = (\text{Ratio of corresponding sides})^3$$

These two rectangular boxes are similar 3D objects, so we can set up an equation about the relationship between certain ratios.

$$\frac{V_2}{V_1} = \left(\frac{s_2}{s_1}\right)^3$$
$$\frac{V_2}{10} = \left(\frac{2}{1}\right)^3 = 8$$
$$V_2 = 80$$

Answer: D

PS33

Let x be the number in question. We can set up an equation based on the information given.

$$\frac{\frac{x}{2}}{\frac{2}{3}} = \frac{9}{2}$$
$$x = \frac{9}{2} \cdot \frac{2}{3} = 3$$

Answer: C

PS34

Imagine cutting the cube in half by a plane so that this cut results in a circle inscribed in a square. The relationship between r and e is clear.

$$e = 2r$$
$$r = \frac{e}{2}$$

Answer: A

PS35

Solving this system of equations for x and y is unnecessary because we need only the value of $x + y$ to answer the question. If we add the given equations together, we get that

$$3x + 3y = 12$$
$$x + y = 4 \implies \frac{x + y}{3} = \frac{4}{3}$$

Answer: B

PS36

To get the full ratio chain, we have to expand the ratio fractions so that they can be joined by a pair of identical numbers.

X	:	Y	:	Z
4	:	1		
		2	:	1
8	:	2		
8	:	2	:	1

Thus, $X : Z = 8 : 1$.

Answer: E

PS37

$$2.2 - (-0.5) = 2.2 + 0.5 = 2.7$$

Answer: E

PS38

$$\text{Min} = 20{,}000 \cdot 2{,}500 \cdot \frac{3}{1{,}000} = \$150{,}000$$
$$\text{Max} = 20{,}000 \cdot 2{,}500 \cdot \frac{5}{1{,}000} = \$250{,}000$$

Answer: D

PS39

Imagine the area of the patio as the difference between the area of a larger rectangle and the area of a smaller rectangle. The corner of the house cuts the smaller rectangle out of the larger rectangle.

$$35 \cdot 40 - 20 \cdot 20 = 1{,}400 - 400 = 1{,}000$$

Answer: C

PS40

A quick refresher: The $ax+b$ linear transformation of every item in a list has the same linear effect on the average, median, and mode, but the standard deviation and range are affected only by the multiplier a.

The linear transformation of the salaries will have the same linear effect on the average salary: the average salary will increase by 10 percent.

$$A_0 = \frac{3{,}250}{5} = \$650 \implies A_1 - A_0 = 650 \cdot \frac{1}{10} = \$65$$

Answer: E

PS41

A quick refresher: The arithmetic average is the ratio of the sum of terms to the number of terms.

Let x be his score on the 5th test. We can set up an equation about his average score on the 5 tests.

$$\frac{78 \cdot 4 + x}{5} = 80$$
$$312 + x = 400$$
$$x = 88$$

Answer: E

Extra notes: We can also use the rule that the sum of the differences from the mean is always zero.

$$4(-2) + y = 0$$
$$y = 8 \implies x = 80 + 8 = 88$$

PS42

We can set up an equation about the total amount he earned last week.

$$40x + (48 - 40) \cdot 22 = 816$$
$$40x + 176 = 816$$
$$40x = 640$$
$$x = 16$$

Answer: D

PS43

The sum of the interior angles of any triangle is 180°. We can set up an equation based on this rule.

$$y + 10 + y + 90 = 180$$
$$2y = 80$$
$$y = 40$$

Thus, the ratio in question is

$$\frac{y}{y+10} = \frac{40}{50} = \frac{4}{5} \implies 4 \text{ to } 5$$

Answer: D

PS44

The question implies that the remainders in question must be the same for all prime numbers greater than 3, so we can simply choose a possible value for n, such as 5, and determine the remainder.

$$5^2 = 25 \implies 25 \bmod 12 = 1$$

Answer: B

PS45

$$\frac{1}{\frac{4}{3}} - \frac{1}{\frac{3}{2}} = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$$

Answer: D

PS46

This is a place value problem. Let x be the 2-digit integer $\overline{ab}$ and let y be the 2-digit integer $\overline{ba}$.

$$x + y = 10a + b + 10b + a = 11a + 11b = 11(a + b)$$

Since a and b are integers, $x + y$ must be a multiple of 11.

Answer: D

PS47

These 8 numbers form an arithmetic sequence with a difference of 1. Since $a_1 = -5$, the last term of the sequence is

$$a_8 = -5 + (8 - 1) \cdot 1 = 2$$

Thus, two terms must be greater than zero.

Answer: C

PS48

We need to find the difference between the total number of boxes to be filled and the number of boxes already filled.

$$\frac{s}{r} - n$$

Answer: E

PS49

I. (X) Counterexample: $a = 1$, $b = 2$, and $c = 3$.

II. ($\checkmark$) $c > b$, which follows from the original information.

III. (X) $c < b$, which contradicts the original information.

Answer: B

PS50

A quick refresher: The midpoint formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Using the midpoint formula, we can set up equations about the x and y coordinates of point Q, respectively.

$$\frac{r + x}{2} = 0 \implies x = -r$$
$$\frac{s + y}{2} = 0 \implies y = -s$$

Answer: E

Extra notes: We can also realize that point Q is the mirror image of point P with respect to the origin, so the coordinates of point Q are simply the coordinates of point P taken with the opposite signs, respectively.

$$P(r, s) \implies Q(-r, -s)$$

PS51

The original equation in distributed form is $10y^2 = x^2 - 4$.

(A) Equivalent: it's a multiple of the original equation.

(B) Equivalent: $20y^2 = 2x^2 - 8$, which is a multiple of the original equation.

(C) Equivalent: it's the original equation with rearranged terms.

(D) Not equivalent: the ratios of the corresponding coefficients are not all equal.

(E) Equivalent: it's the original equation after division by 10.

Answer: D

PS52

A quick refresher: The standard deviation is the quadratic average of the differences from the mean. Since it's a quadratic average, only the absolute differences matter.

(A) $A = 50$, and consequently the absolute differences from the mean are 5, 5, 0, and 0. $\implies 0 < SD < 5$

(B) $A = 20$, and consequently the absolute differences from the mean are 10, 10, 10, and 10. $\implies SD = 10$

(C) This SD is clearly less than the SD for Company B.

(D) This SD is clearly less than the SD for Company B.

(E) $A = 60$, and consequently the absolute differences from the mean are 10, 0, 0, and 10. $\implies 0 < SD < 10$

Answer: B

Extra notes: The quadratic average is always greater than the arithmetic average if the numbers in a list are not all equal. Thus, we can give more precise intervals for the SDs for Company A and Company E.

(A) The arithmetic average of the absolute differences is $\dfrac{5+5+0+0}{4} = 2.5$. $\implies 2.5 < SD < 5$

(E) The arithmetic average of the absolute differences is $\dfrac{10+0+0+10}{4} = 5$. $\implies 5 < SD < 10$

PS53

Let x be the number of units the company needs to produce to reach its goal. We can set up an equation between the total revenue and the total cost.

$$2x = 5{,}040 + \frac{2}{5} \cdot 2x$$
$$\frac{6}{5}x = 5{,}040$$
$$x = \frac{5{,}040 \cdot 5}{6} = 840 \cdot 5 = 4{,}200$$

Answer: A

PS54

One full rotation of the pointer consists of 8 intervals, so we need to determine the remainder of 1,174 after division by 8.

$$1{,}174 \bmod 8 = 6$$

The pointer will stop at the letter that is 6 clockwise rotated intervals after the starting point. Thus, the pointer will stop at letter E.

Answer: E

PS55

To answer the question, we can simply determine the ratio of the excess number for state A to the number for state D.

$$\frac{181-79}{79} \approx \frac{100}{80} = \frac{5}{4} = 125\%$$

Answer: D

PS56

If we combine the given conditions, we get that $xy < 0$. Thus, x and y must have different signs. Only the points in quadrants II or IV have x and y coordinates with different signs.

Answer: D

PS57

Let C be the capacity of the tank, in liters. We can set up an equation based on the information given.

$$\frac{1}{3}C + n = \frac{7}{9}C$$
$$\frac{4}{9}C = n$$
$$C = \frac{9}{4}n$$

Answer: D

PS58

We can set up an equation based on the information given.

$$(100 + w)(150 + w) = 2 \cdot 100 \cdot 150 = 30{,}000$$

Now, we can use the *backwards* method to evaluate the answer choices because the direction of change in the values of the answer choices determines the direction of change in the value of an expression directly related to the evaluation of the answer choices.

Choice	w	Value	Evaluation
(B)	50	$150 \cdot 200 = 30{,}000$	$30{,}000 = 30{,}000$
All others			No need to check.

Answer: B

PS59

$$\frac{1}{-0.25} = -4$$

Answer: A

PS60

On his first investment, he earned $8{,}000 \cdot 0.06 = 80 \cdot 6 = \$480$ interest. The growth factor for his second investment was the product of the growth factors for the two halves of the year. Since 4 percent was compounded for both the first and the second halves of the year, the combined growth factor was $1.04 \cdot 1.04 = 1.0816$. On his second investment, he earned $10{,}000 \cdot 0.0816 = \$816$ interest. Thus, the total interest he earned on the two investments was

$$480 + 816 = \$1{,}296$$

Answer: E

Extra notes: We can also realize that had the 8 percent for the second investment been a simple annual interest rate, he would have earned \$800 interest on that investment. Thus, the total interest he earned would have been \$1,280, and the correct answer choice would be D. However, since the interest on his second investment was compounded semiannually, he must have earned more. Thus, we must choose an answer choice with a value greater than \$1,280. The only option left is answer choice E.

PS61

We have to determine a part-to-whole ratio: the ratio of the yield of these x trees to the total yield of the orchard.

$$\frac{10x}{350} = \frac{x}{35}$$

Answer: A

PS62

Have a quick look at the answer choices. Since n is an integer, the expression in answer choice C is a multiple of 2, and thus it must be even.

Answer: C

PS63

Have a quick look at the answer choices. Since $1/9 < 1/8$, the expression in question must be less than 1. It's also clear that it must be greater than 3/4.

$$\frac{3}{4} = \frac{6}{8} < \frac{7}{8} + \frac{1}{9} < \frac{7}{8} + \frac{1}{8} = 1$$

Answer: B

PS64

Although this is a simple problem, it doesn't take much time to use the *unit conversion factor* method.

$$\text{Car X:} \quad \frac{1 \text{ gal}}{25 \cancel{\text{mi}}} \cdot 12{,}000 \cancel{\text{mi}} = \frac{12{,}000}{25} \text{ gal} = \frac{120 \cdot 100}{25} \text{ gal} = 480 \text{ gal}$$

$$\text{Car Y:} \quad \approx \frac{1 \text{ gal}}{12 \cancel{\text{mi}}} \cdot 12{,}000 \cancel{\text{mi}} = \frac{12{,}000}{12} \text{ gal} = 1{,}000 \text{ gal}$$

Thus, Car Y will use $1{,}000 - 480 = 520$ gallons more than Car X.

Answer: C

PS65

$$|23 - 5y| = 5|4.6 - y|$$

$|4.6 - y|$ measures the distance between 4.6 and y. The smaller this distance, the smaller the value of the original expression. We know that y must be an integer. Although we quickly see that 5 is the integer closest to 4.6, we must be careful not to choose answer choice E because the original question is about the value of the whole expression.

$$|23 - 5 \cdot 5| = |23 - 25| = |-2| = 2$$

Answer: B

PS66

To simplify the expressions, we have to factor out as much as we can from under the square roots.

$$\sqrt{16 \cdot 5} + \sqrt{25 \cdot 5} = 4\sqrt{5} + 5\sqrt{5} = 9\sqrt{5}$$

Answer: A

Extra notes: In a harder version of this problem, we should start by prime factorizing the numbers under the square roots.

PS67

First, we substitute the given x and y values into the equation to get the value of k. Then, we can determine the value of y in the particular situation.

$$\begin{aligned} 17 &= 2k + 3 \\ 2k &= 14 \\ k &= 7 \implies y = 7 \cdot 4 + 3 = 31 \end{aligned}$$

Answer: B

PS68

To select the greatest expression, we can factor in the coefficients under the square roots.

(A) $\sqrt{100 \cdot 3} = \sqrt{300}$

(B) $\sqrt{81 \cdot 4} = \sqrt{324}$

(C) $\sqrt{64 \cdot 5} = \sqrt{320}$

(D) $\sqrt{49 \cdot 6} < \sqrt{300}$

(E) $\sqrt{36 \cdot 7} < \sqrt{300}$

Answer: B

PS69

This is a rate-time-distance problem, so we can use the $R \cdot T = D$ formula. Let t be the time Al drove that day. Ben started 3 hours earlier than Al, so Ben drove $t + 3$ hours that day.

	R (mph)	T (h)	D (mi)
Al	40	t	$40t$
Ben	20	$t+3$	$20t+60$
Total			$60t+60$

We can set up an equation about the total distance driven.

$$\begin{aligned} 60t + 60 &= 240 \\ 60t &= 180 \\ t &= 3 \implies \text{11:00 am} + 3\text{ h} = \text{2:00 pm} \end{aligned}$$

Answer: B

Extra notes: If we can select the category in which we want to introduce a variable, we should always choose T rather than D so that we can avoid using fractions.

PS70

Since 10% of the cleared land is 720 acres, the cleared land is

$$\frac{720}{0.1} = 7{,}200$$

Since 90% of the total land is 7,200 acres, the total land is

$$\frac{7{,}200}{0.9} = \frac{72{,}000}{9} = 8{,}000$$

Answer: D

Extra notes: We can also use the *sequence of events* method to solve this problem. Let x be the area of the total land.

$$x \cdot \frac{9}{10} \cdot \frac{1}{10} = 720$$
$$x = \frac{72{,}000}{9} = 8{,}000$$

PS71

The growth factor was 2 for each month. To get the cumulative effect of the growth factors, we have to multiply them together. Thus, the population size at the end of the 10-month period is $3 \cdot 2^{10}$.

Answer: D

PS72

Looking at the expressions in the given equation, we can realize that at the right-hand side, r is multiplied by 1/3 of the left-hand-side expression. Let x be the left-hand-side expression.

$$x = r \cdot \frac{1}{3}x$$
$$3x = rx$$
$$3 = r$$

Answer: C

PS73

To solve this problem quickly, we have to know the multiple rules very well. Both $4y$ and 200 are multiples of 20, so $3x$ must also be a multiple of 20. Since 20 and 3 do not share any prime factors, the integer x must be a multiple of 20. Being a multiple of 20 is the same thing as being divisible by 20. Only answer choice E is a divisor of 20.

Answer: E

PS74

I. (✓) $\left(2\sqrt{82}\right)^2 = 4 \cdot 82$

II. (X) *Nonzero rational* $\cdot$ *Irrational* = *Irrational*

III. (✓) $\frac{82}{82} = 1$

Answer: E

PS75

Let x be the common multiplier for the parts of the ratio and let S_1, S_2, S_3, and S_4 be the actual numbers of working hours of the four workers, respectively.

$$S_1 : S_2 : S_3 : S_4 : Total = 2x : 3x : 5x : 6x : 16x$$

For each case, we can determine a possible common multiplier and calculate a possible total number of working hours.

$$\begin{aligned} S_1 &\implies 16x = 16 \cdot \frac{30}{2} = 8 \cdot 30 = 240 \\ S_2 &\implies 16x = 16 \cdot \frac{30}{3} = 16 \cdot 10 = 160 \\ S_3 &\implies 16x = 16 \cdot \frac{30}{5} = 16 \cdot 6 = 96 \\ S_4 &\implies 16x = 16 \cdot \frac{30}{6} = 16 \cdot 5 = 80 \end{aligned}$$

D is the only answer choice that is not a possible value of the total number of working hours.

Answer: D

PS76

This is a rate-time-work problem, so we can use the $R \cdot T = W$ formula.

	R	T	W
P_1	$\frac{1}{2 \cdot 3} = \frac{1}{6}$	3	$\frac{1}{2}$
P_2	$\frac{2}{3 \cdot 6} = \frac{1}{9}$	6	$\frac{2}{3}$
$P_1 + P_2$	$\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$	$t = ?$	1

Thus, the time in question is

$$t = 1 \cdot \frac{18}{5} = \frac{36}{10} = 3.6$$

Answer: B

PS77

We can rearrange the given equation into slope-intercept form.

$$3y = -kx + 6$$
$$y = -\frac{k}{3}x + 2$$

Since k doesn't have any effect on the y-intercept of the line, the point $(0, 2)$ must be on the line for every possible value of k.

Answer: B

PS78

We can solve the given inequality for x.

$$x^2 < 2$$
$$|x| = \sqrt{2}$$
$$-\sqrt{2} < x < \sqrt{2}$$

Answer: C

PS79

We first need to determine the number of days and the cumulative number of days for each book.

Book	Number of days	Cumulative number of days
1	6	6
2	3	9
3	3	12
4	4	16
5	4	20
6	1	21
7	5	26
8	2	28

We don't have to evaluate books 9 to 12 because she will just have reached her time limit by the time she finishes book 8.

Answer: B

PS80

We can organize the given data into the following table.

Year	Market share	Sales volume
1990	42%	42% of x
1993	33%	33% of x

Thus, the decrease in sales volume was

$$0.42x - 0.33x = 0.09x = 9\% \text{ of } x$$

Answer: D

Extra notes: The question is not about the decrease in market share, but about the decrease in sales volume.

PS81

The question implies that the remainders in question must be the same for all positive integer values of k, so we can simply choose a possible value for k, such as 1, and determine the remainder.

$$(1+2)(1^3-1) = 3 \cdot 0 = 0 \implies 0 \bmod 6 = 0$$

Answer: A

PS82

The fraction closest to 1/2 is the one with the smallest absolute difference from it. Since the answer choices are all greater than 1/2, we can evaluate them by simple subtractions.

(A) $\frac{4}{7} - \frac{1}{2} = \frac{8-7}{14} = \frac{1}{14}$

(B) $\frac{5}{9} - \frac{1}{2} = \frac{10-9}{18} = \frac{1}{18}$

(C) $\frac{6}{11} - \frac{1}{2} = \frac{12-11}{22} = \frac{1}{22}$

(D) $\frac{7}{13} - \frac{1}{2} = \frac{14-13}{26} = \frac{1}{26}$

(E) $\frac{9}{16} - \frac{1}{2} = \frac{9-8}{16} = \frac{1}{16}$

Answer: D

Extra notes: Since the answer choices are all greater than 1/2, the smallest is the closest to 1/2. If we compare only two fractions at a time, we can quickly choose the smallest one.

PS83

If we multiply both sides of the equation by p, we can isolate r.

$$p^2 - (1 - p^2) = r$$
$$p^2 - 1 + p^2 = r$$
$$r = 2p^2 - 1$$

Answer: D

PS84

A quick refresher: $Range = Greatest - Smallest$

x takes its greatest possible value if x is the greatest element in the data set.

$$12 = x - 3 \implies x_{max} = 15$$

x takes its least possible value if x is the smallest element in the data set.

$$12 = 14 - x \implies x_{min} = 2$$

Thus, the difference in question is $15 - 2 = 13$.

Answer: D

PS85

Let x be the number in question. We can set up an equation based on the information given.

$$x = \frac{2}{3}x + 108$$
$$\frac{1}{3}x = 108$$
$$x = 324$$

Answer: E

PS86

Let's first determine the ratio of the prescribed dosage to the typical dosage.

$$\frac{\frac{18}{120}}{\frac{2}{15}} = \frac{18}{120} \cdot \frac{15}{2} = \frac{9}{8} = 112.5\%$$

Thus, the prescribed dosage was greater than the typical dosage by 12.5%.

Answer: D

PS87

Let x be the number of employees in January. We know that the growth factor for the number of employees was 1.15, and we can set up an equation based on the information given.

$$1.15x = 460$$
$$x = \frac{460}{1.15} = \frac{46{,}000}{115} = \frac{46{,}000}{23 \cdot 5} = \frac{2{,}000}{5} = 400$$

Answer: B

PS88

To get $f(t)$, we can simply substitute t for x in $f(x)$.

$$f(t) = \sqrt{t} - 10$$

We can now solve the equation $u = f(t)$ for t.

$$u = \sqrt{t} - 10$$
$$\sqrt{t} = u + 10$$
$$t = (u + 10)^2$$

Answer: D

PS89

To answer the question, we have to determine the ratio of the evaporated amount to the original amount.

$$\frac{0.01 \cdot 20}{10} = 0.02 = 2\%$$

Answer: D

PS90

We can systematically test possible cases for m and p, trying to maximize mp.

m	p	mp
9	4	36
8	5	40
7	7	49
6	7	Clearly less.
5	8	No need to check.
4	9	No need to check.
3	9	Clearly less.
2	9	Clearly less.
1	9	Clearly less.

Testing more cases is unnecessary because both the given inequality condition and the expression in question are symmetric in m and p.

Answer: D

PS91

We must realize that none of the variables can be equal to zero. If we take reciprocal of the second equation, we get that $c/d = a/b$. We can then combine this transformed equation and the first original equation into the following 3-sided equation.

$$\frac{x}{y} = \frac{c}{d} = \frac{a}{b}$$

I. (✓) It follows from the 3-sided equation.

II. (✓) It follows from the 3-sided equation.

III. (X) It does not follow from the 3-sided equation.

Answer: C

PS92

To simplify the expression, we can rewrite every decimal as the product of an integer with a nonzero units digit and of an integer power of 10.

$$25 \cdot 10^{-4} \cdot 25 \cdot 10^{-3} \cdot 25 \cdot 10^{-5} \cdot 10^{k} = 25^{3} \cdot 10^{k-12}$$

25^3 is an integer with a nonzero units digit. If $k = 12$, then the product in question is the integer 25^3. However, if k is an integer less than 12, then multiplying by 10^{k-12} moves the decimal point to the left in the value of 25^3, and consequently the product in question becomes a decimal. Thus, the smallest possible value of k is 12.

Answer: E

PS93

The two equations have the same roots because they are symmetric in their variables.

$$a^2 + 2a - 24 = 0$$
$$(a+6)(a-4) = 0 \implies a = -6 \text{ OR } a = 4 \implies b = -6 \text{ OR } b = 4$$

Since $a \neq b$, the value of $a + b$ is

$$a + b = -6 + 4 = -2 \text{ OR } a + b = 4 - 6 = -2$$

Answer: B

PS94

The number of votes cast by registered voters is 60 percent of N. Thus, the number of votes she received is

$$8{,}000 + \frac{1}{10} \cdot \frac{3}{5}N = 8{,}000 + 0.06N$$

Answer: E

PS95

A quick refresher: Thales' theorem: If a triangle is inscribed in a circle with a diameter as one of its sides, then its angle opposite this diameter must be a right angle.

Using the Pythagorean triple $3 - 4 - 5$, we can get that $AC = 10$. Thus, the length of arc ABC is half of the circumference of a circle with a radius of $10/2 = 5$.

$$\frac{2 \cdot 5\pi}{2} = 5\pi$$

Answer: E

PS96

A quick refresher: The weighted arithmetic average formula:

$$A = \frac{\sum w_i x_i}{\sum w_i}$$

We know that $Q : P = 2 : 1$. Since we have denominator-type weights, we can use the weighted arithmetic average formula.

$$\frac{2 \cdot 17 + 1 \cdot 20}{3} = \frac{54}{3} = \$18$$

Answer: E

Extra notes: Why is this a weighted average problem, what are the subgroups, and what type of weights are at the heart of this problem, numerator-type or denominator-type? Well, these are questions one has to answer comfortably.

PS97

A quick refresher: $Revenue = Price \cdot Quantity$

Let x be the amount of orange juice used on each day, let g be the volume of a glass, and let p_2 be the price per glass on the second day. We can set up an equation between the daily revenues.

$$0.6 \cdot \frac{2x}{g} = p_2 \cdot \frac{3x}{g}$$
$$3p_2 = 1.2$$
$$p_2 = \$0.4$$

Answer: D

PS98

Since he has carried a total of $17 \cdot 4 = 68$ jugs so far, the number of jugs he has already packed into the last carton is $68 \bmod 7 = 5$. Thus, 2 more jugs are needed to fill the last carton.

Answer: B

PS99

Using that $l = 3w$, we can set up an equation about the area of the painting.

$$3w \cdot 4w = 4{,}800$$
$$12w^2 = 4{,}800$$
$$w^2 = 400$$
$$w = 20$$

Answer: B

PS100

Let A be the number of apples purchased and let B be the number of bananas purchased. We know that A and B are positive integers, and we can set up an equation about the total cost of his purchases.

$$0.7A + 0.5B = 6.3$$
$$7A + 5B = 63$$

Both $7A$ and 63 are multiples of 7, so $5B$ must also be a multiple of 7. Since 7 and 5 do not share any prime factors, B must be a multiple of 7. The only possible value of B is 7, and consequently A must be 4.

$$A + B = 4 + 7 = 11$$

Answer: B

PS101

If we rearrange the equation of the line into slope-intercept form, we can quickly answer the question.

$$7y = -3x + 9$$
$$y = -\frac{3}{7}x + \frac{9}{7} \implies m = -\frac{3}{7}$$

Answer: B

PS102

This is a rate-time-work problem, so we can use the $R \cdot T = W$ formula.

	R	T	W
4 machines	$\frac{x}{6}$	6	x
1 machine	$\frac{x}{24}$		
k machines	$\frac{kx}{24}$		
Plan	$\frac{3x}{4}$	4	$3x$

We can set up an equation between the production rate of k machines and the planned production rate.

$$\frac{kx}{24} = \frac{3x}{4}$$
$$k = \frac{3 \cdot 24}{4} = 18$$

Answer: B

PS103

To get the full ratio chain, we have to expand the ratio fractions so that they can be joined by pairs of identical numbers.

g_1	:	g_2	:	g_3	:	g_4
		8			:	5
3	:	4				
				3	:	2
		16			:	10
				15	:	10
12	:	16				
12	:	16	:	15	:	10

Thus, $g_1 : g_3 = 12 : 15 = 4 : 5$.

Answer: E

PS104

The given inequality is true only if the symbol Δ denotes either subtraction or division.

I. (X) It is not true for division.

II. (X) It is not true for subtraction.

III. $(\checkmark)$ No need to check. [It is true for both subtraction and division.]

Answer: C

PS105

This is a unit conversion factor problem. First, we write down the question: km =? Then, we begin with the factor that has the same type of unit in its numerator. Finally, we simply multiply the first factor by other factors until all unnecessary units cancel out and we reach the required form.

$$\frac{1\text{ km}}{3.9 \cdot 10^4\ \cancel{\text{in}}} \cdot 2.3 \cdot 10^{14}\ \cancel{\text{in}} = \frac{2.3}{3.9} \cdot 10^{10}\text{ km} \approx \frac{24}{40} \cdot 10^{10}\text{ km}$$
$$= \frac{6}{10} \cdot 10^{10}\text{ km} = 0.6 \cdot 10^{10}\text{ km} = 6 \cdot 10^9\text{ km}$$

Answer: B

PS106

We can set up an equation based on the information given, and we can then solve this equation for the expression in question.

$$\frac{1}{4}n = \frac{37.5}{100}m$$
$$n = \frac{37.5}{25}m$$
$$\frac{12n}{m} = \frac{37.5 \cdot 12}{25} = \frac{450}{25} = 18$$

Answer: A

PS107

To determine the radius of the circle, we can use the formula for the distance between two points.

$$r = \sqrt{(5-2)^2 + (0-(-3))^2} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

Thus, the area of the circle is $A = r^2\pi = 18\pi$.

Answer: E

PS108

Sally grew as many inches as Joe did plus 200 percent more.

$$1 + 2 \cdot 1 = 3$$

Answer: D

PS109

Substituting the given removal percents into the cost formula, we can determine the difference in question.

$$\frac{100{,}000 \cdot 90}{10} - \frac{100{,}000 \cdot 80}{20} = 900{,}000 - 400{,}000 = \$500{,}000$$

Answer: A

PS110

To evaluate each option, we can substitute it for y in the given equation and check whether the equation remains true.

I. (X) $\frac{x^2}{4x^2} - \frac{x}{2x} = \frac{1}{4} - \frac{1}{2} \neq 6$

II. (✓) No need to check. [$\frac{4x^2}{x^2} + \frac{2x}{x} = 4 + 2 = 6$]

III. (✓) $\frac{9x^2}{x^2} - \frac{3x}{x} = 9 - 3 = 6$

Answer: E

Extra notes: We could also solve the original quadratic equation for xy and then get y in terms of x directly.

PS111

$$N = \frac{20 \cdot 2 \cdot \frac{1}{2} \cdot 5{,}280}{600 + 1{,}600} = \frac{1{,}056}{22} \approx \frac{1{,}000}{20} = 50$$

Answer: D

PS112

$$\sqrt{48 \cdot 10^8} \approx \sqrt{49 \cdot 10^8} = 7 \cdot 10^4 = 70{,}000$$

Answer: B

PS113

This is a rate-time-work problem, so we can use the $R \cdot T = W$ formula.

	R	T	W
$R + S + T$	$\frac{1}{4}$	4	1
$S + T$	$\frac{1}{5}$	5	1
R	$\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$	$t = ?$	1

Thus, the time in question is $t = 1 \cdot \frac{20}{1} = 20$.

Answer: E

PS114

We can set up an equation between the volume of the new cheese ball and the sum of the volumes of the small cheese balls. Let R be the radius of the new cheese ball and let D be its diameter.

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot 1^3 + \frac{4}{3}\pi \cdot 2^3 + \frac{4}{3}\pi \cdot 3^3$$
$$R^3 = 1 + 8 + 27 = 36$$
$$R = \sqrt[3]{36} \implies D = 2\sqrt[3]{36}$$

Answer: E

PS115

We have to sum the integers $-25, -24, -23, \ldots, 0, \ldots, 23$. Every positive term in the sum can be paired with and canceled out by a negative one. Only three terms remain, and their sum is $-25 - 24 + 0 = -49$.

Answer: D

PS116

The coordinate of point R is a negative number, and the coordinates of points S and T are positive numbers. To get the coordinate of point R, we have to take the absolute value of its coordinate with a negative sign. Thus, the average of the three coordinates is

$$A = \frac{-r + s + t}{3} = \frac{s + t - r}{3}$$

Answer: E

PS117

For each person, we can set up an inequality about his or her sales volume. We can also set up an inequality about the total sales volume.

Mark:	$n - 10 \geq 1$	$\implies$	$n \geq 11$
Ann:	$n - 2 \geq 1$	$\implies$	$n \geq 3$
Total:	$2n - 12 < n$	$\implies$	$n < 12$

The combined solution of the three inequalities above is $11 \leq n < 12$. Thus, the only integer that satisfies all inequalities is $n = 11$.

Answer: A

PS118

We can use one of the formulas for the number of elements of two overlapping sets.

$$Total = All1s - All2 + Neither$$
$$5{,}000 = x + y - z + Neither$$
$$Neither = 5{,}000 - x - y + z$$

Answer: E

Extra notes: We could also use the *cross table* method.

PS119

Let H be the number of stocks closed higher and let L be the number of stocks closed lower. We can set up the following system of equations.

$$H = \frac{6}{5}L \implies L = \frac{5}{6}H$$
$$H + L = 2{,}420$$

We can isolate L from the first equation and substitute it in the second equation.

$$\frac{11}{6}H = 2{,}420$$
$$H = \frac{2{,}420 \cdot 6}{11} = 220 \cdot 6 = 1{,}320$$

Answer: D

PS120

We can organize the given data into a cross table and put a ? mark in the cell directly related to the question. The value in cell NS&NE is 0 because the given information doesn't allow this category of attendees. Solving the problem is faster if we leave the % symbols.

	E	NE	
S		?	62
NS		0	
	47		100

We can quickly get the answer.

	E	NE	
S		**53**	62
NS		0	
	47	53	100

Answer: D

PS121

The number of students is divisible by both $8 = 2^3$ and $12 = 2^2 \cdot 3$, so it must also be divisible by $LCM(8, 12) = 2^3 \cdot 3 = 24$. The smallest positive multiple of 24 is 24 itself.

Answer: B

PS122

For each account, we can calculate a 6 percent threshold based on the amount budgeted and compare this threshold to the absolute difference between the amount spent and the amount budgeted.

Account	6 percent threshold	Absolute difference	Evaluation
Payroll	$110{,}000 \cdot 0.06 = 6{,}600$	7,000	(✓)
Taxes	No need to check.		(X)
Insurance	$2{,}500 \cdot 0.06 = 150$	160	(✓)

Answer: D

PS123

If we connect point P and each vertex of the triangle, then triangle ABC is cut into three identical isosceles triangles. The vertex angle in each of these three triangles is $360/3 = 120°$. Thus, triangle ABC must be rotated clockwise by $120 \cdot 2 = 240°$ to reach the desired position.

Answer: D

PS124

Let f be the number of members voting for the resolution and let a be the number of members voting against the resolution. For the resolution to pass, it must be true that

$$f \geq \frac{2}{3} \cdot 40$$

$$f \geq 26.\overline{6} \implies f_{min} = 27 \implies a_{max} = 40 - 27 = 13$$

Answer: E

PS125

A quick refresher: The percent change formula:

$$\left(\frac{x_1 - x_0}{x_0}\right) \cdot 100\%$$

To calculate faster, we can leave the multiplication by 100%, which forces the result into percent form. Furthermore, we must know that the magnitude of percent change is the absolute value of the percent change.

Days	Magnitude of percent change
$1 \to 2$	$\frac{8}{20} = \frac{40}{100} = 40\%$
$2 \to 3$	$\frac{6}{12} = \frac{1}{2} = 50\%$
$3 \to 4$	$\frac{8}{18} = \frac{4}{9} \approx 44.44\%$
$4 \to 5$	$\frac{6}{10} = 60\%$
$5 \to 6$	$\frac{8}{16} = \frac{1}{2} = 50\%$

Answer: D

PS126

To solve this problem quickly, we have to know the multiple rules very well.

I. (X) Since 20! is a multiple of 15 and 17 is a non-multiple of 15, n must be a non-multiple of 15.

II. (✓) Since both 20! and 17 are multiples of 17, n must also be a multiple of 17.

III. (X) Since 20! is a multiple of 19 and 17 is a non-multiple of 19, n must be a non-multiple of 19.

Answer: C

PS127

Let x and y be negative numbers such that $x < y$. We can set up a system of equations and solve it for y.

$$xy = 160$$
$$x = 2y - 4$$

We can substitute x in the first equation.

$$\begin{aligned}(2y-4)y &= 160\\ 2y^2-4y &= 160\\ y^2-2y-80 &= 0\\ (y-10)(y+8) &= 0 \implies \cancel{y=10} \text{ OR } y=-8\end{aligned}$$

Only $y=-8$ satisfies the original condition.

Answer: D

PS128

The maximum depth is reached 5 hours past 2:00 am because $t=5$ eliminates the first term in the expression for N. Any other value for t would make the first term negative and consequently result in a smaller depth.

$$\text{2:00 am} + 5 \text{ h} = \text{7:00 am}$$

Answer: B

PS129

This is a rate-time-distance problem, so we can use the $R \cdot T = D$ formula. Be careful with his speed: 8 minutes per mile is equivalent to a speed of 1/8 mile per minute. Let x be the number of miles he can run farther south. Before returning to his car, he will run this x miles south, and then he will run back north this x miles plus the initial 3.25 miles.

R (mi/min)	T (min)	D (mi)
1/8	50	$3.25+2x$

$$\begin{aligned}3.25+2x &= \frac{1}{8}\cdot 50 = \frac{25}{4}\\ 13+8x &= 25\\ 8x &= 12\\ x &= \frac{12}{8} = \frac{3}{2} = 1.5\end{aligned}$$

Answer: A

PS130

The growth factor for his investment was 1.08 in each of the two years. The value of his investment at the end of the first year was $1.08x$, and he then invested an additional x amount of money. We can set up an equation about the value of his investment at the end of the second year.

$$(1.08x + x)1.08 = w$$
$$1.08^2 x + 1.08x = w$$
$$x(1.08^2 + 1.08) = w \implies x = \frac{w}{1.08^2 + 1.08}$$

Answer: D

PS131

M is the sum of 100 reciprocals because there are 100 integers from 201 to 300, inclusive. Using that the smallest of these reciprocals is 1/300 and that the greatest of them is 1/201, we can determine upper and lower bounds for M.

$$\frac{1}{300} \cdot 100 < M < \frac{1}{201} \cdot 100 < \frac{1}{200} \cdot 100$$
$$\frac{1}{3} < M < \frac{1}{2}$$

Answer: A

PS132

This is a rate-time-work problem, so we can use the $R \cdot T = W$ formula.

	R	T	W
$A+B$	$\frac{800}{x}$	x	800
A	$\frac{800}{y}$	y	800
B	$\frac{800(y-x)}{xy}$	$t=?$	800

The rate of Machine B is the difference of the rate of Machines $A+B$ and the rate of Machine A.

$$\frac{800}{x} - \frac{800}{y} = \frac{800y - 800x}{xy} = \frac{800(y-x)}{xy}$$

Thus, the time in question is

$$t = 800 \cdot \frac{xy}{800(y-x)} = \frac{xy}{y-x}$$

Answer: E

PS133

Let x be the common multiplier for the parts of the ratio. For the actual amounts, the following must be true.

$$H : F : M : Total = 5x : 2x : x : 8x$$

If we determine the common multiplier, we can calculate the amount allocated to food.

$$8x = 1{,}800$$

$$x = \frac{1{,}800}{8} \implies F = 2x = 2 \cdot \frac{1{,}800}{8} = \frac{1{,}800}{4} = \$450$$

Answer: D

PS134

The number of blue marbles in bag P is 4 because $37 \cdot 0.108 \approx 4$. The number of blue marbles in bag Q is $(2/3)x$ because $66.7\% \approx 2/3$. The number of blue marbles in bag R is 16 because $32 \cdot 0.5 = 16$. We can set up an equation about the ratio of the total number of blue marbles to the total number of marbles.

$$\frac{4 + \frac{2}{3}x + 16}{69 + x} = \frac{1}{3}$$

$$60 + 2x = 69 + x$$

$$x = 9$$

Answer: B

PS135

We can rewrite each decimal as an integer multiplied by an integer power of 10.

$$\frac{36 \cdot 28 \cdot 10^{-5}}{4 \cdot 1 \cdot 3 \cdot 10^{-6}} = 12 \cdot 7 \cdot 10 = 840$$

Answer: A

PS136

We can test possible cases and evaluate each answer choice whether it is divisible by 3.

	$n = 7$	$n = 8$
(A)	(✓)	(✓)
(B)	(✓)	(X)
(C)	(X)	No need to check.
(D)	(X)	No need to check.
(E)	(✓)	(X)

Answer: A

PS137

Since the ordinal number for the median is $(161 + 1)/2 = 81$, the median is the 81st term in the increasingly ordered list of the 161 ages. The cumulative number of employees for the first category is 29, and the cumulative number of employees for the second category is $29 + 58 = 87$. Thus, the median must be in the second category.

Answer: A

PS138

First, we can substitute $1 - y$ for x in the expression in question.

$$100(1 - y) + 200y = 100 + 100y$$

Then, we can set up an inequality about y and transform it into an inequality about the expression in question.

$$\begin{aligned} 0 &< y < 1 \\ 0 &< 100y < 100 \\ 100 &< 100 + 100y < 200 \end{aligned}$$

I. (X)

II. (✓)

III. (✓)

Answer: E

PS139

Since $0.11 \leq 0.1X \leq 0.19$ and $0.021 \leq 0.02Y \leq 0.029$, the greatest possible value of the fraction is

$$\frac{0.1X}{0.02Y} = \frac{0.19}{0.021} = \frac{190}{21} \approx 9$$

Answer: D

PS140

There are 10 possibilities for the first place in the list. For each of these possibilities, there are 9 possibilities for the second place in the list. Etc.

$$10 \cdot 9 \cdot 8 \cdot 7 = 5{,}040$$

Answer: D

PS141

We know that n is a positive integer, and we need to find the least possible value of n such that $n!$ is divisible by

$$990 = 2 \cdot 3^2 \cdot 5 \cdot 11$$

The greatest prime factor of 990 is 11, so n must be at least 11. Since 11! is divisible by 2, 3^2, 5, and 11, it must also be divisible by 990.

Answer: D

PS142

We can organize the given data into a joint probability table. The value in cell M&R is 0 because the corresponding event is impossible.

	R	NR	
M	0		
NM			0.8
		0.6	1

Then, we can fill the empty cells.

	R	NR	
M	**0**	**0.2**	0.2
NM	**0.4**	0.4	0.8
	0.4	0.6	1

To answer the question, we have to add the corresponding probabilities together.

$$0.4 + 0 + 0.2 = 0.6 = \frac{3}{5}$$

Answer: C

PS143

A quick refresher: $Profit = Revenue - Cost$

The average profit is the ratio of profit to output.

$$\frac{20{,}000 \cdot 8 - 10{,}000 - 20{,}000 \cdot 3}{20{,}000} = \frac{2 \cdot 8 - 1 - 2 \cdot 3}{2} = \frac{16 - 7}{2} = \frac{9}{2} = \$4.5$$

Answer: C

PS144

These Q consecutive integers form an arithmetic sequence with a difference of 1. Since the number of terms of the sequence is odd, the median is the middle term, so $(Q-1)/2$ terms are greater than the median. Thus, the largest of these consecutive integers is

$$120 + \frac{Q - 1}{2}$$

Answer: A

PS145

A quick refresher: The corresponding sides of a 30-60-90 degree special right triangle are in a ratio of $1 : \sqrt{3} : 2$.

Let point A be the base of the ladder and let point B be the top of the ladder. Imagine that through point B, we draw a line perpendicular to the plane determined by the top of the fire truck. Let point C be the intersection of this line and this plane.

Triangle ABC is a 30-60-90 degree special right triangle with a hypotenuse of 70 feet. Side BC is opposite the 60° angle, so its length must be

$$\frac{70}{2} \cdot \sqrt{3} = 35\sqrt{3}$$

Since the length of BC is the height of the ladder above the top of the truck, the height of the ladder above the ground is

$$7 + 35\sqrt{3}$$

Answer: D

PS146

The radius of the semicircle is 2, so the length of the rectangle is $10 - 2 = 8$. Thus, the area of the window is

$$4 \cdot 8 + \frac{2^2\pi}{2} = 32 + 2\pi$$

Answer: E

PS147

Using the exponent rules, we can rewrite the given expression as $t^4 \cdot 10^{-12}$. The factor 10^{-12} just moves the decimal point 12 places to the left.

I. (X) $3^4 = 81$, which is a 2-digit integer. $\implies$ $12 - 2 = 10$ zeros

II. (X) $5^4 = 625$, which is a 3-digit integer. $\implies$ $12 - 3 = 9$ zeros

III. (X) $9^4 = 81 \cdot 81 \approx 6400$, which is a 4-digit integer. $\implies$ $12 - 4 = 8$ zeros

Answer: A

PS148

The second digit must be either 0 or 1. If the second digit is 0, we have

8 possible choices for the first digit
1 possible choice for the second digit
9 possible choices for the third digit

If the second digit is 1, we have

8 possible choices for the first digit
1 possible choice for the second digit
10 possible choices for the third digit

Thus, the total number of different codes is

$$8 \cdot 1 \cdot 9 + 8 \cdot 1 \cdot 10 = 72 + 80 = 152$$

Answer: B

PS149

Since the question is about a denominator-type weight and since we have only two subgroups, we can use the quick formula to solve this weighted average problem.

$$w_1 = \frac{A - x_2}{x_1 - x_2} = \frac{5 - 12}{2 - 12} = \frac{-7}{-10} = \frac{7}{10} \implies \frac{7}{10} \cdot 60 = 42 \text{ liters}$$

Answer: E

Extra notes: Why is this a weighted average problem, what are the subgroups, and what type of weights are at the heart of this problem, numerator-type or denominator-type? Well, these are questions one has to answer comfortably.

PS150

Le J be Jake's current weight and let S be his sister's weight. We can set up a system of equations and solve it for J.

$$J + S = 278$$
$$J - 8 = 2S$$

Let's subtract the second equation from the first equation.

$$S + 8 = 278 - 2S$$
$$3S = 270$$
$$S = 90 \implies J = 188$$

Answer: E

Extra notes: We could also realize that an 8-pound-loss is not significant and could quickly get an approximate value for Jake's current weight.

$$J : S : Total \approx 2 : 1 : 3 \implies J = 278 \cdot \frac{2}{3} \approx 90 \cdot 2 = 180$$

PS151

A quick refresher: The $ax+b$ linear transformation of every item in a list has the same linear effect on the average, median, and mode, but the standard deviation and range are affected only by the multiplier a.

The linear transformation of the test scores had the following effect on the standard deviation.

$$SD_1 = a \cdot SD_0$$
$$SD_1 = 0.8 \cdot 20 = 16$$

Answer: B

PS152

We can use one of the formulas for the number of elements of three overlapping sets. However, only part of the formula is needed because we are not interested in $Neither$.

$$\begin{aligned} Total &= All1s - All2s + All3 + Neither \\ At\ least\ in\ one &= All1s - All2s + All3 \end{aligned}$$

We know that no members of the club traveled to both England and France, so the number of members who traveled to all three countries must also be zero.

$$All1s - All2s + All3 = 26 + 26 + 32 - 0 - 6 - 11 + 0 = 67$$

Answer: B

PS153

A quick refresher: The percent increase formula:

$$\left(\frac{x_1 - x_0}{x_0}\right) \cdot 100\%$$

To calculate faster, we can leave the multiplication by 100%, which forces the result into percent form.

$$\frac{385 - 320}{320} = \frac{65}{320} \approx \frac{65}{65 \cdot 5} = \frac{1}{5} = 20\%$$

Answer: C

PS154

A quick refresher: If the dividend and the divisor are both positive, then

$$Decimal\ remainder = \frac{Remainder}{Divisor}$$

We can set up an equation about the decimal remainder.

$$\begin{aligned} 0.12 &= \frac{9}{y} \\ y &= \frac{9}{0.12} = \frac{900}{12} = \frac{300}{4} = 75 \end{aligned}$$

Answer: B

PS155

A quick refresher: A product is zero if and only if at least one of the factors is zero.

The solution of the first equation is $x = 0$ OR $x = -1/2$, and the solution of the second equation is $x = -1/2$ OR $x = 3/2$. Thus, the solution of the given system of equations is $x = -1/2$.

Answer: B

Extra notes: A solution of a system of equations must also be a solution of each equation.

PS156

The eight identical triangles are 45-45-90 degree special right triangles with a ratio of $1 : 1 : \sqrt{2}$ for their corresponding sides. If a leg of one of these triangles is one unit, then the ratio in question is

$$\frac{P_X}{P_Y} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3} = 2\sqrt{2} : 3$$

Answer: C

PS157

Let x be the number of teachers who taught only one class and let y be the number of teachers who taught two classes. We can set up a system of equations: one equation about the total number of teachers and another equation about the total number of classes to be taught.

$$\begin{aligned} x + y + n &= 37 \\ 1x + 2y + 3n &= 64 \end{aligned}$$

If we subtract the first equation from the second one, we can get a solution for n.

$$\begin{aligned} y + 2n &= 27 \\ n &= \frac{27 - y}{2} \end{aligned}$$

Since x, y, and n must be non-negative integers,

$$\begin{aligned} y_{max} = 27 \quad &\Longrightarrow \quad n_{min} = 0 \\ y_{min} = 1 \quad &\Longrightarrow \quad n_{max} = 13 \end{aligned}$$

We can see that x is also a non-negative integer in each of the two cases above.

Answer: A

Extra notes: y cannot be equal to 0 because n must be an integer.

PS158

$$A = \frac{5n+15}{5} = n+3$$
$$Me = n+2$$
$$A - Me = n+3-n-2 = 1$$

Answer: B

PS159

Let k be the common multiplier for the parts of the ratio. For the actual edge lengths, the following must be true.

$$L : W : H = 3k : 2k : 2k$$

We can set up an equation about the volume.

$$3k \cdot 2k \cdot 2k = x$$
$$12k^3 = x$$
$$k = \sqrt[3]{\frac{x}{12}} \implies H = 2k = 2 \cdot \sqrt[3]{\frac{x}{12}} = \sqrt[3]{\frac{8x}{12}} = \sqrt[3]{\frac{2x}{3}}$$

Answer: B

PS160

Let x be the common multiplier for the parts of the ratio. For the actual numbers of people, the following must be true.

$$S : T = 30x : x$$

We can set up an equation about the new ratio.

$$\frac{30x+50}{x+5} = \frac{25}{1}$$
$$30x+50 = 25x+125$$
$$5x = 75$$
$$x = 15 \implies T = 15$$

Answer: E

PS161

We can use the exponent rules and the inequality rules.

$$\left(5^2\right)^n > 5^{12}$$
$$5^{2n} > 5^{12}$$
$$2n > 12$$
$$n > 6$$

Since n must be an integer, $n_{min} = 7$.

Answer: B

PS162

We can organize the given data into a cross table and put a ? mark in the cell directly related to the question. Solving the problem is faster if we leave the % symbols.

	L	NL	
W	?		60
M			40
			100

The probability in question is simply the decimal form of the value in cell W&L.

$$60 \cdot 0.45 = 27 \implies 27\% = 0.27$$

Answer: C

PS163

Let x be the number of trees at the beginning of the 4-year period. Since the growth factor was $1 + 1/4 = 5/4$ for each year, we can set up the following equation about x.

$$x \cdot \left(\frac{5}{4}\right)^4 = 6{,}250$$
$$x \cdot \frac{5^4}{4^4} = 5^4 \cdot 10$$
$$x = 4^4 \cdot 10 = 256 \cdot 10 = 2{,}560$$

Answer: D

PS164

The number of years for which we have data is $n = 2000 - 1990 + 1 = 11$. Since the ordinal number for the median is $(11+1)/2 = 6$, the median is the 6th term in the increasingly ordered list of the 11 yearly data points. Thus, the median is the data point for 1994, which is approximately 310,000.

Answer: C

PS165

The rephrased question: How many times is 72 divisible by 2 ? If we determine the prime factored form of 72, the exponent of the prime factor 2 will be our answer.

$$72 = 2^3 \cdot 3^2 \implies k = 3$$

Answer: B

PS166

We know that 68 percent of a certain distribution lies between $m-d$ and $m+d$. Since the distribution is symmetric about the mean, half of the remaining 32 percent must lie above $m+d$. Thus, $100 - 32/2 = 84$ percent of the distribution lies below $m+d$.

Answer: D

PS167

She ate 3 small pieces and 1 big piece, so the total amount she ate was the following fraction of a whole sandwich.

$$3 \cdot \frac{1}{m} + \frac{1}{m-4} = \frac{3(m-4)+m}{m(m-4)} = \frac{3m-12+m}{m(m-4)} = \frac{4m-12}{m(m-4)}$$

Answer: E

PS168

If $1+\sqrt{2}$ is a root, then it must satisfy the corresponding equation. Since x^2 is a common term of the answer choices, we can first substitute the given root into it.

$$\left(1+\sqrt{2}\right)^2 = 1 + 2\sqrt{2} + 2 = 2\sqrt{2} + 3$$

To eliminate the irrational expression $2\sqrt{2}$, we must subtract it with the help of another term in the equation. Only answer choices B and D make this subtraction possible. However, the integer terms in B don't cancel out each other.

Answer: D

PS169

A quick refresher: The growth factor for a ratio is the ratio of the growth factor for the numerator to the growth factor for the denominator.

For the unemployment rate, which is the ratio of the number of unemployed workers to the labor force (the total number of employed or unemployed workers), the growth factor can be calculated indirectly with the above formula. However, it can also be calculated directly from the unemployment rates for the two dates. Let x be the growth factor for the unemployed workers.

$$\frac{\frac{x}{6}}{5} = \frac{9}{16}$$

$$x = \frac{9}{16} \cdot \frac{6}{5} = \frac{9}{8} \cdot \frac{3}{5} = \frac{27}{40} = 0.675 \implies 32.5\% \text{ decrease}$$

Answer: B

Extra notes: We could also determine the approximate percent change by comparing the growth factor 27/40 to the answer choices.

PS170

Some probability problems are easy to solve if we imagine the selection process as a sequence of events and multiply the corresponding probabilities together.

$$\frac{9}{12} \cdot \frac{8}{11} = \frac{3 \cdot 2}{11} = \frac{6}{11}$$

Answer: C

Extra notes: If we had time, we could also phrase the question with probability notation. Let n_i be the event that his ith selection is a non-defective pen.

$$P(n_1 \cap n_2) = P(n_1) \cdot P(n_2|n_1) =?$$

PS171

To solve this problem, we can use a 2-step approach. Let x be the original number of oranges. We can set up an equation about the average price of her original selection.

$$\frac{40(10-x)+60x}{10} = 56$$
$$40 + 2x = 56$$
$$2x = 16$$
$$x = 8 \implies 10 - x = 2$$

Let y be the number of oranges she must put back. We can set up another equation about the average price of her new selection.

$$\frac{40 \cdot 2 + 60(8-y)}{10-y} = 52$$
$$80 + 480 - 60y = 520 - 52y$$
$$40 = 8y$$
$$y = 5$$

Answer: E

Extra notes: There is a more elegant way to solve this problem. Let n be the number of oranges she must put back.

$$\frac{56 \cdot 10 - 60n}{10-n} = 52$$
$$560 - 60n = 520 - 52n$$
$$40 = 8n$$
$$n = 5$$

PS172

A quick refresher: The percent change formula with the growth factor:

$$\left(\frac{x_1}{x_0} - 1\right) \cdot 100\%$$

We can directly determine the growth factor for the ratio of royalties to sales.

$$\frac{\frac{9}{108}}{\frac{3}{20}} = \frac{9}{108} \cdot \frac{20}{3} = \frac{3 \cdot 5}{27} = \frac{5}{9} = 0.\overline{5} \implies 45\% \text{ decrease}$$

Answer: C

PS173

Off-period	Length (min)
8:46 - 8:54	8
9:26 - 9:29	3
9:46 - 10:00	14
Total	25

Answer: B

PS174

A quick refresher: The corresponding sides of a 30-60-90 degree special right triangle are in a ratio of $1 : \sqrt{3} : 2$.

The parallelogram in the figure is a rhombus because its sides are equal. The diagonals of a rhombus are perpendicular bisectors of each other and cut the rhombus into four identical right triangles. Using that the adjacent angles in a parallelogram are supplementary angles, we can see that the angles adjacent to the 60° angle are 120° angles. Since each interior angle in a rhombus is bisected by the corresponding diagonal, the four identical triangles are 30-60-90 degree special right triangles.

If the shorter leg of one of these identical right triangles is 1 unit, the longer leg must be $\sqrt{3}$ units. Thus, for the rhombus, the ratio of the shorter diagonal to the longer diagonal is

$$\frac{2 \cdot 1}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Answer: D

PS175

To answer the question, we have to determine the exponent of 3 in the prime factored form of 30!. In other words, we have to determine how many times 30! is divisible by 3. We can use the following counting method.

$$\left[\frac{30}{3}\right] + \left[\frac{30}{3^2}\right] + \left[\frac{30}{3^3}\right] = \left[\frac{30}{3}\right] + \left[\frac{30}{9}\right] + \left[\frac{30}{27}\right] = 10 + 3 + 1 = 14$$

Answer: C

PS176

We can use the difference of squares formula to factorize n.

$$\begin{aligned} n = 3^8 - 2^8 = \left(3^4\right)^2 - \left(2^4\right)^2 &= (3^4 + 2^4)(3^4 - 2^4) \\ &= (3^4 + 2^4)(3^2 + 2^2)(3^2 - 2^2) \\ &= (3^4 + 2^4)(3^2 + 2^2)(3 + 2)(3 - 2) \end{aligned}$$

$3^4 + 2^4 = 81 + 16 = 97$	Thus, 97 is a factor of n.
$3^2 + 2^2 = 9 + 4 = 13$	Thus, 13 is a factor of n.
$3 + 2 = 5$	Thus, 5 is factor of n.
$13 \cdot 5 = 65$	Thus, 65 is a factor of n.

Answer: C

PS177

Let R and r be the radii of the big and the small circles, respectively. We can set up an equation about the area of the shaded region.

$$\begin{aligned} A_{\text{shaded}} &= A_{\text{big}} - A_{\text{small}} \\ 3r^2\pi &= R^2\pi - r^2\pi \\ 4r^2\pi &= R^2\pi \\ \frac{R^2}{r^2} &= 4 \\ \frac{R}{r} &= 2 \implies \frac{2R\pi}{2r\pi} = 2 \end{aligned}$$

Answer: C

PS178

A quick refresher: The LCM-based remainder formula is $N = LCM(d_1, d_2)q + R_0$, where R_0 is the common remainder.

Let N be the number of members of the club.

$$\begin{aligned} N &= 4q_1 + 3 \\ N &= 5q_2 + 3 \\ N &= LCM(4,5)q_3 + 3 = 20q_3 + 3 \end{aligned}$$

The only possible value for N such that $10 < N < 40$ is 23. Thus,

$$23 \bmod 6 = 5$$

Answer: E

PS179

Let x be the number of days he had to complete the assignment. We can set up an equation between his original plan and his actual progress.

$$\begin{aligned} 90x &= 75(x-6)+690 \\ 90x &= 75x-450+690 \\ 15x &= 240 \\ 5x &= 80 \\ x &= 16 \end{aligned}$$

Answer: B

PS180

We can simply square both sides of the given equation and isolate r.

$$\begin{aligned} \frac{r}{s} &= s^2 \\ r &= s^3 \end{aligned}$$

Answer: D

PS181

Let p be a prime number. We can set up an equation about x and solve it with the constraint $3 < x < 100$.

$$\begin{aligned} \frac{x}{3} &= p^2 \\ x &= 3p^2 \end{aligned}$$

p	p^2	x	
2	4	12	(✓)
3	9	27	(✓)
5	25	75	(✓)
7	49	$3 \cdot 49$	(X)

Only three values for x satisfy the given conditions.

Answer: B

PS182

The number of ways to select a single letter from n distinct letters is $\binom{n}{1} = n$, and the number of ways to select two distinct letters from n distinct letters is $\binom{n}{2}$. We can realize that two distinct letters always have a unique alphabetical order. Thus, the total number of possible codes is

$$n + \binom{n}{2}$$

Now, we can use the *backwards* method to evaluate the answer choices because the direction of change in the values of the answer choices determines the direction of change in the value of an expression directly related to the evaluation of the answer choices.

Choice	n	Value	Evaluation
(B)	5	$5 + \frac{5!}{2! \cdot 3!} = 15$	$15 \geq 12$
(A)	4	$4 + \frac{4!}{2! \cdot 2!} = 10$	$10 < 12$
All others			No need to check.

Answer: B

PS183

First, we can determine the slope of line l.

$$m = \frac{\Delta y}{\Delta x} = \frac{2-0}{0-(-3)} = \frac{2}{3}$$

Next, we can rearrange each answer choice into slope-intercept form and choose the one whose slope is also 2/3. We can quickly see that the correct answer choice is A.

$$3y - 2x = 0$$
$$3y = 2x \implies y = \frac{2}{3}x$$

Answer: A

PS184

The maximum height is reached after 3 seconds because $t = 3$ eliminates the first term in the expression for h. Any other value for t would make the first term negative and consequently result in a lower height. 2 seconds after reaching the maximum height, $t = 3 + 2 = 5$ and

$$h = -16(5-3)^2 + 150 = -16 \cdot 4 + 150 = -64 + 150 = 86$$

Answer: B

PS185

We can isolate x from each of the given inequalities.

$$x > 4 \text{ AND } x \leq 8 \implies 4 < x \leq 8$$

Answer: D

PS186

Name	Info	Number of books
David		d
Jeff	$d = 3j$	$d/3$
Paula	$d = p/2$	$2d$

Thus, the total number of books, in terms of d, is

$$d + \frac{d}{3} + 2d = \frac{10}{3}d$$

Answer: C

PS187

To solve this combinatorics problem, we can use the simple combination formula.

$$\binom{8}{2} = \frac{8!}{2! \cdot 6!} = \frac{8 \cdot 7}{2} = 28$$

Answer: C

PS188

Let t be his estimated time and let w be his regular hourly rate. We can set up two equations: one about his expected situation and another about his actual situation.

$$tw = 336 \implies w = \frac{336}{t}$$
$$(t+4)(w-2) = 336$$

We can isolate w from the first equation and substitute it in the second equation.

$$(t+4)\left(\frac{336}{t} - 2\right) = 336$$
$$336 + \frac{4 \cdot 336}{t} - 2t - 8 = 336$$
$$-2t^2 - 8t + 4 \cdot 336 = 0$$
$$t^2 + 4t - 672 = 0$$

How do we solve this quadratic equation, which doesn't look nice at all? We can use the *complete the square* method.

$$(t^2 + 4t + 4) - 4 - 672 = 0$$
$$(t+2)^2 = 676$$
$$|t+2| = 26 \implies t = 24 \text{ OR } \cancel{t = -28}$$

Since t is the length of a time period, only the positive solution applies to the context of the problem.

Answer: B

Extra notes: What if you don't know that $26^2 = 676$? You may think about learning it.

PS189

A quick refresher: Taking the reciprocal of both sides of an inequality will flip the inequality sign if the sides are both positive or both negative.

Since p and q are positive, both sides of the given inequality are positive as well.

$$\frac{p}{q} < 1$$
$$\frac{q}{p} > \frac{1}{1} = 1$$

Answer: E

PS190

To mail the two packages, we have to pay per pound. Whether mailing them separately or combined, we are charged for 8 pounds in total. Mailing them combined is obviously cheaper because in this case, we are charged only once, not twice, the higher rate of x cents for the first pound. Thus, the money saved is $x - y$ cents.

Answer: A

Extra notes: The detailed, but unnecessary calculations are the following. Mailing them separately would cost $(x + 2y) + (x + 4y) = 2x + 6y$, and mailing them combined would cost $x + 7y$. Thus, mailing them combined is cheaper by

$$(2x + 6y) - (x + 7y) = x - y$$

PS191

At 8 percent interest compounded annually, his investment value will double in approximately $70/8 \approx 9$ years. Since $18/9 = 2$, his investment value will double approximately twice in 18 years. Thus, the value of his \$5,000 investment will be approximately

$$\$5{,}000 \cdot 2^2 = \$20{,}000$$

Answer: A

PS192

Let m be the actual number of miles and let g be the actual number of gallons. We know that $285 \le m < 295$ and $11.5 \le g < 12.5$. Thus,

$$\frac{285}{12.5} < \frac{m}{g} < \frac{295}{11.5}$$

Answer: D

PS193

A quick refresher: The inequality of a line segment on the number line:

$$|x - \textit{midpoint}| \le \textit{half-length}$$

Midpoint: $\dfrac{3-5}{2} = -1$

Half-length: $\dfrac{3-(-5)}{2} = 4$

Thus, any x on the shaded part must satisfy the following inequality.

$$|x-(-1)| \leq 4$$
$$|x+1| \leq 4$$

Answer: E

PS194

A quick refresher: The arithmetic average is the ratio of the sum of terms to the number of terms.

Let x be the average daily revenue for the last 4 days. The sum of the total revenues for the first 6 days and for the last 4 days must be equal to the total revenue for the 10-day period.

$$360 \cdot 6 + 4x = 400 \cdot 10$$
$$90 \cdot 6 + x = 100 \cdot 10$$
$$x = 1{,}000 - 540 = \$460$$

Answer: D

PS195

A quick refresher: In the prime factored form of a perfect square, each distinct prime factor must have an even exponent.

For some integer n and for some positive integer y,

$$n^2 = 3{,}150 \cdot y$$
$$n^2 = 2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot y$$

Multiplying by y, we must introduce at least one more 2 and one more 7 into the product of primes to make every exponent even. Thus,

$$y_{min} = 2 \cdot 7 = 14$$

Answer: E

PS196

$$[-1.6] + [3.4] + [2.7] = -2 + 3 + 2 = 3$$

Answer: A

PS197

A quick refresher: The sum of terms of an arithmetic sequence:

$$S = \frac{2a_1 + (n-1)d}{2} \cdot n$$

Her consecutive savings form an arithmetic sequence with a difference of 1, so we can use the sum of terms formula.

$$\frac{2 \cdot 1 + (52-1) \cdot 1}{2} \cdot 52 = 53 \cdot 26 = \$1{,}378$$

Answer: C

PS198

Using the given recursive formula, we can get x_2 and subsequently x_3.

$$x_2 = 2 \cdot x_1 - \frac{1}{2} \cdot x_0 = 2 \cdot 2 - \frac{1}{2} \cdot 3 = \frac{5}{2}$$
$$x_3 = 2 \cdot x_2 - \frac{1}{2} \cdot x_1 = 2 \cdot \frac{5}{2} - \frac{1}{2} \cdot 2 = 4$$

Answer: C

PS199

A quick refresher: The weighted harmonic average formula:

$$A = \frac{\sum w_i}{\sum \frac{w_i}{x_i}}$$

We usually draw a table to solve a rate-time-distance problem, but we now have a simple situation in which we have to determine the global average. Since we have numerator-type weights, we can use the weighted harmonic average formula.

$$\frac{100}{\frac{x}{40} + \frac{100-x}{60}} = \frac{100}{\frac{3x + 200 - 2x}{120}} = \frac{12{,}000}{x + 200}$$

Answer: E

Extra notes: Why is this a weighted average problem, what are the subgroups, and what type of weights are at the heart of this problem, numerator-type or denominator-type? Well, these are questions one has to answer comfortably.

PS200

The units digits of positive integer powers of 3 form a repeating sequence with a 4-term-long pattern.

n	Units digit of 3^n	n mod 4
1	**3**	**1**
2	**9**	**2**
3	**7**	**3**
4	**1**	**0**
5	3	1
6	9	2
⋮	⋮	⋮

n mod 4 is the remainder of n after division by 4. We can see that the remainders themselves also form a repeating sequence with a 4-term-long pattern.

$$43 \bmod 4 = 3 \implies \text{The units digit of } 33^{43} \text{ is } 7.$$
$$33 \bmod 4 = 1 \implies \text{The units digit of } 43^{33} \text{ is } 3.$$

Since units digits can be added together, the units digit of $33^{43} + 43^{33}$ must be equal to the units digit of $7 + 3 = 10$, which is 0.

Answer: A

PS201

3 positions for the boys are fixed in the lineup, and thus there are 3! ways to arrange the 3 boys. 3 positions for the girls are also fixed in the lineup, and again there are 3! ways to arrange the 3 girls. Thus, the total number of ways to arrange the lineup is

$$3! \cdot 3! = 6 \cdot 6 = 36$$

Answer: D

PS202

Let b be the width of the border. We can set up an equation about the area of the border.

$$\begin{aligned} A_{\text{big}} - A_{\text{small}} &= A_{\text{border}} \\ (8+2b)(10+2b) - 8 \cdot 10 &= 144 \\ (8+2b)(10+2b) &= 224 \end{aligned}$$

Now, we can use the *backwards* method to evaluate the answer choices because the direction of change in the values of the answer choices determines the direction of change in the value of an expression directly related to the evaluation of the answer choices.

Choice	b	Value	Evaluation
(B)	4	$16 \cdot 18 = 288$	$288 > 224$
All others			No need to check.

Answer: A

PS203

$$d = \frac{1}{5^4} \cdot \frac{1}{2^3 \cdot 5^3} = \left(\frac{1}{5}\right)^4 \cdot \frac{1}{10^3} = 0.2^4 \cdot 10^{-3} = 0.0016 \cdot 10^{-3}$$

Since 10^{-3} just moves the decimal point three places to the left, d has two nonzero digits.

Answer: B

PS204

The consecutive even integers between 99 and 301 form an arithmetic sequence with a difference of 2.

$$n = \frac{a_n - a_1}{d} + 1 = \frac{300 - 100}{2} + 1 = 101$$

$$S = \frac{a_1 + a_n}{2} \cdot n = \frac{100 + 300}{2} \cdot 101 = 101 \cdot 200 = 20{,}200$$

Answer: B

PS205

We first have to determine the prime factored form of 7,150.

7,150	$2 \cdot 5$
715	5
143	11
13	13

Since 7,150 is equal to $2 \cdot 5^2 \cdot 11 \cdot 13$, it has four distinct prime factors between 1 and 100.

Answer: D

Extra notes: How to determine whether n, an integer greater than 1, is a prime? If n does have some prime factors between 2 and $\sqrt{n}$, inclusive, then it is not a prime. However, if n does not have any prime factors between 2 and $\sqrt{n}$, inclusive, then it is a prime.

PS206

$$\begin{aligned} a_n &= a_1 \cdot a_2 \dots a_{n-1} = t \\ a_{n+1} &= (a_1 \cdot a_2 \dots a_{n-1}) \cdot a_n = t \cdot t = t^2 \\ a_{n+2} &= (a_1 \cdot a_2 \dots a_{n-1}) \cdot a_n \cdot a_{n+1} = t \cdot t \cdot t^2 = t^4 \end{aligned}$$

Answer: D

PS207

A quick refresher: The percent change formula with the growth factor:

$$\left(\frac{x_1}{x_0} - 1\right) \cdot 100\%$$

Another refresher: The growth factor for a ratio is the ratio of the growth factor for the numerator to the growth factor for the denominator.

For the P/E ratio, we can determine the percent change by using the above formula.

$$\begin{aligned} \left(\frac{1 + \frac{k}{100}}{1 + \frac{m}{100}} - 1\right) \cdot 100\% &= \left(\frac{100 + k}{100 + m} - 1\right) \cdot 100\% \\ &= \frac{100 + k - 100 - m}{100 + m} \cdot 100\% = \frac{100(k - m)}{100 + m}\% \end{aligned}$$

Answer: D

PS208

We can use the formulas for the number of elements of three overlapping sets. If we first use the percentages given, we can do the calculations faster. Furthermore, we can infer that $Neither = 0$.

$$\begin{aligned} Total &= All1s - Only2s - 2 \cdot All3 + Neither \\ 100 &= 40 + 30 + 75 - 35 - 2 \cdot All3 + 0 \\ 100 &= 110 - 2 \cdot All3 \\ 2 \cdot All3 &= 10 \\ All3 &= 5 \end{aligned}$$

$$\begin{aligned} Total &= Only1s + Only2s + All3 + Neither \\ 100 &= Only1s + 35 + 5 + 0 \\ Only1s &= 60 \end{aligned}$$

Finally, we can determine $Only1s$ in natural units.

$$Only1s = 300 \cdot 0.6 = 180$$

Answer: D

PS209

We can simply square both sides of the given equation.

$$\begin{aligned} m^{-1} &= -\frac{1}{3} \\ m^{-2} &= \left(-\frac{1}{3}\right)^2 = \frac{1}{9} \end{aligned}$$

Answer: D

PS210

A quick refresher: $Profit = Revenue - Cost$

Let c be the initial cost for each camera.

$$\frac{6}{5}c = 250 \implies c = 250 \cdot \frac{5}{6}$$

To answer the question, we have to determine the ratio of profit to cost.

$$\frac{P}{C} = \frac{R-C}{C} = \frac{R}{C} - 1 = \frac{54 \cdot 250 + 6 \cdot \frac{1}{2} \cdot 250 \cdot \frac{5}{6}}{60 \cdot 250 \cdot \frac{5}{6}} - 1 = \frac{54 + \frac{5}{2}}{50} - 1$$
$$= \frac{113}{100} - 1 = \frac{13}{100} = 13\%$$

Answer: D

PS211

Using the information given, we can imagine the increasingly ordered list of the piece lengths. Let S be the length of the shortest piece and let L be the length of the longest piece.

$$\underline{S}\ \_\_\ \_\_\ \underline{Me = 84}\ \_\_\ \_\_\ \underline{L = 4S + 14}$$

We can see that the total length is $68 \cdot 7 = 476$. Since the total length is given, we can maximize S and consequently L if we minimize the other unknown piece lengths. This goal is reached if the second and the third shortest pieces have the same lengths as the length of the shortest piece and if the second and the third longest pieces have the same lengths as the median length.

$$\underline{S}\ \underline{S}\ \underline{S}\ \underline{84}\ \underline{84}\ \underline{84}\ \underline{4S + 14}$$

We can now set up an equation about the total length.

$$7S + 266 = 476$$
$$7S = 210$$
$$S = 30 \implies L_{max} = 4 \cdot 30 + 14 = 134$$

Answer: D

PS212

$$a_5 = 5 + 2^4 = 21$$
$$a_6 = 6 + 2^5 = 38$$
$$a_6 - a_5 = 38 - 21 = 17$$

Answer: E

PS213

We can see that the product of these randomly selected integers has the least possible value if it is a negative number with the greatest possible absolute value.

Since the list of possible numbers contains both positive and negative numbers, we can use the following strategy. First, we can select the number with the greatest possible absolute value for each place in the product except one. Then, we must be careful to select the right number for the last place because we have to consider not only its absolute value but also its sign.

Although -10 and 10 have the same absolute value, we should choose 10 for each of the first 19 places because negative numbers always complicate things.

Since 10^{19} is positive, we must select a negative number with the greatest possible absolute value for the last place in the product.

$$10^{19}(-10) = (-)10^{19}(10) = (-)10^{20} = -10^{20}$$

-10^{20} is the same number as $-(10)^{20}$ in answer choice E.

Answer: E

Extra notes: If the given list did not contain both positive and negative numbers, we would have to invent a different strategy.

PS214

To solve this combinatorics problem, we can use the indirect formula.

$$\textit{Favorable} = \textit{Total} - \textit{Unfavorable}$$

We can determine the total number of outcomes by using the permutation with repetition formula. To determine the number of unfavorable outcomes, we have to treat the two letters I as a single object, which can form permutations with the three other letters.

$$\frac{5!}{2!} - 4! = 5 \cdot 4 \cdot 3 - 24 = 60 - 24 = 36$$

Answer: D

PS215

Let n be the total number of newspapers sold. We know that r percent of the store's total newspaper revenue was from Newspaper A, and we also know that p percent of the newspapers sold were copies of Newspaper A. We can set up an equation about the ratio of the revenue from Newspaper A to the total newspaper revenue.

$$\frac{r}{100} = \frac{1 \cdot \frac{p}{100} \cdot n}{1 \cdot \frac{p}{100} \cdot n + 1.25 \cdot \frac{100-p}{100} \cdot n}$$

$$\frac{r}{100} = \frac{p}{p + 125 - 1.25p}$$

$$r = \frac{100p}{125 - 0.25p}$$

$$r = \frac{400p}{500 - p}$$

The last step was necessary because none of the answer choices have a decimal in the denominator.

Answer: D

PS216

To simplify the expressions, we can use the difference of squares formula.

$$\frac{1-0.00000001}{1+0.0001}-\frac{1-0.00000009}{1+0.0003}=\frac{1^2-0.0001^2}{1+0.0001}-\frac{1^2-0.0003^2}{1+0.0003}=$$

$$\frac{(1+0.0001)(1-0.0001)}{1+0.0001}-\frac{(1+0.0003)(1-0.0003)}{1+0.0003}=$$

$$1-0.0001-(1-0.0003)=0.0002=2\cdot 10^{-4}$$

Answer: D

PS217

A quick refresher: The arithmetic average is the ratio of the sum of terms to the number of terms.

We can set up an equation about the average daily production during the total period of $n+1$ days.

$$\frac{50n+90}{n+1}=55$$

$$50n+90=55n+55$$

$$35=5n$$

$$n=7$$

Answer: E

PS218

The x-intercept of line l is 3, and the y-intercept of line l is 2. For answer choice A, the intercepts have different signs, so A cannot be the correct answer. For answer choice B, the x-intercept is $6/2=3$, and the y-intercept is $6/3=2$. There's no need to check the other answer choices.

Answer: B

PS219

This is a place value problem. Let $\overline{ab}$ be a 2-digit integer such that a is greater than b. We can set up the following equation.

$$\begin{aligned}\overline{ab} - \overline{ba} &= 27\\ 10a + b - 10b - a &= 27\\ 9a - 9b &= 27\\ a - b &= 3\end{aligned}$$

Answer: A

PS220

We can set up an equation about the combined resistance of the two resistors.

$$\begin{aligned}\frac{1}{r} &= \frac{1}{x} + \frac{1}{y}\\ \frac{1}{r} &= \frac{y+x}{xy}\\ r &= \frac{xy}{x+y}\end{aligned}$$

Answer: D

PS221

A quick refresher: The formula for the probability of a complementary event:

$$P(\overline{A}) = 1 - P(A)$$

The probability that Zelda won't solve the problem is $1 - 5/8 = 3/8$. We know that they try to solve the problem independently of each other. To get the probability of the combined event in question, we have to multiply together the probabilities of the corresponding independent events.

$$\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{64}$$

Answer: E

PS222

$$\frac{x+1-x}{x^2+x} = \frac{1}{x+4}$$
$$x+4 = x^2+x$$
$$x^2 = 4 \implies x = 2 \text{ OR } x = -2$$

Answer: C

PS223

If we look at the answer choices, it seems reasonable to rewrite each factor in the product as an exponential expression with a base of 1/2. Using that $1/4 = (1/2)^2$ and $1/16 = (1/2)^4$, we can simplify the given expression.

$$\left(\frac{1}{2}\right)^{-3}\left(\frac{1}{2}\right)^{-4}\left(\frac{1}{2}\right)^{-4} = \left(\frac{1}{2}\right)^{-11}$$

Answer: B

PS224

To simplify the solution, we can suppose that the shaded polygon is a regular 9-sided polygon. In this case, the attached triangles are all identical isosceles triangles, so the supposed case is valid. The measure of an interior angle of a regular 9-sided polygon is

$$180° - \frac{360°}{9} = 140°$$

Since the base angles of each attached isosceles triangle are supplementary to the corresponding interior angles of the 9-sided polygon, the measure of a base angle is

$$180° - 140° = 40°$$

Thus, the measure of the vertex angle of each attached isosceles triangle is

$$180° - 2 \cdot 40° = 100°$$

Answer: A

PS225

The difference $E - S$ is the net effect of all rounding ups and downs. For the ith term in the list, let x_i be the original value and let r_i be the corresponding adjustment value such that the rounded value is $x_i + r_i$.

$$E - S = \sum(x_i + r_i) - \sum x_i = \sum x_i + \sum r_i - \sum x_i = \sum r_i$$

If a number is rounded up, the corresponding adjustment value is positive and must lie in the following interval.

$$0.1 < r_u < 1$$

If a number is rounded down, the corresponding adjustment value is negative and must lie in the following interval.

$$-1 < r_d \leq -0.1$$

Since 10 decimals are rounded up and 20 decimals are rounded down, we can determine an interval for the net effect of all rounding ups and downs.

$$1 < 10r_u < 10$$
$$-20 < 20r_d \leq -2$$

$$1 - 20 < 10r_u + 20r_d < 10 - 2$$
$$-19 < E - S < 8$$

I. (✓)

II. (✓)

III. (X)

Answer: B

Extra notes:

	Lower bound	Upper bound
r_u	e.g., $2 - 1.899 \approx 0.1$	e.g., $2 - 1.001 \approx 1$
r_d	e.g., $1 - 1.999 \approx -1$	e.g., $1 - 1.1 = -0.1$

PS226

$$5 - \frac{6}{x} = x$$
$$5x - 6 = x^2$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0 \implies x = 3 \text{ OR } x = 2$$

Answer: C

PS227

Since the question is about a relative denominator-type weight and since we have only two subgroups, we can use the quick formula to solve this weighted average problem.

$$w_X = \frac{A - x_Y}{x_X - x_Y} = \frac{30-25}{40-25} = \frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$$

Answer: B

Extra notes: Why is this a weighted average problem, what are the subgroups, and what type of weights are at the heart of this problem, numerator-type or denominator-type? Well, these are questions one has to answer comfortably.

PS228

Since x cannot be equal to 2, we have two cases.

- Case 1: $x > 2$

  If $x > 2$, then $x-2$ is positive and thus multiplying both sides of the given inequality by $x-2$ will not flip the inequality sign.

  $$(x+2)(x+3) \geq 0 \implies x \leq -3 \text{ OR } x \geq -2$$

  The solution for Case 1 is

  $$x > 2$$

- Case 2: $x < 2$

  If $x < 2$, then $x-2$ is negative and thus multiplying both sides of the given inequality by $x-2$ will flip the inequality sign.

  $$(x+2)(x+3) \leq 0 \implies -3 \leq x \leq -2$$

  The solution for Case 2 is

  $$-3 \leq x \leq -2$$

The combined solution for the two cases is

$$-3 \leq x \leq -2 \text{ OR } x > 2$$

Of the integer solutions, only four are less than 5, namely -3, -2, 3, and 4.

Answer: D

Extra notes: If a parabola opens upward, it takes positive values outside its roots and negative values between its roots. Where is the parabola in this problem, why does it open upward, and what are its roots? Well, these are questions one has to answer comfortably.

PS229

We can use one of the formulas for the number of elements of three overlapping sets.

$$\begin{aligned} Total &= All1s - Only2s - 2 \cdot All3 + Neither \\ 150 &= 90 + 75 + 45 - Only2s - 2 \cdot 5 + 5 \\ 150 &= 210 - Only2s - 5 \\ Only2s &= 55 \end{aligned}$$

Answer: D

PS230

We simply have to divide the given fraction by 2^{-17}.

$$\frac{2^3 + 2^2 + 2 + 1}{5} = \frac{8 + 4 + 2 + 1}{5} = \frac{15}{5} = 3$$

Answer: C