How to Solve: Inequality Problems

Algebra and Sine Wave/Wavy Method

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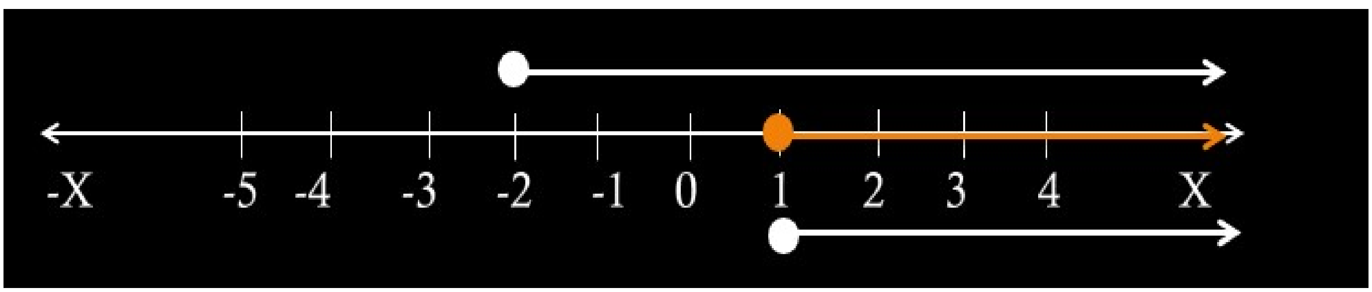
YouTube Video Link to this Post is Here

Following is covered in this post

- **Combining Inequalities**
- **Recap of 4 types of Inequality Problems**
- **Solving Linear Inequalities: Method 1: Algebra**
- **Solving Linear Inequalities: Method 2: Sine Wave Method / Wave Method / Wavy method**

Combining Inequalities

We are discussing this because we will use this in solving problems using the algebra method



If after solving an inequality equation we are getting $x \geq -2$ and $x \geq 1$ as two solutions then our final solution will be $x \geq 1$
As it is the intersection/common part of both the inequalities
(As shown in orange in above figure)

4 types of Inequality Problems solved using Algebra and Sine Wave Method

There are mainly four types of inequality problems which you would need to solve:--

TYPE 1
$x*y > 0$
When $xy > 0$ then we know that both x and y can be either positive or both can be negative
i.e. both x and y have the same sign
so, we have
$x>0, y>0$ or $x<0,y<0$

Example Problem
(x-1)*(x-2) > 0

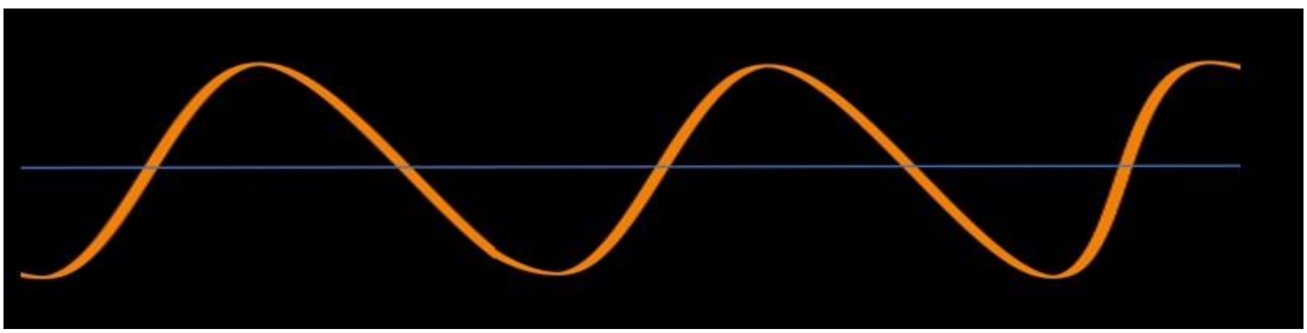
Method 1: Algebra
So, we have two cases
Case 1
both (x-1) and (x-2) are positive
so, x-1 > 0 => x > 1
and x-2 > 0 => x > 2
Intersection of the two cases is x >2

Case 2
both (x-1) and (x-2) are negative
so, x-1 < 0 => x < 1
and x-2 < 0 => x < 2
Intersection of the two cases is x < 1

So, Solution to the question is x < 1 or x > 2

Method 2: Sine Wave Method / Wave Method / Wavy method

In this method we are going to use a sine wave method to solve the problem. Just a quick preview, sine wave is a continuous curve which oscillates between a minimum and a maximum value below and above the base line respectively. Sample image below:

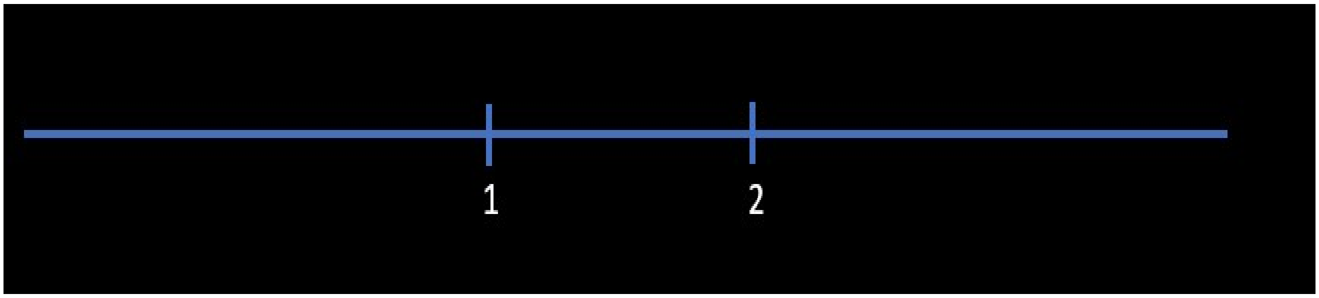


Let's attempt to solve (x-1)*(x-2) > 0 using Sine Wave Method

* Remember that in order to solve the problems using the sine wave method we need to have the coefficient of x positive. [Check out the last part of the video to go through this]

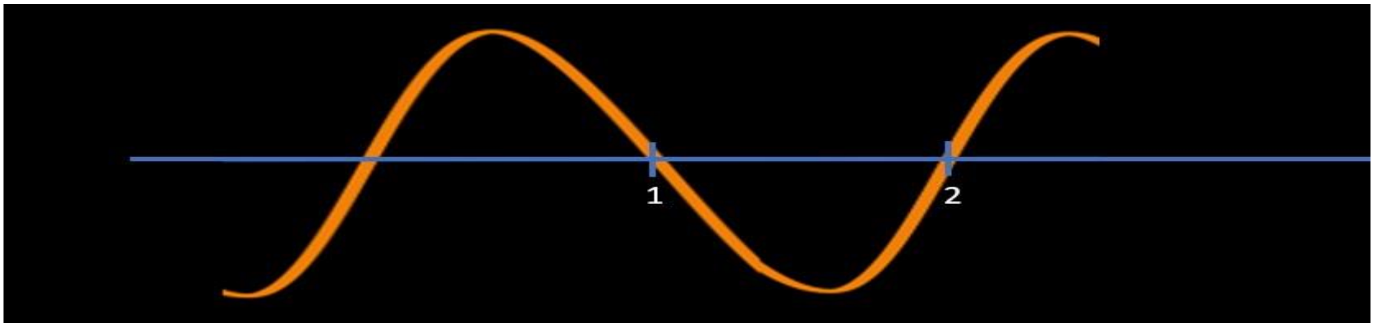
To solve an inequality using this method we find out the intersection points by equating the inequality to 0
=> (x-1)*(x-2) = 0
=> x = 1 or 2

Now, we plot these two points on the number line as shown below



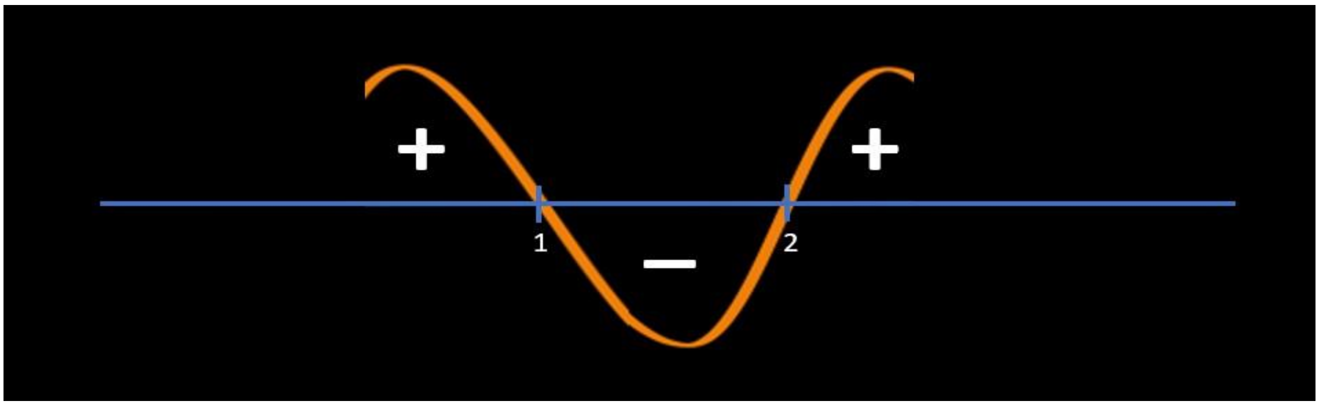
Then we are going to draw a sine curve

- Starting from right top
- Going down at the first solution which is 2 in this case and then
- Coming up in the second solution which is 1 in this case and
- Going down in the third solution if it is there (in this it is not there



Now we will start marking + and - as mentioned below:
Any Area (in-between) above the number line and below the sine curve is marked as "+" and

Any Area (in-between) below the number line and above the sine curve is marked as "-" as shown below



Now, get your answer as below:

- If the inequality in the question is > 0 then pick all the ranges which are "+"
- If the inequality in the question is < 0 then pick all the ranges which are "-"

In our case the question was (x-1)*(x-2) > 0 so we will pick all "+" areas which are
x > 2 and x < 1

If the question was (x-1)*(x-2) < 0 then we will pick all "-" areas which are
1 < x < 2

Note that if the question has ≥ or ≥ then we need to check for the border conditions too
Ex: if question was (x-1)*(x-2) ≥ 0 then we need to check the border condition of x = 1 and x = 2 manually and see if we want to include it in the answer or not.

<u>TYPE 2</u>
x/y > 0
When x/y > 0 then we know that both x and y can be either positive or both can be negative
i.e. both x and y have the same sign
so, we have
x>0, y>0 or x<0,y<0

Example Problem
(x−3)(x−4) > 0

<u>Method 1: Algebra</u>
So, we have two cases
Case 1
Both (x-3) and (x-4) are positive
=> x-3 > 0 => x>3
And x-4 > 0 => x>4
Intersection of the two cases is x > 4

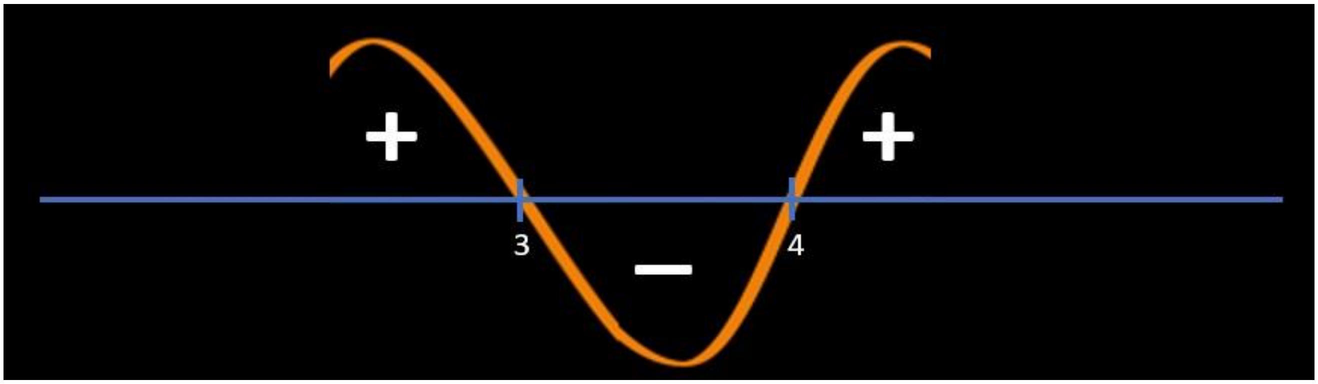
Case 2
Both (x-3) and (x-4) are negative
=> x-3 < 0 => x < 3
and x-4 < 0 => x < 4
Intersection of the two cases is x < 3

So, solution to the question is x < 3 or x > 4

<u>Method 2: Sine Wave Method / Wave Method / Wavy method</u>

Point of intersections:
x - 3 = 0 and x-4 = 0
=> x = 3, 4

Refer below image



Since question is (x−3)(x−4) > 0
So, we will pick "+" area regions
So, answer is x < 3 and x > 4

TYPE 3
x*y < 0
When x*y < 0 then we know that that
(x can be positive and y will be negative) or (x can be negative and y will be positive)
i.e. x and y have opposite signs
so, we have
x>0, y<0 or x<0,y>0

Example Problem
(x+1)(x-1) < 0

Method 1: Algebra
So, we will have two cases
Case 1
(x+1) is positive and (x-1) is negative
=> x + 1 > 0 => x > -1
And x - 1 < 0 => x < 1
Intersection of the two cases is
-1 < x < 1

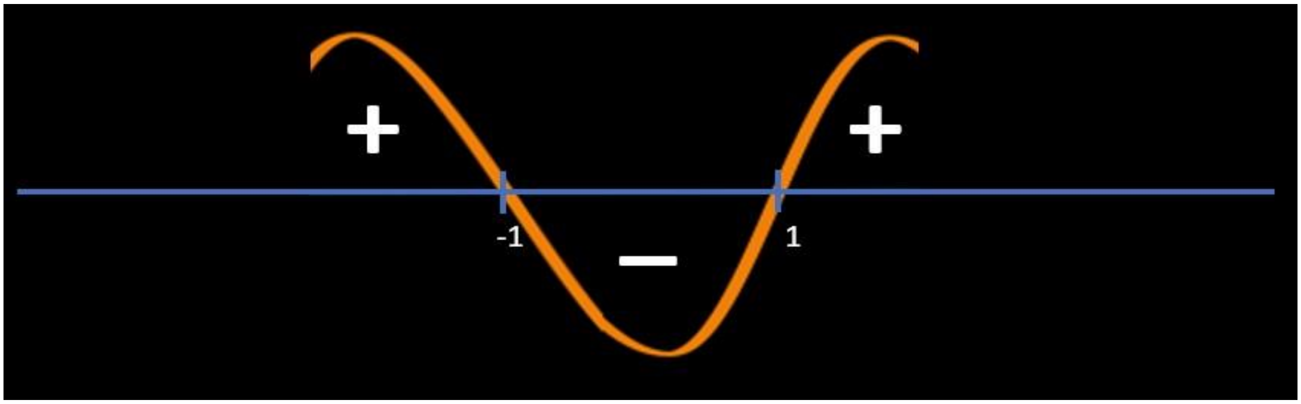
Case 2
(x+1) is negative and (x-1) is positive
=> x+1 < 0 => x < -1
And x-1 > 0 => x > 1
The two cases have no intersection. So, no solution from this case

So, solution of the problem is -1 < x < 1

Method 2: Sine Wave Method / Wave Method / Wavy method

Point of intersections:
x + 1 = 0 and x - 1 = 0
=> x = -1, 1

Refer below image



Since question is (x+1)(x-1) < 0
So, we will pick "-" area regions
So, answer is -1 < x < 1

TYPE 4
x/y < 0
When x/y < 0 then we know that that
(x can be positive and y will be negative) or (x can be negative and y will be positive)
i.e. x and y have opposite signs
so, we have
x>0, y<0 or x<0,y>0

Example Problem
(x−2)(x+3) < 0

Method 1: Algebra
So, we will have two cases
Case 1
(x-2) is positive and (x+3) is negative
=> x-2 > 0 => x > 2
And x+3 < 0 or x < -3
There is no intersection of the two cases. So, no solution from this case

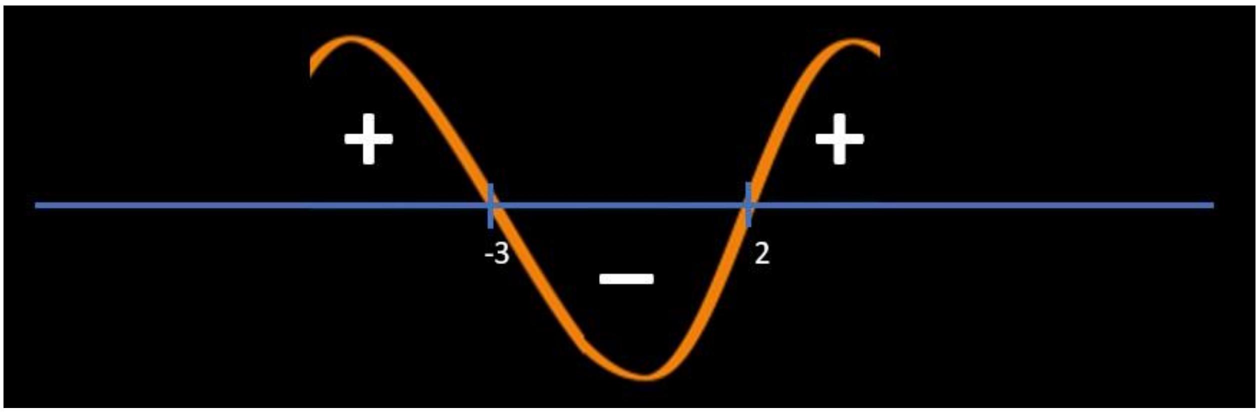
Case 2
(x-2) is negative and (x+3) is positive
=> x-2 < 0 => x < 2
And x+3 > 0 => x > -3
Intersection of the two cases is -3 < x < 2

So, Solution of the question is -3 < x < 2

Method 2: Sine Wave Method / Wave Method / Wavy method

Point of intersections:
x - 2 = 0 and x + 3 = 0
=> x = 2, -3

Refer below image



Since question is (x−2)(x+3)(x−2)(x+3) < 0
So, we will pick "-" area regions
So, answer is -3 < x < 2

Sample Problems

1. x(x-1) > 0. Then value of x will be?

A. x > 0 and x > 1
B. x < 0 and x > 1
C. x < 0 and x < 1
D. x > 0 and x < 1

Solution:
x*(x-1) > 0
this is of the form xy>0 i.e. x and y have the same sign
so,
(1) either, x > 0 and x-1 >0
i.e. x >0 or x>1
taking intersection of the two possibilities we have x >1

(2)or x <0, and x-1 < 0
i.e. x <0 or x<1
taking intersection of the two possibilities we have x < 0

So, Answer will be B

Link to the Problem

2. Which of the following describes all the values of y for which y < y^2 ?

A. 1 < y
B. −1 < y < 0
C. y < −1
D. 1/y < 1
E. 0 < y < 1

Solution:
The question can be written as
y^2 - y > 0
s=> y*(y-1) > 0
It is of the form xy > 0
So, we will have two cases
Case 1
Both y and y-1 are positive
=> y > 0
And y-1 > 0 => y > 1
Intersection of the two cases is y > 1

Case 2
Both y and y-1 are negative
=>y < 0
And y -1 < 0 => y < 1
Intersection of the two cases is y <0

So, solution to the problem is y < 0 or y > 1

So, Answer will be D
(As option D can be written as
1/y - 1 < 0
or, (1-y)/y < 0
or (y-1)/y > 0
And solution to this will be same as that of y*(y-1) > 0)

Link to the Problem

3. Which of the following describes all values of x for which 1–x^2 >= 0?

(A) x >= 1
(B) x <= –1
(C) 0 <= x <= 1
(D) x <= –1 or x >= 1
(E) –1 <= x <= 1

Solution:
Question can be written as
x^2 - 1 <=0
=> (x+1)*(x-1) <=0
Case 1
x+1 is positive or 0 and x-1 is negative or 0
=> x+1 >= 0 => x >= -1
And x-1 <= 0 => x <= 1
Intersection is -1 <= x <= 1

Case 2
x+1 is negative or 0 and x-1 is positive or 0
x+1 <=0 => x <= -1
And x-1 >= 0 => x >= 1
No intersection in this case

So, solution to the problem is -1 <= x <= 1
So, Answer will be E

Link to the Problem

4. If y>0>x, and (3+5y)/(x−1) < −7, then which of the following must be true?

A. 5y−7x+4 < 0
B. 5y+7x−4 > 0
C. 7x−5y−4 < 0
D. 4+5y+7x > 0
E. 7x−5y+4 > 0

Solution:
(3+5y)/(x−1) < −7
=> (3+5y)/(x−1) + 7 < 0
=> ((3+5y) + 7*(x-1))/ (x-1) < 0
=> (7x + 5y -4)/ (x-1) < 0
Now, we know that x < 0 so, x- 1 < 0
in (7x + 5y -4)/ (x-1) < 0
we know that x - 1 < 0
=> (7x + 5y -4) > 0
So, Answer will be B

Link to the problem

5. Is k^2 + k - 2 > 0 ?

(1) k < 1
(2) k < -2

Solution:
k^2 + k - 2 > 0
=> (k+2)*(k-1) > 0
So, we will have two cases
Case 1
Both k+2 and k -1 positive
k+2 > 0 and k-1 > 0
=> k > -2 and k > 1
Intersection is k > 1

Case 2
Both k+2 and k-1 negative
k+2 < 0 and k -1 < 0
=> k < - 2 and k < 1
intersection is k < -2
So, Solution to the problem is k> 1 or k < -2

So, STAT1 is not SUFFICIENT and STAT2 is SUFFICIENT => Answer will be B

Link to the problem