How to Solve: Inequality Basics

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YouTube Video Link to this Post is Here

Following is covered in this post

- What is Inequality and Types of Inequalities
- Graphing Inequalities
- Properties of Inequalities
- Types of Inequality Problems
    - x*y > 0
    - x/y > 0
    - x*y < 0
    - x/y < 0
- Basic Problems on Inequalities

What is Inequality and Types of Inequalities

Usually we are given discrete values of variables like x=2, y=3 etc. In case of inequalities we are given a range of values. Let's take some examples to understand this:

x > 3 => x can take all real values which are greater than 3, i.e. 3.001, 4, 5, 6, 8, 100, etc...
So, instead of giving a single value in case of inequalities we are given a set of values for the variables.

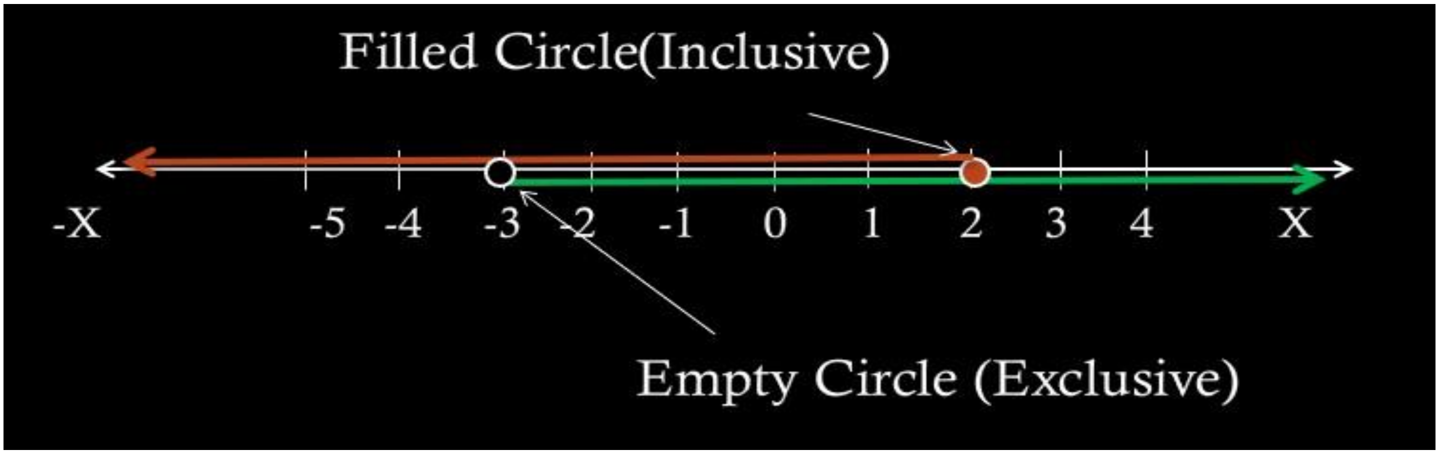
Let's understand various types of inequalities now:

Greater Than Inequality (>) : Ex: x > 3 (We have seen above)
Less Than Inequality (<) : Ex: y < 2 => Y can take all read values which are less than 2
Greater Than or Equal to Inequality (≥): Ex: x ≥ 5 (x can take all real values greater than or equal to 5)
Less Than or Equal to Inequality (≤): Ex: y ≤ 3 (y can take all real values less than or equal to 3)
In-Between Inequality (-2 ≤ x < 5): Ex: x can take all values which are greater than or equal to -2 and less than 5

Graphing Inequalities

Now let's talk about how to plot and inequality on the number line

Ex 1: Graph a ≤ 2 and b > -3 on a number line
Sol:
To plot a ≤ 2 we need to draw a line starting at 2 and extending till -∞ on the left hand side (Refer Orange line in below image). Note that we need to darken point 2 because it is included (as a ≤ 2, so 2 is included)
To plot b > -3 we need to draw a line starting at -3 and extending till +∞ on the right hand side. (Refer Green line in below image). Note that we DO NOT darken point -3 as it is excluded (As b > -3 and not ≥ -3)



Properties of Inequalities

PROP 1: Adding or Subtracting the same number from both the sides of the inequality DOES NOT change the sign of the inequality.

Ex 1: 7 > 3
Add 4 on both the sides we get
7 + 4 > 3 + 4 => 11 > 7 [Which is True and Note that sign of inequality which was > is still >]

Ex 2: 8 > 4
Subtract 9 from both the sides we get
8 -9 > 4 -9 => -1 > -5 [Which is True and Note that sign of inequality which was < is still <]

Ex 3: a > b
Add k on both the sides we get
a + k > b + k [which is true and sign of inequality did not change]

PROP 2: Multiplying / Dividing an inequality equation with a positive number DOES NOT change the sign of the inequality

Ex 1: 7 > 3
Multiply both the sides by +2 we get
7 * 2 > 3 * 2 => 14 > 6 [which is true and sign of inequality did not change]

Ex 2: 8 > 4
Divide both the sides by +2 we get
8282 > 4242 => 4 > 2 [which is true and sign of inequality did not change]

Ex 3: a > b
Multiply both the sides with a positive variable k we get
ak > bk

PROP 3: Multiplying / Dividing an inequality equation with a negative number REVERSES the sign of the inequality

Ex 1: 7 > 3
Multiply both the sides by -2 we get
7 * -2 < 3 * -2 => -14 < -6 [note the sign of inequality has changed from > to <]

Ex 2: 8 > 4
Divide both the sides by -2 we get
8−28−2 < 4−24−2 => -4 < -2 [note the sign of inequality has changed from > to <]

Ex 3: a > b
Multiply both the sides with a negative variable t we get
at < bt [note the sign of inequality has changed from > to <]

PROP 4: We can add two inequalities which have the same inequality sign

Ex 1:
7 > 3 and
8 > 2, Since the two inequalities have same sign of (>) so we can add both of them to get
7 + 8 > 3 + 2 => 15 > 5

Ex 2:
a > b and
c > d
Since the two inequalities have same sign of (>) so we can add both of them to get
a + c > b + d [Note that this is true irrespective of the signs of a, b, c and d]

Ex 3:
If two inequalities have different signs then we can multiply one of them to make the signs same and then add them
a > b
c < d
we can multiple c < d with -1 to get
-c > -d and now we can add a > b and -c > -d to get
a - c > b - d

PROP 5: Taking Square Root on both sides of an inequality DOES NOT Change the sign of the inequality (provided it is possible to take square root on both the sides and get real values).

Ex 1: $a^2 > b^2$ [given that a and b are positive numbers]
Taking square root on both the sides we will get
a > b

PROP 6: Square of a number is always non-negative

Ex 1: $a^2 \geq 0$ [this is true for all real values of a]
this will be equal to 0 only when a itself is zero

Types of Inequality Problems

Type 1: x * y > 0
If product of two variables > 0 that means that the two variables have SAME SIGN
Either Both are Positive => x > 0 and y > 0
Or Both are Negative => x < 0 and y < 0

Type 2: x / y > 0
If division of two variables > 0 that means that the two variables have SAME SIGN
Either Both are Positive => x > 0 and y > 0
Or Both are Negative => x < 0 and y < 0

Type 3: x * y < 0
If product of two variables < 0 that means that the two variables have DIFFERENT SIGN
Either x > 0 and y < 0
Or x < 0 and y > 0

Type 4: x / y < 0
If division of two variables < 0 that means that the two variables have DIFFERENT SIGN
Either x > 0 and y < 0
Or x < 0 and y > 0

Basic Problems on Inequalities

Q1. Is bd > 0 ?
A. ab > 0
B. cd > 0

Sol:
Stat A: ab > 0
There are two cases
a>0 and b>0
a<0 and b<0
In both the cases we don't know anything about the sign of d so NOT sufficient

Stat B: cd > 0
There are two cases
c>0 and d>0
c<0 and d<0
In both the cases we don't know anything about the sign of b so NOT Sufficient

Combining both the statements we will have four cases
(1) a>0 b>0 c> 0 d>0 (2) a>0 b>0 c< 0 d<0
(3) a<0 b<0 c>0 d>0 (4) a<0 b<0 c<0 d<0
In case 1 and 4 bd > 0 and in case 2 and 3 bd < 0
So, Together also NOT sufficient. So, Answer will be E

Q2. Is bd>0 ?
A. ab > 0
B. ad > 0

Sol:
Stat A: ab > 0
There are two cases
a>0 and b>0
a<0 and b<0
In both the cases we don't know anything about the sign of d so NOT sufficient

Stat B: ad > 0
There are two cases
a>0 and d>0
a<0 and d<0
In both the cases we don't know anything about the sign of b so NOT Sufficient

Combining both the statements we will have two cases
(Since we have a common variable "a" in both the statements so we will combine the two statements based on the sign of the common variable
First case of STAT A will be combined with the first case of Stat B and

Second case of STAT A will be combined with the second case of Stat B
(1) $a>0$ $b>0$ $d>0$ (2) $a<0$ $b<0$ $d<0$
In both the cases $bd > 0$
So, Together the two statements are sufficient. So, Answer will be C

Q3. Given that $y = 4 + (1-x)2(1-x)2$. Find yMin(Minimum Value of y) and find the value of x for which y = yMin

Sol:
$y = 4 + (1-x)2(1-x)2$
We know that square of a number can never be negative, so Min value of $(1-x)2(1-x)2 = 0$ when $1-x = 0$ or when $x = 1$
So, yMin = 4 when $x = 1$