How to Solve: Remainders (Basics)

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YouTube Video Link to this Post is Remainders Basics, Remainders Advanced

Following is covered in this post

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- **Remainder of numbers when divided by 2, 3, 5, 9, 10**
- **Remainder of numbers when divided by other numbers**
- **Binomial Theorem**
- **Application of Binomial Theorem in finding remainders**

Remainders Basics

• Definition: What is "Remainder"?

Remainder is the integer which is left over in a division, when the divisor cannot evenly divide the dividend

Dividend = Divisor * Quotient + Remainder

Ex: 13 when divided by 3 gives 1 remainder. (13 = 3*4 + 1)
13 -> Dividend
3 -> Divisor
4 -> Quotient
1 -> Remainder

• **Remainder Notation**

1. A number when divided by 5 gives 3 as remainder.
Let the number be n
using Dividend = Divisor * Quotient + Remainder
n = 5k + 3 [where k is quotient and is an integer]

2. A number when divided by 13 gives 5 as remainder.
Let the number be n
using Dividend = Divisor * Quotient + Remainder
n = 13k + 5 [where k is quotient and is an integer]

3. A number when divided by 16 gives 3 as remainder.
Let the number be n
using Dividend = Divisor * Quotient + Remainder
n = 16k + 3 [where k is quotient and is an integer]

• **How to find Remainders : Long Division Method -> Check the video link**

• **Remainder Range**

"A number when divided by a number k can give remainder from 0 to k-1"

Ex: If we are trying to divide a number by 6 then possible values of remainder are from 0-5

Q1. x when divided by "a" gives 3 as remainder.
y when divided by "b" gives 4 as remainder.
Find min(a+b)

Sol: As a and b are positive numbers so min(a+b) = min(a) + min(b)
x when divided by "a" gives 3 as remainder -> Since a is giving 3 remainder that means that a >= 4. So, min a = 4
y when divided by "b" gives 4 as remainder -> Since b is giving 4 remainder that means that b >= 5. So, min b = 5
=> min(a+b) = min(a) + min(b) = 4+ 5 =9

• **PT1: Basic Problems on Finding Remainders -> Check out the video for examples on long division method**

Q1. Find the remainder when 200 is divided by 3.
Q2. Find the remainder when 80 is divided by 7.
Q3. Find the remainder when 100 is divided by 9.

Ans:
Q1. 2, Q2 3, Q3. 1

• **PT2: Find divisor when remainder is given**

Q1. 20 when divided by which number will give 4 as remainder?

Sol: 20 = nk + 4
=> nk= 20-4 = 16
=> n = 16/k

Now, n is giving 4 remainder that means that n>=5
Let's start putting values of k and get values of n
k=1 => n=16
k=2 => n=16/2 = 8
k=3 => n not integer
k=4 => n= 16/4 not possible as n>=5
So, possible values of the number (n) are 8 and 16

Q2. 50 when divided by which number will give 7 as remainder?

Sol: 50 = nk + 7
=> nk = 50-7 = 43 [note 43 is prime so it has only two factors 1 and 43 itself]
=> n = 43/k
k= 1 => n = 43
k = 43 not possible as n becomes 1 [but we know that n>=8 as n is giving 7 remainder]
So, Answer is 43

• PT3: Find dividend when one divisor and remainder are given

Q1. n when divided by 7 gives 4 as remainder. Find the possible values of the number.

Sol: n when divided by 7 gives 4 as remainder.
n = 7k + 4

We will start taking values of k starting from k=0 and find values of n correspondingly
k=0 n=7*0 + 4 = 4 k=5 n=7*5 + 4 = 39
k=1 n=7*1 + 4 = 11 k=6 n=7*6 + 4 = 46
k=2 n=7*2 + 4 = 18 k=7 n=7*7 + 4 = 53
k=3 n=7*3 + 4 = 25 k=8 n=7*8 + 4 = 60
k=4 n=7*4 + 4 = 32 k=9 n=7*9 + 4 = 67

Q2. n when divided by 5 gives 2 as remainder. Find the possible values of n.

Sol: n when divided by 5 gives 2 as remainder.
n = 5k + 2

We will start taking values of k starting from k=0 and find values of n correspondingly
k=0 n=5*0 + 2 = 2 k=5 n=5*5 + 2 = 27
k=1 n=5*1 + 2 = 7 k=6 n=5*6 + 2 = 32
k=2 n=5*2 + 2 = 12 k=7 n=5*7 + 2 = 37
k=3 n=5*3 + 2 = 17 k=8 n=5*8 + 2 = 42
k=4 n=5*4 + 2 = 22 k=9 n=5*9 + 2 = 47

• PT4: Find dividend when two divisors and remainders are given

Q1. n when divided by 7 gives 3 remainder and when divided by 5 gives 3 remainder. Find first 2 non-negative values of n

Sol: Method -1
n when divided by 7 gives 3 remainder
n = 7k + 3

We will start taking values of k starting from k=0 and find values of n correspondingly
k = 0 , 1, 2, 3, 4, 5, 6, 7, 8, 9
n = 3 , 10, 17, 24, 31, 38, 45, 52, 59, 66

n when divided by 5 gives 3 remainder
n = 5t + 3
We will start taking values of t starting from t=0 and find values of n correspondingly
t = 0 , 1, 2, 3, 4, 5, 6, 7, 8, 9
n = 3 , 8, 13, 18, 23, 28, 33, 38, 43, 48

First two common values are 3 and 38

Sol: Method-2
n when divided by 7 gives 3 remainder
n = 7k + 3

n when divided by 5 gives 3 remainder
n = 5t + 3

7k+3 = 5t+3 => t = 7k/5
So, only those values of k will give us common values of n for which t is integer too.
k = 0 => t=7*0/5 = 0
k = 5 => t=7*5/5 = 7
So, k = 0 => n = 7*0 + 3 = 3
k = 5 => n = 7*5 + 3 = 38

Q2. n when divided by 6 gives 4 remainder and when divided by 4 gives 2 remainder. Find first 2 non-negative values of n

Sol: Method-1
n when divided by 6 gives 4 remainder
n = 6k + 4
We will start taking values of k starting from k=0 and find values of n correspondingly
k = 0 , 1, 2, 3, 4, 5, 6, 7, 8, 9
n = 4, 10, 16, 22, 28, 34, 40, 46, 52, 58

n when divided by 4 gives 2 remainder
n = 4t + 2
We will start taking values of t starting from t=0 and find values of n correspondingly
t = 0 , 1, 2, 3, 4, 5, 6, 7, 8, 9
n = 2, 6, 10, 14, 18, 22, 26, 30, 34, 38

First two common values are 10 and 22

Sol: Method-2
n when divided by 6 gives 4 remainder
n = 6k + 4

n when divided by 4 gives 2 remainder
n = 4t + 2

6k+4 = 4t+2 => t = (6k+2)/4 = (3k+1)/2
So, only those values of k will give us common values of n for which t is integer too.
k = 1 => t=(6*1 + 2)/4 = 2
k = 3 => t=(6*3 + 2)/4 = 5
So, k = 1 => n = 6*1 + 4 = 10
k = 3 => n = 6*3 + 4 = 22

Note: When the numerator is smaller than the denominator then the remainder is the numerator itself
Ex: If 2 is divided by 3 then remainder is 2

• Remainder of sum of two numbers by a number

13 when divided by 3 gives us 1 remainder

When we split 13 as 8 and 5 and divide 8 and 5 individually by 3 then we still get the same remainder
13/3 = (8+5)/3 = 8/3 + 5/3
8/3 will give 2 remainder
5/3 will give 2 remainder

Total remainder is 2+2 = 4, but remainder can't be greater than 3 so remainder will be 4-3 = 1 which is same as the remainder for 13/3

Q1. "A" when divided by 12 gives 3 as remainder. What is the remainder when A is divided by 4?

Sol: A = 12k + 3
When A is divided by 4 then 12k will give 0 remainder and 3 will give 3 remainder. Total remainder is 3

Q2. "B" when divided by 15 gives 6 as remainder. What is the remainder when B is divided by 5?

Sol: B = 15k + 6
When B is divided by 5 then 15k will give 0 remainder and 6 will give 1 remainder. Total remainder is 1

• Remainder of difference of two numbers by a number

13 when divided by 3 gives us 1 remainder

When we split 13 as 15 - 2 and divide 15 and 2 individually by 3 then we still get the same remainder
13/3 = (15-2)/3 = 15/3 - 2/3
15/3 will give 0 remainder
2/3 will give 2 remainder
Total remainder = 0-2 = -2
But remainder cannot be negative so we are going to keep on adding 3 to -2 till the time the sum comes in the range of 0 and 2 [3-1]
=> -2 + 3 = 1
So, remainder is 1

• **Remainder of product of two numbers by a number**

21 when divided by 5 gives us 1 remainder. Now if we break 21 into product of two numbers and find the remainder of these individual numbers by 5 and multiply the remainders then we are going to get the same remainder as we got when we divided 21 by 5

Let's write 21 = 3*7 and divided 3 and 7 by 5
3/5 * 7/5 will give us 3 * 2 remainder respectively = 6
But we divided set of numbers by 5 so remainder cannot be more than 5, so we divide 6 again by 5 to get final remainder as 1

Q1. Find the remainder of 136 * 148 * 298 by 5.

Sol: 136/5 remainder is 1
148/5 remainder is 3
298/5 remainder is 3
Total remainder = 1*3*3 = 9 [>=5]
=> Final remainder = remainder of 9/5 = 4

Q2. Find the remainder of 1205 * 1208 * 2404 by 12.

Sol: 1205/12 remainder is 5
1208/12 remainder is 8
2404/12 remainder is 4
Total remainder = 5*8*4 = 160 [>=12]
=> Final remainder = remainder of 160/12 = 4

More Examples of sample problems and their solution in this post

Remainders by 2, 3, 5, 9, 10

In next section I have just listed down the theory. For problems and Binomial theorem please refer the video

Check above video for examples

- **Remainder of numbers when divided by 2**

When we try to find remainder of a number by 2 then we just need to know if the number is odd or even.
If the number is odd then remainder is 1
If the number is even then the remainder is 0

- **Remainder of numbers when divided by 3**

Remainder of a number by 3 is same as the remainder of sum of digits of the number by 3

- **Remainder of numbers when divided by 5**

Remainder of a number by 5 is same as the remainder of the unit's digit of the number by 5

- **Remainder of numbers when divided by 9**

Remainder of a number by 9 is same as the remainder of sum of digits of the number by 9

- **Remainder of numbers when divided by 10**

Remainder of a number by 10 is same as the unit's digit of the number

- Remainder of numbers when divided by other numbers
- Binomial Theorem
- Application of Binomial Theorem in finding remainders -> Please check Video for this