Working with Sets

There are two ways to solve problems involving overlapping sets

1. Tabulation
2. Venn Diagrams

I'll go through both quickly here with some examples.

Tabulation

Problems that involve 2 or more given sets of data that partially intersect each other are termed overlapping sets.

Problems that involve 2 sets can be solved using a table as shown below:

	B	!B	Total
A			
!A			
Total			

The example below shows how you can utilize this method.

Example 1: (*Question taken off GMATClub Math Forum)*

Of the students at a certain school, 50% of the students are in class A, 30% of the students are in class B, and 20% of the students are in class C. Of the students in class A, 20% are wearing eyeglasses. Of the students in class B, 10% are wearing eyeglasses. Of the students in class C, 10% are wearing eyeglasses.
If a student is selected at random, what is the probability that the student is in class B not wearing eyeglasses?

	Eye Glass	No Eyeglass	Total
A	10	40	50
B	3	27	30
C	2	18	20
Total	15	85	100

Since we're working with percentages, one good method is to work with a total of 100 students.

50% are in class A, so 50 students are in class A
30% are in class B, so 30 students are in class B
20% are in class C, so 20 students are in class C

We fill in the values under the total column, since these numbers represent the total number of students for each individual class.

Now, we're also told in class A, 20% wear eyeglasses. Therefore, 10 students in class A wear eyeglasses.
Also, we're told 10% of the students in class B wear eyeglasses. Therefore, 3 students in class B wear eyeglasses.

Finally, we're told 10% of the students in class C wear eyeglasses. Therefore, 2 students in class C wear eyeglasses.

We can now fill in these values under the column 'eye glasses for each individual class. We can also sum up the total number of people who wear eyeglasses (10+3+2 = 15)

Now all we need to do is to fill up the remaining spaces in the table.

Number of students in Class A who do not wear eye glasses = 50-10 = 40
Number of students in Class B who do not wear eye glasses = 30-3 = 27
Number of students in Class C who do not wear eye glasses = 20-2 = 18

Therefore total number of students who do not wear eyeglasses = 40+27+18 = 85 (Note: You can also calculate the number of students who do not wear eyeglasses by subtracting 15 from 100)

Since we're asked for the probability of a student from class B not wearing eyeglasses, we will need the total number of students who do not wear eyeglasses (85) and the number of students in class B who do not wear eyeglasses (27)

The probability is therefore 27/85

Example 2

30 people are in a room. 20 of them play golf. 15 of them play golf and tennis. If everyone plays at least one sport, how many of the people play tennis only?

Again, we can tabulate the values in order to solve the problem.

	Plays Tennis	Do Not Play Tennis	Total
Plays Golf	15	5	20
Do Not Play Golf	10	0	10
Total	25	5	30

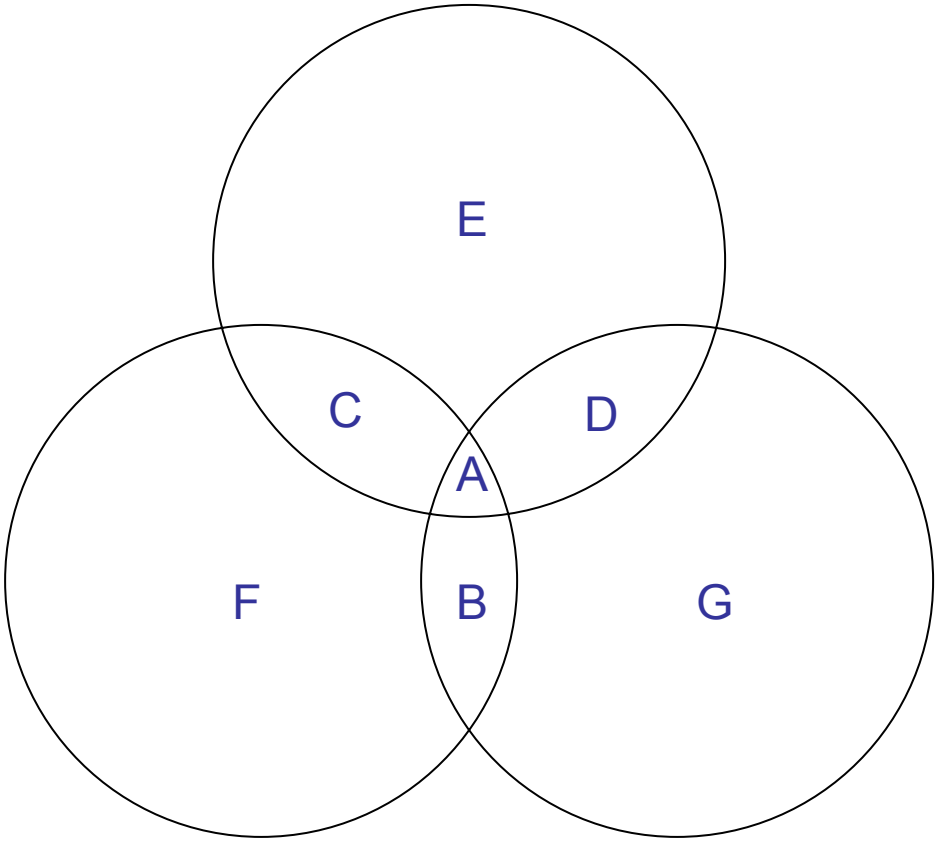
Set the value of total to 30 (since 30 people are in the room)
Set the value of plays golf to 20 (since 20 of them plays golf)
Set the value of 15 to the intersection of 'plays golf' and 'plays tennis' (since 15 of them play both)

We also know that everyone plays at least one sport. So the intersection of 'do not play golf' and 'do not play tennis' should be 0.

We can now proceed to fill in the rest:
Number of people who do not plays golf = 30-20 = 10
Number of people who plays golf but do not plays tennis = 20-15 = 5
Total number of people who do not play tennis = 5+0=5
Total number of people who plays tennis = 30-5 = 25
Number of people who do not play golf but plays tennis = 25-15=10

So now we know how many people play tennis only: = 10

Venn Diagrams
Problems involving 3 overlapping sets can be solved using a Venn diagram.
Venn diagrams should be worked from inside out.



Venn diagrams should be worked from inside out.
So A should be filled in first, followed by B, C and D, and finally E, F and G.

The example below will illustrate how to solve a problem involving 3 overlapping sets using a Venn diagram.

Example 1 (Question taken off GMATClub Math Forum)

Of the 60 students of the IT class of 2003 at UC Berkeley, 30 use a Mac, 20 use a SPARC workstation, and 20 use a SGI. 13 use both Mac and SPARC, 5 uses both MAC and SGI, and 8 uses both SGI and SPARC. If 5 of the students use all three, how many don't use any of the three (not a Mac, not a SPARC, not a SGI, but maybe a crappy Intel machine)?

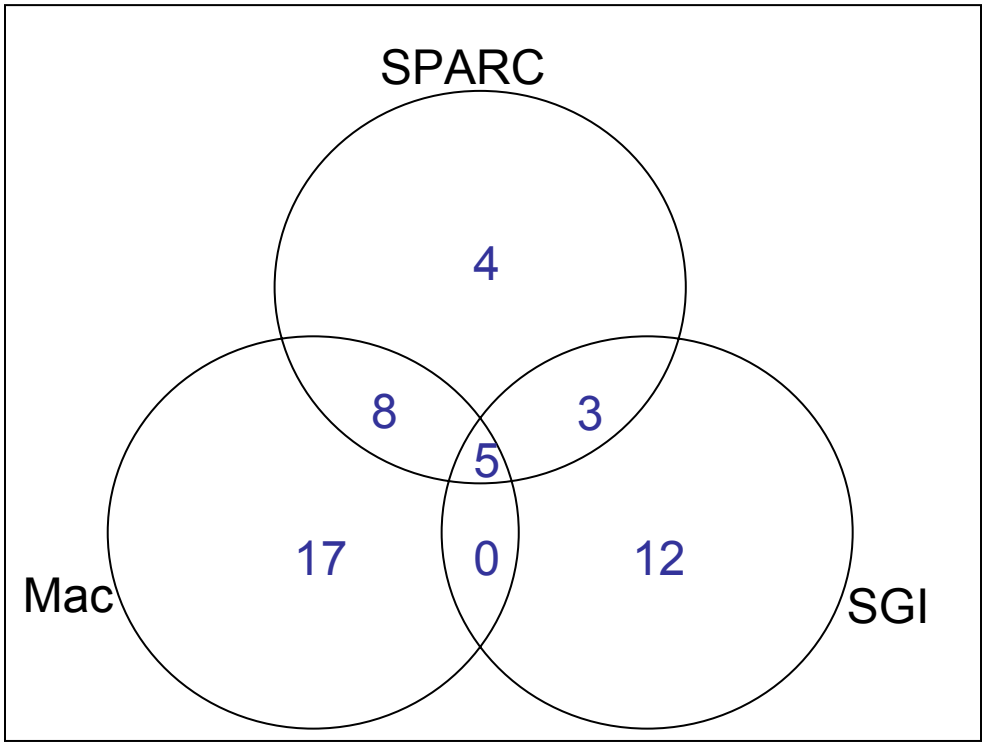
O 0
O 5
O 10
O 11
O 12

Using the Venn diagram, we can now proceed to fill in the values.

5 students use all three machines, so A = 5
13 students use both Mac and SPARC, so C = 13-5 = 8
5 students use both Mac and SGI, so B = 5-5=0
8 students use both SGI and SPARC, so D=8-5 =3
30 students use a MAC, so F = 30-8-5=17
20 students use a SPARC, so E = 20-8-5-3=4
20 students use a SGI, so G = 20-5-3=12

So now your Venn diagram should look this way:

Use none of 3



To solve for the number of students who don't use either of these machines, take 60 and subtract off from this number the sum of all the numbers in the Venn diagram.

So number of students who use none of the 3 computers is: 60-(17+8+5+4+3+12) = 11

So Answer = D